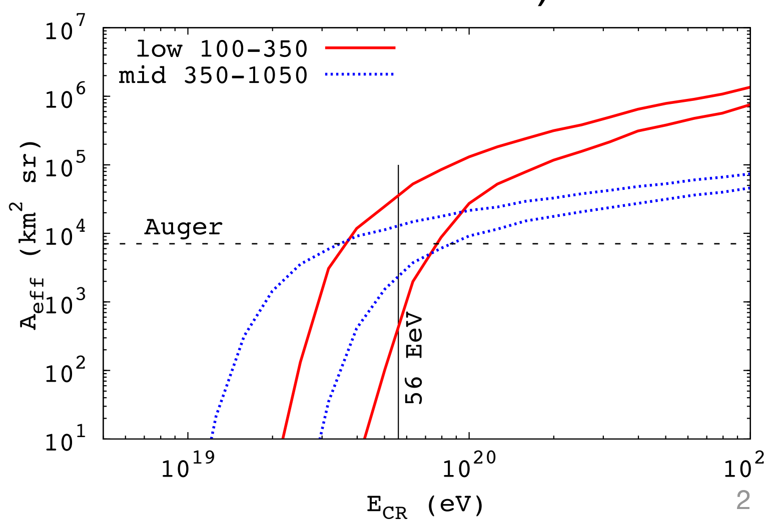
The effects of surface roughness on lunar Askaryan pulses

Clancy W. James, ECAP
University of Erlangen-Nuremberg
ARENA 2014, Annapolis, Maryland

ERLANGEN CENTRE FOR ASTROPARTICLE PHYSICS
FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG

Motivation

- Lunar surface roughness
  - The Moon is not smooth. How to model this?
  - Current treatment: ignores small-scale roughness. Let's do this properly!
- History of the work:
  - Work initially presented in Nantes (ARENA 2010), over 3 slides
  - Poster at ICRC 2013 (one nice person even asked me about it!)
- Renewed ('newed'?) interest:
  - ANITA roughness
  - Observing with the SKA

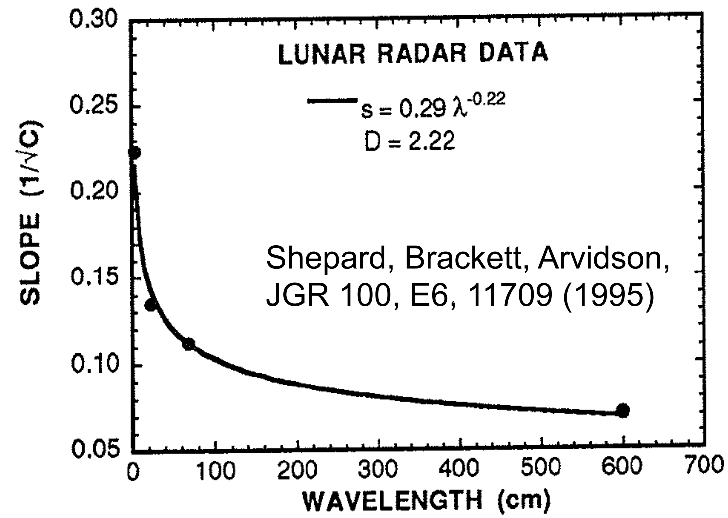


Current treatment in simulations*

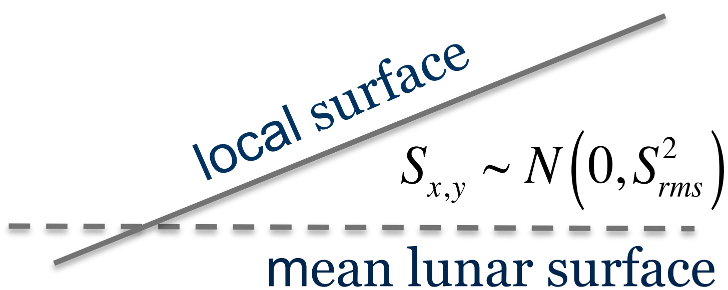
- Surface roughness: use radar backscatter

$$\tan S_{\mathrm{rms}} = 0.29\left(\frac{\lambda}{1\ \mathrm{cm}}\right)^{-0.22}$$

  - 7.8 degrees at 1 GHz
  - 4.7 degrees at 100 MHz



- Deviate local surface and apply ray tracing:



$$S_{x,y} \sim N\left(0, S_{rms}^2\right)$$

$$t_{\perp} = \frac{2n_t \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

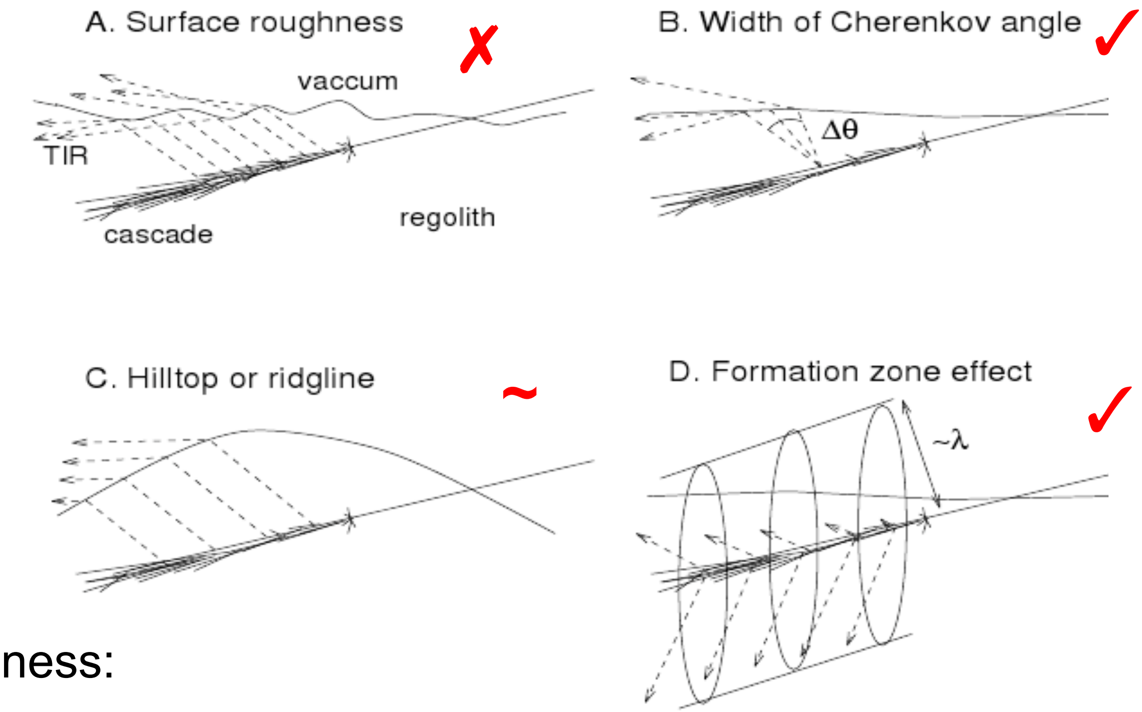
$$t_{\parallel} = \frac{2n_t \cos\theta_t}{n_t \cos\theta_t + n_i \cos\theta_i}$$

Fresnel with 'solid-angle stretching'

*Sims for Goldstone, Kalyazin, & LUNASKA (similar for ANITA)

Lunar surface effects

- B: Treated in simulations
- D: D.N.E.
- C: ~treated statistically
- A: ??? Let's do this!



P.W. Gorham et al., RADHEP, 2001

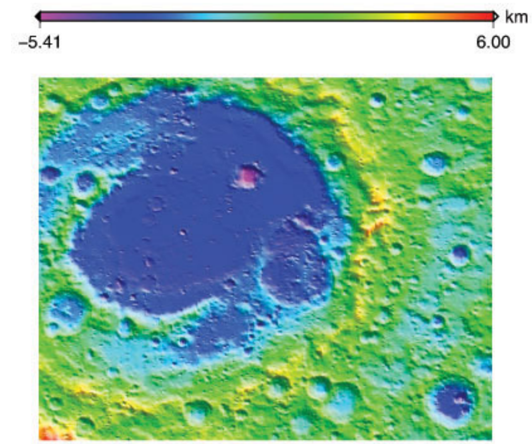
- Effects of small-scale roughness:
    - Decoherence – increases detection threshold
    - Scattering – increases effective area

- *Let's do this properly!*

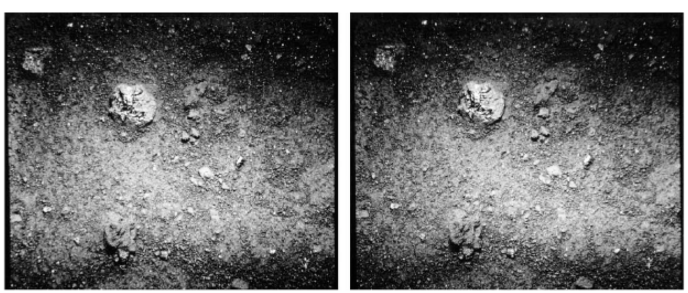
Making a surface

What we know of the lunar surface

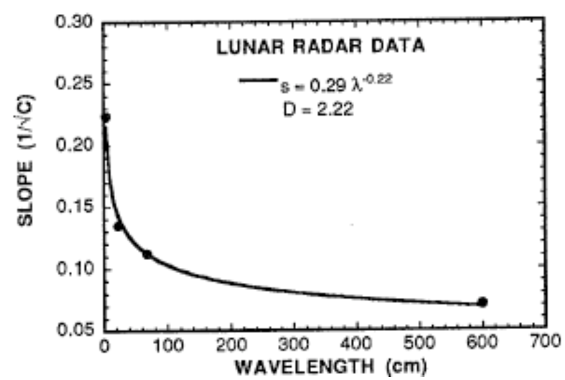
- Largest scales: satellite laser altimiters
  - Lunar Reconnaissance Orbiter (LRO) global maps to 500m
  - LRO North/South pole maps (to 5m)
- Intermediate scale: radar reflections
  - Arecibo backscatter (statistical data) from 1cm to 6m
  - Radio polarisation at lunar limb
- Smallest scales: stereo photography
  - Apollo missions: 1mm to 10 cm



Credit: NASA/LRO/LOLA (2010)
http://lunar.gsfc.nasa.gov/

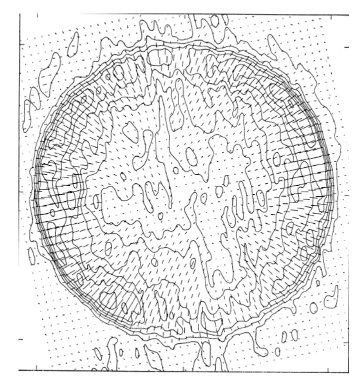


Helfenstein & Shepard, Icarus 141, 107 (1999)



