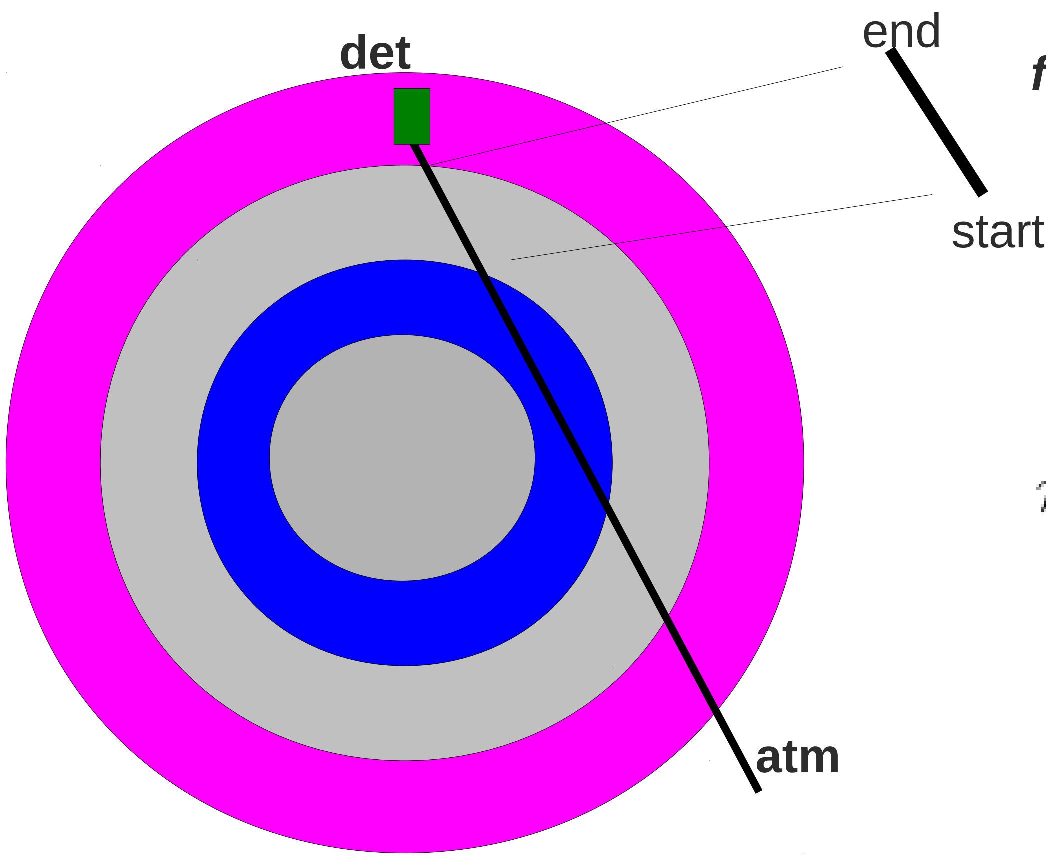
Measuring the neutrino mass hierarchy (with Orca).

Aart Heijboer - Nikhef, Amsterdam

- calculation is not orca specific (assumptions in stead of simulation input)
- but will try to make connection to ongoing Orca simulations at some points

Computing Oscillation Probabilities (numerically)



for one piece of constant matter density:

$$i\frac{\partial}{\partial t}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix} = \mathcal{H}_m(\rho)\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix}$$

$$\mathcal{H}_m(\rho) = \begin{bmatrix}E_1 & & \\ & E_2 & \\ & & E_3\end{bmatrix} + \mathbf{U}^{\mathbf{T}}\begin{bmatrix}A(\rho) & & \\ & 0 & \\ & & 0\end{bmatrix}\mathbf{U}$$

$$\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix}_{\mathrm{end}} \equiv \mathbf{T}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix} = e^{-i\mathcal{H}_m L}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix}_{\mathrm{start}}$$

for traversing the full Earth:

$$\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix}_{\mathrm{det}} = \mathbf{U}\ \mathbf{T}_n\ \mathbf{T}_{n-1}\ \ldots\ \mathbf{T}_1\ \mathbf{U}^{\mathbf{T}}\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix}_{\mathrm{atm}}$$

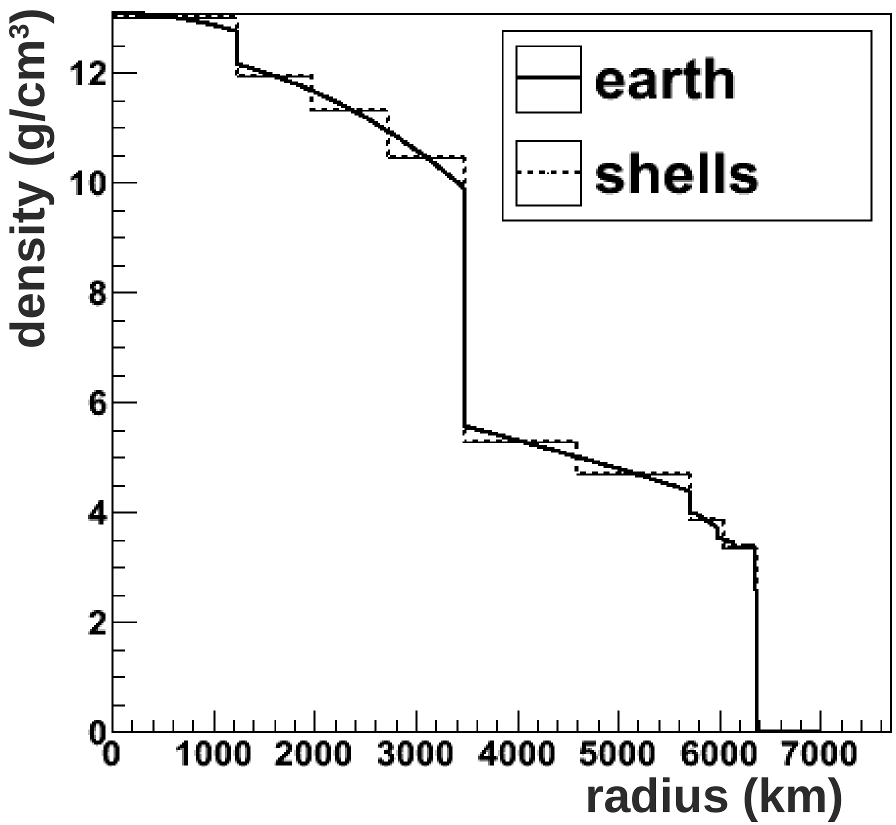
square to get probability

Transition matrix T involves exponent of complex 3x3 matrix
(diagonalize, power series, Cayleigh Hamilton..)

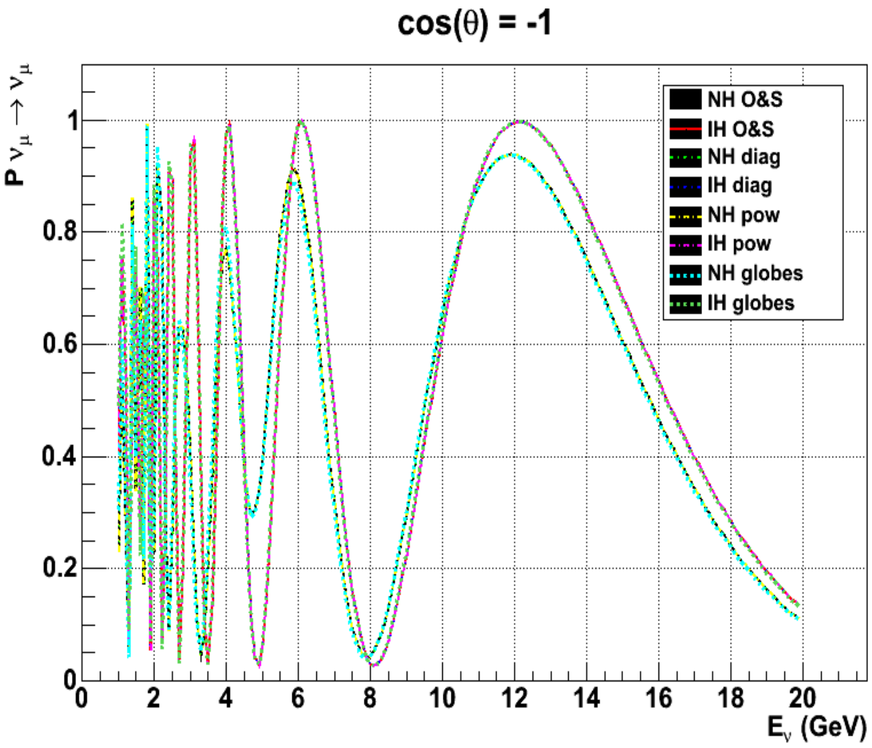
implementation must be fast (for fitting)

Computing Oscillation Probabilities

example of discretization of continuous Earth model



all methods agree



smart choice of a few shells → *high speed* and good accuracy

Comparing Oscillation Probabilities

Arxiv:1205.5254

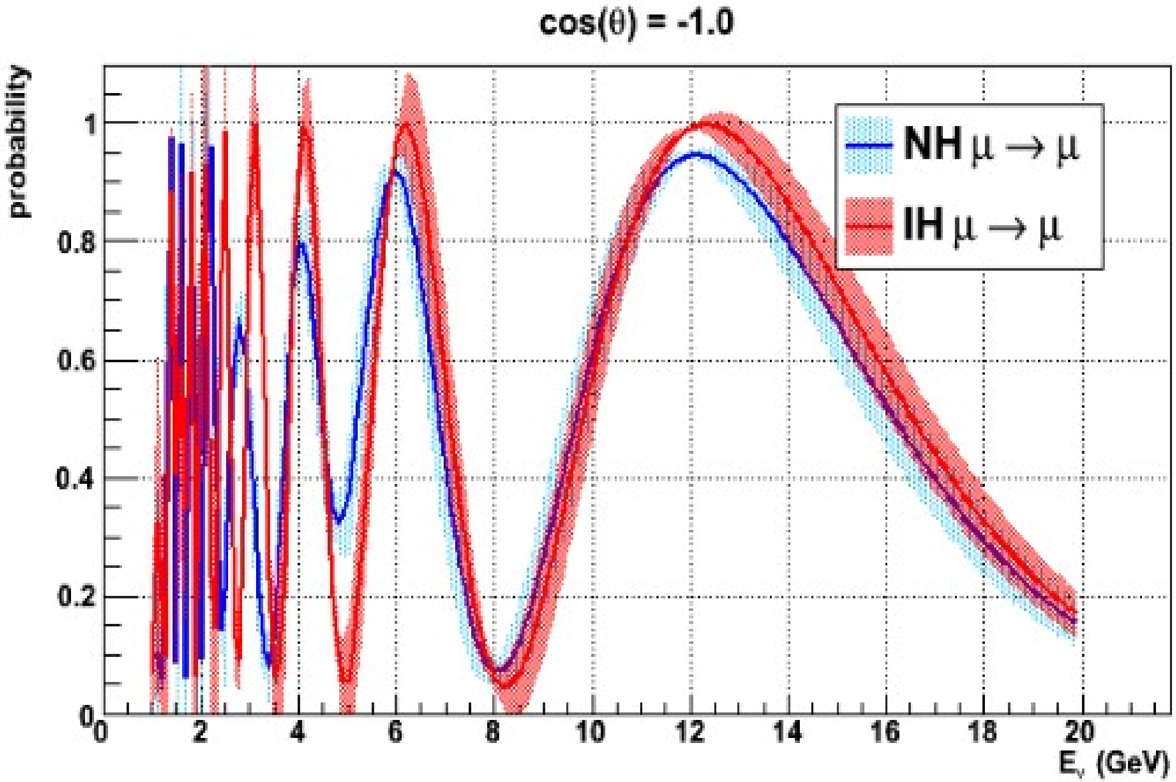
TABLE I: Results of the global 3ν oscillation analysis, in terms of best-fit values and allowed 1, 2 and 3σ ranges for the 3ν mass-mixing parameters. We remind that Δm^2 is defined herein as $m_3^2 - (m_1^2 + m_2^2)/2$, with $+\Delta m^2$ for NH and $-\Delta m^2$ for IH.

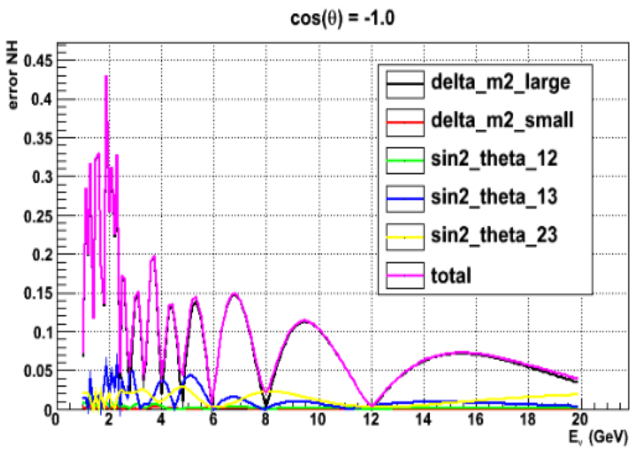
Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.07	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.33 – 2.49	2.27 – 2.55	2.19 – 2.62
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.42	2.31 – 2.49	2.26 – 2.53	2.17 – 2.61
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.41	2.16 – 2.66	1.93 – 2.90	1.69 – 3.13
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.44	2.19 – 2.67	1.94 – 2.91	1.71 – 3.15
$\sin^2\theta_{23}/10^{-1}$ (NH)	3.86	3.65 – 4.10	3.48 – 4.48	3.31 – 6.37
$\sin^2\theta_{23}/10^{-1}$ (IH)	3.92	3.70 – 4.31	3.53 – 4.84 ⊕ 5.43 – 6.41	3.35 – 6.63
~~δ/π (NH)~~	~~1.08~~	~~0.77 – 1.36~~	—	—
~~δ/π (IH)~~	~~1.09~~	~~0.83 – 1.47~~	—	—

Used in the following, but so far we set δ=0

Comparing Oscillation Probabilities

with uncertainties



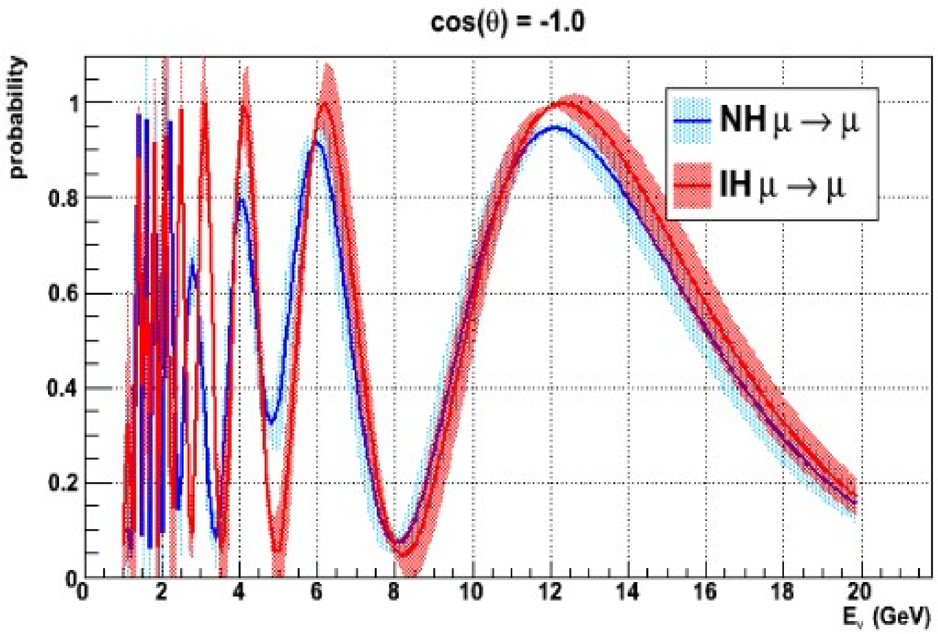


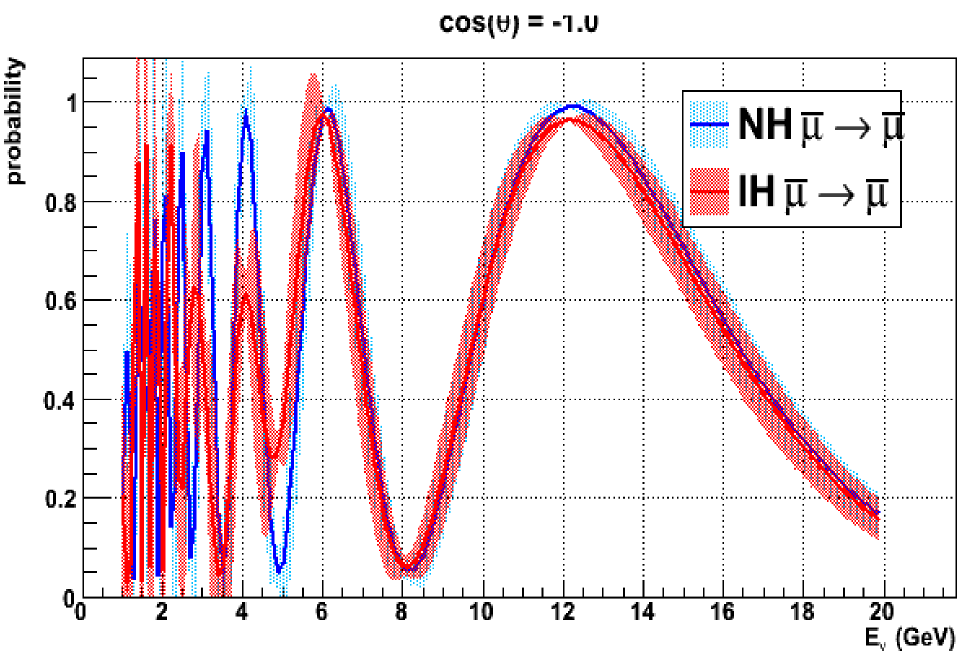
- NH/IH difference above ~13 GeV is degenerate with Δm^2_{large}
- regions around 5 GeV where genuine NH/IH difference remains (but not 5σ)
- not hopeless ← can use data to constrain this parameter

