Particle Astrophysics & Neutrinos

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What's beyond the observable universe?

"I'll tell you what's beyond the observable universe – lots and lots of un-observable universe."

What *is* the observable universe?

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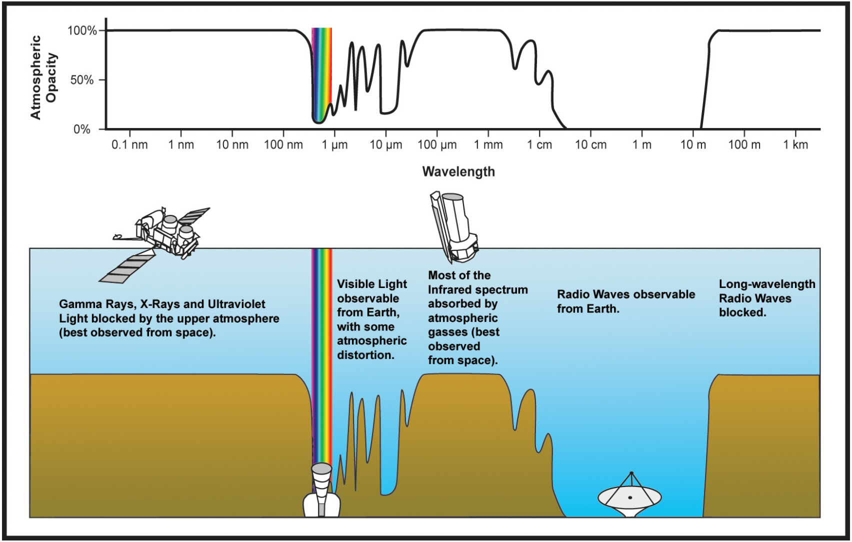
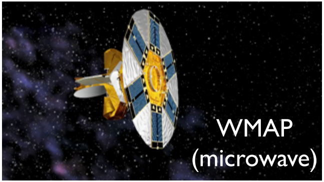
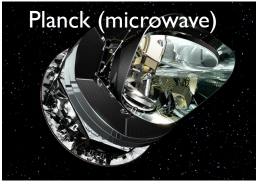
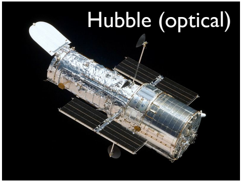


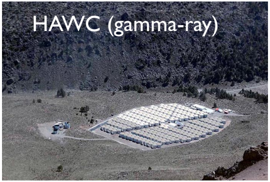
[NASA/IPAC]

Multi-Wavelength Astronomy



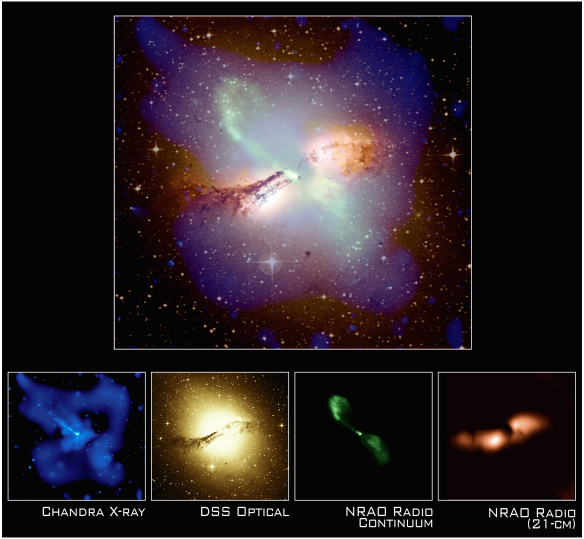






Multi-Wavelength Astronomy

radio galaxy **Centaurus A** at different wavelengths



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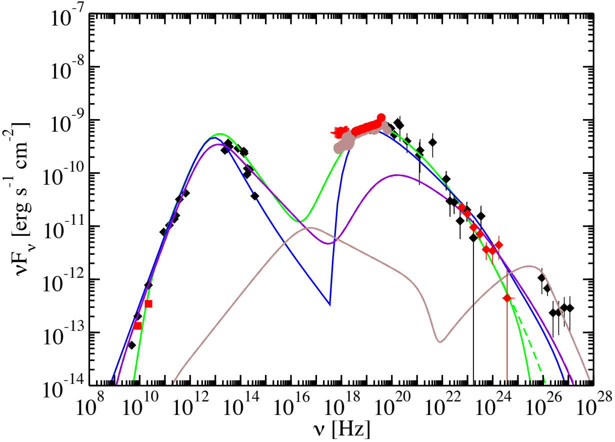
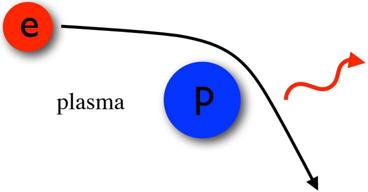


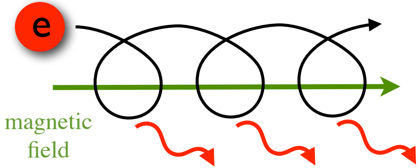
- multi-wavelength spectrum of the core region in Centaurus A [Fermi'10]
- frequency-to-energy conversion: $E \simeq 415\,\mathrm{GeV}(\nu/10^{26}\mathrm{Hz})$
- successful fit to the data via a synchrotron/synchrotron-self-Compton model (green line)
- requires a high-energy population of electrons

High-Energy γ-Radiation

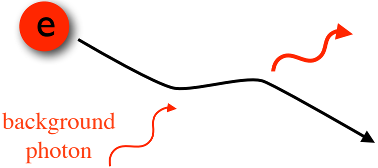
Bremsstrahlung



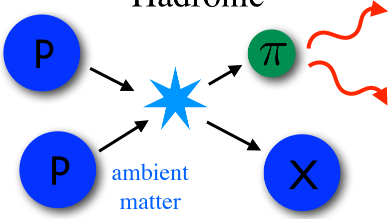
Synchrotron



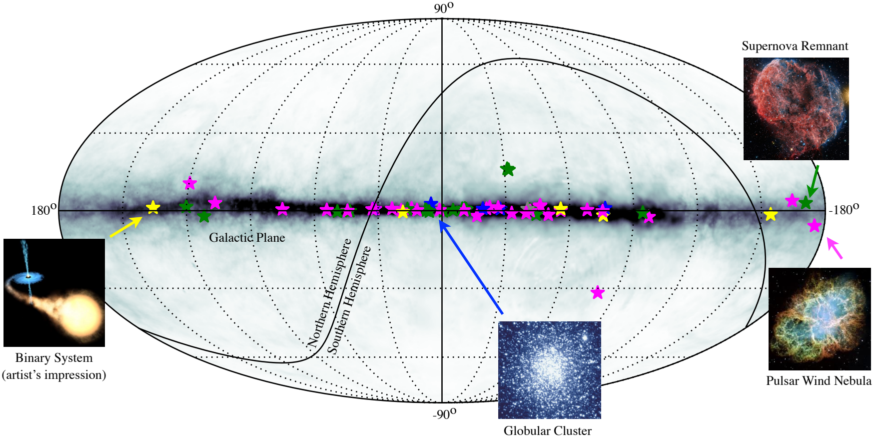
(inverse-)Compton



Hadronic

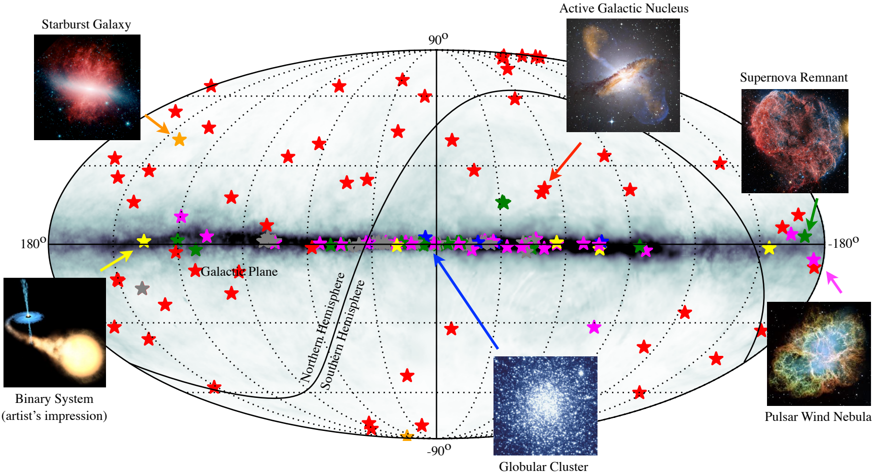


High-Energy γ-Radiation



LBL, IBL, LBL, FRI, FSRQ   Globular Cluster, Star Forming Region, Massive Star Cluster
Binary   PWN   Shell, SNR/Molec.Cloud, Composite SNR   Starburst   Others   [TeVCat'14]

High-Energy γ-Radiation



LBL, IBL, LBL, FRI, FSRQ   Globular Cluster, Star Forming Region, Massive Star Cluster
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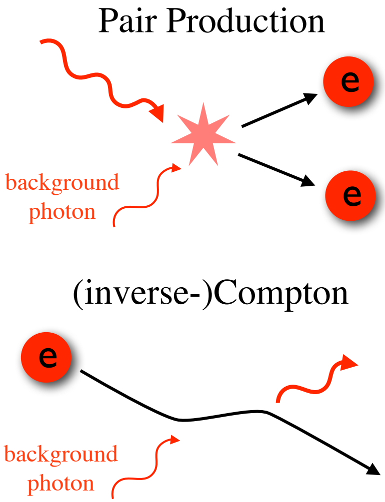
Universe's γ-ray Opacity

- very-high energy γ-rays can interact with background photons to produce $e^{\pm}$-pairs
- inverse Compton scattering "recycle" photons

➜ **repeated cycles initiate cascades**

- main driver is cosmic microwave background (CMB)
- also extragalactic background light (EBL) relevant for distant sources
- rapid cascade interactions produce background of GeV-TeV emission

➜ Universe is opaque to γ-ray emission beyond TeV scales!