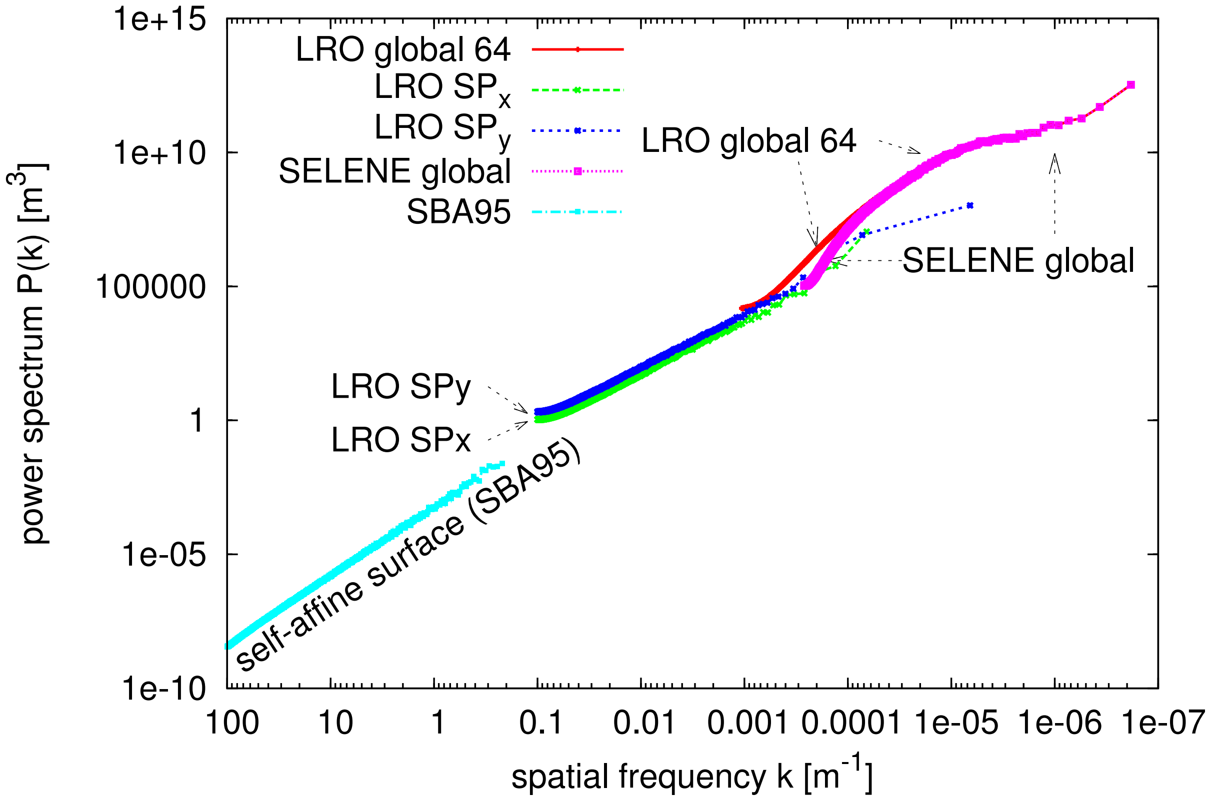
Shepard et al, J. Geo.Res.
100 E6 11709 (1995)

P.H.Moffat, MNRAS 160, 139 (1972)



What we know of the lunar surface

- Lunar surface data: power spectrum over the entire Moon

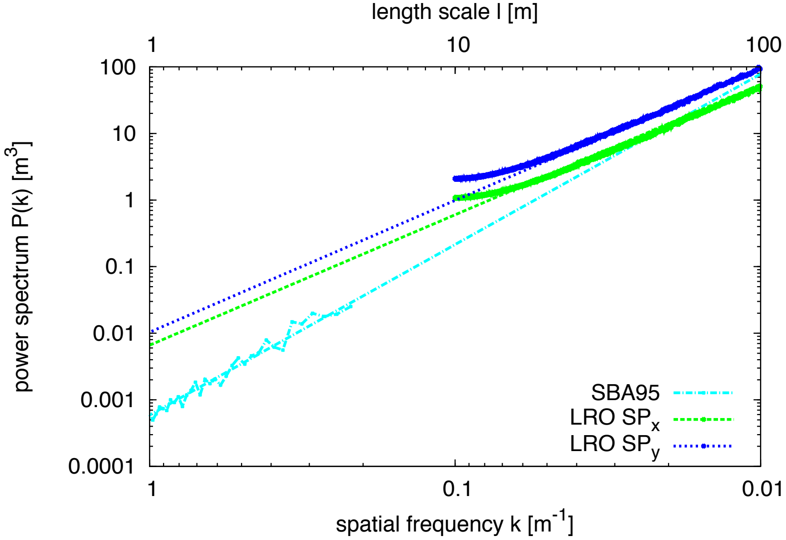


Combining LRO SP data and random roughness?

- Strange upturn in LRO polar spectrum
  - Instrumental errors: add artificial power?
  - DFT properties – FFT over finite length
  - Data processing (usually would cause smoothing…)

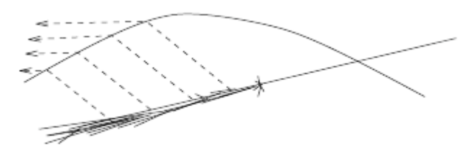
- Future work:
  - Use exact lunar maps to 5m
  - Add random component at smaller scales
  - Not yet complete

- Here: use only random rough surface

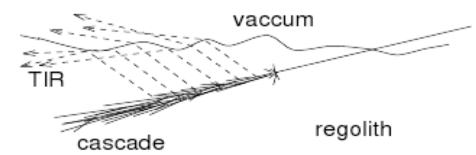


Example: random rough surface

C. Hilltop or ridgeline

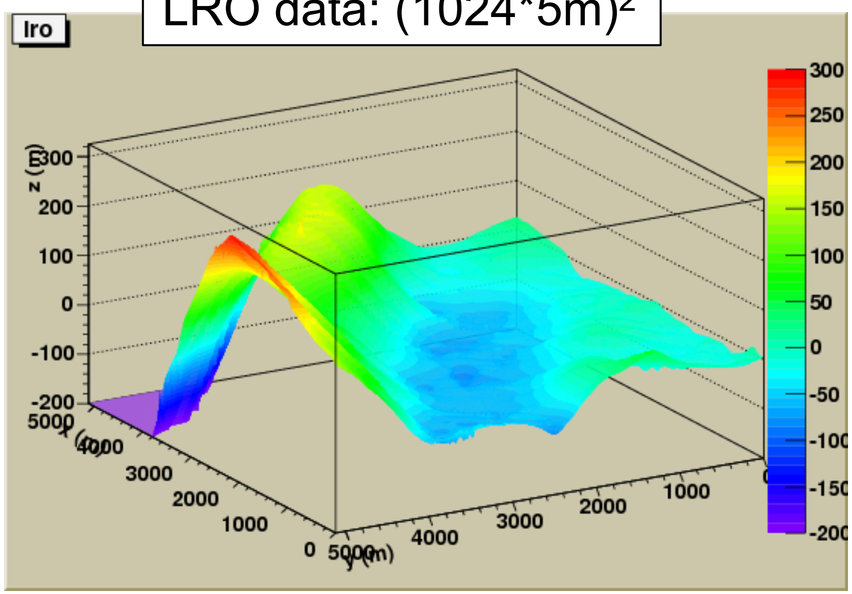


A. Surface roughness

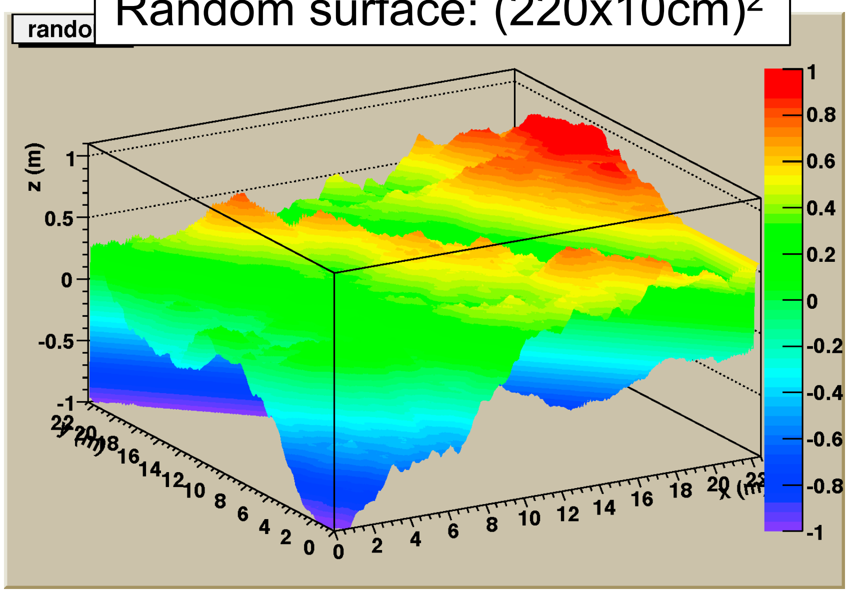


- Generate random surface using power-spectrum method
  - Create Fourier Transform of the surface
  - FFT to spatial domain
  - Cut away edges