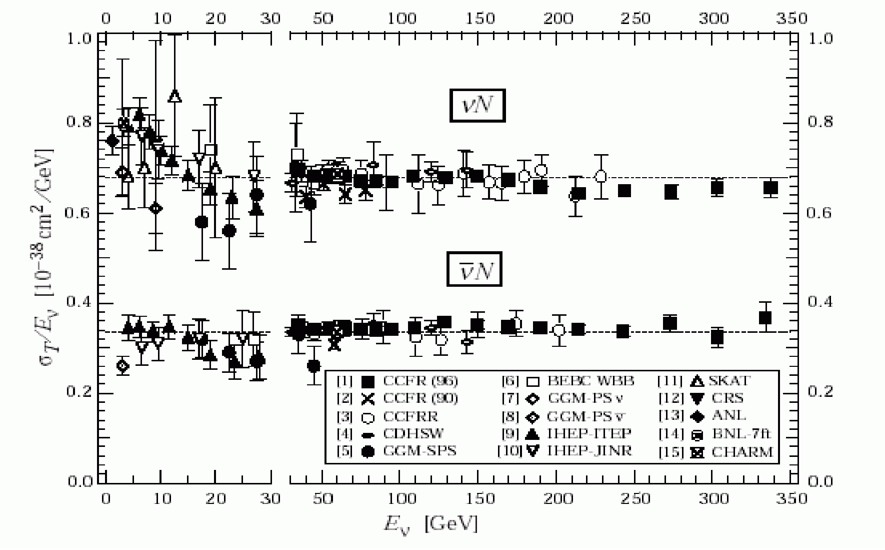
- does not make sense to speak of NH vs IH for a given set of mixing parameters
- Only when we can distinguish all allowed IH models from all allowed NH models can we determine the mass hierarchy.

Comparing Oscillation Probabilities

with uncertainties



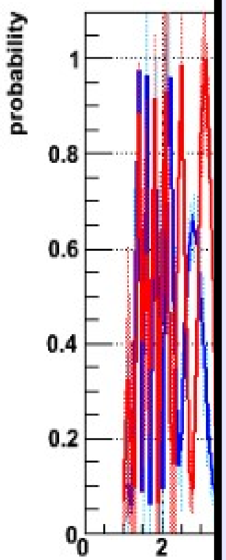


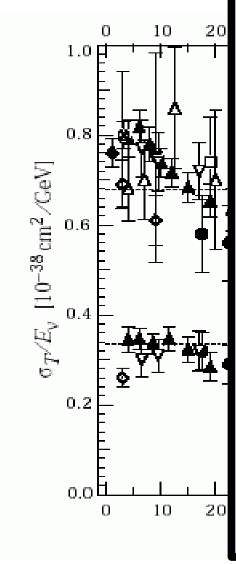


Effect survives because anti-neutrino cross-section is factor ~2 smaller than neutrino cross-section.

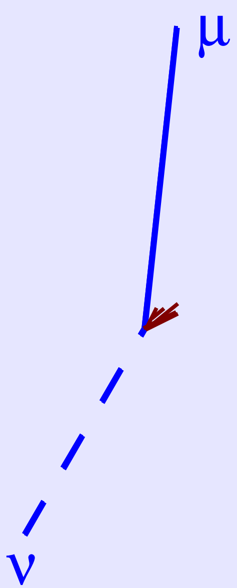
however...

Comparing Oscillation Probabilities
with uncertainties

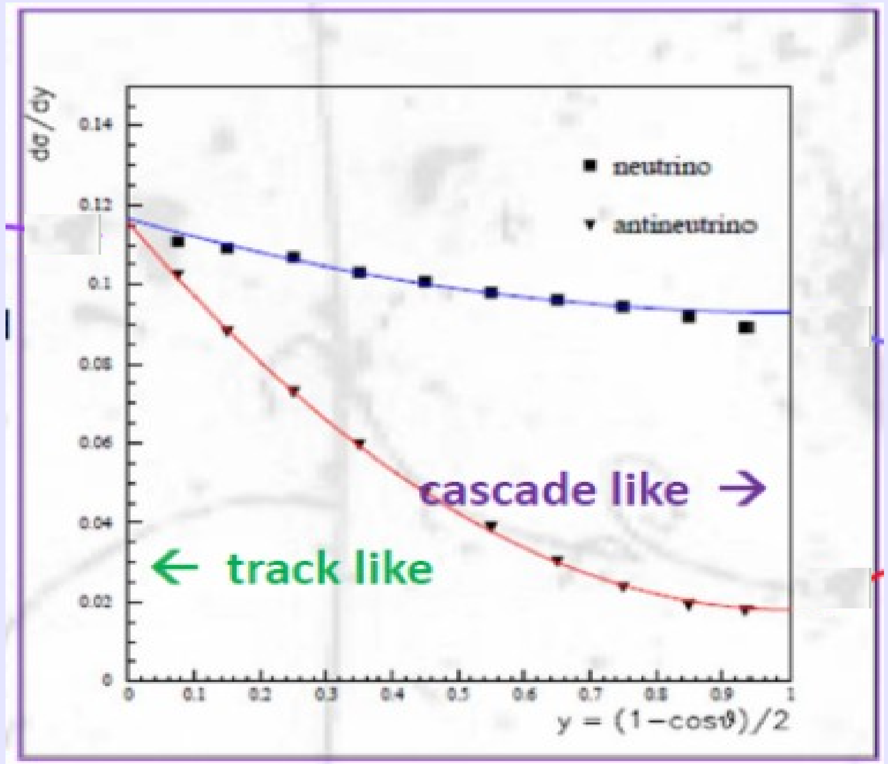


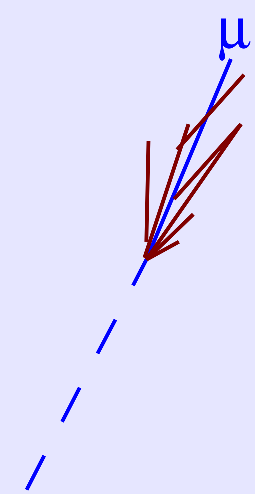


Rolf Nahnhauer / Juergen Brunner



nice events,but small asymmetry





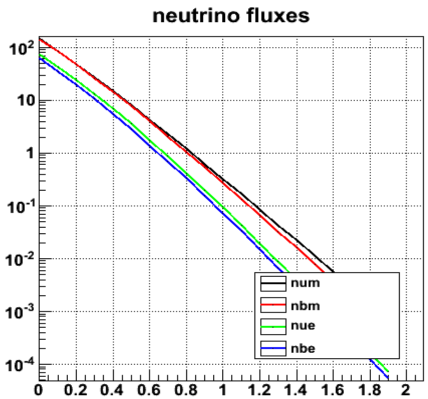
large asymmetry, but beware of ν_e ν_τ background (must tag the μ!)

- Different topologies contribute to same E_ν bin
  → systematics of acceptance and energy resolution need to be stringently controlled
  - events at highest y will not be usable (-)
  - … but ultimately can put y in the fits (+)

Toy Analysis

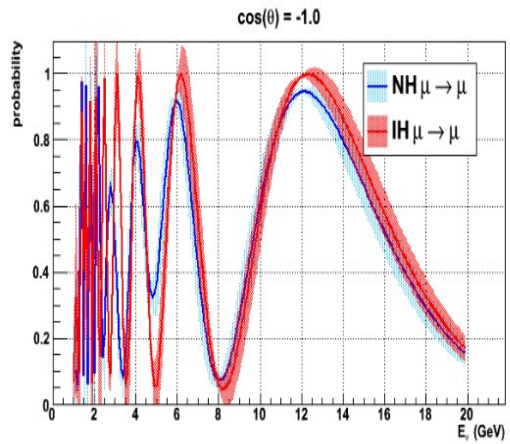
stage 1: compute event rates

bartol flux



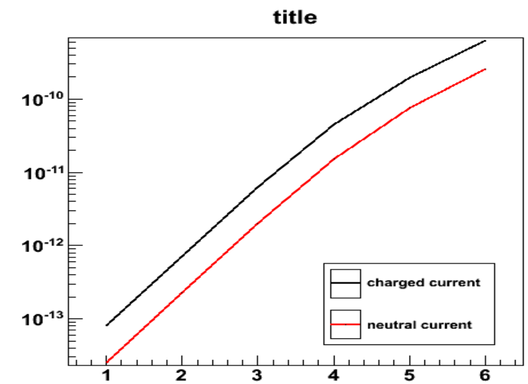
$\otimes$

Posc



$\otimes$

cross section



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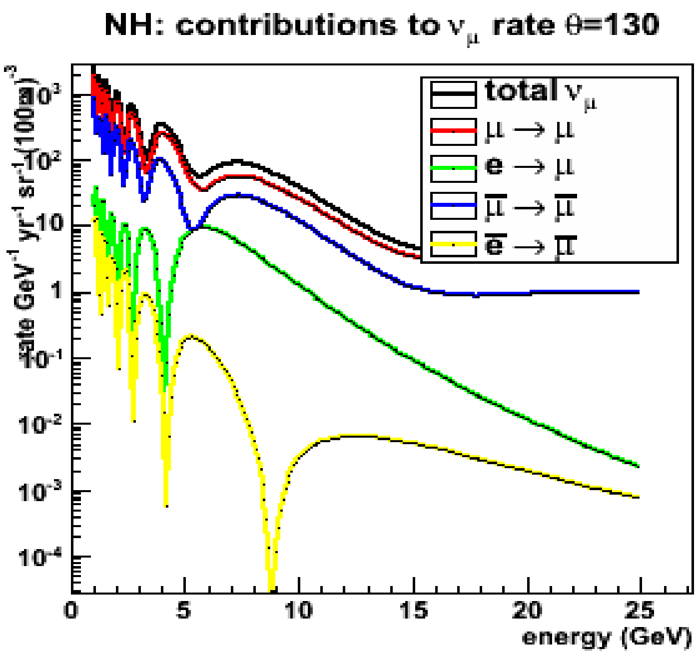
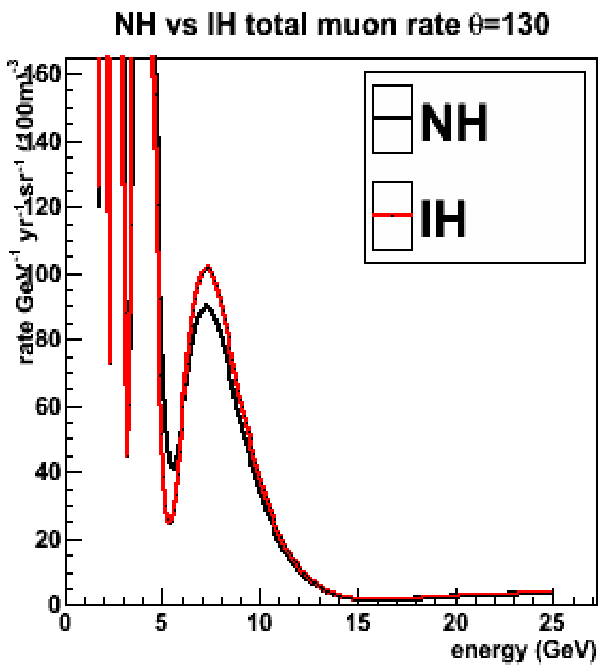