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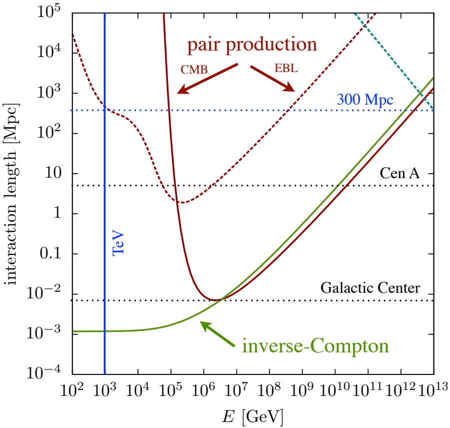

What else is *observable* in the universe?

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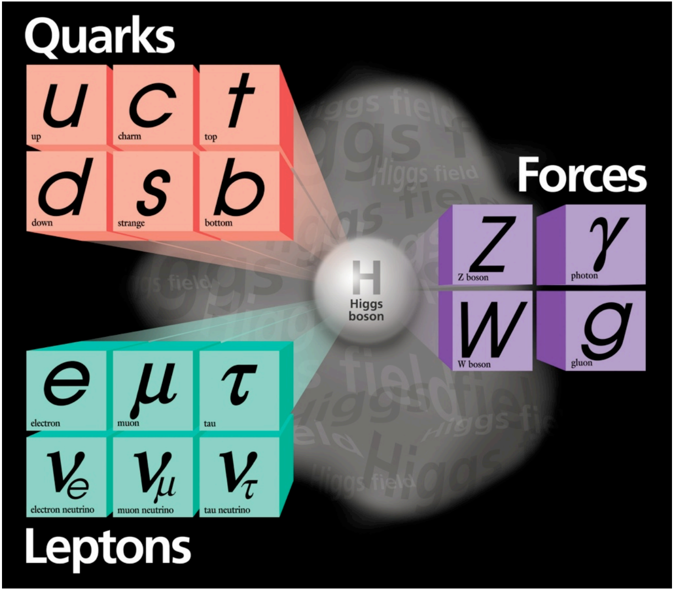

[FNAL]

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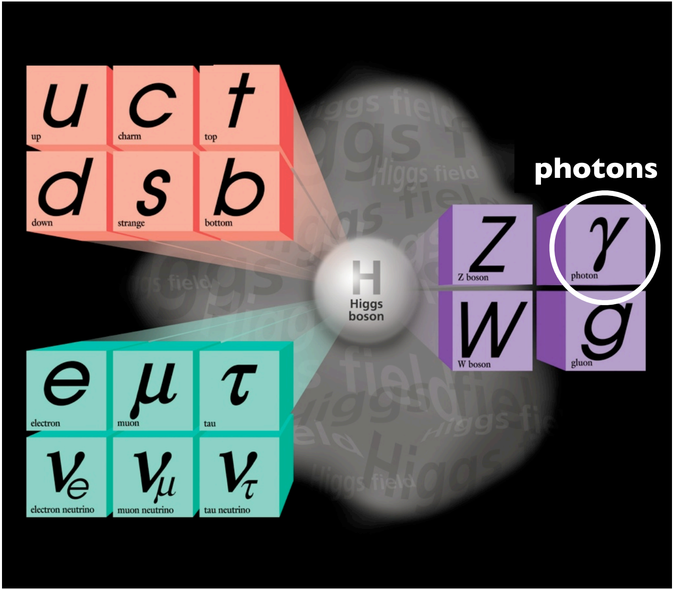

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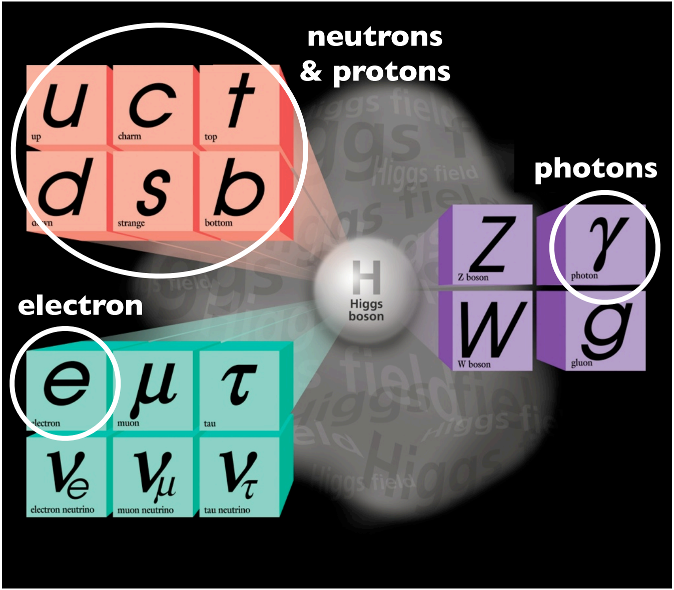
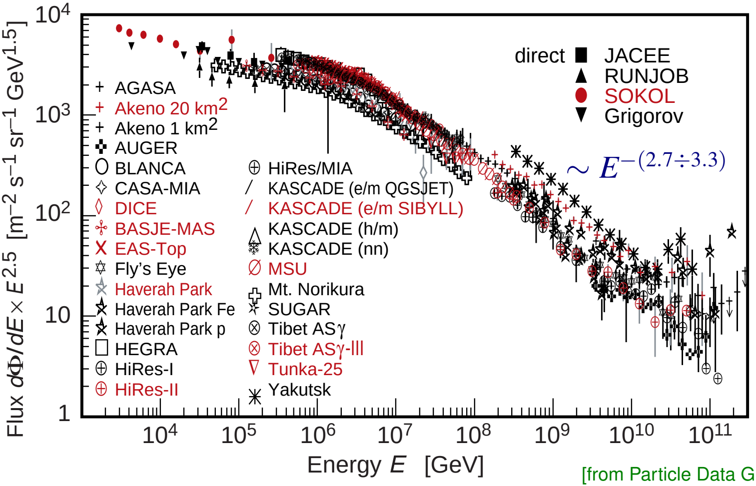


[FNAL]

The cosmic leg

The all-particle spectrum (as $E^{2.5} \times J$) of cosmic rays.



[from Particle Data Group '05]

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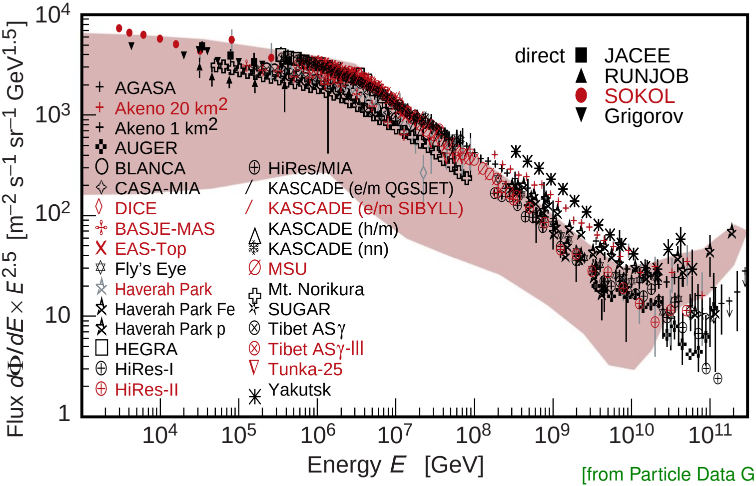