LRO data: $(1024*5m)^2$

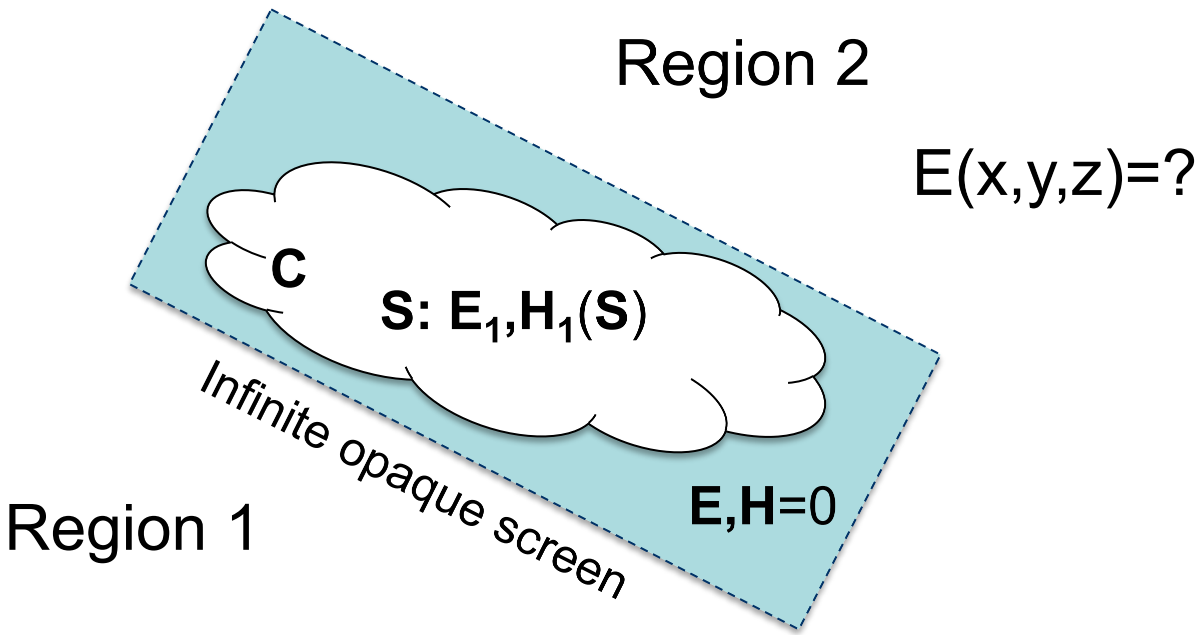


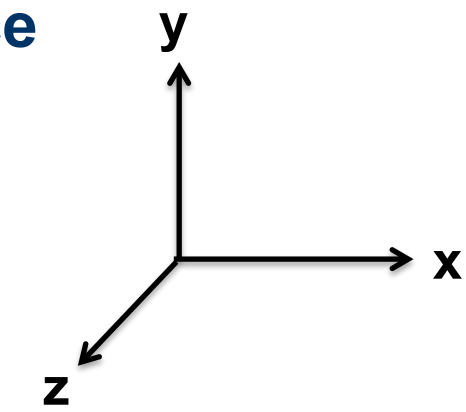
Random surface: $(220x10cm)^2$



Radio transmission through a rough surface

Transmission of fields through an interface



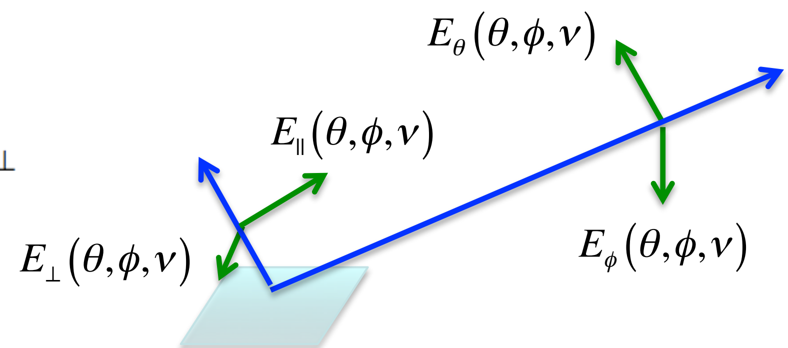


- Field in region 2: (Stratton & Chu. 1939, Eq. 24):

$$4\pi E\left(x,y,z\right) = -\frac{1}{i\omega\epsilon}\oint_C \nabla\psi\mathbf{H}_1\cdot d\mathbf{s} - \iint_{S_1}\left[i\omega\mu(\mathbf{n}\times\mathbf{H}_1)\psi + (\mathbf{n}\times\mathbf{E}_1)\times\nabla\psi + (\mathbf{n}\cdot\mathbf{E}_1)\nabla\psi\right]da$$

Plane wave solutions on rectangular screen

- Assume:
  - E1, H1 are incident plane waves (far-field Askaryan radiation)
  - E2, H2 are outgoing plane waves (at Earth or lunar satellite)
  - S is a rectangular surface (a facet!)
- Solutions: 4 coefficients between incoming and outgoing parallel and perpendicular polarisations.



$$\mathbf{E}_{\hat{\theta}\perp} = \frac{ikA}{4\pi}(1+\cos\theta\cos\alpha)(\sin\phi\cos\beta - \cos\phi\sin\beta)E_\perp$$

$$\mathbf{E}_{\hat{\phi}\perp} = \frac{ikA}{4\pi}(\cos\alpha+\cos\theta)(\sin\phi\sin\beta + \cos\phi\cos\beta)E_\perp$$

$$\mathbf{E}_{\hat{\theta}\|} = \frac{ikA}{4\pi}(\cos\alpha+\cos\theta)(\sin\phi\sin\beta + \cos\phi\cos\beta)E_\|$$

$$\mathbf{E}_{\hat{\phi}\|} = \frac{ikA}{4\pi}(1+\cos\theta\cos\alpha)(\sin\beta\cos\phi - \sin\phi\cos\beta)E_\|$$

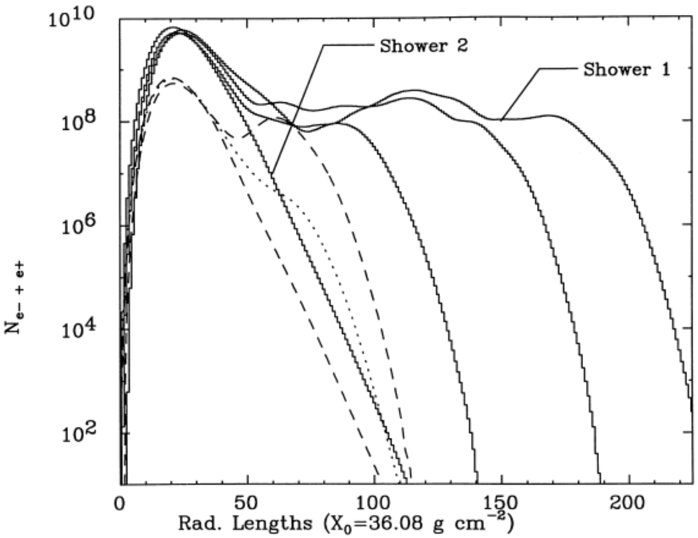
$$A = 4\frac{e^{ikR}}{R}\frac{\sin\left(\frac{a}{2}(\mathbf{k}_t-\mathbf{k}_i)_x\right)}{(\mathbf{k}_t-\mathbf{k}_i)_x}\frac{\sin\left(\frac{b}{2}(\mathbf{k}_t-\mathbf{k}_i)_y\right)}{(\mathbf{k}_t-\mathbf{k}_i)_y}$$

- Full diffractive solution – each facet radiates in all directions

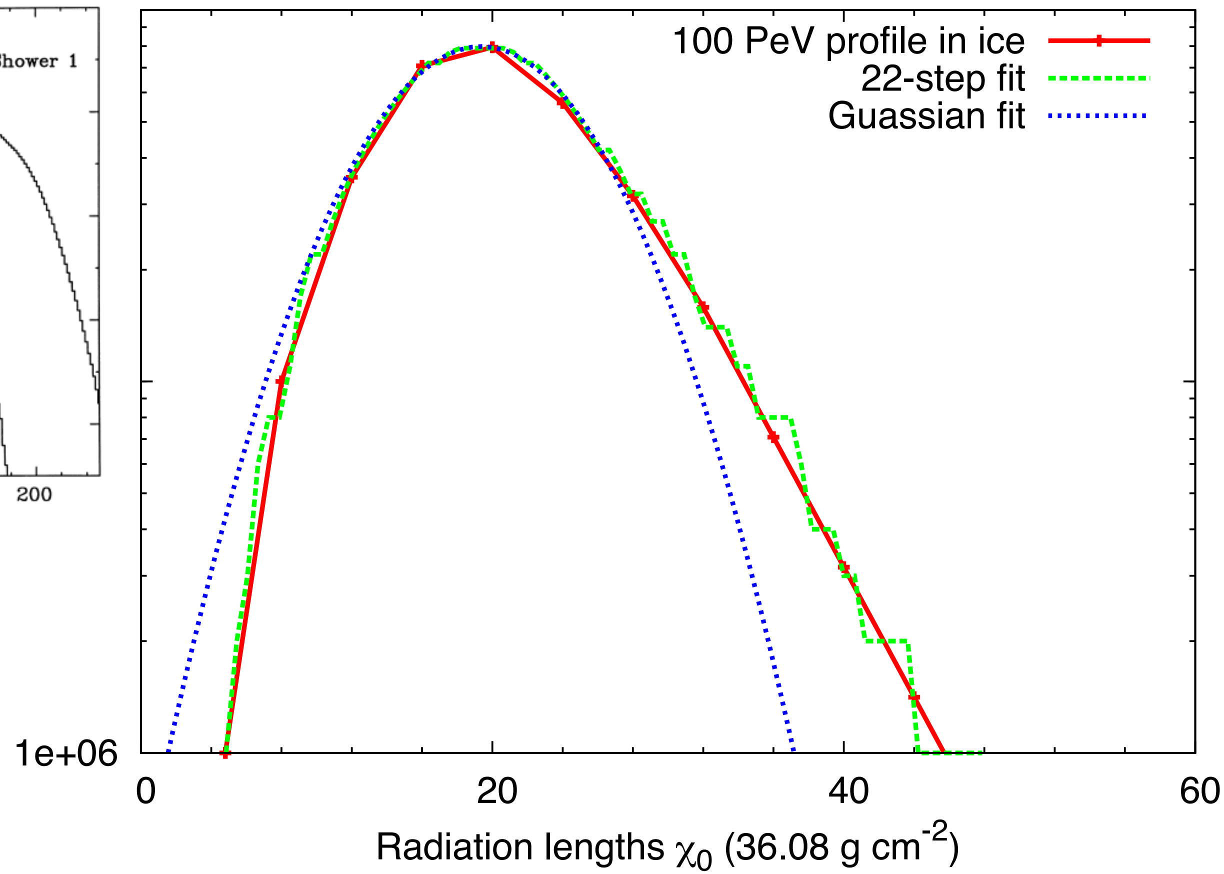
Modelling a hadronic cascade

Hadronic cascade profiles in the Moon

- Fit cascade shape using total charge from simulations in ice:



Alvarez-Muniz, Zas,
Phys.Lett. B 434 (1998) 396



Emission from particle cascades

- Emission from entire cascade:

$$R\,E(f,\mathrm{E}_S) = 8.45\times10^{-8}\,\frac{E_S}{1\ \mathrm{TeV}}\,\frac{f}{1\ \mathrm{GHz}}\,\varepsilon(\mathrm{E}_S)\,\exp\left(-\left(\frac{\theta-\theta_C}{\Delta\theta}\right)^2\right)\left(\frac{1}{1+\left(f/\mathrm{f}_0\right)^{1.23}}\right)^{-1}\quad(\mathrm{V/MHz})$$