-bartol flux
-Posc(E,θ)
-simple Dis total cross-section

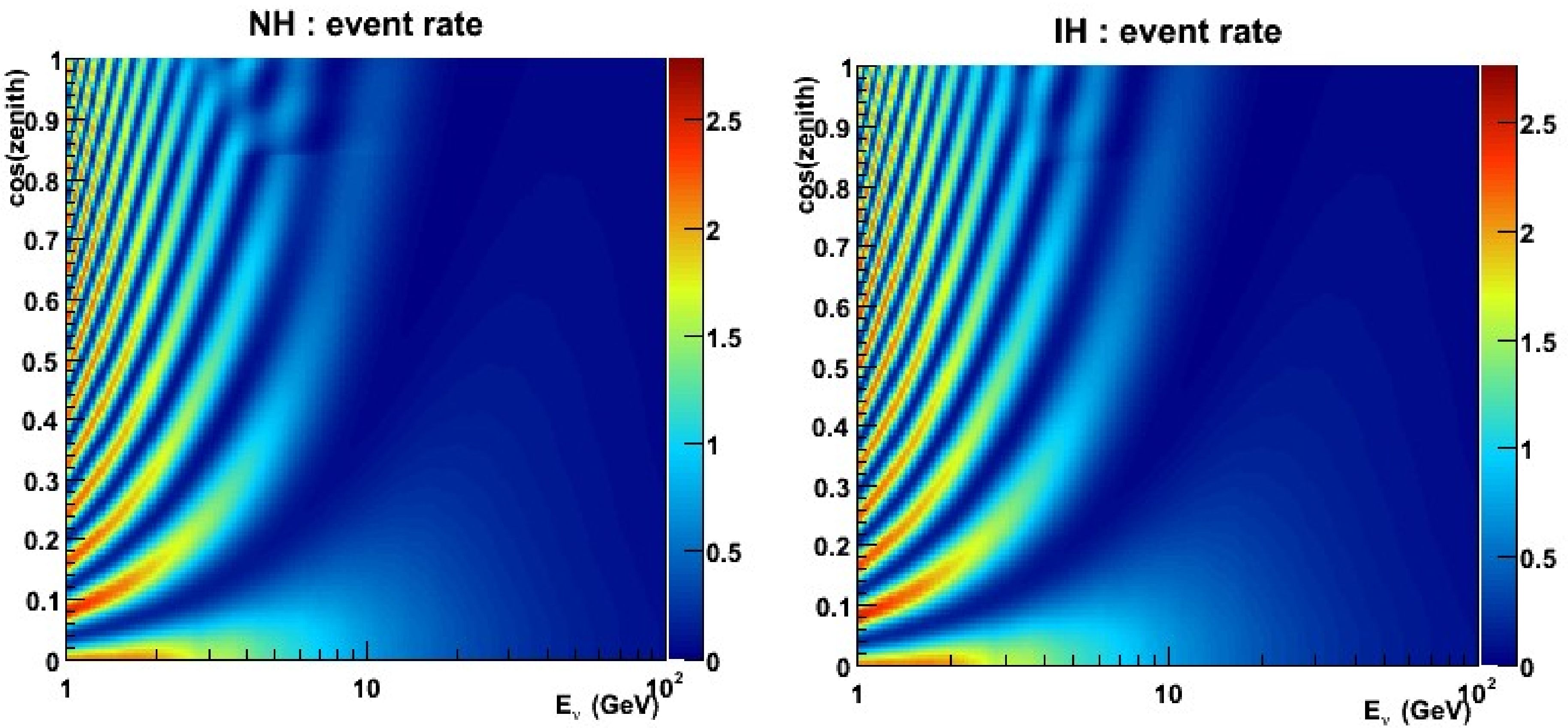
included : ν_ε
not yet : NC, ν_τ

so far: nothing assumed about any detector

Toy Analysis

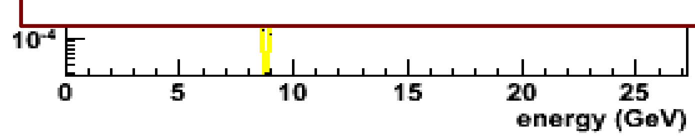
stage 1: compute event rates

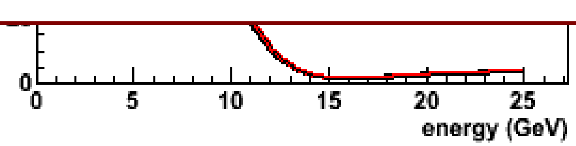
Posc



can spot the difference by eye in these plots

ss-



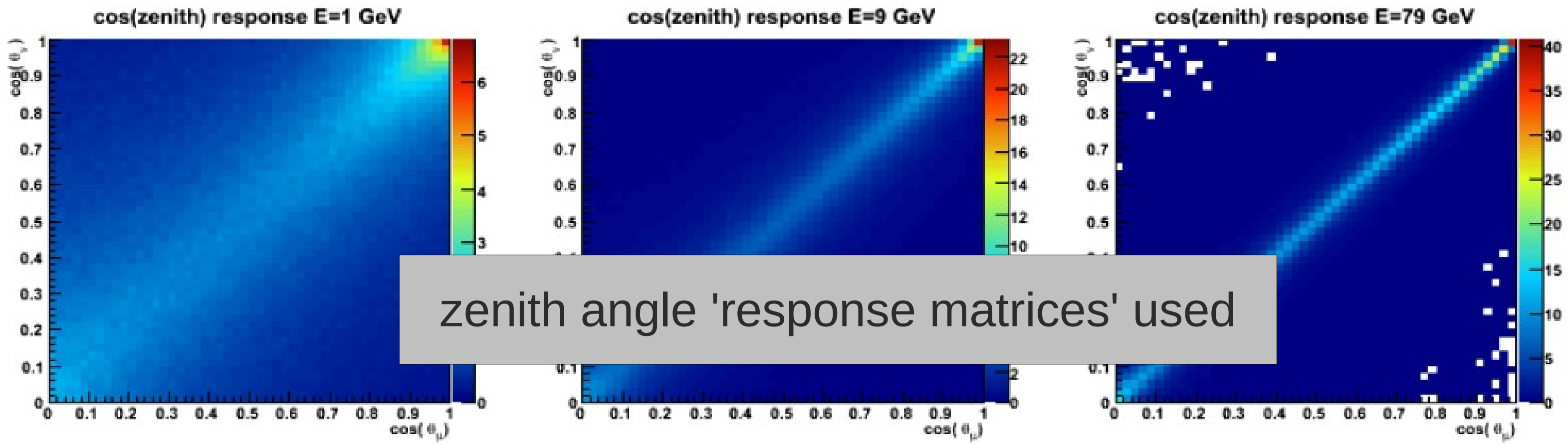


so far: nothing assumed about any detector

Toy Analysis

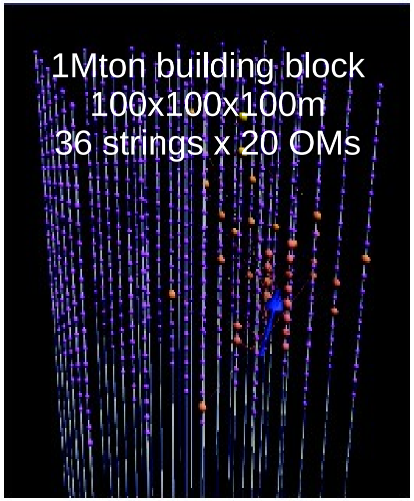
stage 2: acceptance and resolutions

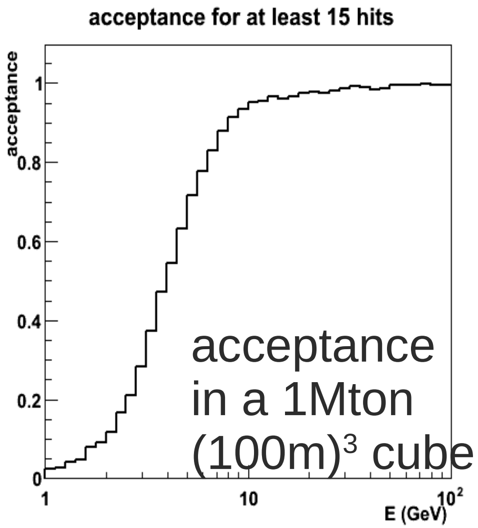
- Neutrino energy resolution : make assumptions: 10,20,25,30%
  - shortly: switch to full response matrix from simulations
- effective volume : don't assume anything (plot results vs Tobs x Mass)
- efficiency derived from simple MC, requiring 15 hits
  - *agrees very well with curve for full simulation + cuts (see next slide)*
- direction resolution: assume we measure the muon perfectly
  - *indeed full simulation shows the interaction dominates the resolution (see next slide)*
- atmospheric muon background not included in the simulation
  - *seems realistic (see next-to-next slide)*

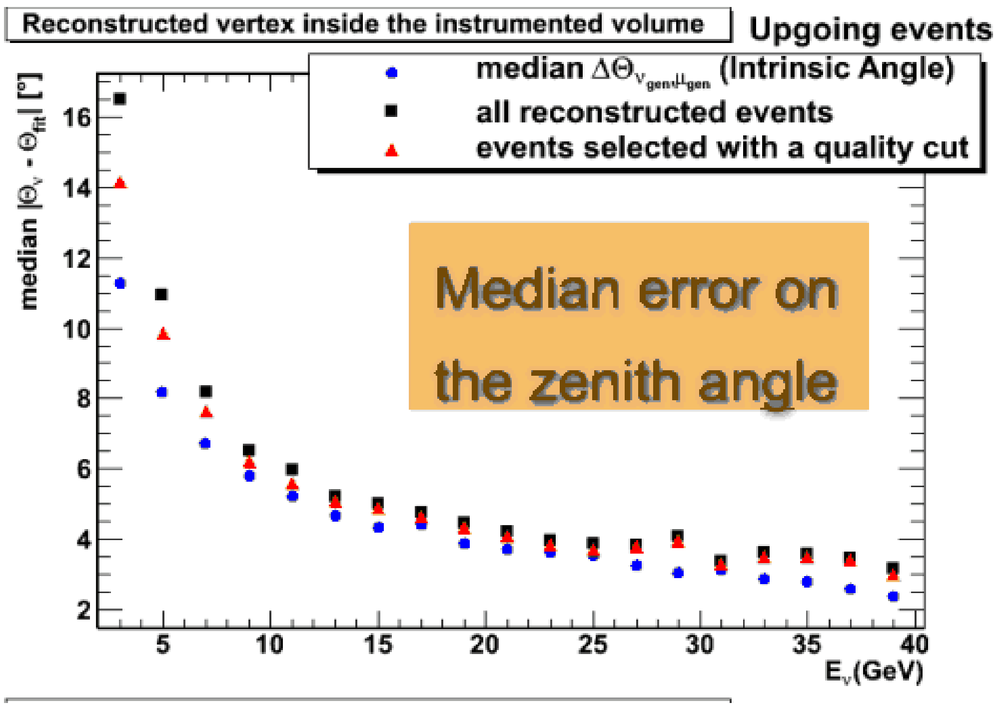


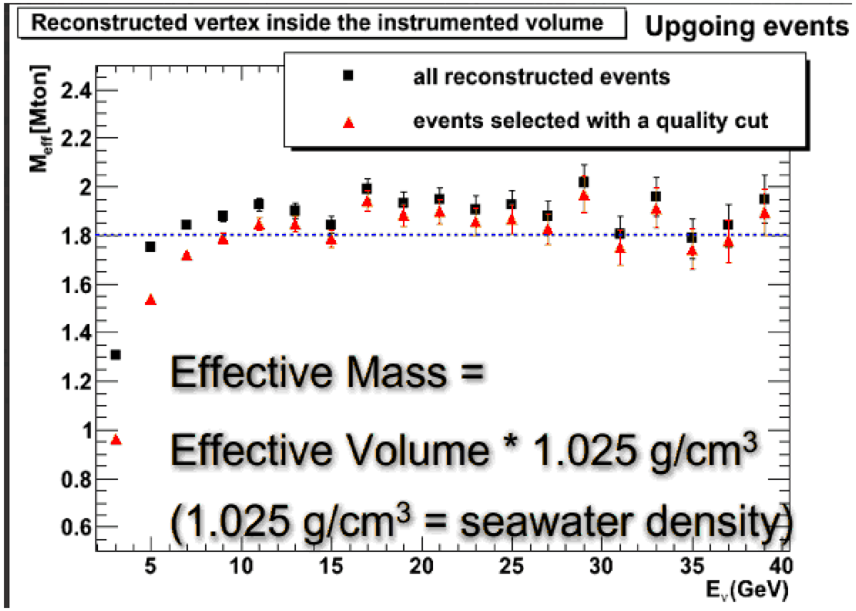
Toy Analysis
acceptance & angular resolution

see talk by A. Trovato tomorrow morning







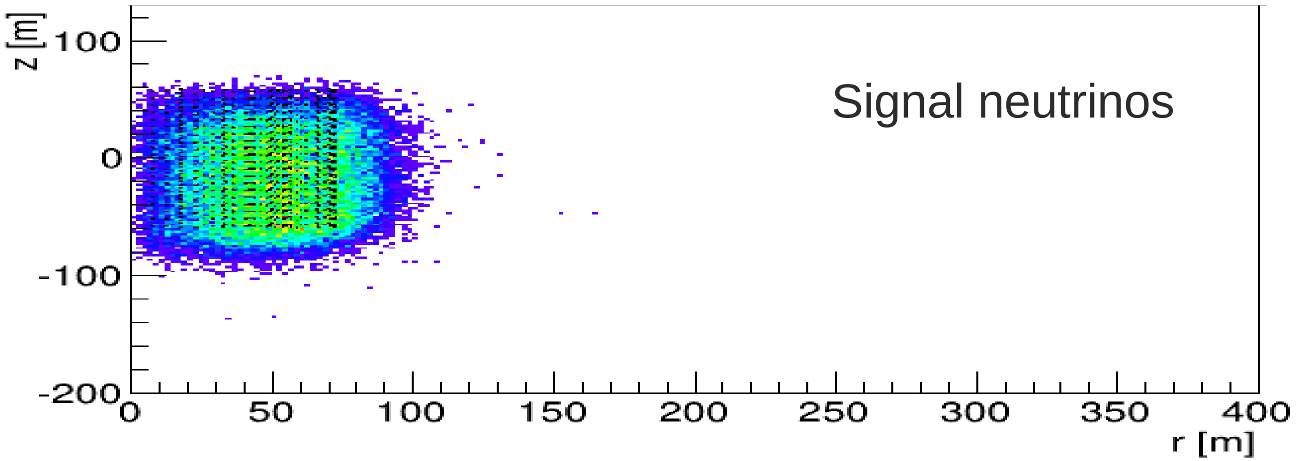


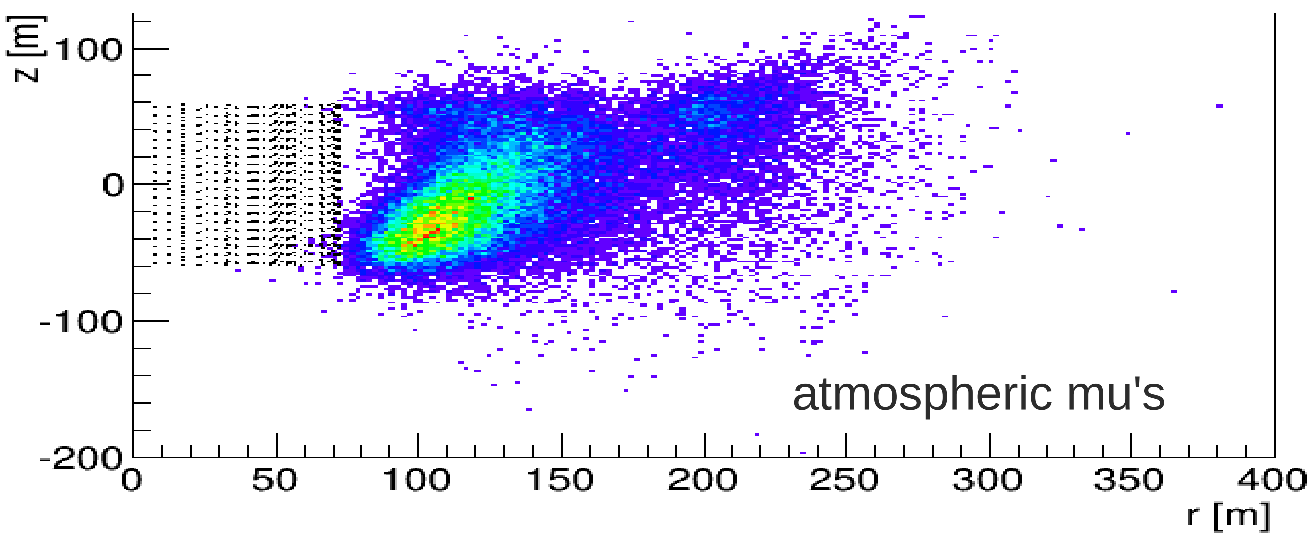
- assumed turn-on curve agrees well with the one from full simulation (after scaling to same assumed M_{eff})
- angular resolution in full chain already limited by physics

Muon contamination (intermezzo)

Contamination from atmospheric muons not included in computation.
Reason to assume muons might indeed be veto'd

- vertex estimate from projection of first hit
- require estimated vertex inside the detector
- combine





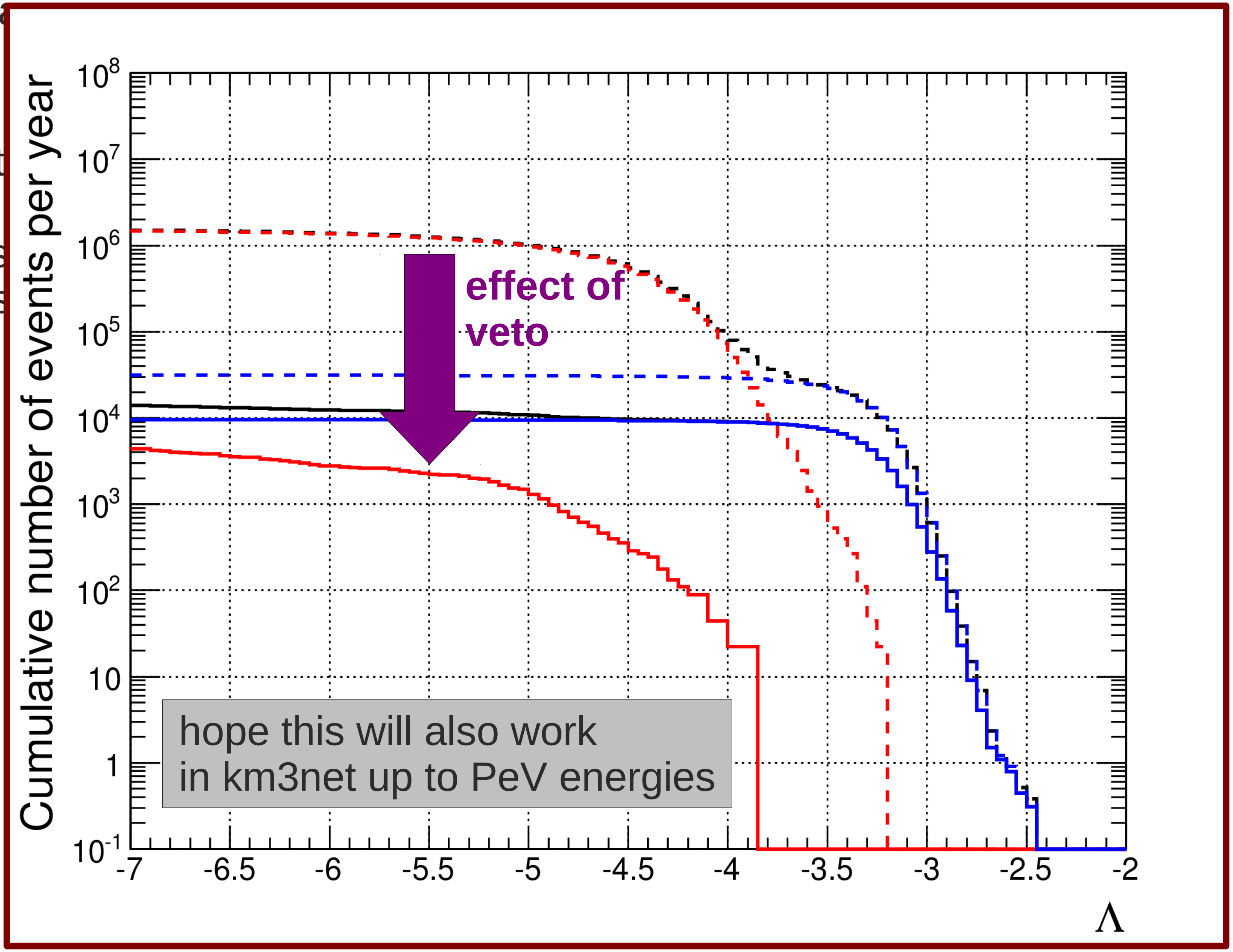
see talk by L. Fusco this morning

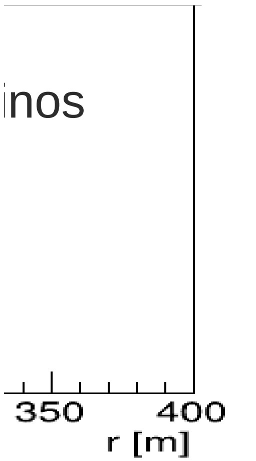
L. Fusco

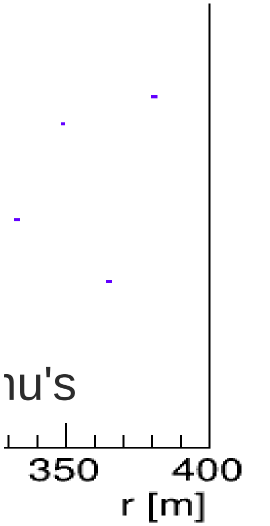
Muon contamination (intermezzo)