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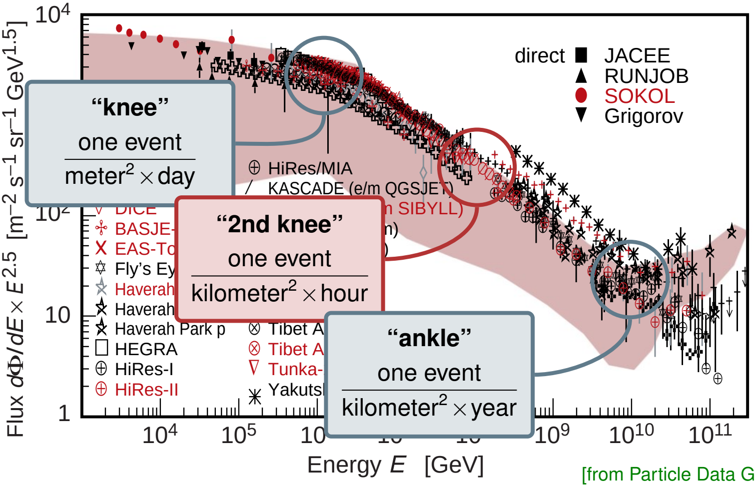


[from Particle Data Group '05]

Electronvolt ???

$$10^6 \text{ eV} = 1 \text{ MeV} \qquad m_e c^2 \simeq \frac{1}{2} \text{ MeV}$$

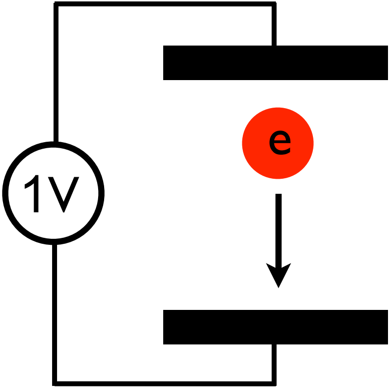
$$10^9 \text{ eV} = 1 \text{ GeV} \qquad m_p c^2 \simeq 1 \text{ GeV}$$

$$10^{12} \text{ eV} = 1 \text{ TeV} \qquad \sqrt{s_{\rm LHC}} \simeq 7 \text{ TeV}$$

$$10^{15} \text{ eV} = 1 \text{ PeV} \qquad E_{\rm max,Earth} \simeq 2 \text{ PeV}$$

$$10^{18} \text{ eV} = 1 \text{ EeV} \qquad \text{Joule} \simeq 6 \text{ EeV}$$

$$10^{21} \text{ eV} = 1 \text{ ZeV} \qquad ???$$



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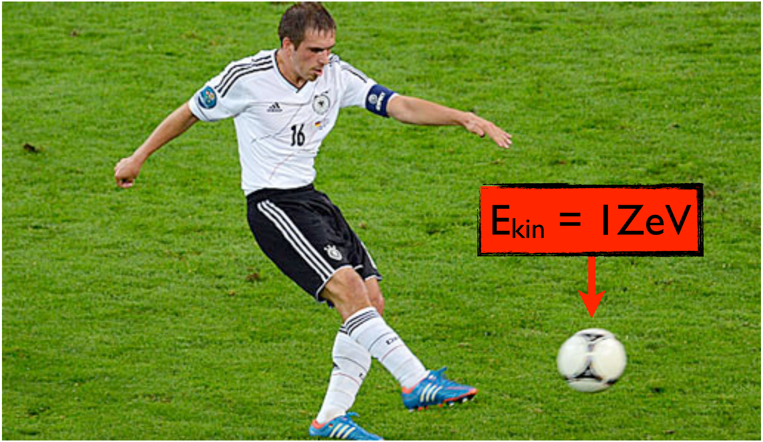


Intermission: Units & Conventions

- **natural units**: $c = \hbar = k_{\rm B} = \epsilon_0 = \mu_0 = 1$
- conversion factors:

$$\hbar c \simeq 2 \times 10^{-7}{\rm eVm} \qquad c \simeq 3 \times 10^8 \frac{\rm m}{\rm s}$$

$$k_{\rm B} \simeq 8.6 \times 10^{-5} \frac{\rm eV}{\rm K} \qquad \alpha_{\rm EM} \simeq \frac{1}{137} = \frac{{\rm e}^2}{4\pi}$$

- example

$$1{\rm Tesla} = 1\frac{\rm Vs}{\rm m^2} = \frac{c}{\rm m/s}\frac{\hbar c}{\rm eVm}\frac{1}{\sqrt{4\pi\alpha_{\rm EM}}}({\rm eV})^2 \simeq 195({\rm eV})^2$$

- other important relations/definitions:

$$1{\rm erg} \simeq 624 \text{ GeV} \qquad 1{\rm eV} \simeq 1.8 \times 10^{-36} \text{ kg} \qquad 1{\rm pc} \simeq 3.26{\rm ly} \simeq 3.09 \times 10^{16} \text{ m}$$

Galactic Cosmic Rays

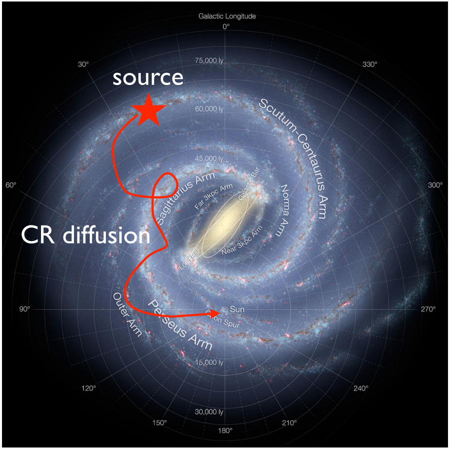
- "**Supernova remnants**" with $E_{\rm CR} \simeq 10^{-3} \times {\rm M}_\odot$ and a rate of 3 SNe per century? [Baade & Zwicky'34]
- galactic CRs via diffusive shock acceleration? (more on this later)

$$\frac{{\rm d}N}{{\rm d}E} \propto E^{-2.2} \qquad \text{(at source)}$$

- energy-dependent **diffusive escape** from Galaxy

$$\frac{{\rm d}N}{{\rm d}E} \propto E^{-2.7} \qquad \text{(observed)}$$

- maximal energy $E_{\rm max} \sim 4$ PeV ("**CR knee**")



Direct & Indirect Evidence

- direct evidence from "pion bump" signatures [Fermi'13]

- pion production in CR-gas interactions

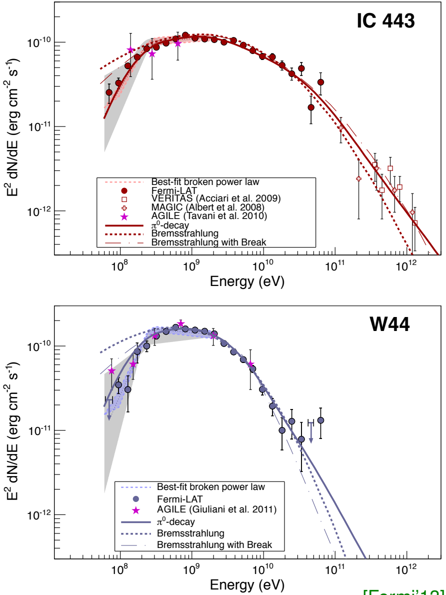
$$p + p \to \pi^0 + \text{other particles}$$

- γ-ray energy in rest frame of pion:

$$E_\gamma = \frac{1}{2} m_{\pi^0} = 67.5\text{MeV}$$

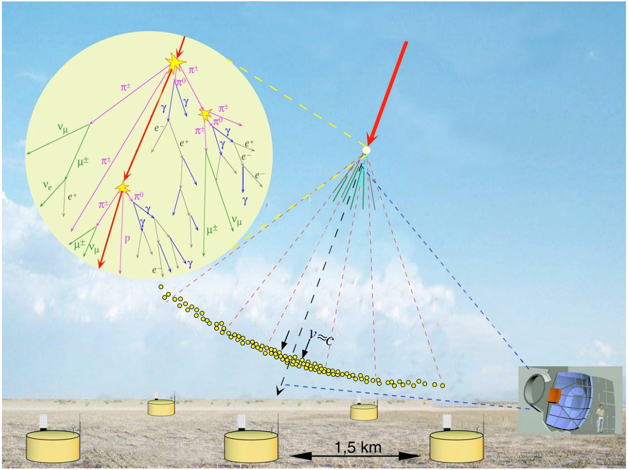
- kinematics of the interaction produces a break at about $E_\gamma \simeq 200$ MeV

- indirect evidence of Galactic CRs via diffuse hadronic emission from the Galaxy



[Fermi'13]

Ultra-High Energy Cosmic Ray Observations

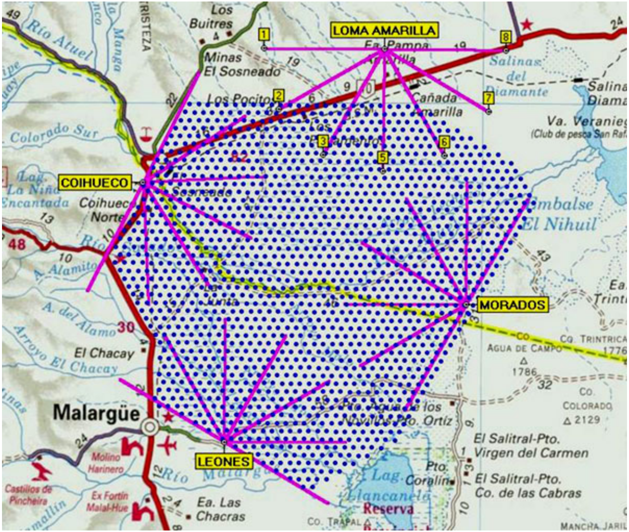


- Cosmic rays interact in the upper atmosphere
- Cascade of particle production and repeated interactions produce a shower

➜ Particle number (electrons/positrons, muons/anti-muons, gamma-rays) grows exponentially

- Observation by fluorescence light (nitrogen excitation) and by surface detectors (Cherenkov light).