$f_0 = 2.32\ \mathrm{GHz}$ (lateral decoherence)

$\Delta\theta_0 \approx 2.4^\circ - 0.18^\circ \log(E_S/10\ \mathrm{EeV})$ (longitudinal decoherence)

$\varepsilon(\mathrm{E}_S) \approx 90\%$ (electromagnetic fraction)

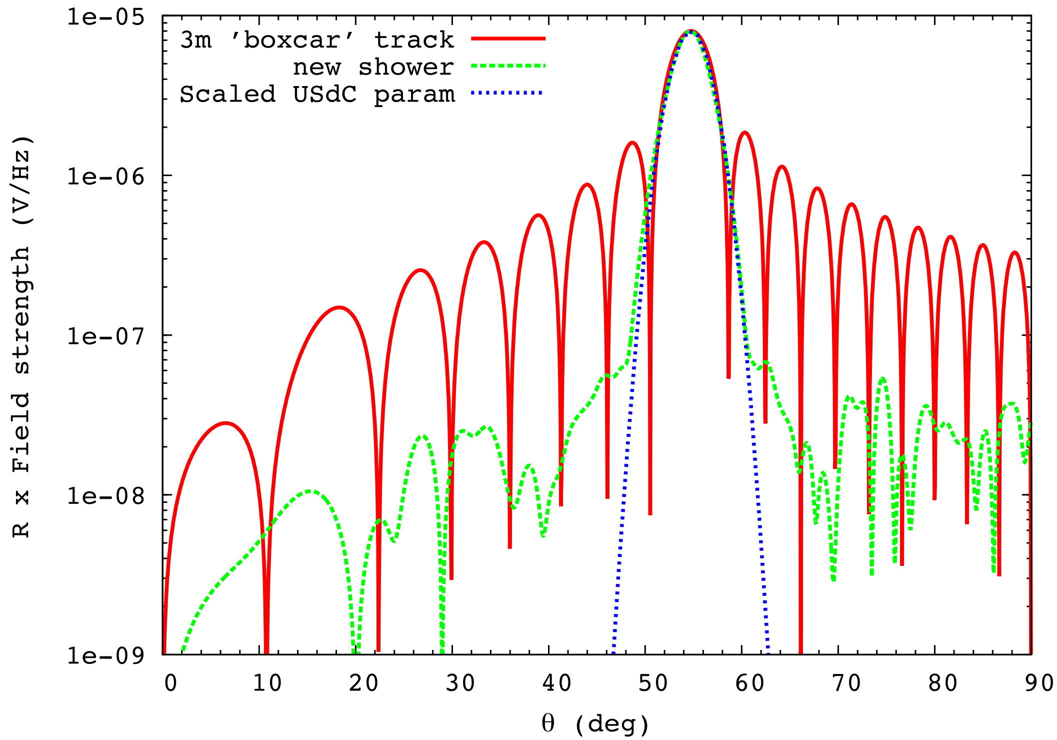
  - Model longitudinal profile only, assume lateral decoherence ~ independent of surface roughness

- Emission from small piece of longitudinal profile: ZHS formula

$$\left|\vec{E}(\theta,R,\nu)\right| = \frac{q}{c}\,\frac{e^{i(kR)}}{R(t)}\,e^{2\pi i\nu t_1}\left(\frac{1-e^{2\pi i\nu(1-n\beta\cos\theta)(t_2-t_1)}}{1-n\beta\cos\theta}\right)\beta\sin\theta$$

Angular spectrum of emission in the Moon

- 3m boxcar vs 43-track spectrum vs analytic parameterisation

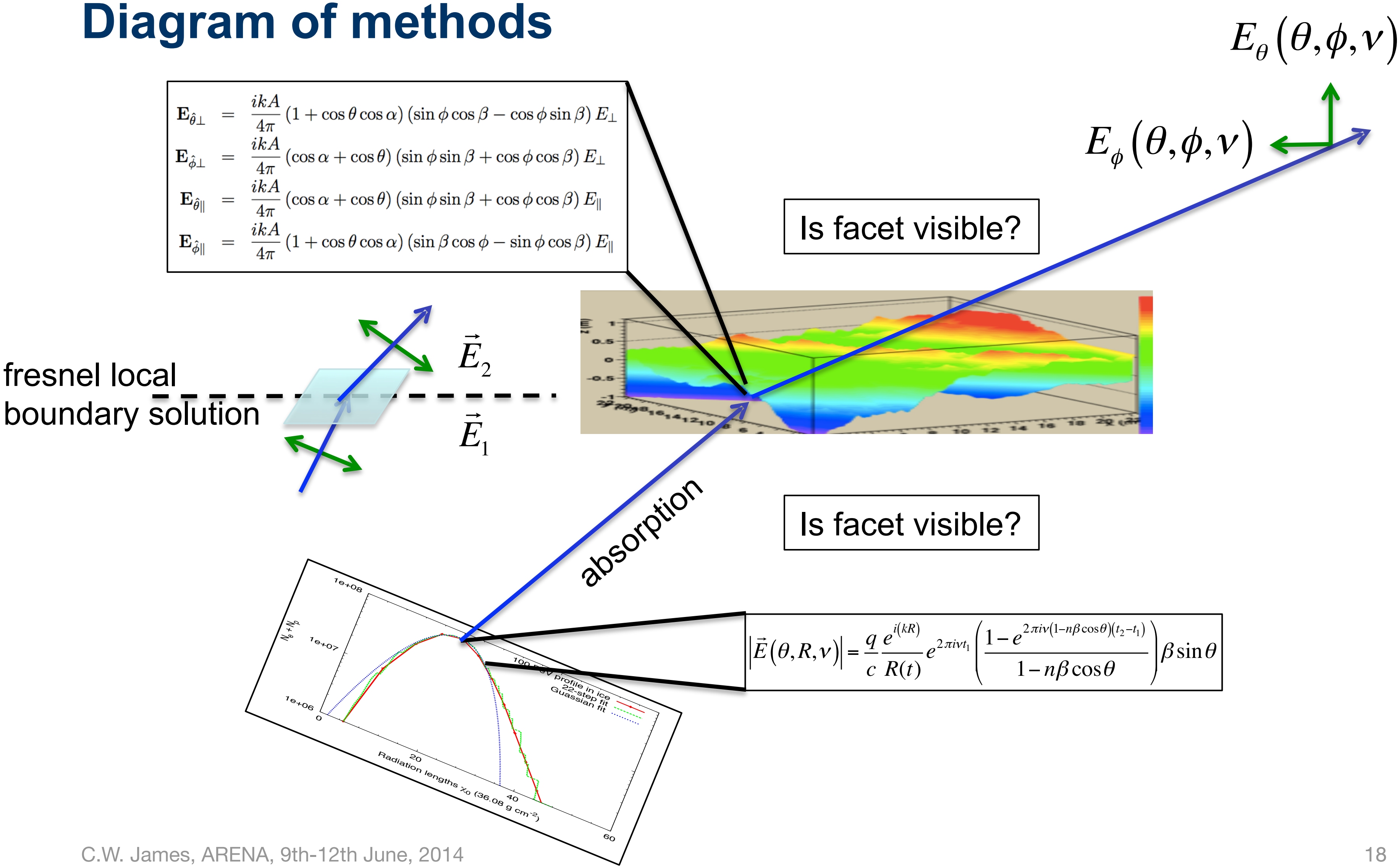


- Here: results for both a 3m boxcar and a 43-piece model

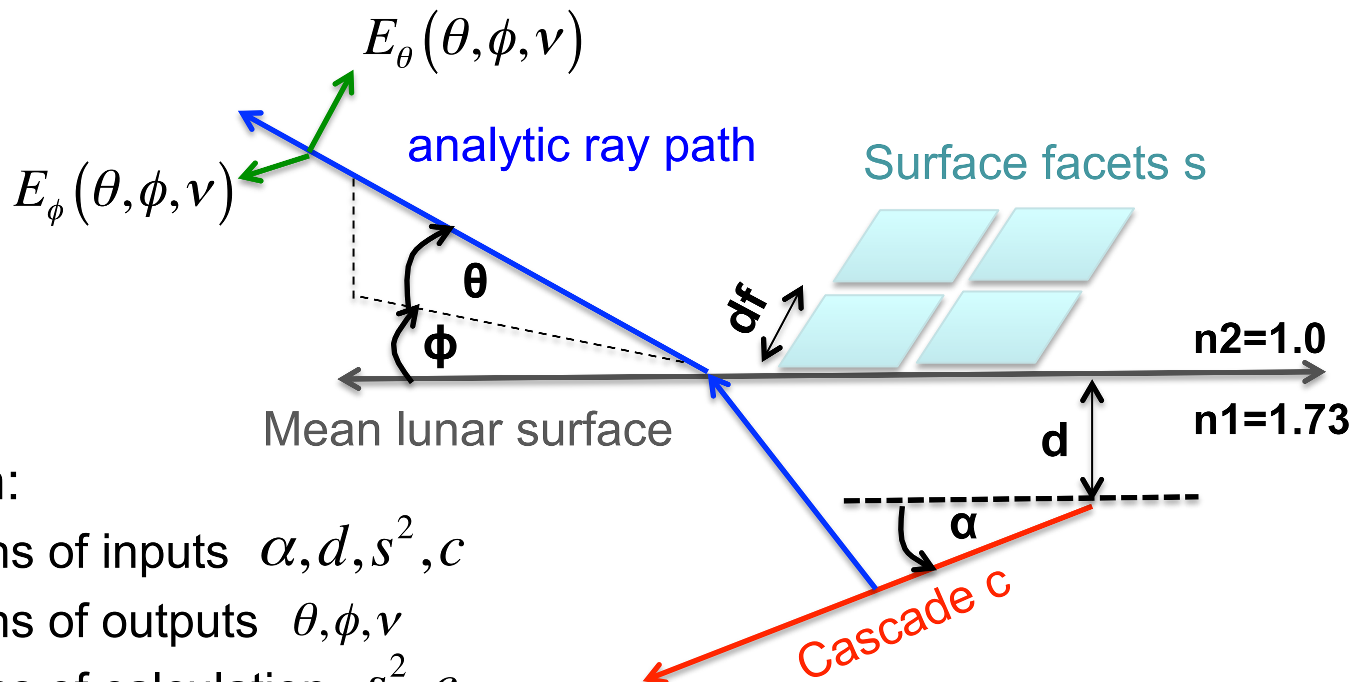
Scaling: Alvarez-Muniz, Marques, Vazquez, Zas, Phys Rev D 74, 023007 (2006)