Contamination from atmospheric muons not included in computation.
Reason to a

- vertex est
  projection
- require es
  vertex ins
- combine







L. Fusco

Muon contamination (intermezzo)

Contamination from atmospheric muons not included in computation.
Reason to a

- vertex est
  projection
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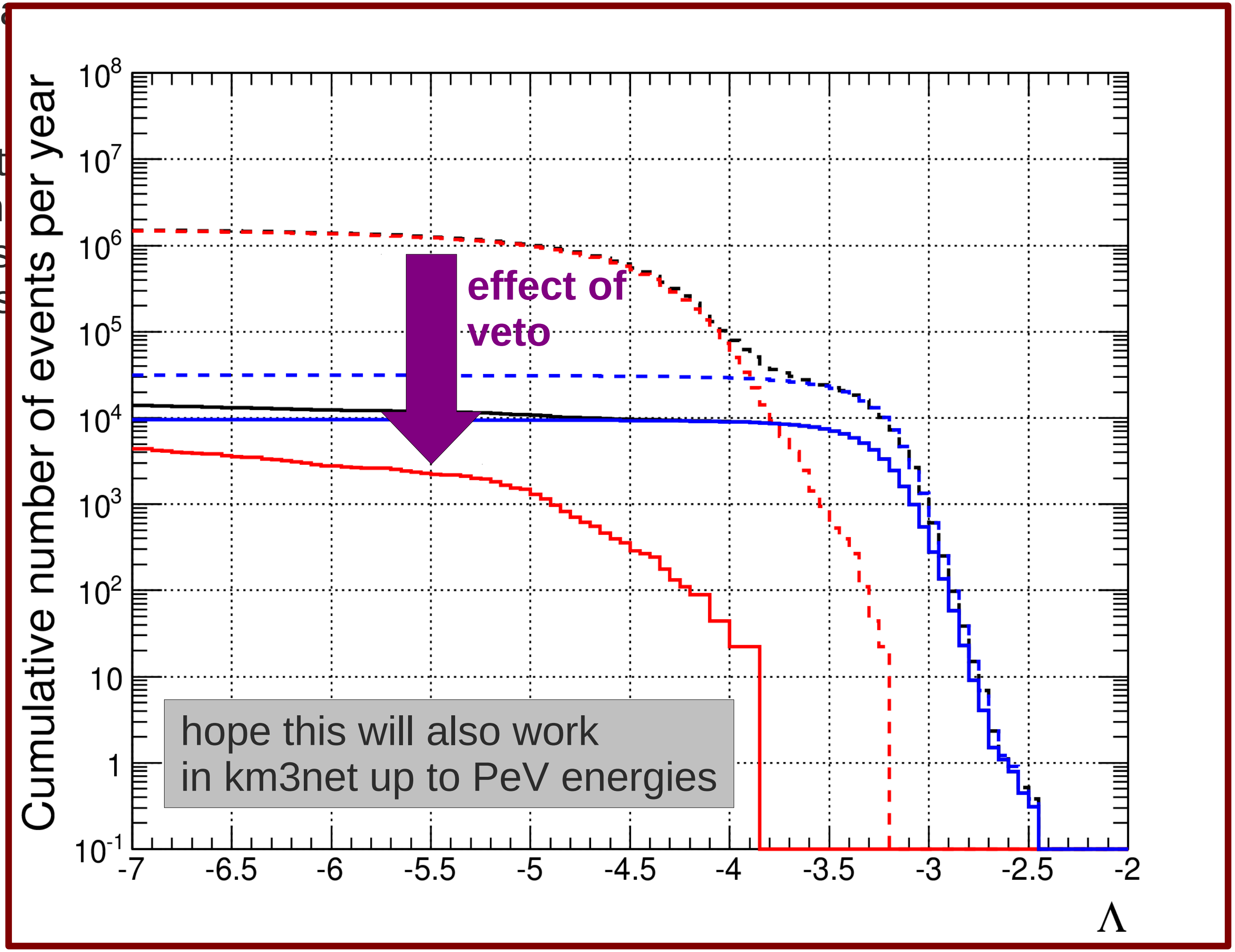
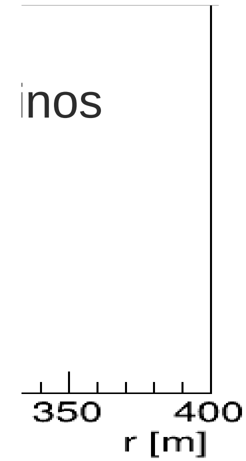
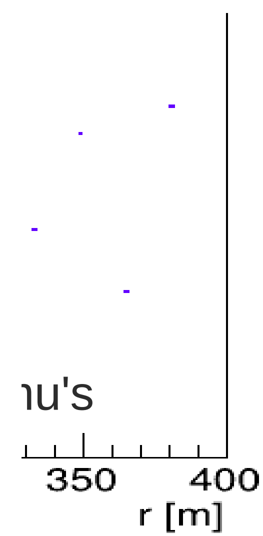
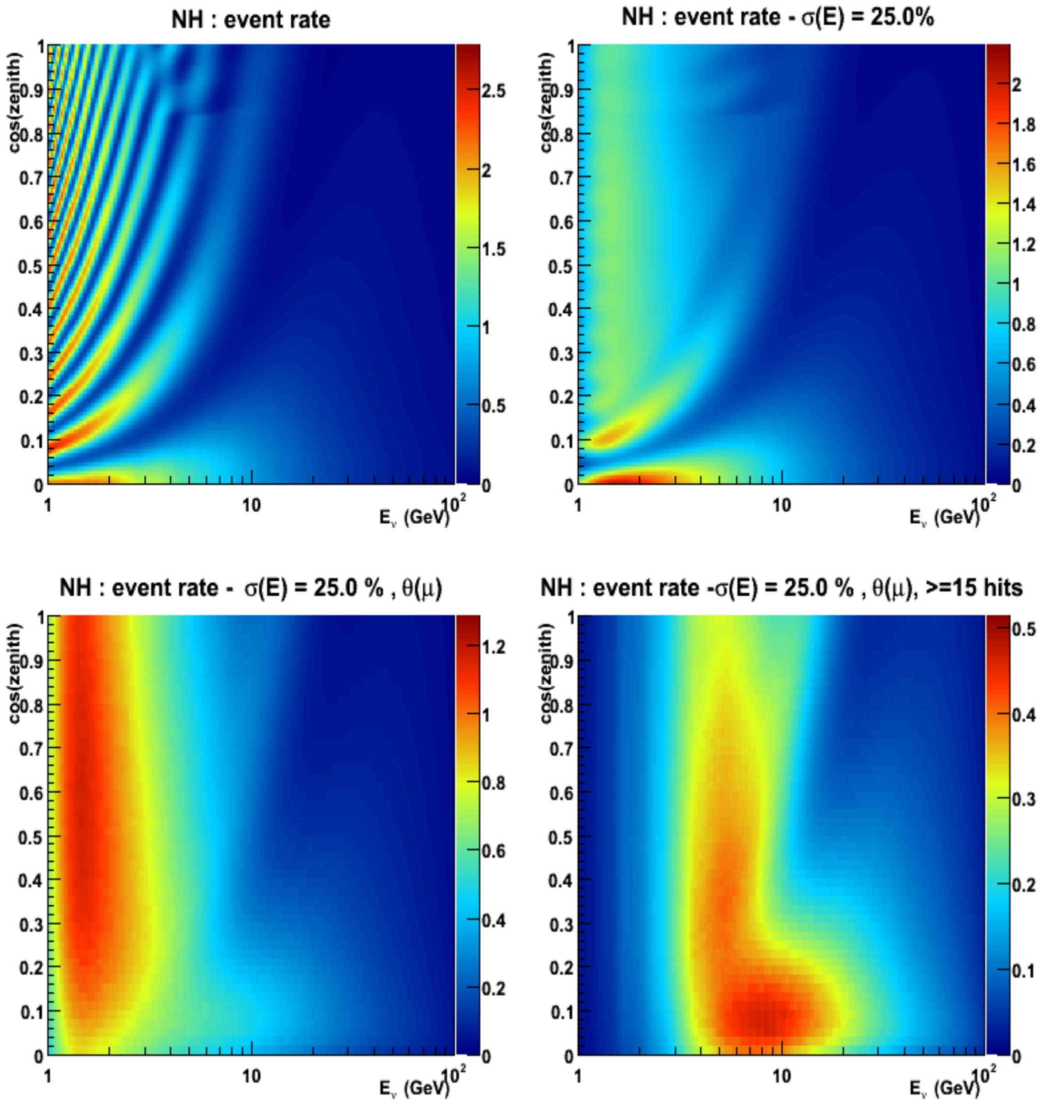
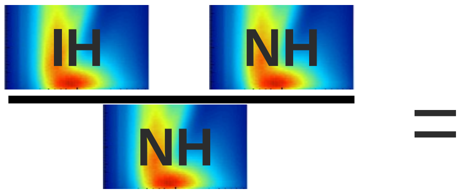


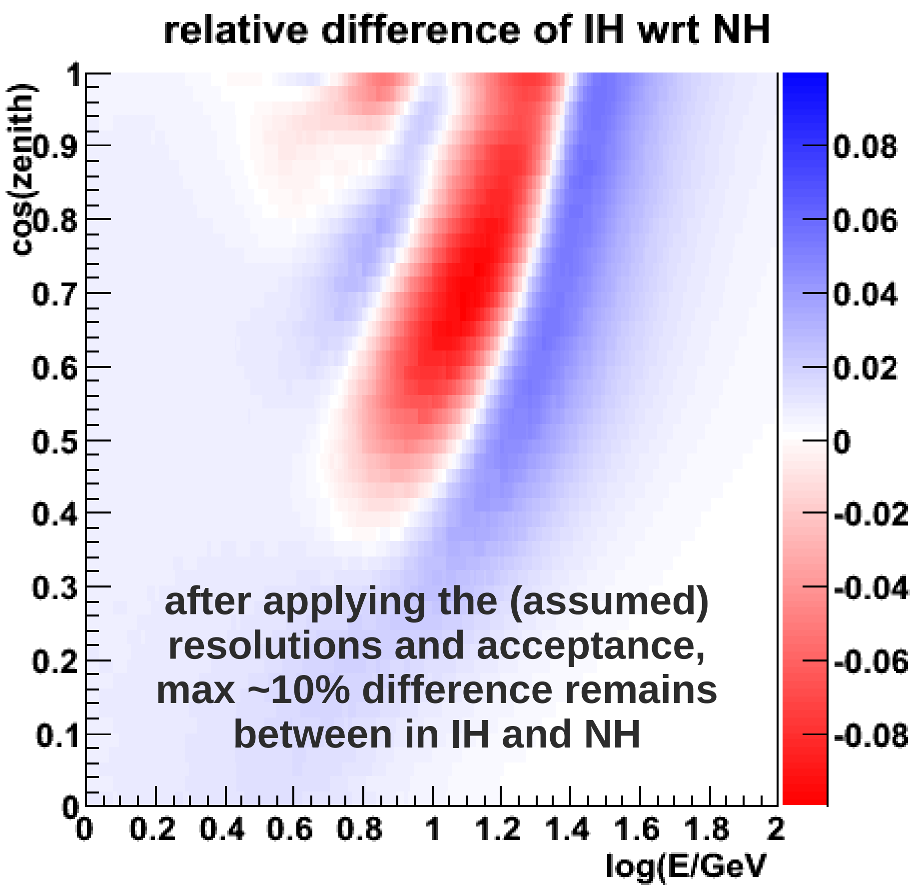




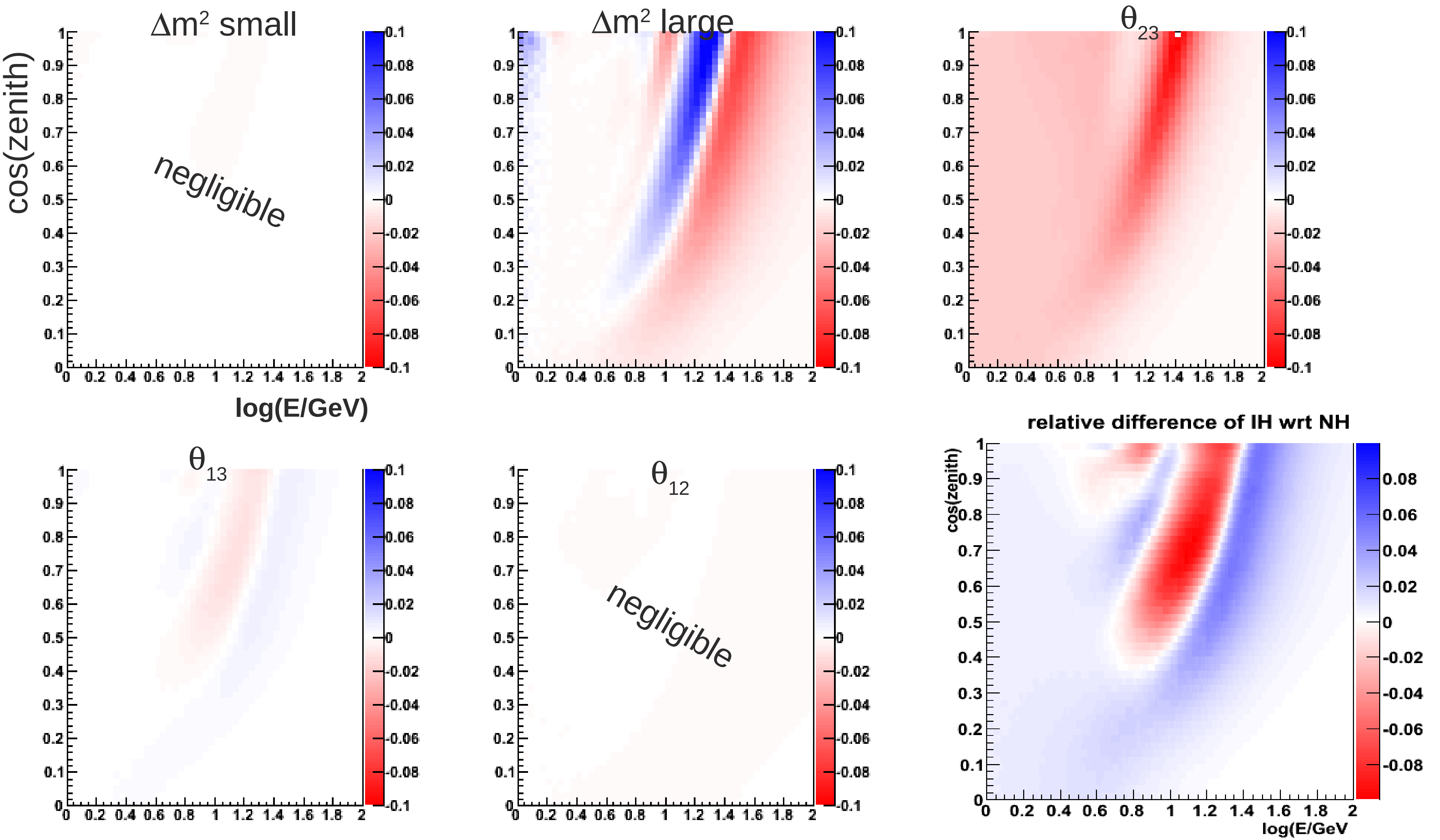
Toy Analysis – effect of resolutions & acceptance