Ultra-High Energy Cosmic Ray Observations



- State-of-the-art: Pierre Auger Observatory in Argentina
- surface detector: $\simeq 3000\ \mathrm{km}^2$, four fluorescence detector stations

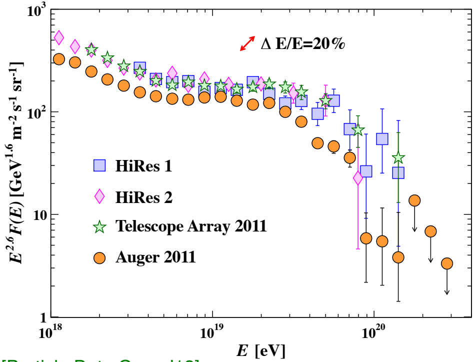
UHE CR and GZK cutoff

- UHE CR spectrum expected to show "cutoff" due to interactions with cosmic radiation background. [Greisen&Zatsepin'66;Kuzmin'66;Berezinsky&Zatsepin'70]

➜ resonant proton interaction $p\gamma \to \Delta \to n\pi^+$ with CMB: $E_{\rm CR} < E_{\rm GZK} \simeq 40{\rm EeV}$

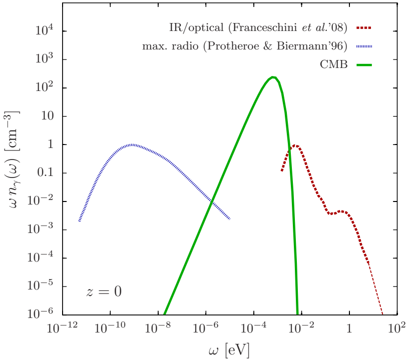
- UHE CR propagation limited to "only" 200 Mpc.

UHE CR spectrum

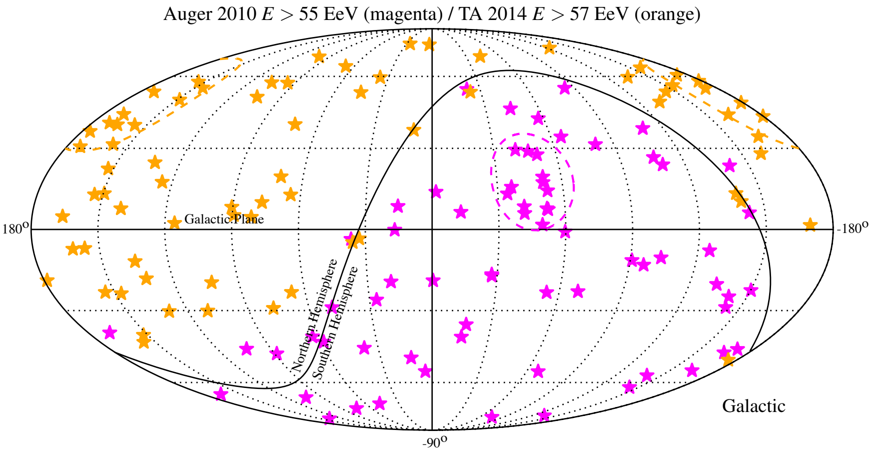


[Particle Data Group'12]

radiation background



UHE CR arrival direction



- $\theta_{\rm rms} \simeq 1^\circ \, (D/\lambda_{\rm coh})^{1/2}(E/55{\rm EeV})^{-1}(\lambda_{\rm coh}/1{\rm Mpc})\,(B/1{\rm nG})$ [Waxman & Miralda-Escude'96]
- "hot spots" (dashed), but no significant auto-correlation in Auger and Telescope Array data
- no significant cross-correlation with source catalogs [Auger'10;TA'14]

Particle acceleration in the Universe

- Acceleration is a continuous process.

➜ Accelerators need to confine the particle by magnetic fields.

- Larmor radius:

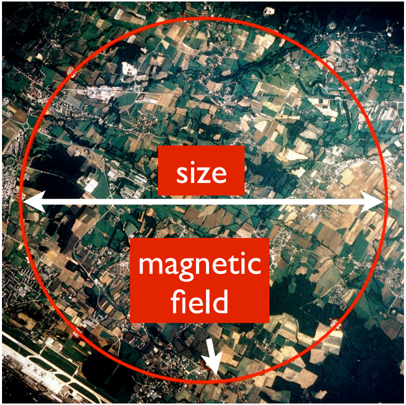
$$R_L = \frac{E}{ZeB} \simeq \frac{1.1}{Z}\left(\frac{E}{\mathrm{EeV}}\right)\left(\frac{B}{\mu\mathrm{G}}\right)^{-1}\mathrm{kpc}\,.$$

- maximal energy from $R_L = R_{acc}$:

$$E_{max} \simeq 0.9Z\left(\frac{B_{acc}}{\mu\mathrm{G}}\right)\left(\frac{R_{acc}}{\mathrm{kpc}}\right)\mathrm{EeV}\,.$$

- for example, the LHC:

$$E_{max} \simeq 9\left(\frac{B_{acc}}{8\mathrm{T}}\right)\left(\frac{R_{acc}}{4\mathrm{km}}\right)\mathrm{TeV}\,.$$



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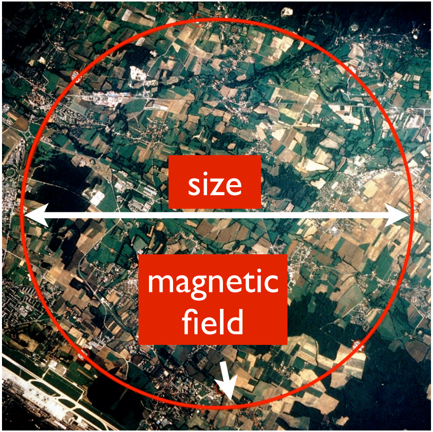
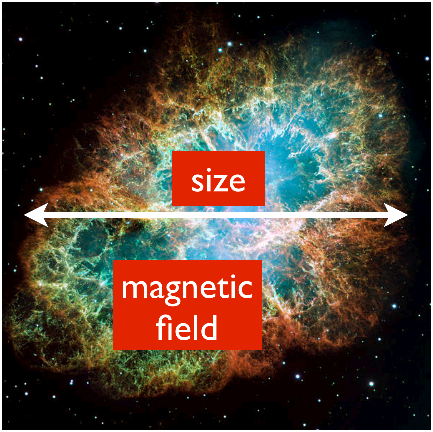


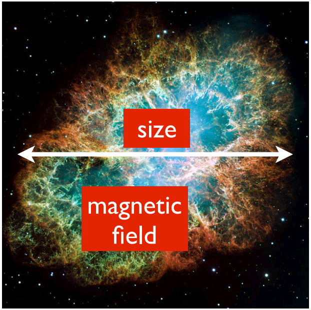


Sources of UHE CRs?

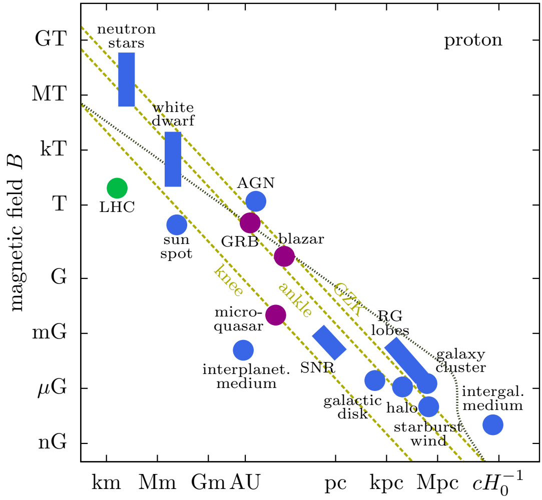
- fundamental energy bound on cosmic accelerators

➜ accelerators with size **R** and magnetic field strength **B**:

$$E_{\max} \simeq 0.9\beta Z\left(\frac{B}{\mu\mathrm{G}}\right)\left(\frac{R}{\mathrm{kpc}}\right)\mathrm{EeV}$$



"Hillas plot"



[after Hillas'84]

Acceleration mechanism?