Putting it all together

Diagram of methods



Description of the problem

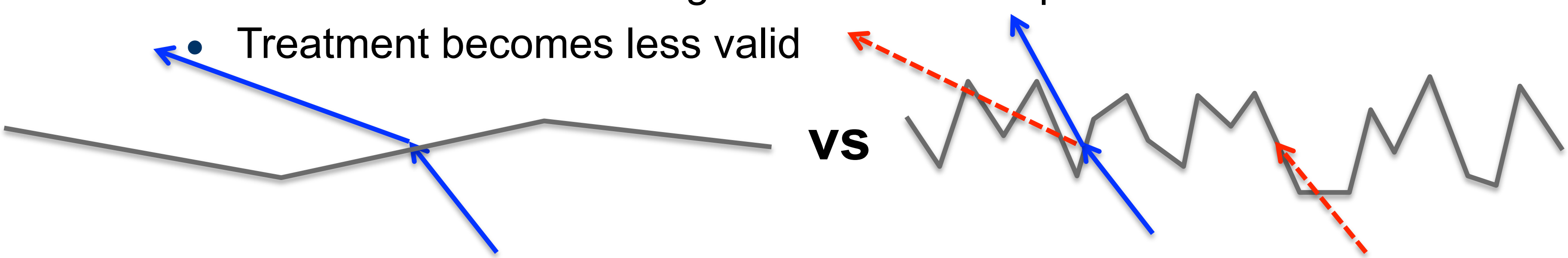


- Full calculation:
  - 5 dimensions of inputs α, d, s^2, c
  - 3 dimensions of outputs θ, ϕ, ν
  - 3 dimensions of calculation s^2, c

- Today, only $\phi = 0$, one rough surface/cascade
- Look at θ, ν dependence for select geometries

Numerical dials

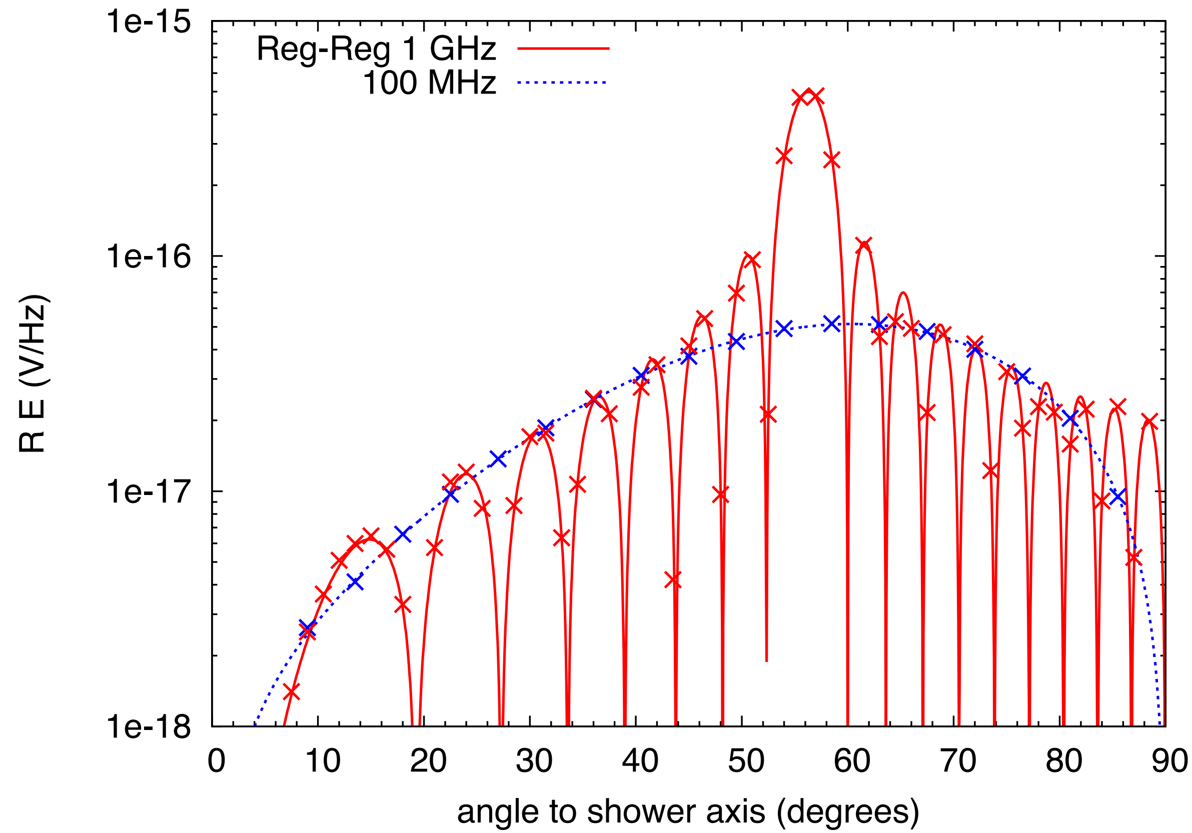
- Length scales: tune using numerical convergence
  - Surface size: how large a surface do we need to describe?
  - Track division: how short do we need the tracks?
  - Facet division: how small do we need the facets?
- Self shadowing
  - Roughness increases at small length scales
  - Effects of self-shadowing become more important
  - Treatment becomes less valid

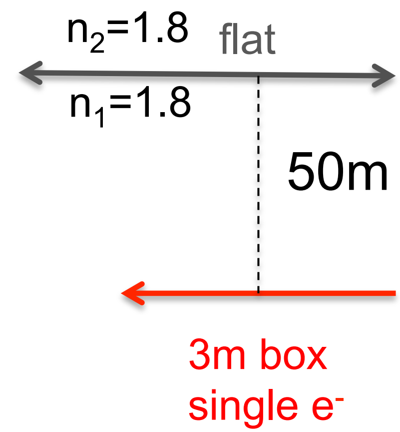


  - Current treatment: describe roughness to ~1cm, track fraction of self-shadowed facets, make sure it's not 'too high' (whatever that is).