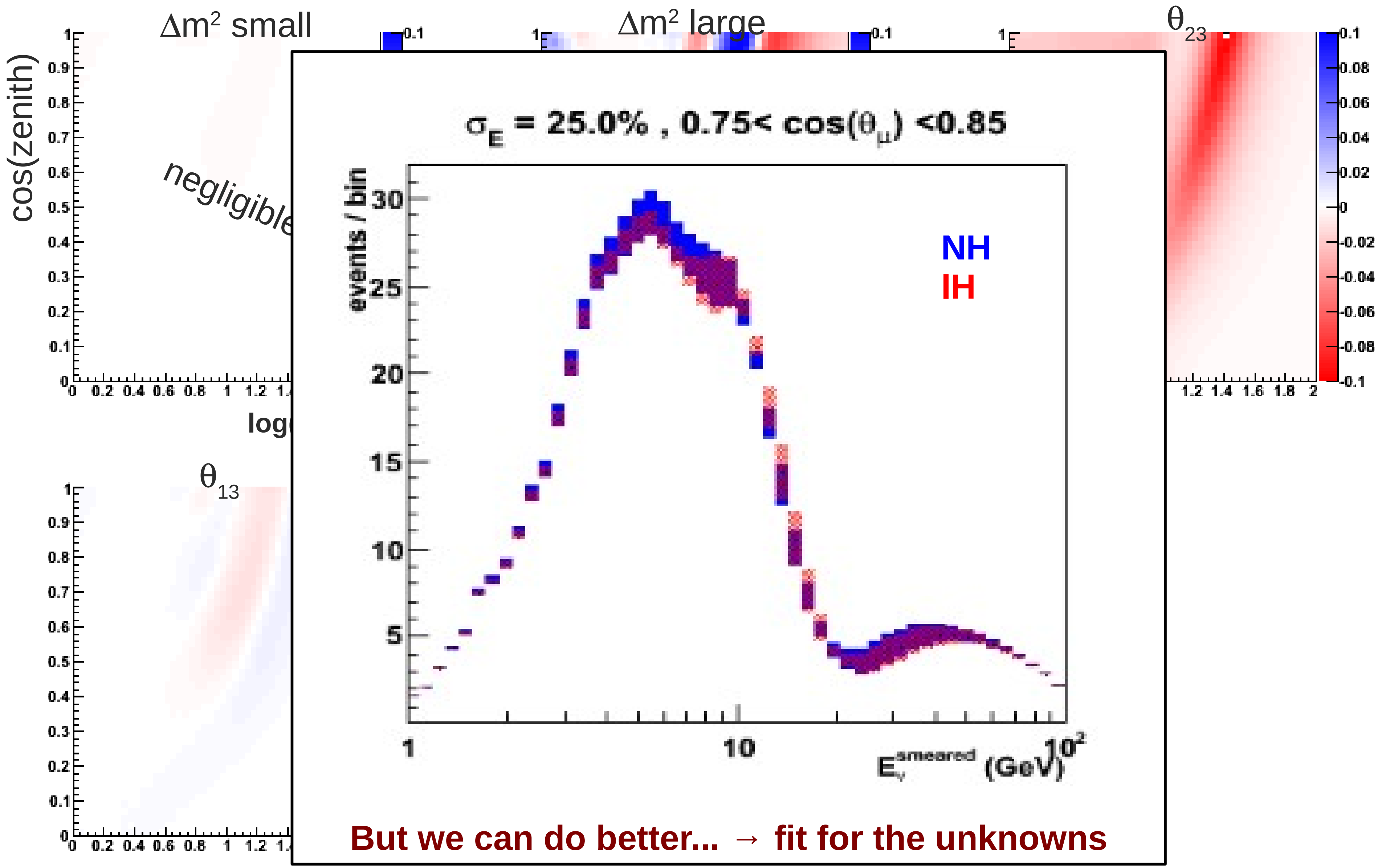




Toy Analysis – 'degeneracies'

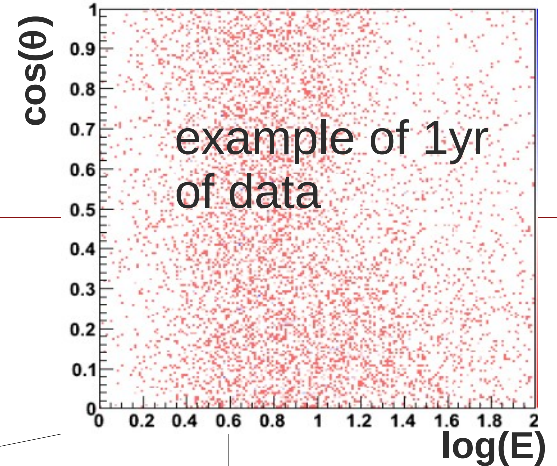


Toy Analysis – 'degeneracies'



But we can do better... → fit for the unknowns

Distinguishing NH and IH

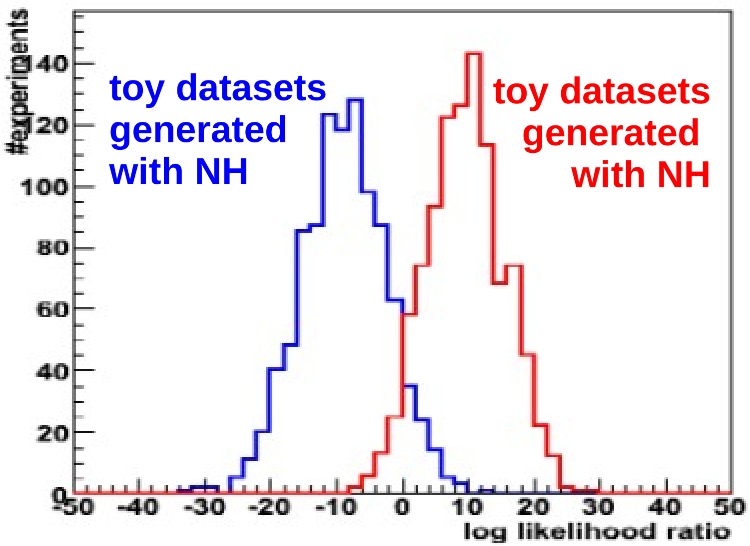


to optimally distinguish between IH and NH:
likelihood ratio test with nuisance parameters

$$\Delta \log(L^{\max}) = \sum_{\text{bins}} \log P(\text{data}|\hat{\theta}^{\text{NH}}, \text{NH}) - \log P(\text{data}|\hat{\theta}^{\text{IH}}, \text{IH})$$

$\hat{\theta}^{\text{H}} =$ maximum-likelihood estimates for the Δm^2's and angles using both data and constraints from global fit.
nb: constraints are different for H=IH and H=NH

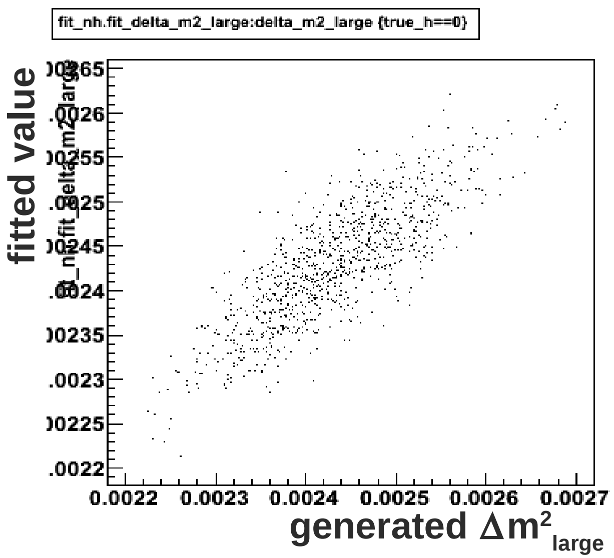
1) fit mixing parameters assuming NH
2) fit mixing parameters assuming IH
3) compute ΔlogL = log(L(NH)/L(IH))

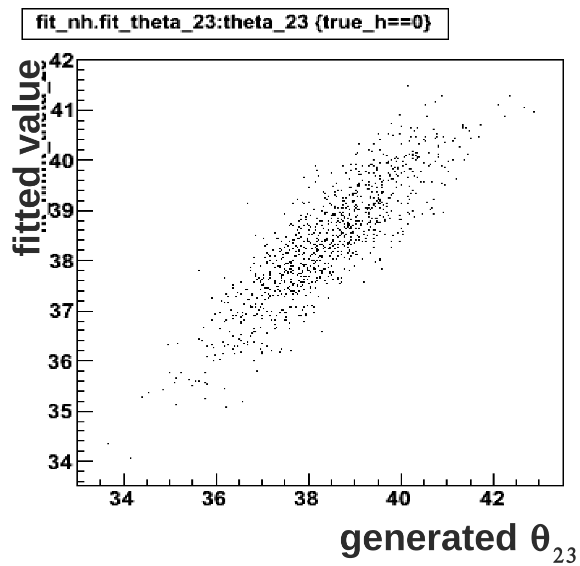


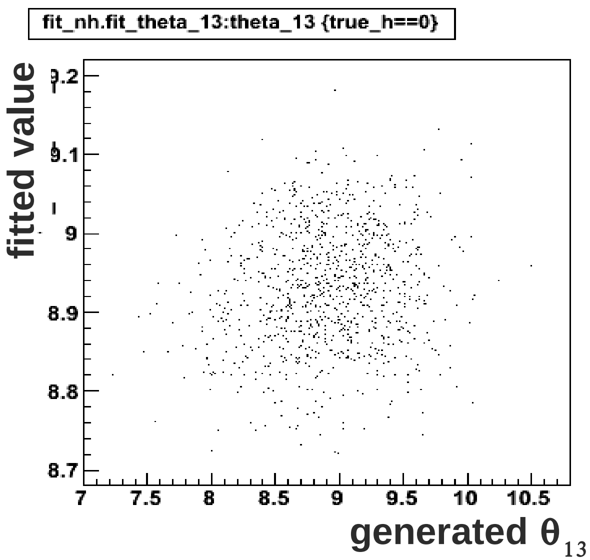
- Only when we can distinguish all allowed IH models from all allowed NH models can we determine the mass hierarchy.
- Fit involves computing many oscillograms for each Pseudo-experiment → should be fast

Results of parameter fit

1 Mton*year (NHtrue, NHfit)





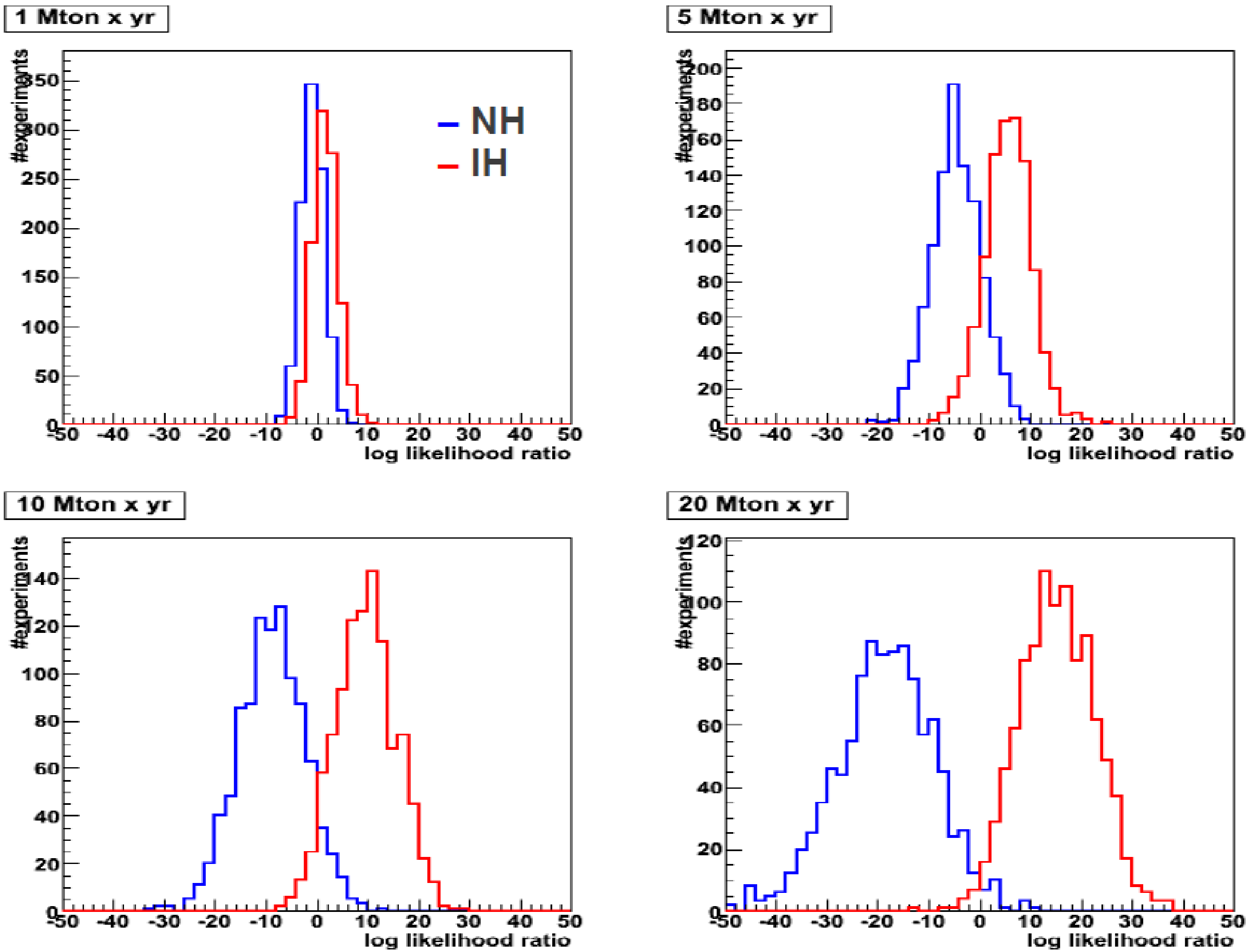


Eres = 25%, 1-100 GeV

Mton x yr	$\sigma(\Delta m^2_{large})$ (eV^2)	$\sigma(\theta_{23})$ (deg)	$\sigma(\theta_{13})$ (deg)
0(now)	8.0e-5	1.3	0.45
1	4.3e-05	0.61	0.42
5	2.3e-05	0.32	0.44
10	1.8e-05	0.22	0.39
20	1.4e-05	0.16	0.39
30	1.2e-05	0.13	0.37