- There is a problem with this analogy.

✘ Universe is a "perfect conductor"

→ It is unlikely to build up large potentials on long time-scales that accelerate charged particles.

- astrophysical environments are described (to leading order) as an ideal magneto-hydrodynamical (MHD) system:

$$\partial_t \rho = -\nabla(\rho \mathbf{v}) \qquad \text{(continuity)}$$

$$\rho(\partial_t + \mathbf{v}\nabla)\mathbf{v} = (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p \qquad \text{(momentum)}$$

$$\partial \mathbf{B} = -\nabla \times \mathbf{E} \qquad \text{(Faraday's law)}$$

$$\nabla \mathbf{B} = 0 \qquad \text{(no divergence)}$$

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} \qquad \text{(Ohm's law)}$$

- in particular, Ohm's law gives $\mathbf{E} \perp \mathbf{v}$

→ **no acceleration** along electric fields

→ exceptions (NLO effects): magnetic reconnections, double layers, relativistic motion,. . .

Fermi's idea

PHYSICAL REVIEW VOLUME 75, NUMBER 8 APRIL 15, 1949

On the Origin of the Cosmic Radiation

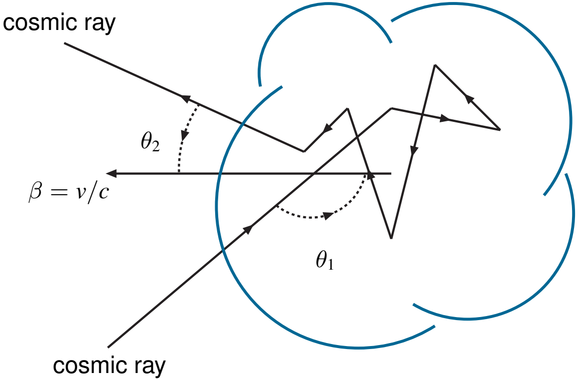
ENRICO FERMI
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magnetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

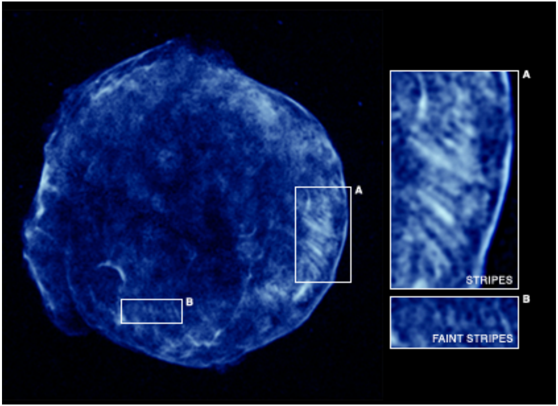
- *Try to get this paper on the web!*
- hints:
  - `http://inspirehep.net/` (type in "`f a fermi and t cosmic`")
  - `http://adsabs.harvard.edu/abstract_service.html`
  - `http://arxiv.org/`

Fermi's original idea

"collisionless" scattering of charged particles with "magnetic clouds"



Diffuse Shock Acceleration

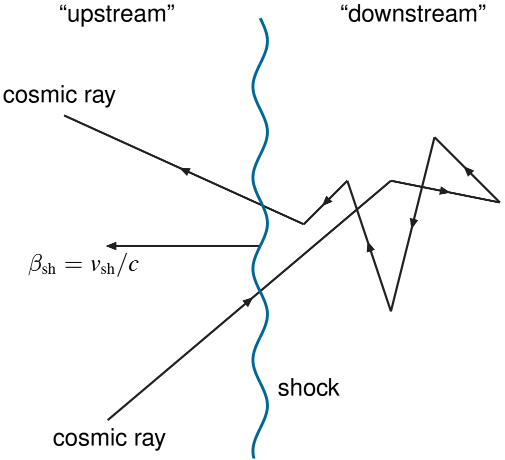


- Danish astronomer Tycho Brahe (1546–1601) observed supernova in 1572
- Chandra observation of supernova remnant reveals high-energy X-ray emission near shock

➜ interpreted as synchrotron radiation of electrons spiraling in magnetic field enhanced by cosmic rays

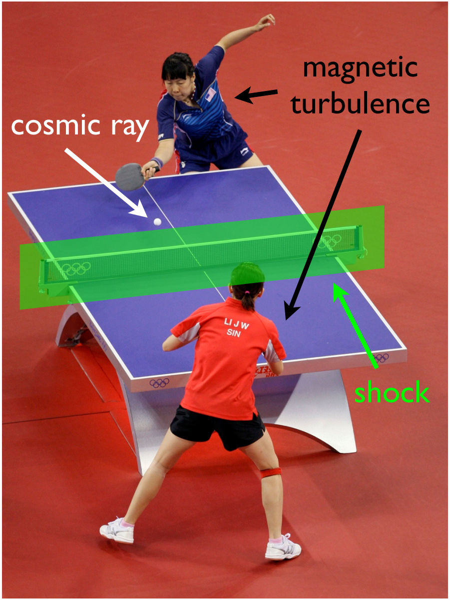
Diffuse shock acceleration

"collisionless" scattering of charged particles across shocks



Cosmic Ray Acceleration

Cosmic Ray Acceleration



Spectrum

- evolution of energy and particle number

$$\partial_t E = \frac{1}{t_{\rm acc}} E \qquad \partial_t N = -\frac{1}{t_{\rm esc}} N$$

➜ dividing

$$\partial_E N = -\frac{t_{\rm acc}}{t_{\rm esc}} \frac{N}{E}$$

➜ re-arranging

$$\frac{{\rm d}N}{N} = -\frac{t_{\rm acc}}{t_{\rm esc}} \frac{{\rm d}E}{E}$$

➜ integrating

$$\int_{N_0}^{N(E)} \frac{{\rm d}N'}{N'} = -\int_{E_0}^{E} \frac{t_{\rm acc}}{t_{\rm esc}} \frac{{\rm d}E'}{E'}$$

➜ final spectrum

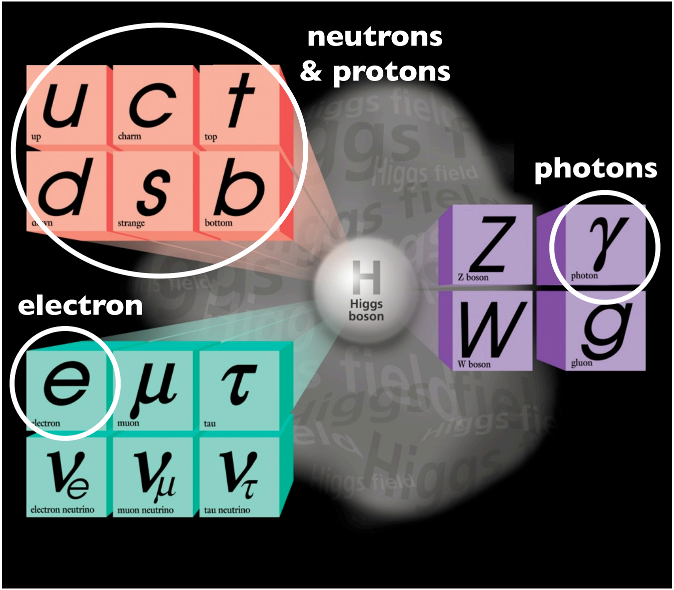
$$N(E) = N_0 (E/E_0)^{-\Gamma}$$

- power index for non-relativistic plasma and strong shocks: $\Gamma = t_{\rm acc}/t_{\rm esc} \simeq 1$

➜ differential spectrum

$$\frac{{\rm d}N}{{\rm d}E} \propto E^{-2}$$

What else is *observable* in the universe?