Testing against analytic formula: flat surfaces

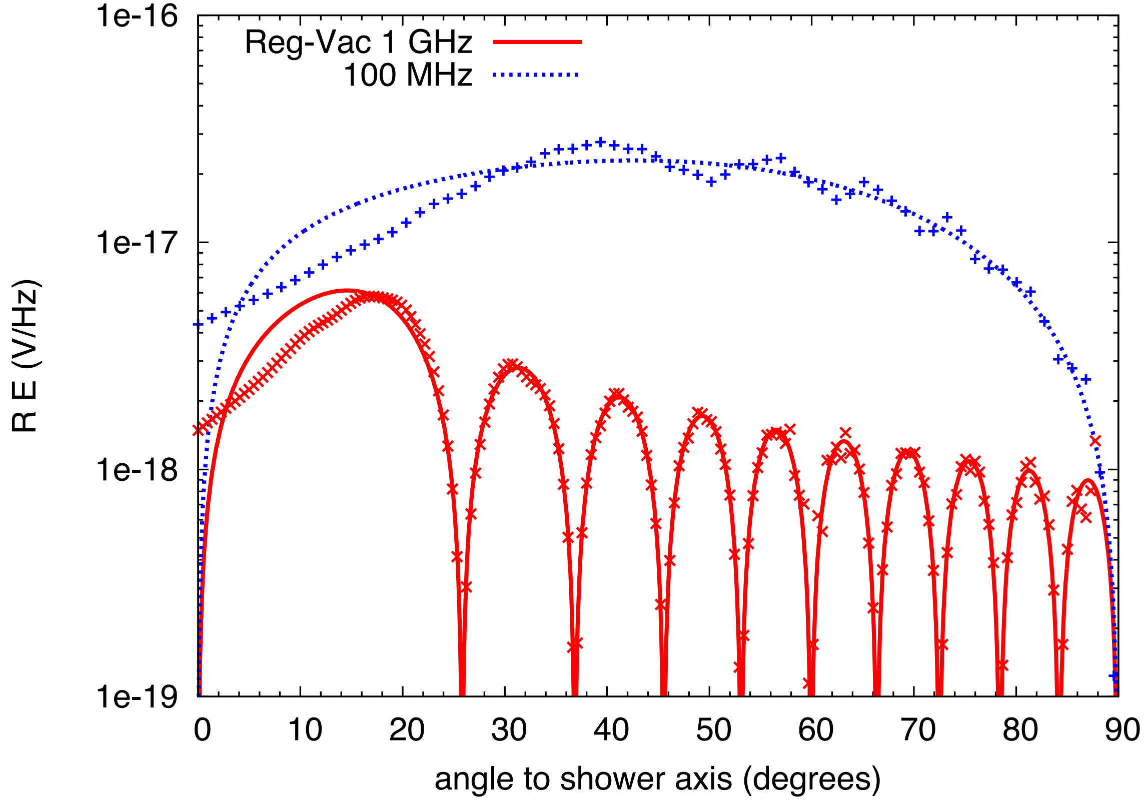
3m boxcar: simulation vs theory

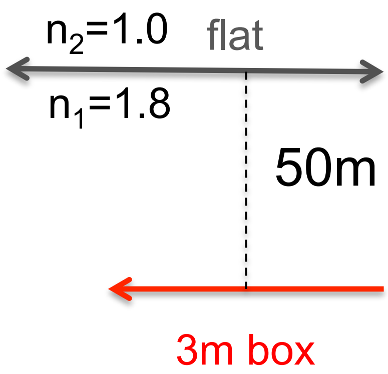




- Lines: 5 lines gnuplot code
- Points: each a sum of ~8x10^8 track/facet pieces

Flat-surface transmission: 50m depth

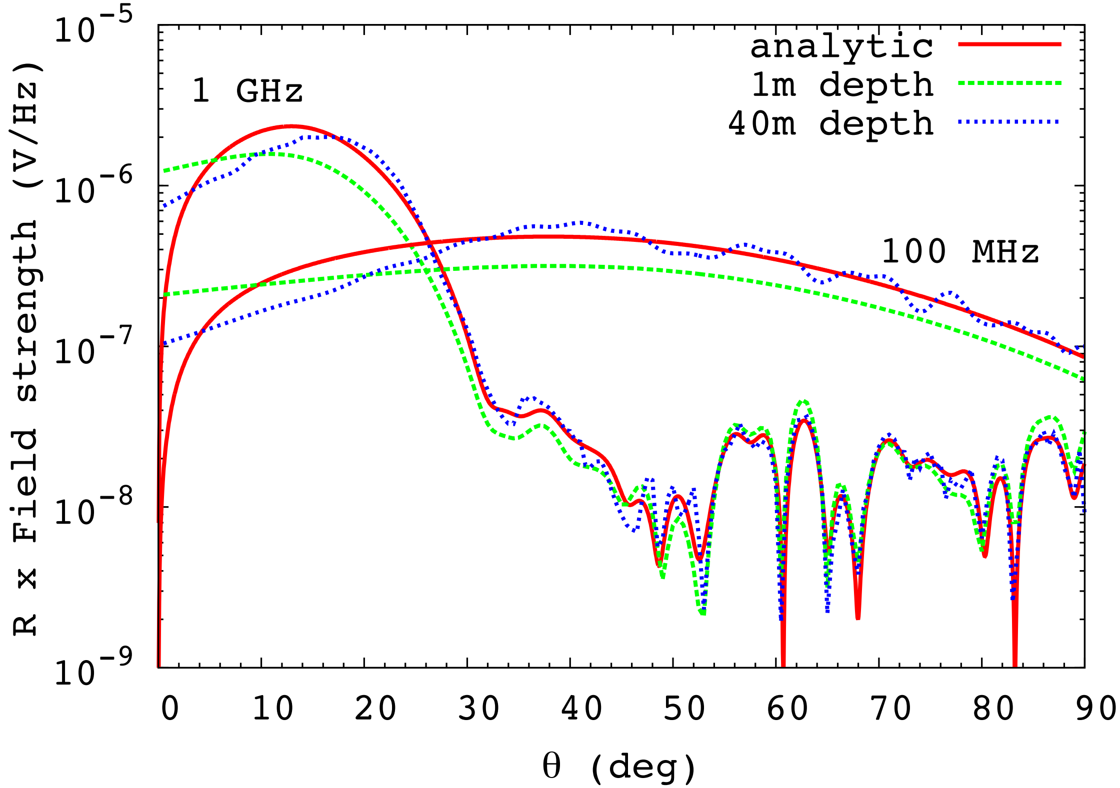


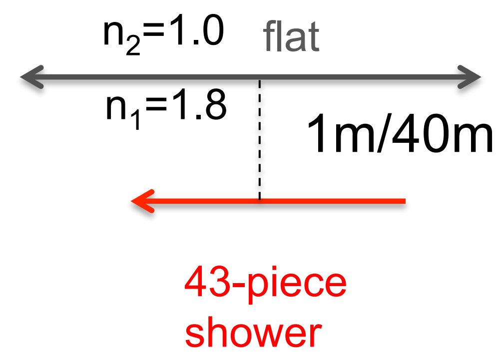


- Analytic model does not include diffraction!
- When estimates differ, the numerical model is (probably) correct

Testing for near-surface showers (CR)

PRELIMINARY
(numerical dials still need turning)



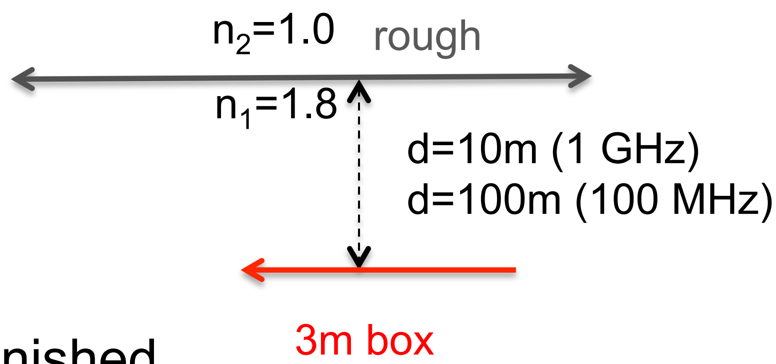


- ZHS @ 1m: OK when transmitting to infinity?
- Caution: analytic results ignore diffraction

Preliminary results: ~neutrinos

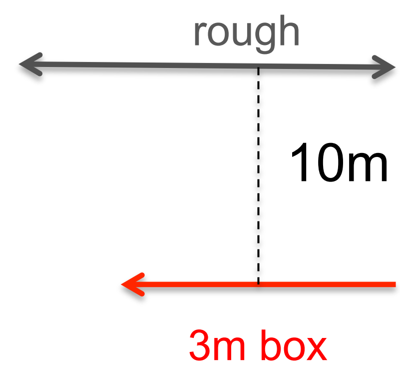
Typical neutrino geometries

- Neutrinos: detect 'skimming' events (parallel to the surface)
- Field attenuation length: $\ell_{abs} = 104\lambda_n$
  - 1 GHz: probe to ~10m
  - 100 MHz: probe to ~100m

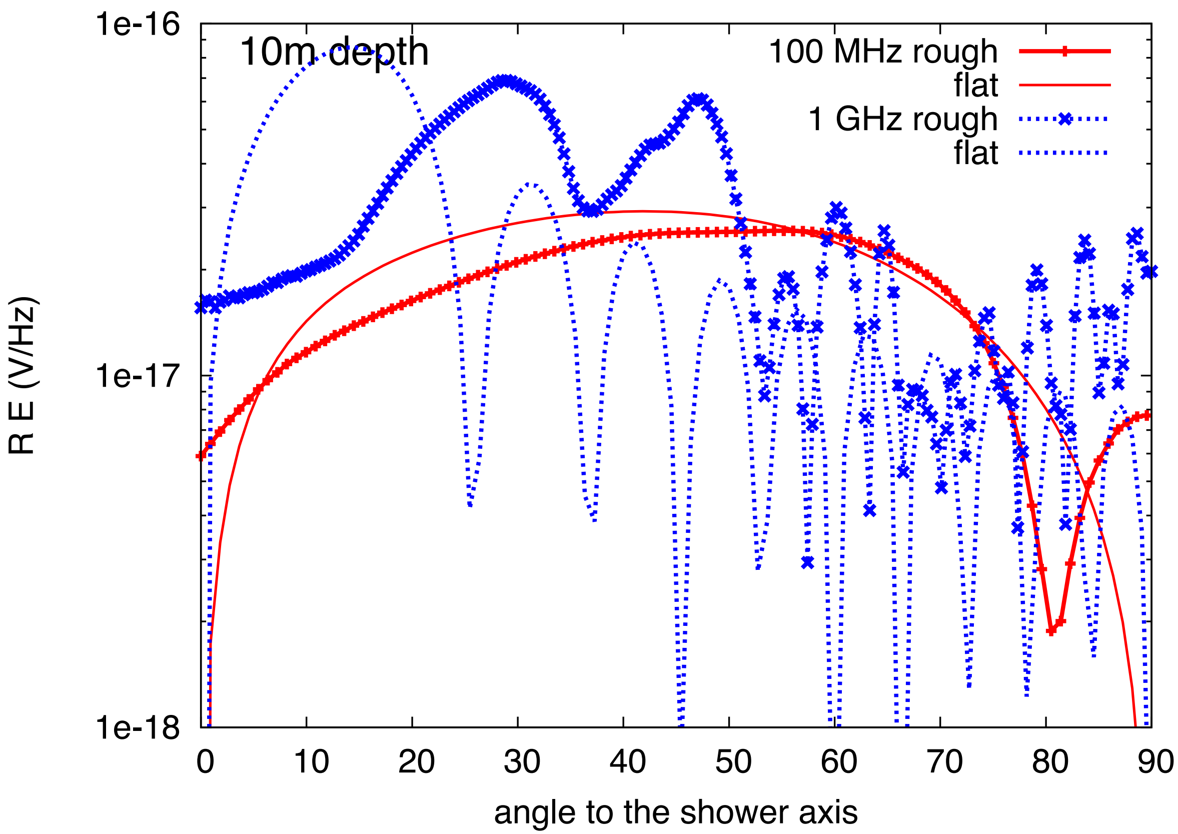


- Simulations for shower profile not yet finished…
- What are the effects of surface roughness at high and low frequencies?
  - Angular distribution
  - Pulse coherence

Results with simple 3m, 1 e^- track model

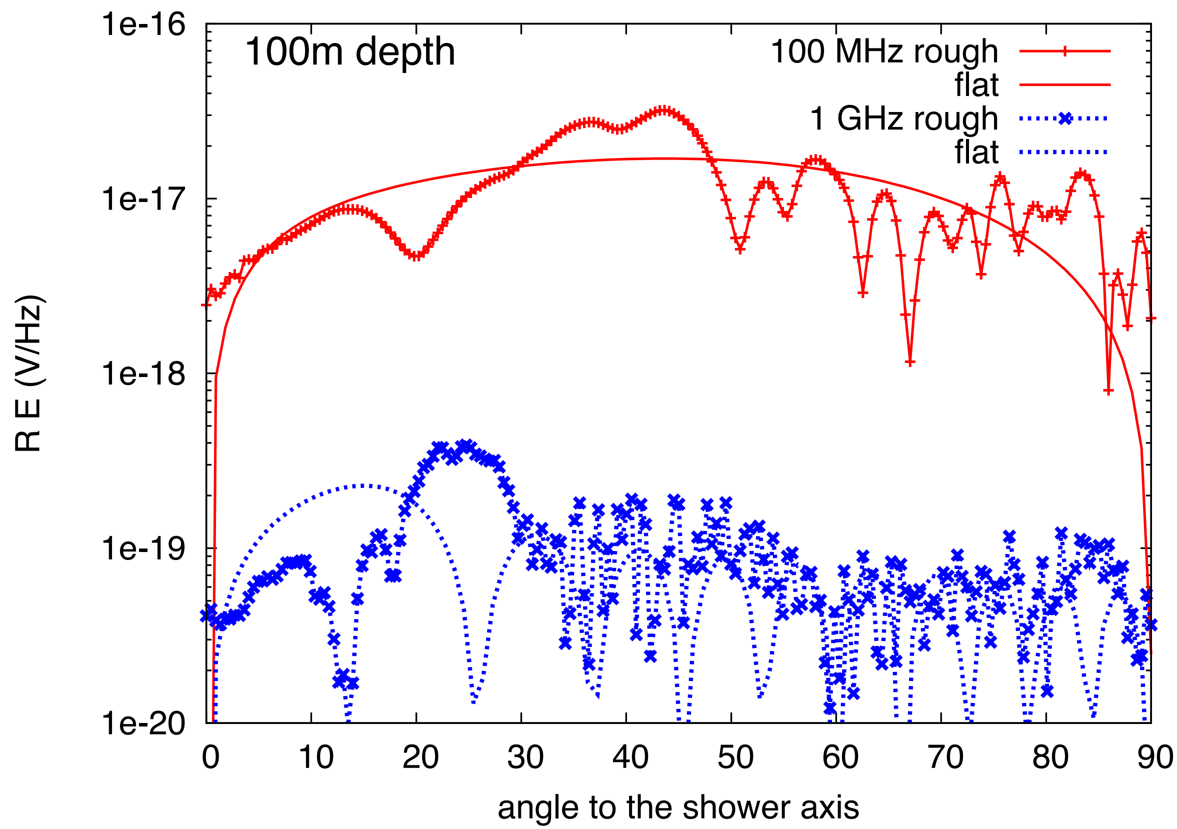


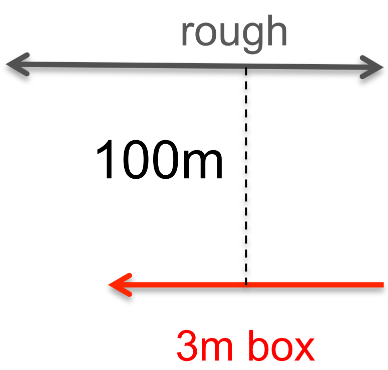
- '3m track' model, 10m depth, parallel to surface