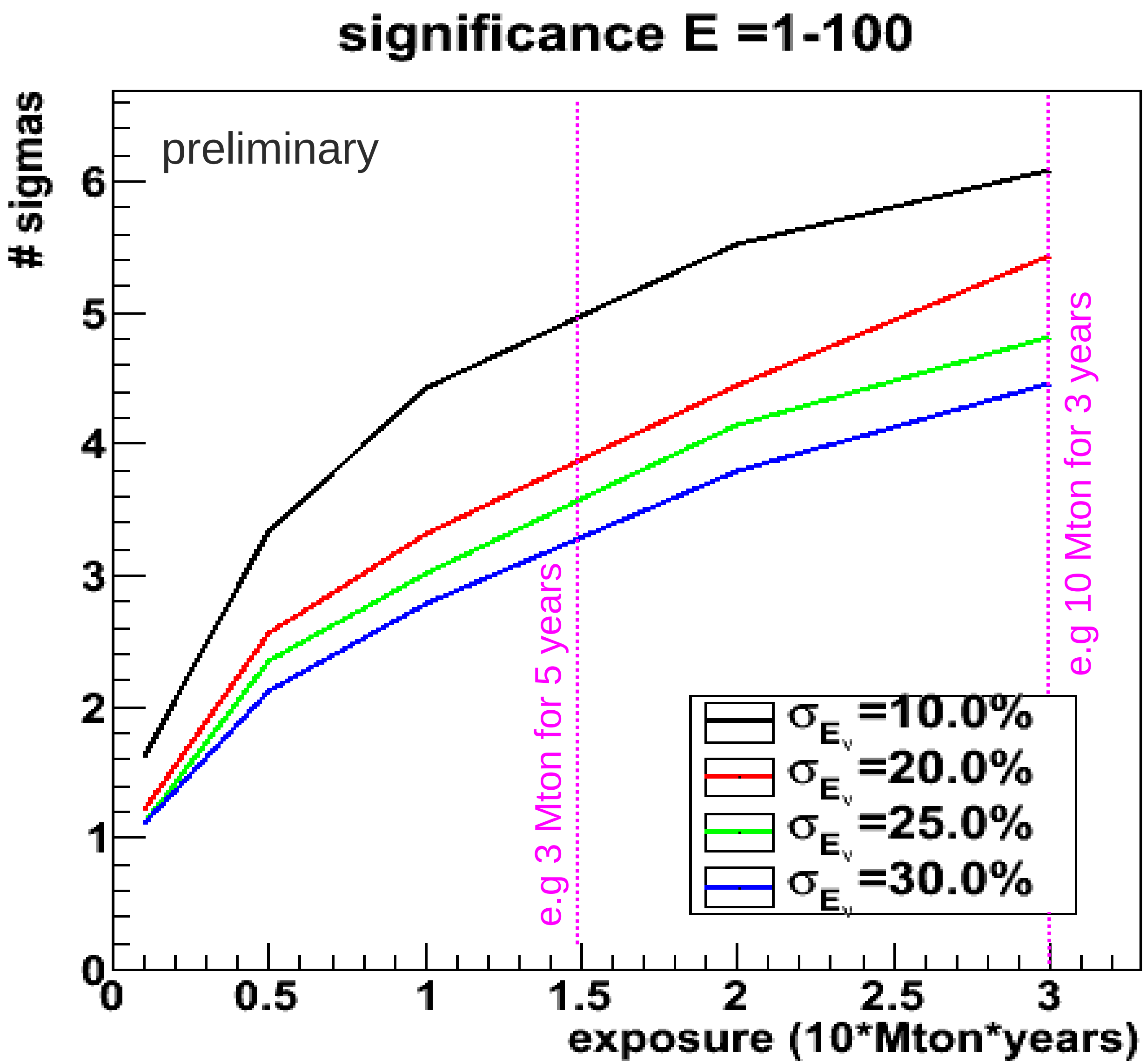
Fit working well.
Good sensitivity to Δm^2_{large} & θ_{23} !

Mass hierarchy significance



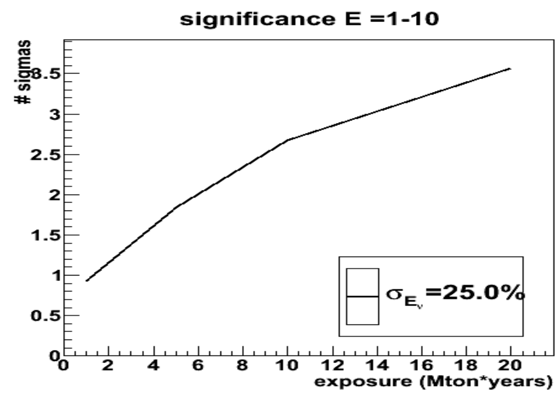
Mass hierarchy significance

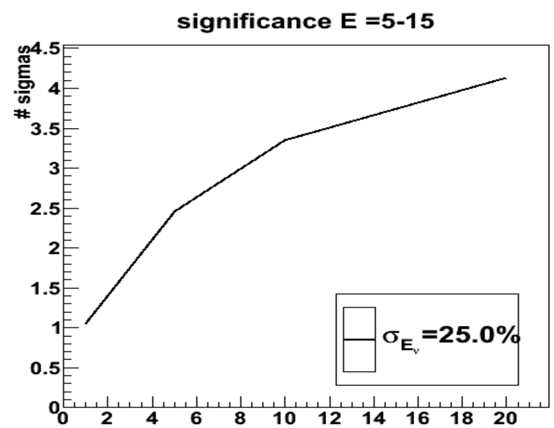
separation between
log-likelihood-ratio
test statistic distributions

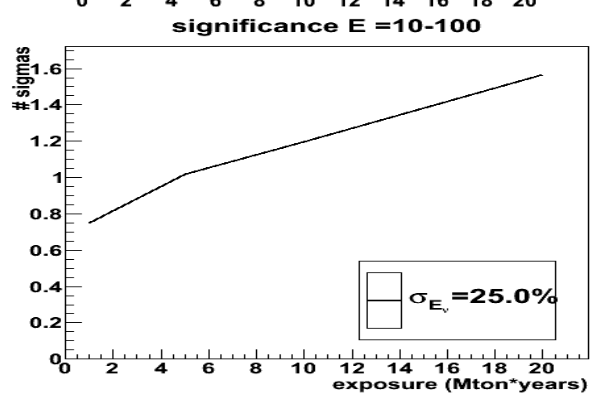


relavant energy range?

To get feeling for relevant energy range, run full analysis in different ranges







Eres = 25%, 1-10 GeV

Mton x yr	delta_m2_large	theta_23	theta_13
0(now)	8.0e-5	1.3	0.45
1	6.3e-05	0.72	0.47
5	4.3e-05	0.4	0.43
10	3.3e-05	0.3	0.44
20	2.6e-05	0.22	0.39
30	2.1e-05	0.17	0.4

Eres = 25%, 5-15 GeV

Mton x yr	delta_m2_large	theta_23	theta_13
0(now)	8.0e-5	1.3	0.45
1	5.8e-05	0.82	0.44
5	3.3e-05	0.5	0.45
10	2.6e-05	0.36	0.4
20	1.9e-05	0.25	0.39
30	1.7e-05	0.21	0.37

Eres = 25%, 10-100 GeV

Mton x yr	delta_m2_large	theta_23	theta_13
0(now)	8.0e-5	1.3	0.45
1	4.2e-05	0.87	0.47
5	2.5e-05	0.48	0.43
10	2e-05	0.35	0.45
20	1.6e-05	0.27	0.46
30	1.4e-05	0.22	0.46

relevant energy range?

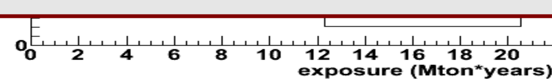
To get feeling f…
energy range, r…
analysis in diffe…

Eres = 25%, 1-10 GeV

Mton x yr	delta_m2_large	theta_23	theta_13
			0.45
			0.47
			0.43
			0.44
			0.39
			0.4

theta_13
0.45
0.44
0.45
0.4
0.39
0.37

			theta_13
			0.45
			0.47
			0.43
			0.45
			0.46
30	1.4e-05	0.22	0.46



results for 20 Mton yr

range	MH significance	uncertainty on Δm^2-large (10^{-5} eV^2)
1 - 100 GeV	4.2 sigma	1.4
1 - 10 GeV	3.5 sigma	2.6
5 - 15 GeV	4.2 sigma	1.9
10 - 100 GeV	1.6 sigma	1.6

conclusion:
- sweet spot between 5 and 15 GeV
- mixing parameters can also be measured in region >10 GeV but not MH

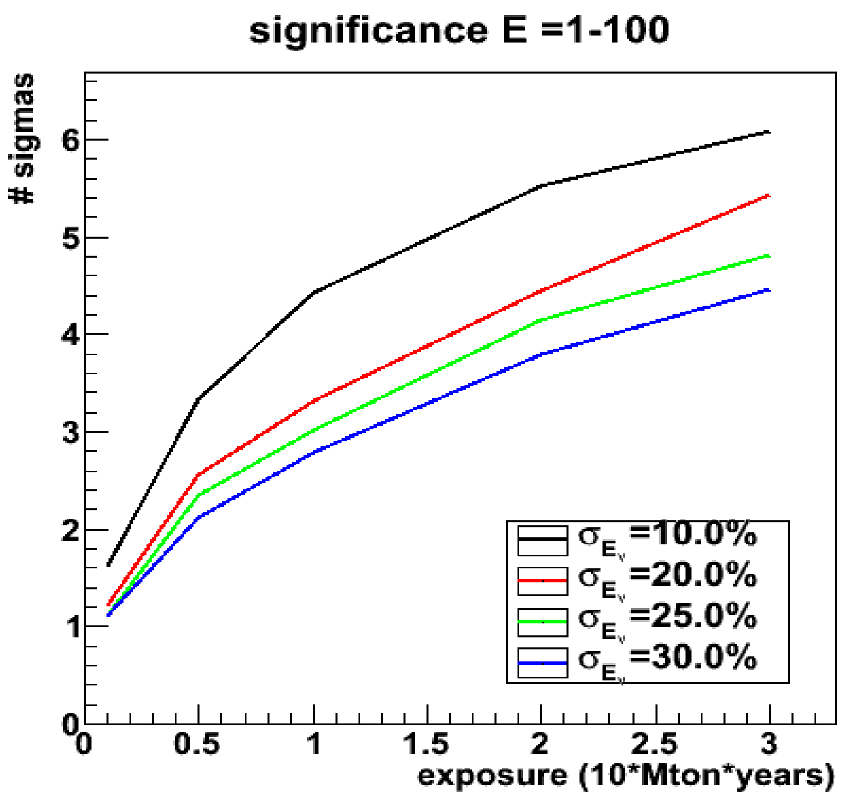
Conclusions

- Measurement is challenging
  - relies on very stringent control of systematics on acceptance, energy measurement, backgrounds etc...
  - many aspects still to be understood
  - but we are get a handle on some of them in our full simulations

- Likelihood fit (floating the unknows) is a good way to deal with 'degeneracies'

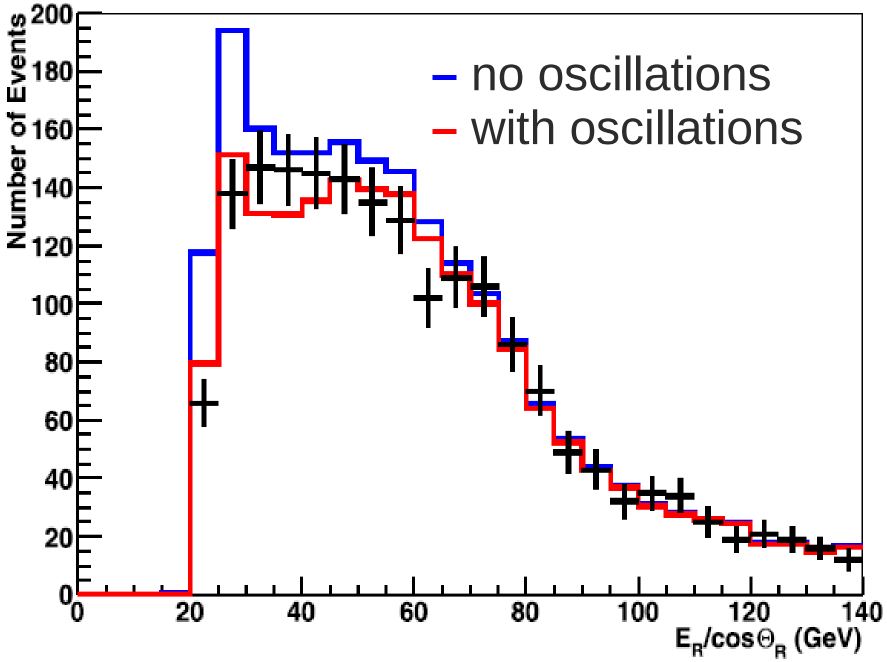
- Excellent sensitivity to Δm^2_{large} & θ_{23} !
  - factor 2 improvement over current uncertainty with only 1 Mton yr

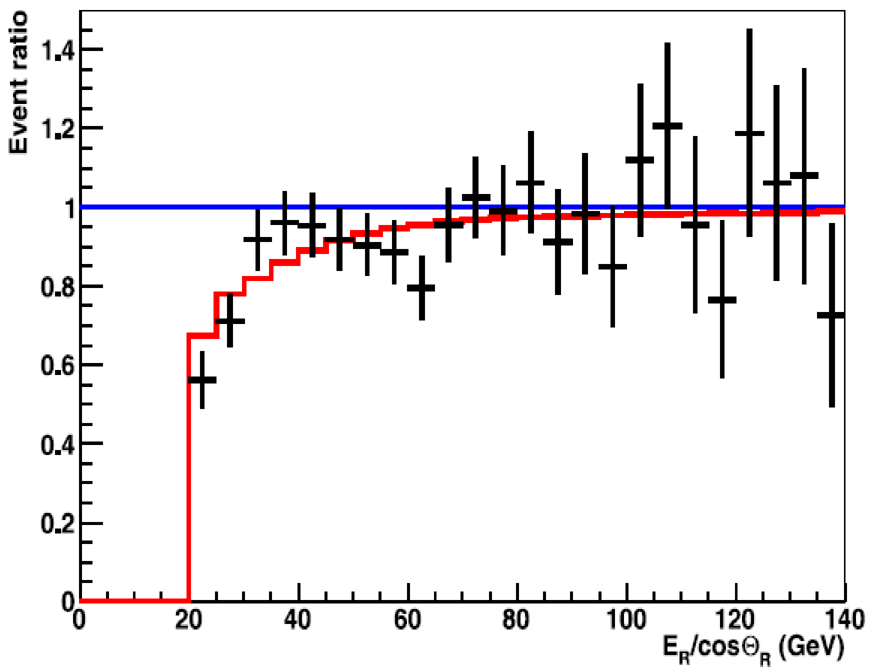
- MH Measurement:
  With a 10 Mton detector:
  3σ after one year, 5σ after 3 years
  provided we can achieve good energy resolution