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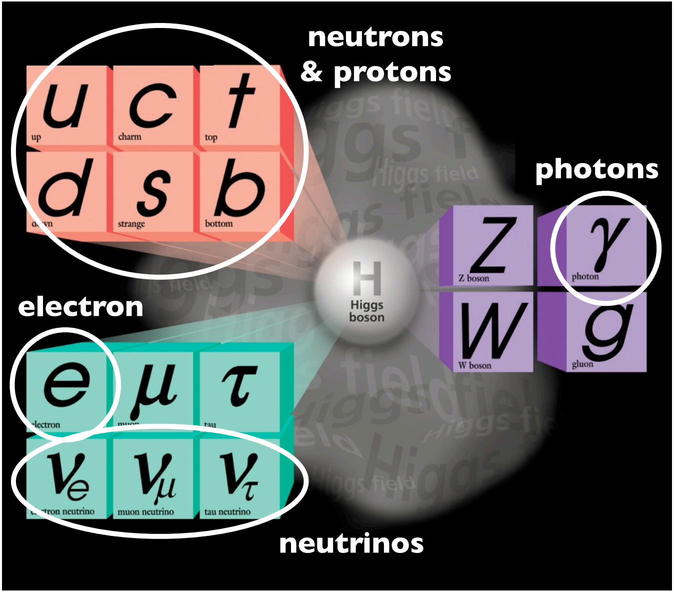


[FNAL]

Cosmic Neutrinos

- produced in **collisions of cosmic rays with gas and radiation**, *e.g.*

$$p + p\,\text{(gas)} \to X\,\text{(rest)} + \pi^-\,\text{(pion)} \qquad p + \gamma\,\text{(radiation)} \to X\,\text{(rest)} + \pi^-\,\text{(pion)}$$

$$\pi^-\,\text{(pion)} \to \mu^-\,\text{(muon)} + \bar{\nu}_\mu$$

$$\mu^-\,\text{(muon)} \to e^-\,\text{(electron)} + \nu_\mu + \bar{\nu}_e$$

- **"smoking-gun"** of cosmic ray sources
- **no deflection** in magnetic fields (➜ point source detection)
- (practically) **no absorption** (➜ distant sources)
- **flavor oscillation** creates (nearly) equal mix between ν_e, ν_μ and ν_τ

Example: GZK neutrinos

- Greisen-Zatsepin-Kuzmin (GZK) interactions of ultra-high energy CRs with cosmic microwave background (CMB) [Greisen'66;Zatsepin/Kuzmin'66]

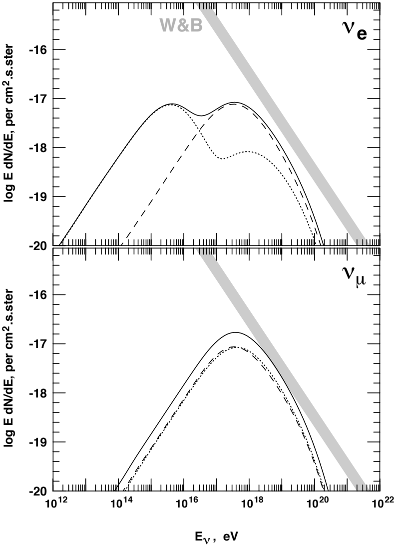
- "GZK"-neutrinos at EeV energies from pion decay [Berezinsky/Zatsepin'69]

- three neutrinos ($\nu_\mu/\bar{\nu}_\mu/\nu_e$) from π^+:

$$E_{\nu_\pi} \simeq \frac{1}{4}\langle x\rangle E_p \simeq \frac{1}{20}E_p$$

- one neutrino from neutron decay:

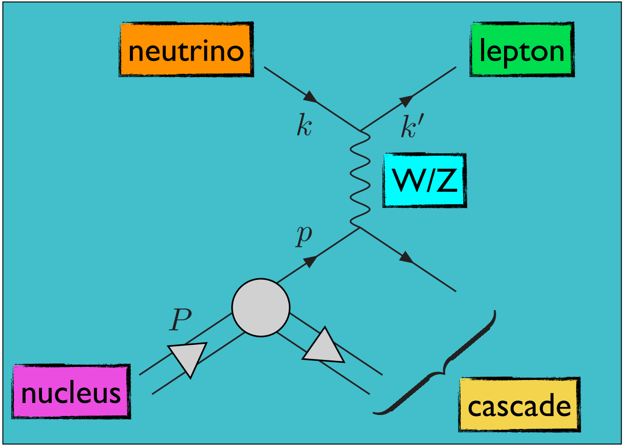
$$E_{\bar{\nu}_e} \simeq \frac{m_n - m_p}{m_n}E_p \simeq 10^{-3}E_p$$



[Engel, Stanev & Seckel'01]

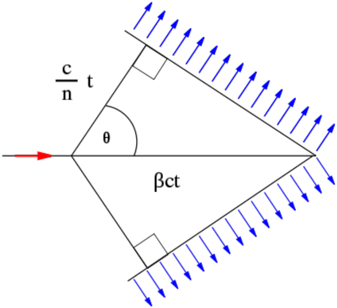
What does a neutrino look like?

- High energy neutrinos collide with nuclei – **rarely, but very violently**.
- "Charged" (W) and "neutral" (Z) current interactions with quarks.

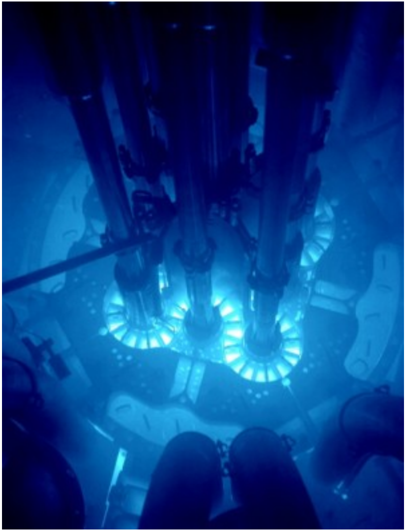


Cherenkov Radiation

- neutrino interaction creates high-energetic charged particle
- charged particles have velocity faster than the speed of light (in water or ice)
- Cherenkov light is emitted along the particle tracks



[source: Wikipedia]



[Advanced Test Reactor (Idaho)]

High-energy neutrino detection

- High energy neutrino collisions with nuclei are **rare** ➜ huge detectors needed!

➜ Secondary charged particles can be detected by their **Cherenkov radiation** in transparent media, *e.g.* ice or water

back-of-the-envelope ($E_\nu \sim 10^{15}$ eV):

- **flux of neutrinos** : $\frac{\mathrm{d}^2 N_\nu}{\mathrm{d}t\,\mathrm{d}A} \sim \frac{1}{\mathrm{cm}^2 \times 10^5 \mathrm{yr}}$
- **cross section** : $\sigma_{\nu N} \sim 10^{-33} \mathrm{cm}^2$
- **targets**: $N_N \sim N_A \times V/\mathrm{cm}^3$

➜ **rate of events** :

$$\dot{N}_\nu \sim N_N \times \sigma_{\nu N} \times \frac{\mathrm{d}^2 N_\nu}{\mathrm{d}t\,\mathrm{d}A} \sim \frac{1}{\mathrm{year}} \times \frac{V}{1\mathrm{km}^3}$$

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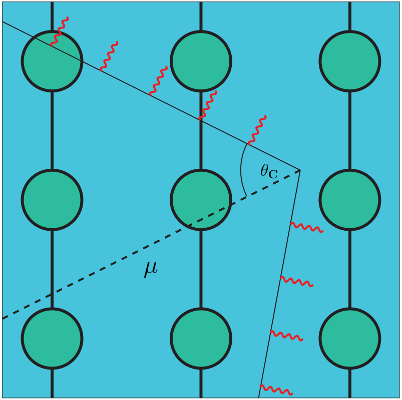
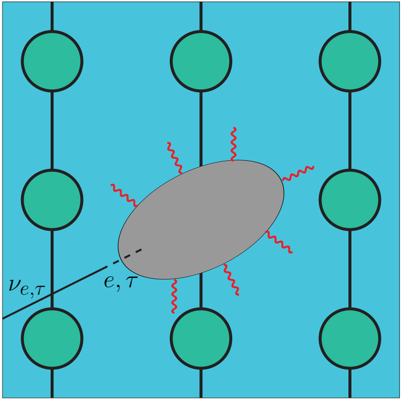
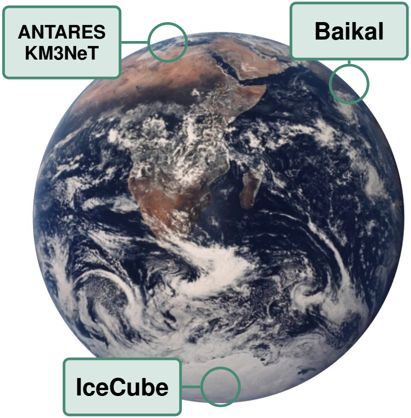




Neutrino Cherenkov Telescopes

Astrophysical neutrinos are an important addition to **multi-messenger** astronomy (no deflection & absorption in space; "smoking-gun" of cosmic rays)



detector requirements:

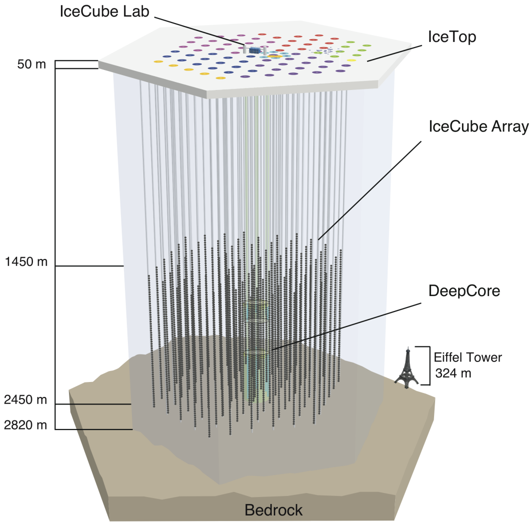
$$N_N \times \sigma_{\nu N} \times \frac{\mathrm{d}^2 N_\nu}{\mathrm{d}t\,\mathrm{d}A} \sim \frac{1}{\text{year}} \times \frac{V}{1\text{km}^3}$$

$$\rightarrow M_{\text{det}} \simeq V \times m_p \sim 1\ \text{Gton}$$

realization:

Observation of **Cherenkov light** in km^3-volumes of deep ocean water (Mediterranean), fresh water (Lake Baikal) or ice (Antarctic).