Results with simple 3m track model

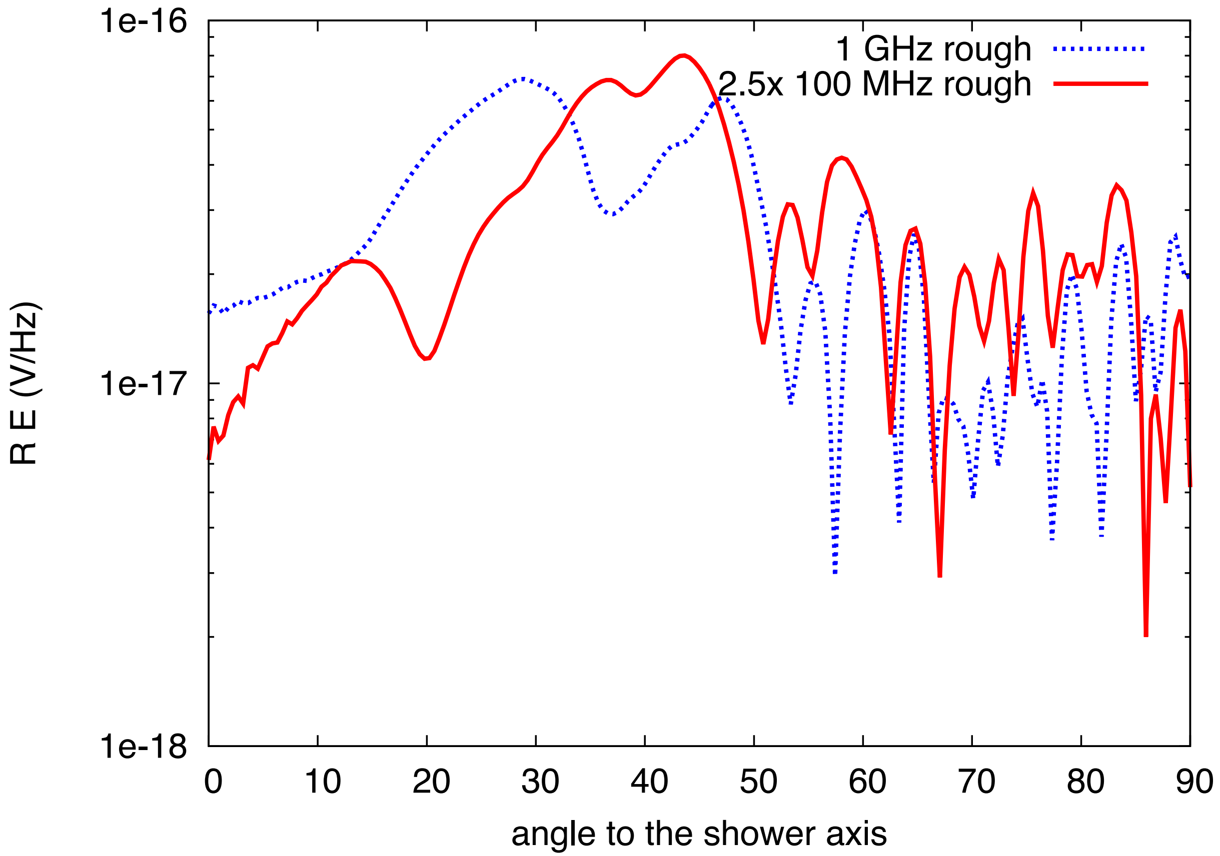
- 100 MHz and 1 GHz at 100m





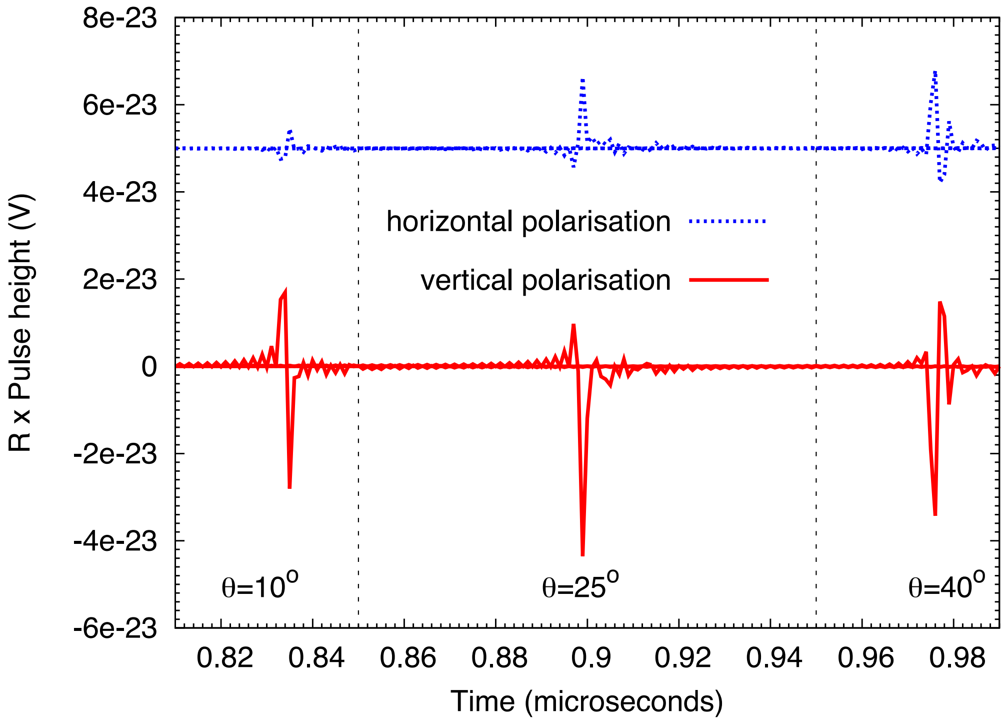
Roughness has similar effects at both freqs!

- 100 MHz at 100m vs 1 GHz at 10m



Time-domain pulses with 3m boxcar at 10m depth

- 1.0-2.0 GHz bandwidth: time-domain pulses ‘mostly coherent’



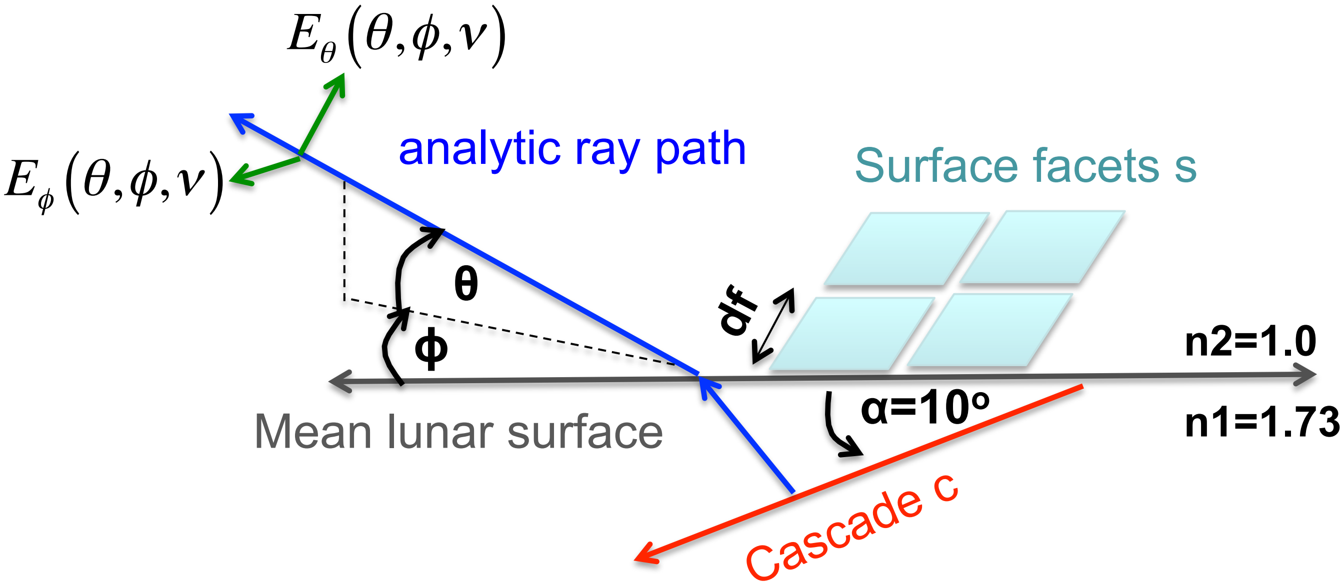
- Horizontal polarisation: analytic model predicts none

Preliminary results: ~cosmic rays

ERLANGEN CENTRE FOR ASTROPARTICLE PHYSICS

Cosmic rays with the SKA

- Pierre Auger runs out of statistics
- SKA might detect ~10x Auger cosmic rays above 5×10^{19} eV
- Most promising frequency range: 100-350 MHz (SKA LOW band)