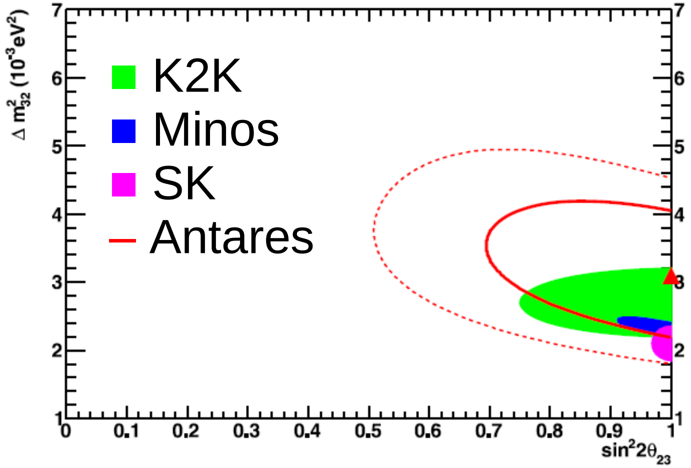


end

(vacuum) Oscillation physics with Antares





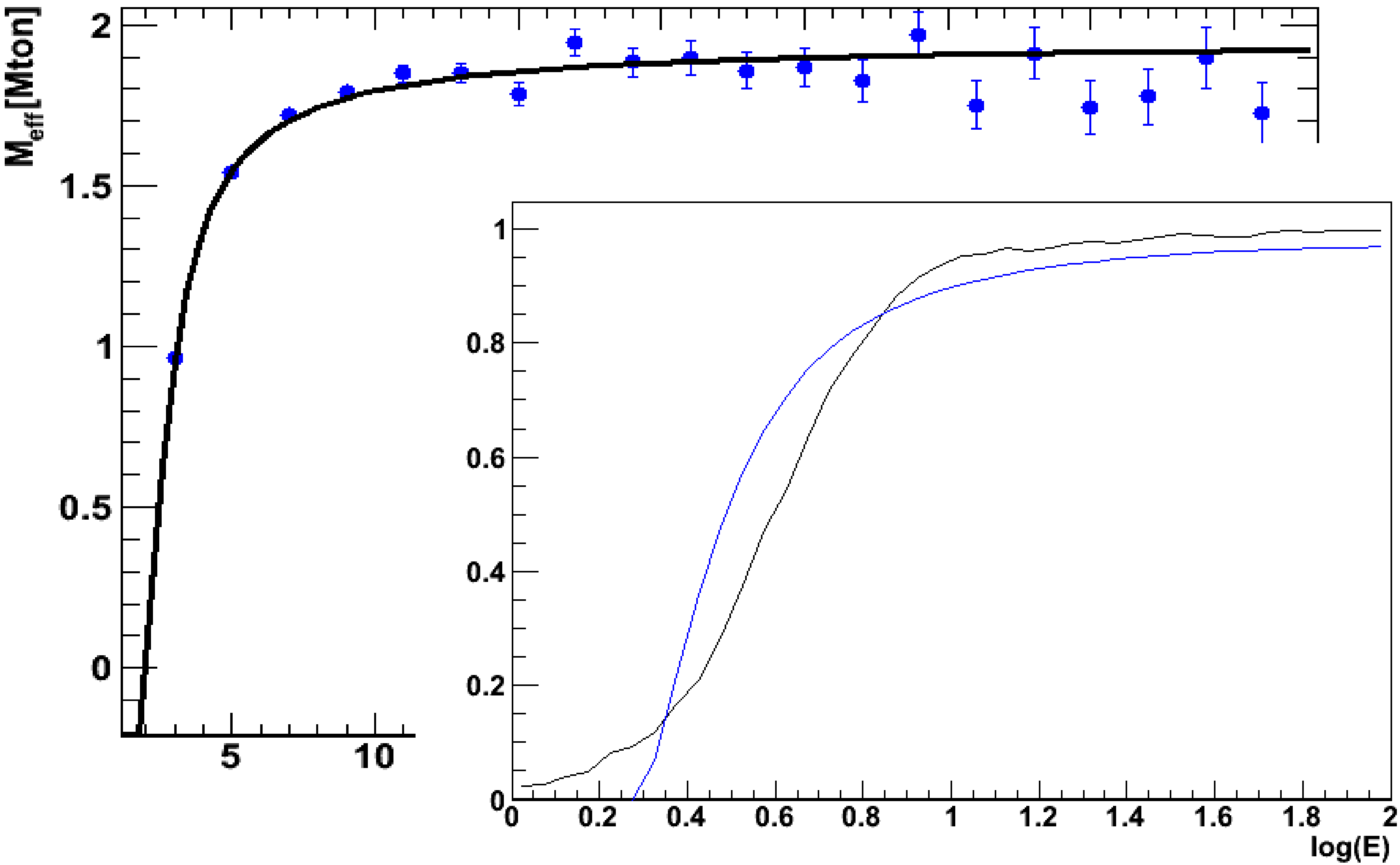


p-value of no-oscillation hypothesis: 2.1%

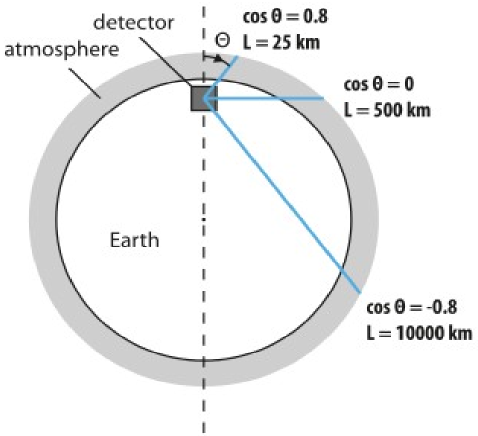
assuming maximal mixing:
$\Delta m^2 = (3.1 \pm 0.9)\ 10^{-3}\ \mathrm{eV}^2$

Reconstructed vertex inside the instrumented volume

Upgoing events



ORCA?



- Even if it is hard for us, it seems also hard for others

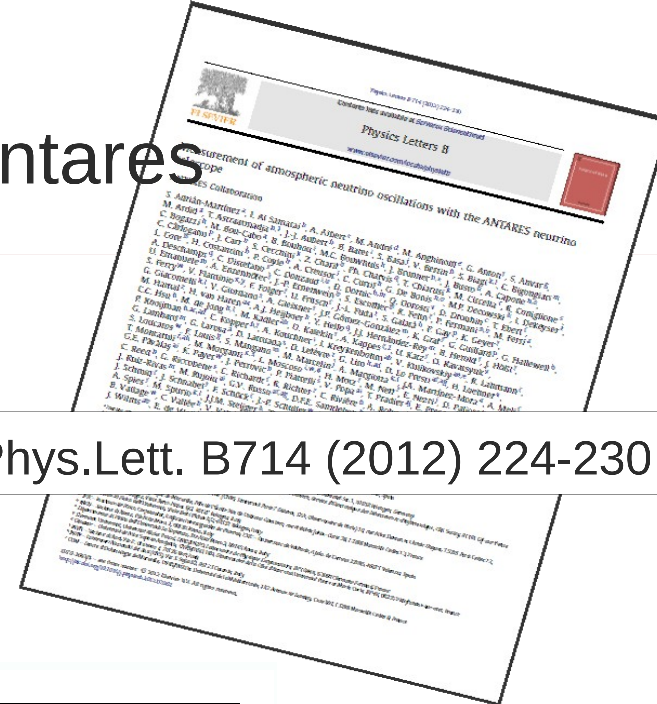
Mass hierarchy measurements

From J. Brunner

Project	Neutrino source	Detector	Goal	Problem
NOvA	LBL 810 km	14 kt tracking calorimeter	2σ for some values of δ ; 2020	Parameter degeneracy
Daya Bay II Reno II	Reactor 60 km	50 kt liquid scintillator	3σ in 2023	E_ν resolution & absolute scale
PINGU / ORCA	Atmosphere	1-10 Mt	$3\text{-}5\sigma$ in ?	E_ν resolution Systematics
INO	Atmosphere	50 kt magnetized iron calorimeter	3σ in 2030	Low statistics 10 years needed
T2 Hyper Kamiokande	LBL 295 km	1 Mt water	3σ in 2030	Parameter degeneracy
LBNE	LBL 1300 km	10 kt Liquid Argon	$2\text{-}5\sigma$ in 2030	Parameter degeneracy
LAGUNA Glacier	LBL 2300 km	20 kt Liquid Argon	5σ in 2030	Beam line from CERN
LAGUNA LENA	LBL 2300 km	50 kt Liquid scintillator	5σ in 2030	Beam line from CERN

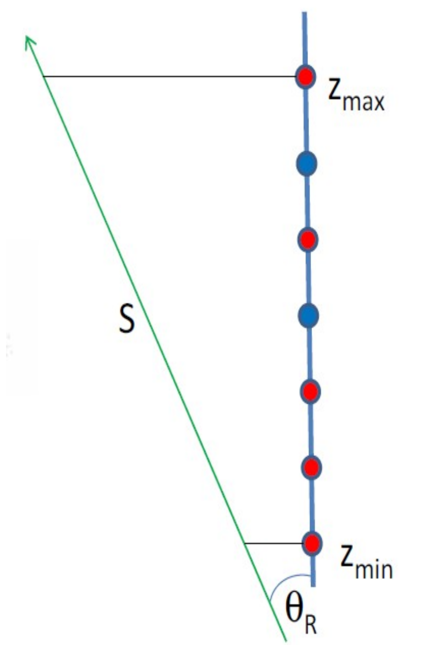
- Can ORCA/KM3NeT do this measurements with ~available funds ?
- We are in the process of answering this question.
- Decision should be taken this year.

(vacuum) Oscillation physics with Antares

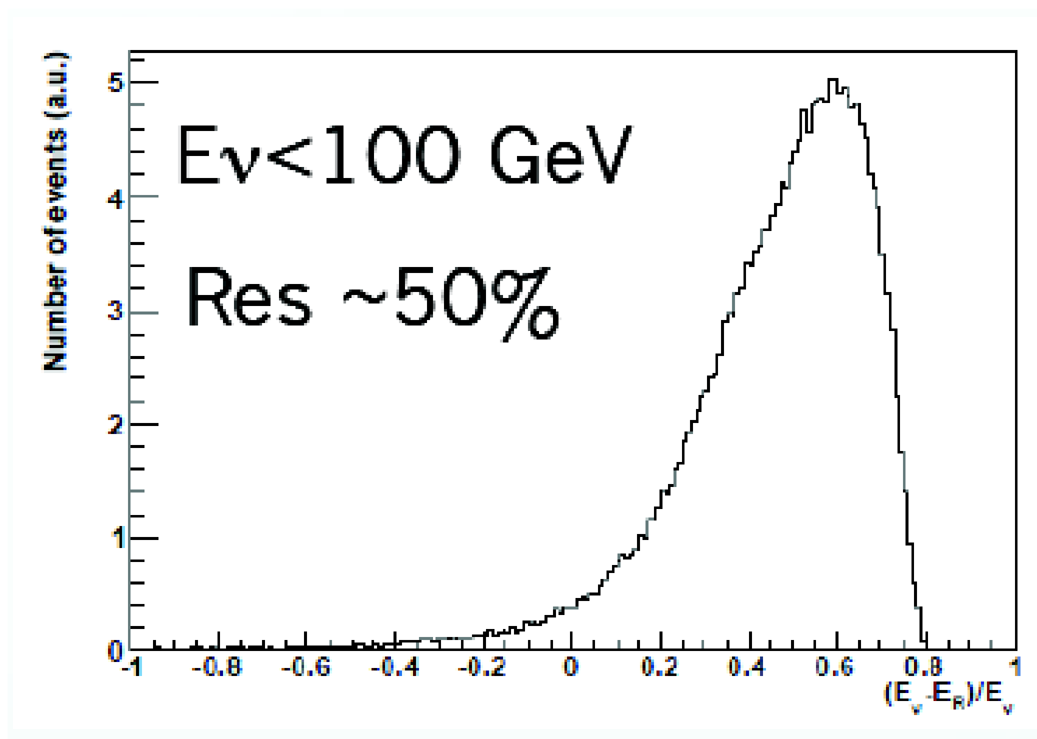


Phys.Lett. B714 (2012) 224-230

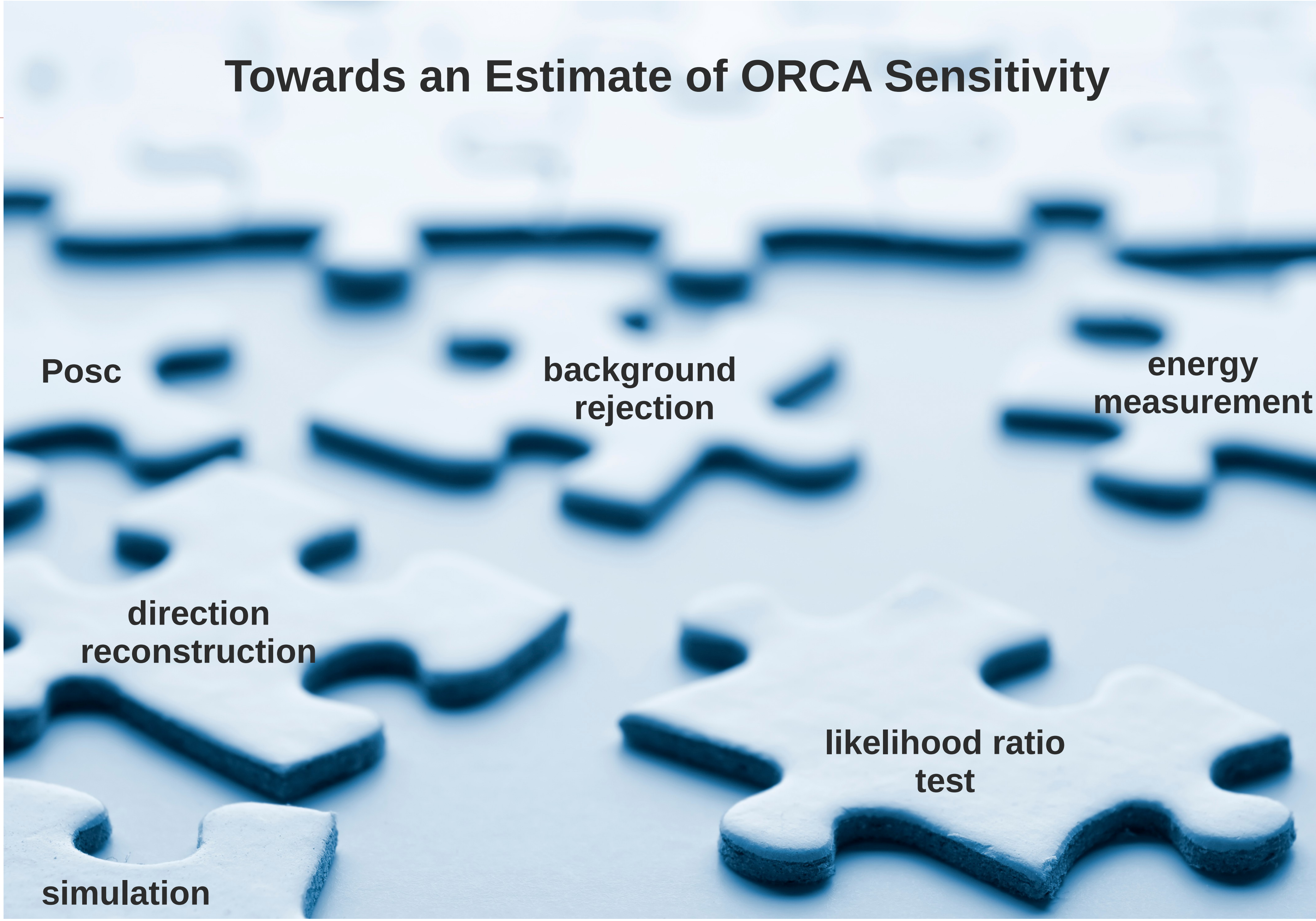
- for Antares: low-energy domain
- 2007 to 2010 : 863 days of active time
- 25 % of events reconstructed on only one line
- energy estimated from muon length