The IceCube Observatory

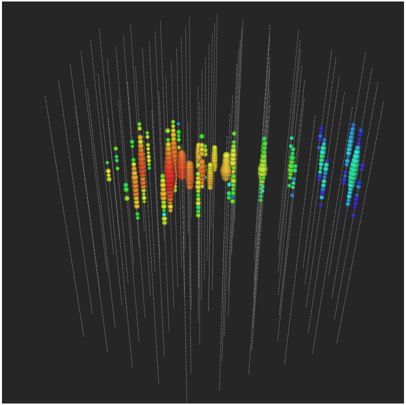


- Giga-ton telescope at the South Pole
- Collaboration of about 250 people at 43 intl. institutions
- 60 digital **optical modules** (DOMs) per string
- **78 IceCube strings**
  125 m apart on triangular grid
- **8 DeepCore strings**
  DOMs in particularly clear ice
- **81 IceTop stations**
  two tanks per station, two DOMs per tank
- 7 year construction phase (2004-2011)
- price tag: **30 Cents per ton**

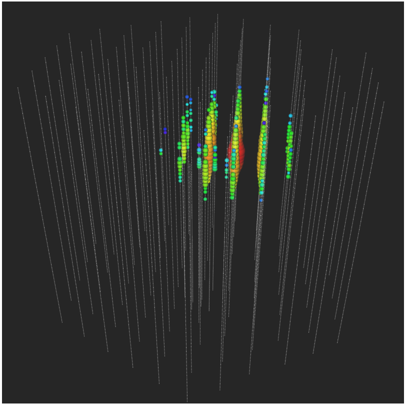
The IceCube Observatory

- “cascades”: **good** energy, but **poor** angular resolution ($\Delta\theta > 10^\circ$)
- “tracks”: **poor** energy, but **good** angular resolution ($\Delta\theta \lesssim 1^\circ$)
- **time-dependent** signal: early to late light detection

track event (IC-79)



cascade event (IC-86)



[two examples from the high-energy starting event (HESE) analysis; IceCube Science 342 (2013)]

Atmospheric neutrino flux and diffuse limit

- high-energy atmospheric ν_μ/ν_e-spectrum as seen by **IC-40** & **IC-79/DC**

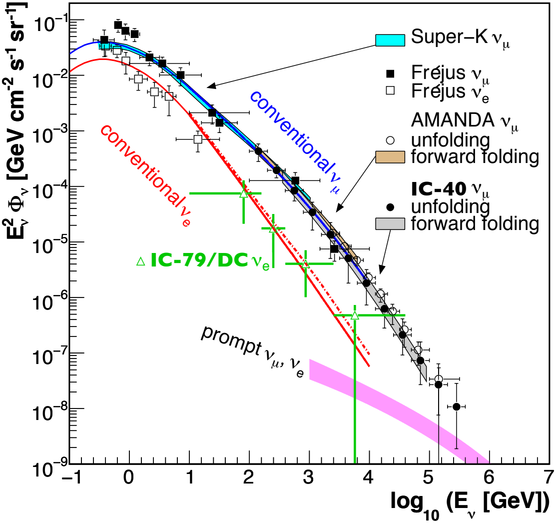
  [IceCube'11,'12]

- predicted prompt atmospheric ν-fluxes (charmed meson decay)

  [Enberg *et al.*'08]

➜ high-energy starting event (HESE) analysis

[IceCube Science'13]



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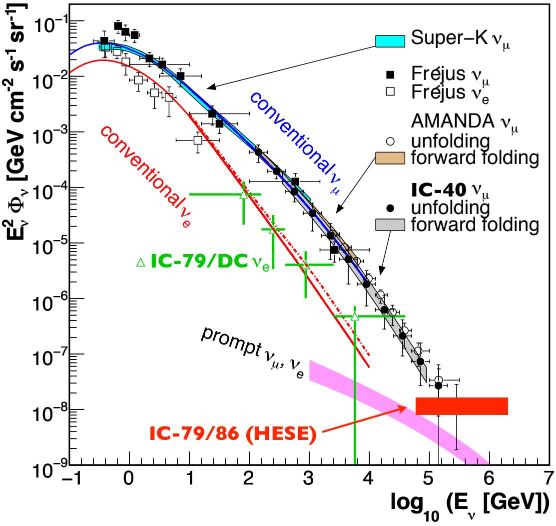
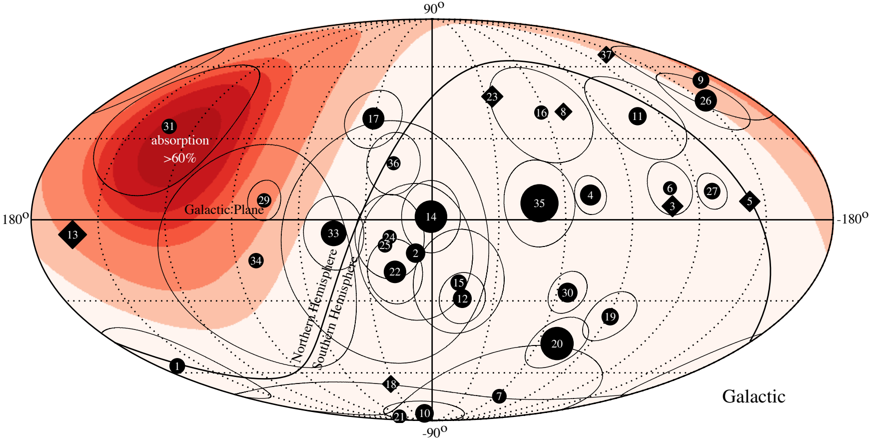


Arrival Directions



- **28 “cascade events” (circles) and 7 “tracks events” (diamonds)**; size of symbols proportional to deposited energy (30 TeV to 2 PeV) [IceCube PRL 113 (2014)]

✘ **no significant spatial or temporal correlation of events**

Appendix

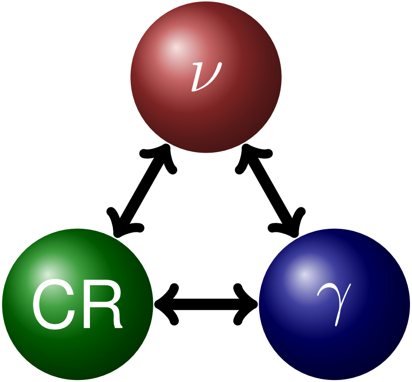
Multi-messenger paradigm

- Neutrino production is closely related to the production of cosmic rays (CRs) and γ-rays.