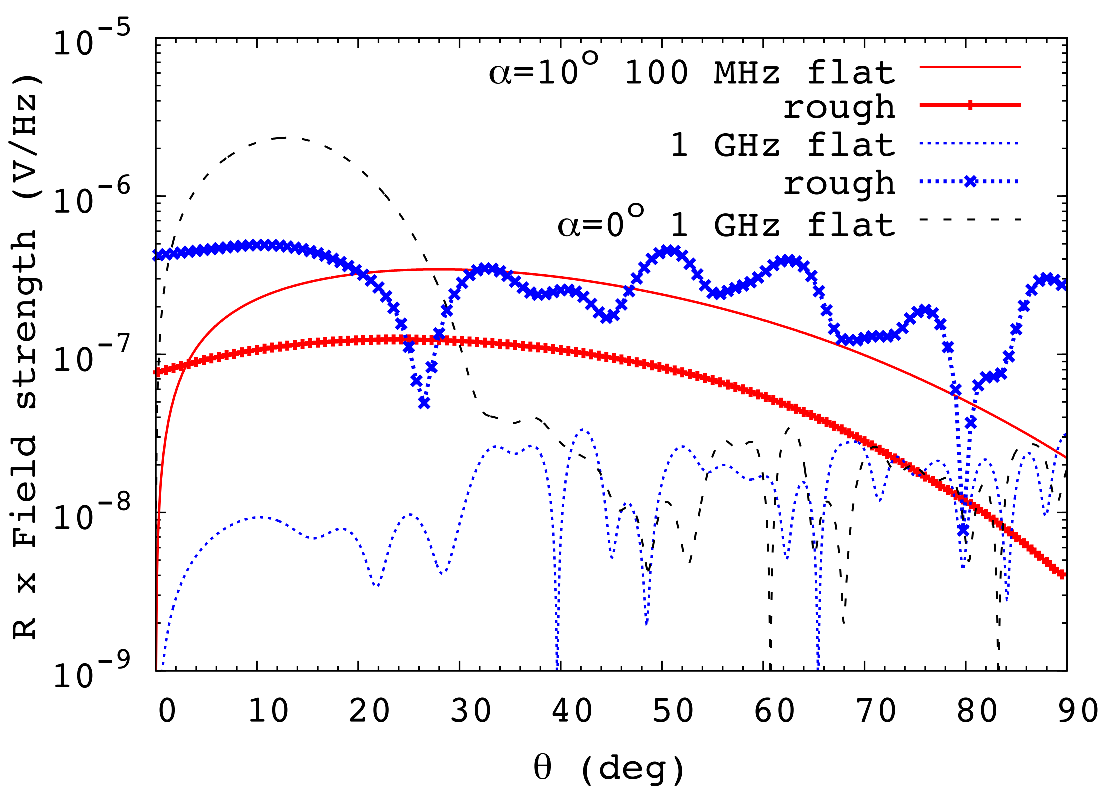


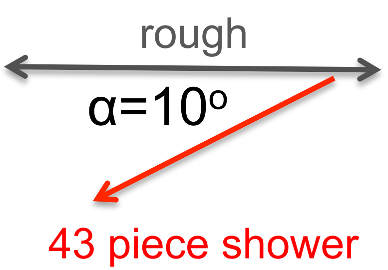
- Key questions: does the signal maintain coherence?

Tentative results: cosmic rays

PRELIMINARY
(numerical dials still need turning)

- 10^{20} eV hadronic cascade, 10^{o} angle of incidence, shower max 4.6m after initial point





Conclusion

- First calculation of the effects of small-scale roughness on coherent pulses – it works!
  - Outputs: spectrum, angular distribution, and time-domain pulses
- Simulations slow; numeric tuning an art. Comprehensive study still needs to be done.
- First results: roughness ~equally important over all frequencies for neutrinos. May be positive for high-freq CR detection?
- Next steps:
  - Automate numerical tuning
  - Characterise/quantify effects
  - Adapt to Antarctic surfaces?
  - Add lateral spread
  - Use in sensitivity calculations

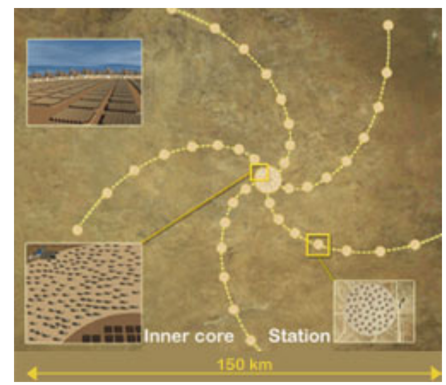
Backup

Germany announces intent to leave SKA

- *"On Thursday 5th of June, 2014, in an official letter signed by Dr. Georg Schütte, the State Secretary of BMBF, the German federal science ministry, Germany officially informed the SKA Director-General of its intent to leave the SKA Organisation."*

Federal Ministry of Education and Research

VS



- Priorities: E-ELT, CTA
- SKA data access policy: no SKA data for Germany
- SKA grant applications: no chance for BMBF funding

Track/facet division

- Determine errors in plane wave assumption:

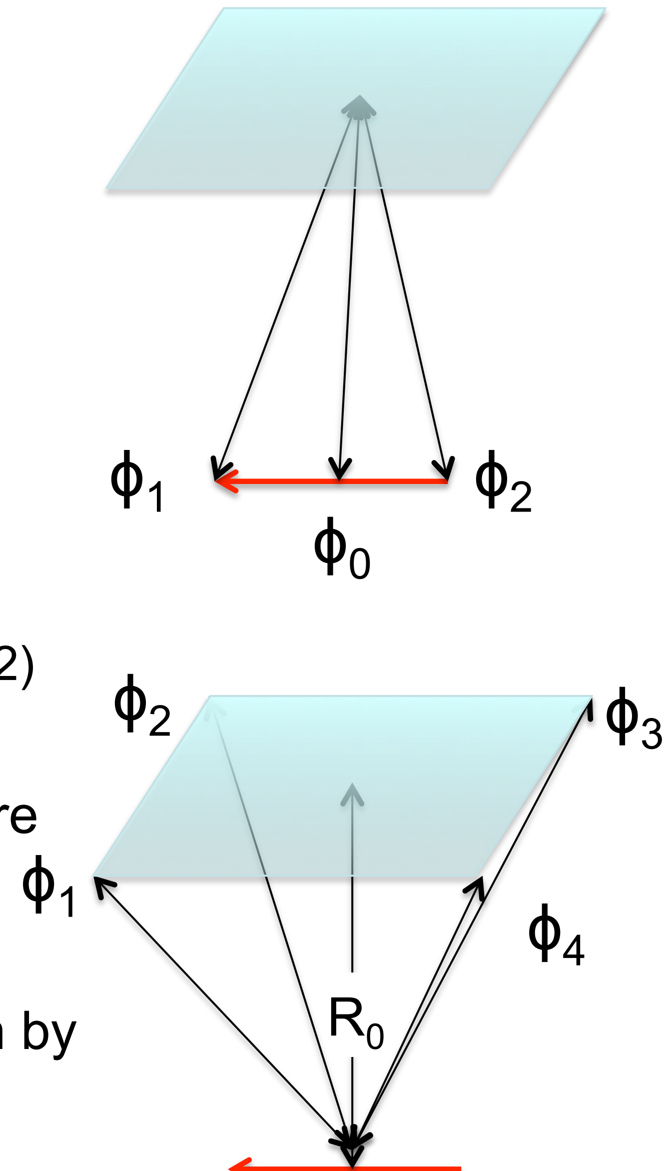
$$\Delta\phi_i=|\phi_i-\phi_0|$$

  - Calculate phase errors from track length
  - If $\Delta\phi$ > [error_margin], divide track in two

  - Calculate phase errors from facet size
  - If $\Delta\phi$ > [error_margin], divide facet in four (2x2)

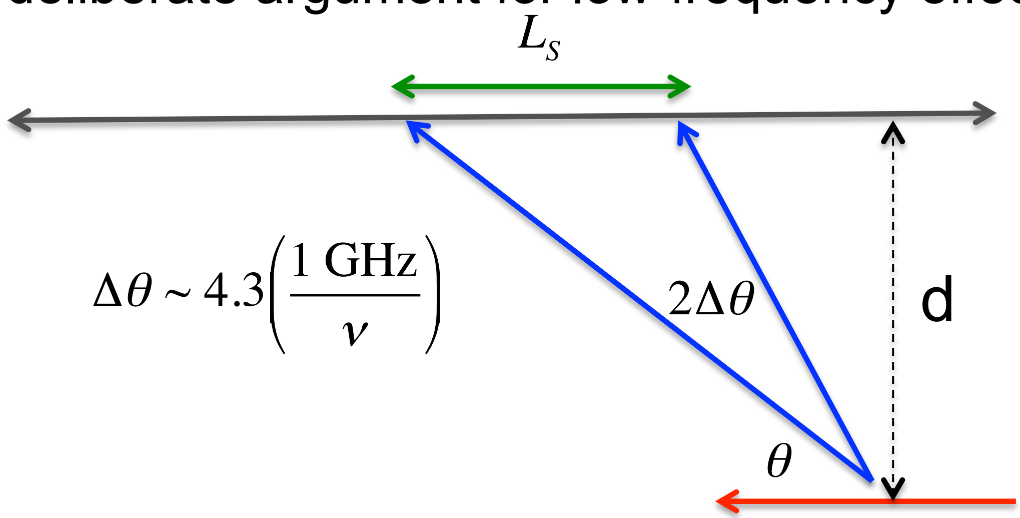
- The current checks on numerical accuracy are far from optimal (e.g. no check in 1/R)

- Results presented here required optimisation by hand



Frequency-dependence of roughness effects?

- A deliberate argument for low-frequency effects



$$\Delta\theta \sim 4.3\left(\frac{1\ \text{GHz}}{\nu}\right)$$

$$\ell_{abs}(1\ \text{GHz}) = 18\ \text{m}$$

$$\ell_{abs}(100\ \text{MHz}) = 180\ \text{m}$$

$$E \sim E_o e^{-d/(\ell_{abs}\sin\theta)}$$

- Characteristic surface length seen by emission: $L_S \sim d\Delta\theta \sim \nu^{-2}$
- Surface roughness: power-law scaling
- Effect on the signal:

$$S_{rms}\left(\frac{\Delta z}{\Delta L}\right) \sim L^{-0.22},\ \Delta z \sim L^{0.78}$$

$$\Delta z / \lambda \sim \nu\Delta z \sim \nu^{-0.56}$$

Putting it all together

- Ingredients:
  - Longitudinal cascade profile (10^{20} eV hadronic cascade)
  - Random rough surface
- For each facet x track x frequency:
  - Subdivide facet and track to meet far-field requirements
  - Calculate sub-track radiation on sub-facet using ZHS formula
  - Apply standard Fresnel transmission to get E1
  - Calculate outgoing radiation for each outgoing direction
  - Sum fields coherently, retaining full polarisation/phase information
- Numerous numerical dials to turn:
  - When to subdivide tracks/surfaces
  - How to subdivide surfaces ($z(x_{1,2}, y_{1,2})$ is not rectangular…)
  - Intrinsic roughness of surface