- 1 PeV neutrinos correspond to 20 PeV CR nucleons and 2 PeV γ-rays

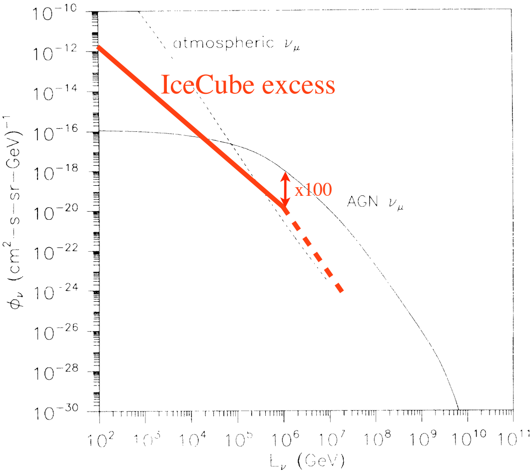
➜ **very interesting** energy range:

  - galactic or extragalactic?
  - isotropic or point-sources?
  - PeV γ-ray counterparts?
  - Glashow resonance visible? ($\bar{\nu}_e e^- \to W^-$ at $E_\nu \simeq 6.3$ PeV)



Active Galactic Nuclei

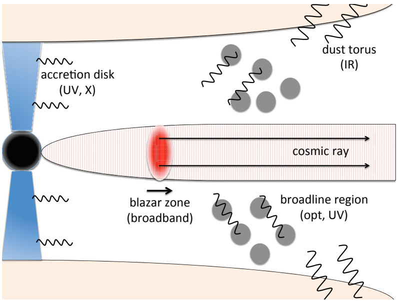
- neutrino interactions from $p\gamma$ interactions in AGN cores [Stecker *et al.*'91]
- AGN diffuse emission normalized to X-ray background
- revised model predicts 5% of original estimate [Stecker'05;'13]

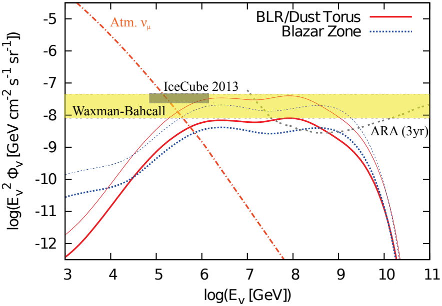


[Stecker *et al.*'91]

Active Galactic Nuclei

- neutrino from $p\gamma$ interactions in AGN jets [Mannheim'96; Halzen & Zas'97]
- complex spectra due to various photon backgrounds
- typically, deficit of sub-PeV and excess of EeV neutrinos

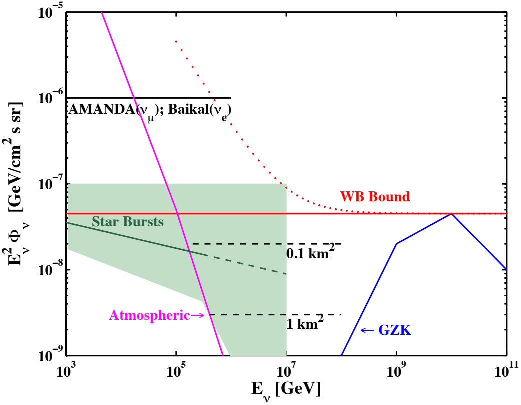




[Murase, Inoue & Dermer 1403.4089]

Starburst galaxies

- intense CR interactions (and acceleration) in dense starburst galaxies
- cutoff/break feature $(0.1 - 1)$ PeV at the CR knee (of these galaxies), but very uncertain
- plot shows muon neutrinos on production $(3/2$ of total)



[Loeb & Waxman'06]

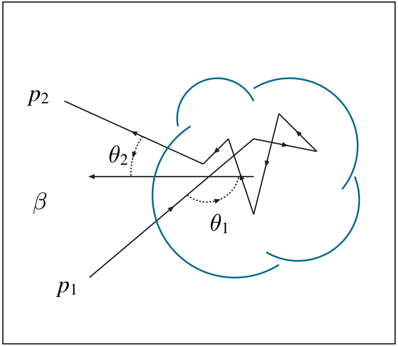
Fermi acceleration (second order)

- “magnetic cloud” with velocity β.
- momentum in rest frame

$$E_1' = \gamma E_1(1 - \beta\cos\theta_1)$$

- elastic scattering within cloud conserves energy ($E_2' = E_1'$) but isotropizes the emission direction θ_2'
- emitted energy

$$E_2 = \gamma E_2'(1 + \beta\cos\theta_2')$$



- energy gain per scatter:

$$\frac{\Delta E}{E_1} = \frac{E_2 - E_1}{E_1} = \gamma^2(1 + \beta\cos\theta_2')(1 - \beta\cos\theta_1) - 1$$

➜ can be **positive or negative** depending on scattering angle

Fermi acceleration (second order)

- distribution of θ_2' is (appr.) isotropic $\frac{\mathrm{d}n}{\mathrm{d}\cos\theta_2'} \propto 1$
- averaging over θ_2':

$$\frac{\langle \Delta E \rangle_{\theta_2'}}{E_1} = \int_{-1}^{1} \mathrm{d}\cos\theta_2' \frac{\mathrm{d}n}{\mathrm{d}\cos\theta_2'} \frac{\Delta E}{E_1} = \frac{1}{2}\int_{-1}^{1} \mathrm{d}\cos\theta_2' \frac{\Delta E}{E_1} = \gamma^2(1-\beta\cos\theta_1) - 1$$

- distribution of θ_1 follows number of particles per second in direction θ_1

$$\frac{\mathrm{d}n}{\mathrm{d}\cos\theta_1} \propto (1-\beta\cos\theta_1)$$

- further averaging over θ_1

$$\begin{aligned}\frac{\langle \Delta E \rangle_{\theta_1 \& \theta_2'}}{E_1} &= \int_{-1}^{1} \mathrm{d}\cos\theta_1 \frac{\mathrm{d}n}{\mathrm{d}\cos\theta_1} \frac{\langle \Delta E \rangle_{\theta_2'}}{E_1} \\ &= \frac{1}{2}\int_{-1}^{1} \mathrm{d}\cos\theta_1 (1-\beta\cos\theta_1)[\gamma^2(1-\beta\cos\theta_1) - 1] \\ &= \gamma^2\left(1+\frac{\beta^2}{3}\right) - 1 = \frac{1+\frac{\beta^2}{3}}{1-\beta^2} - 1 \simeq 1 + \frac{\beta^2}{3} + \beta^2 - 1 = \frac{4}{3}\beta^2\end{aligned}$$

➜ **on average energy gain** with $\Delta E/E \propto \beta^2$

➜ slow for $\beta \ll 1$; these days called *second order* Fermi acceleration

Fermi acceleration (first order)

- “downstream” relative velocity β (relative to “upstream”)
- momentum in “downstream” rest frame

$$E'_1 = \gamma E_1(1 - \beta\cos\theta_1)$$

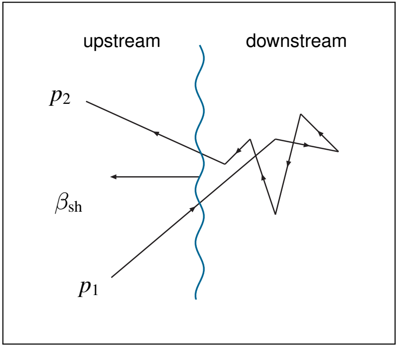
- elastic scattering in “downstream” region conserves energy ($E'_2 = E'_1$) but isotropizes direction
- emitted energy

$$E_2 = \gamma E'_2(1 + \beta\cos\theta'_2)$$

- energy gain per scatter:

$$\frac{\Delta E}{E_1} = \frac{E_2 - E_1}{E_1} = \gamma^2(1 + \beta\cos\theta'_2)(1 - \beta\cos\theta_1) - 1$$

➜ **always positive** since $\cos\theta_1 < 0$ and $\cos\theta'_2 > 0$



Fermi acceleration (first order)

- distributions of θ_1 and θ_2' follow projection onto the shock:

$$\frac{\mathrm{d}n}{\mathrm{d}\cos\theta_1} \propto \cos\theta_1 \ \ (\cos\theta_1 < 0) \qquad\qquad \frac{\mathrm{d}n}{\mathrm{d}\cos\theta_2'} \propto \cos\theta_2' \ \ (\cos\theta_2' > 0)$$

- averaging over θ_2':

$$\frac{\langle \Delta E \rangle_{\theta_2'}}{E_1} = \int_{-1}^{1} \mathrm{d}\cos\theta_2' \frac{\mathrm{d}n}{\mathrm{d}\cos\theta_2'} \frac{\Delta E}{E_1} = 2\int_0^1 \mathrm{d}\cos\theta_2' \cos\theta_2' \frac{\Delta E}{E_1}$$
$$= \gamma^2 (1 - \beta\cos\theta_1 + \frac{2}{3}\beta - \frac{2}{3}\beta^2\cos\theta_1) - 1$$

- also averaging over θ_1

$$\frac{\langle \Delta E \rangle_{\theta_1 \& \theta_2'}}{E_1} = \int_{-1}^{1} \mathrm{d}\cos\theta_1 \frac{\mathrm{d}n}{\mathrm{d}\cos\theta_1} \frac{\langle \Delta E \rangle_{\theta_2'}}{E_1}$$
$$= -2\int_{-1}^{0} \mathrm{d}\cos\theta_1 \cos\theta_1 [\gamma^2(1 - \beta\cos\theta_1 + \frac{2}{3}\beta - \frac{2}{3}\beta^2\cos\theta_1) - 1]$$
$$= \gamma^2\left(1 + \frac{2}{3}\beta\right)^2 - 1 \simeq 1 + \frac{4}{3}\beta - 1 = \frac{4}{3}\beta$$

➜ **on average energy gain** with $\Delta E / E \propto \beta$

➜ *first order* Fermi acceleration more efficient