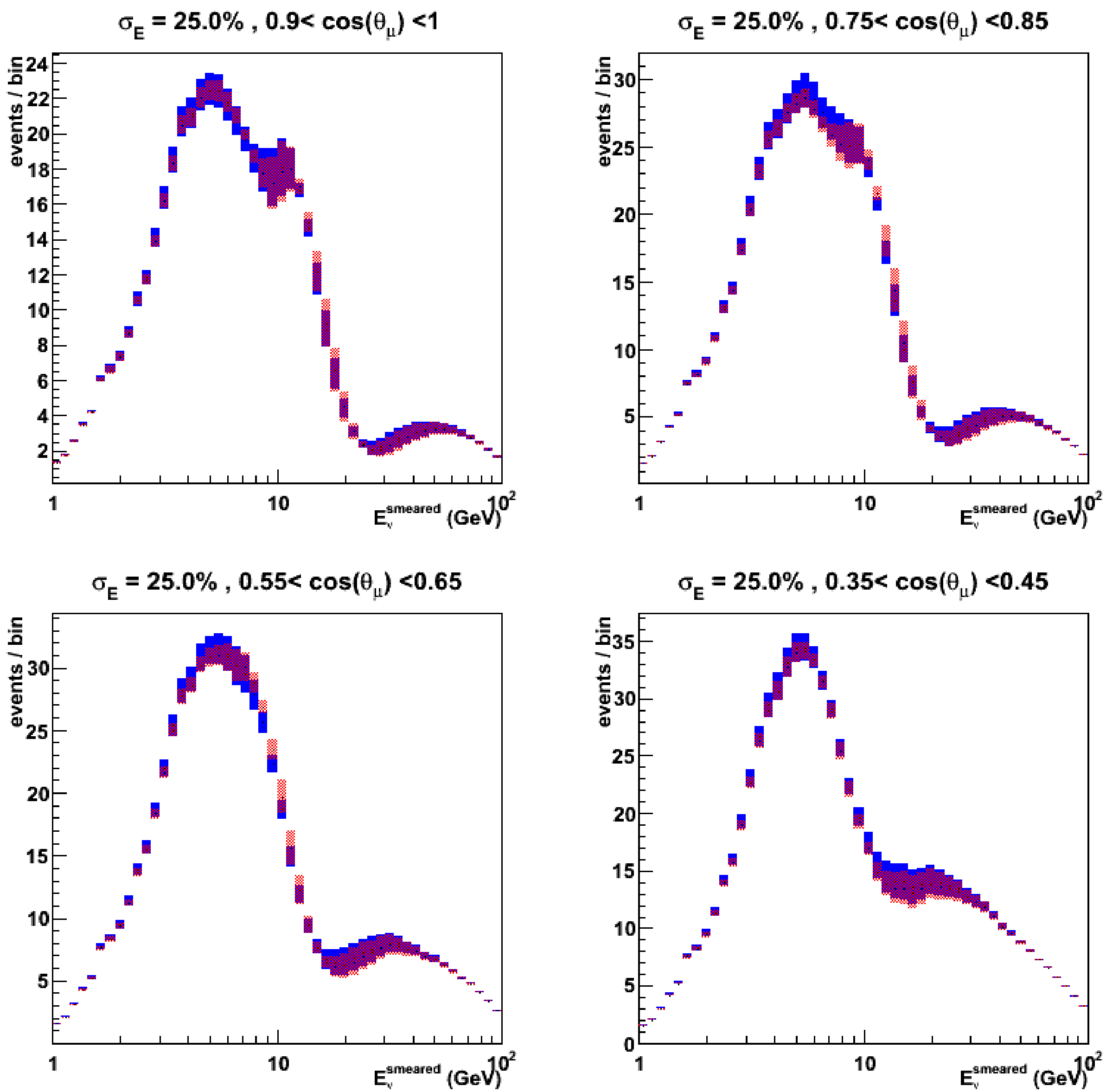
$$E_R = S \cdot 0.2\ \mathrm{GeV/m}.$$

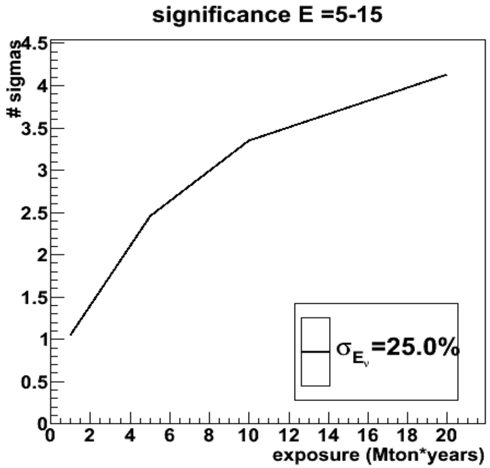


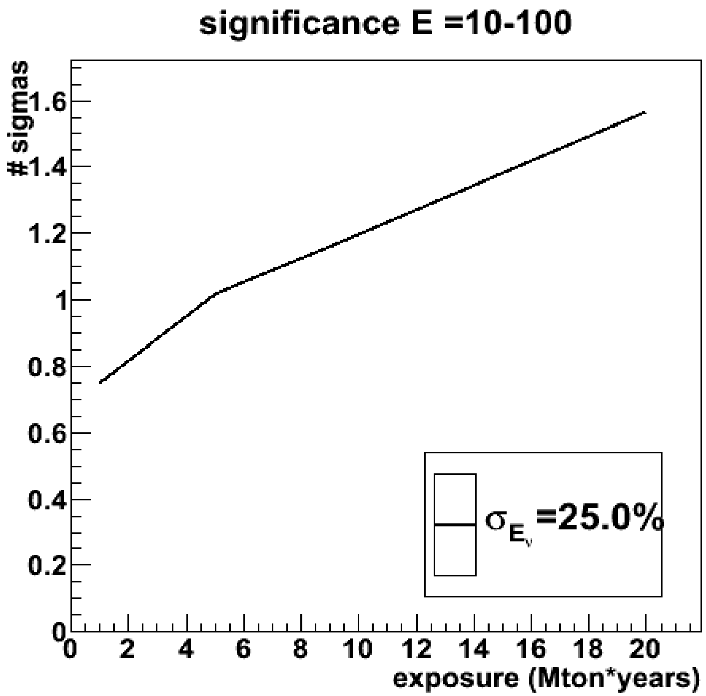
Towards an Estimate of ORCA Sensitivity

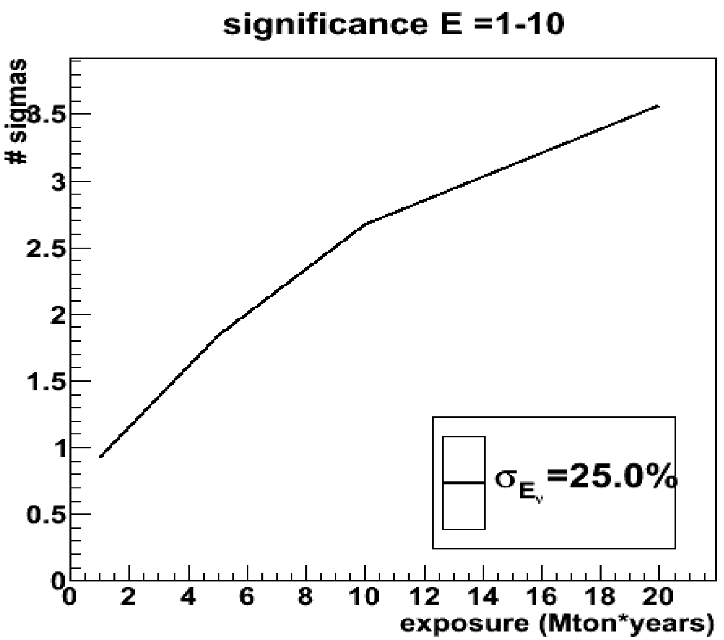


all pieces needed to get realistic estimate of physics potential


σE = 25.0% , 0.9< cos(θμ) <1
σE = 25.0% , 0.75< cos(θμ) <0.85
σE = 25.0% , 0.55< cos(θμ) <0.65
σE = 25.0% , 0.35< cos(θμ) <0.45
events / bin
Eνsmeared (GeV)

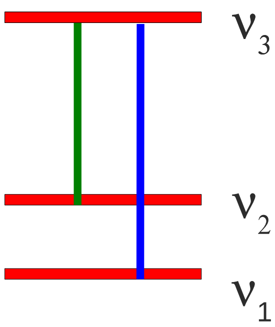

significance E =5-15
sigmas
exposure (Mton*years)
σ_{E_ν} =25.0%

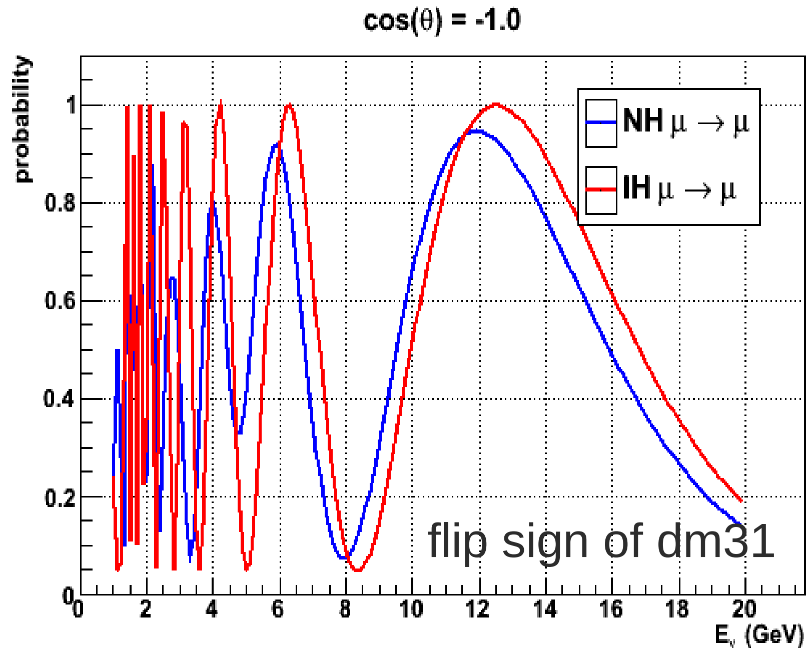

significance E =10-100
sigmas
exposure (Mton*years)
σ_{E_ν} =25.0%

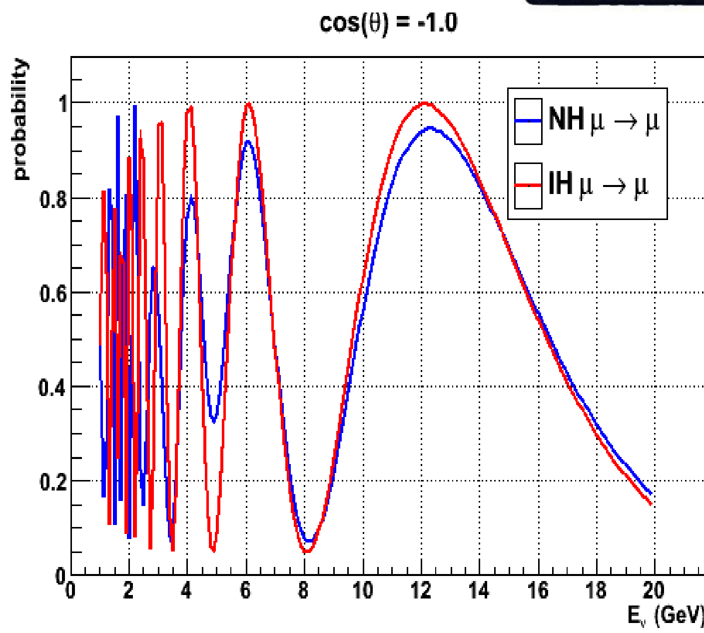

significance E =1-10
sigmas
exposure (Mton*years)
σ_{E_ν} =25.0%

Comparing Oscillation Probabilities

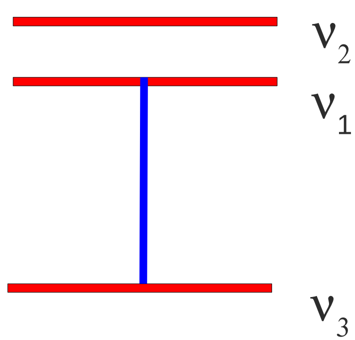
NH



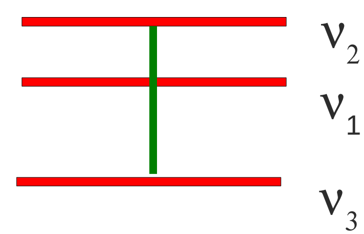




IH flip sign of Δm^2_{31}



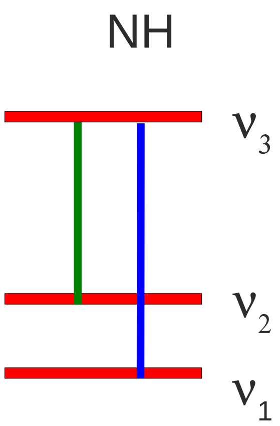
IH : flip sign of Δm^2_{32}

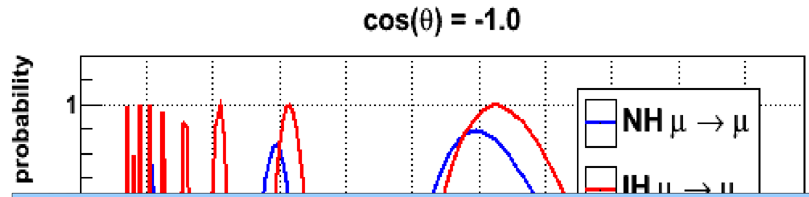


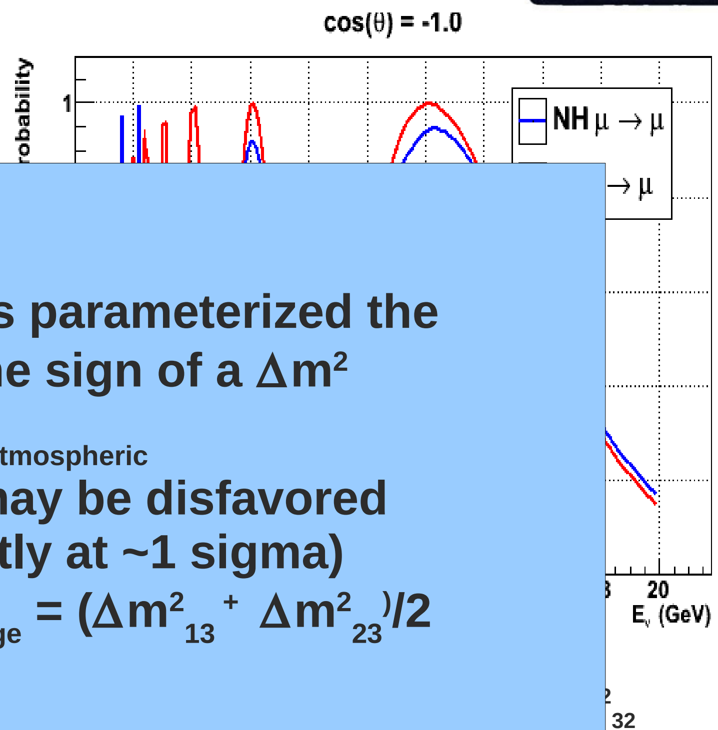
Depending on how one has parameterized the problem, simply flipping the sign of a Δm^2 may result in a IH model that is disfavored (~1σ) in the global fits.

and include parameter uncertainties.

Comparing Oscillation Probabilities







- **Depending on how one has parameterized the problem, simply flipping the sign of a Δm^2 changes the effective $\Delta m^2_{atmospheric}$**
  - **The resulting IH model may be disfavored by the global fits (currently at ~1 sigma)**
  - **best to flip sign of $\Delta m^2_{large} = (\Delta m^2_{13} + \Delta m^2_{23})/2$**

- **Still any NH → IH difference can be degenerate with change of Δm2**
- **One should compare all allowed NH models with all allowed IH models**

…meterized
…ign of a Δm2
may result in a IH model that is disfavored
(~1σ) in the global fits.

Measuring the neutrino mass hierarchy (with Orca).

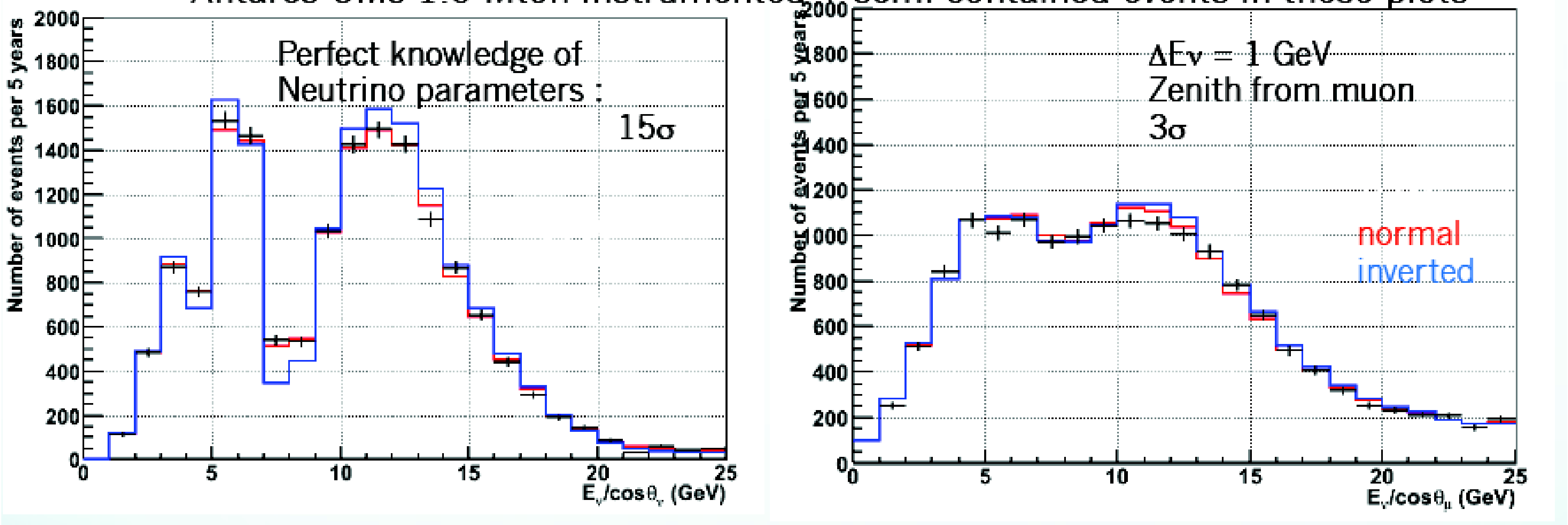
Aart Heijboer - Nikhef, Amsterdam

but a rather general computation based on some simple assumptions about detector performance

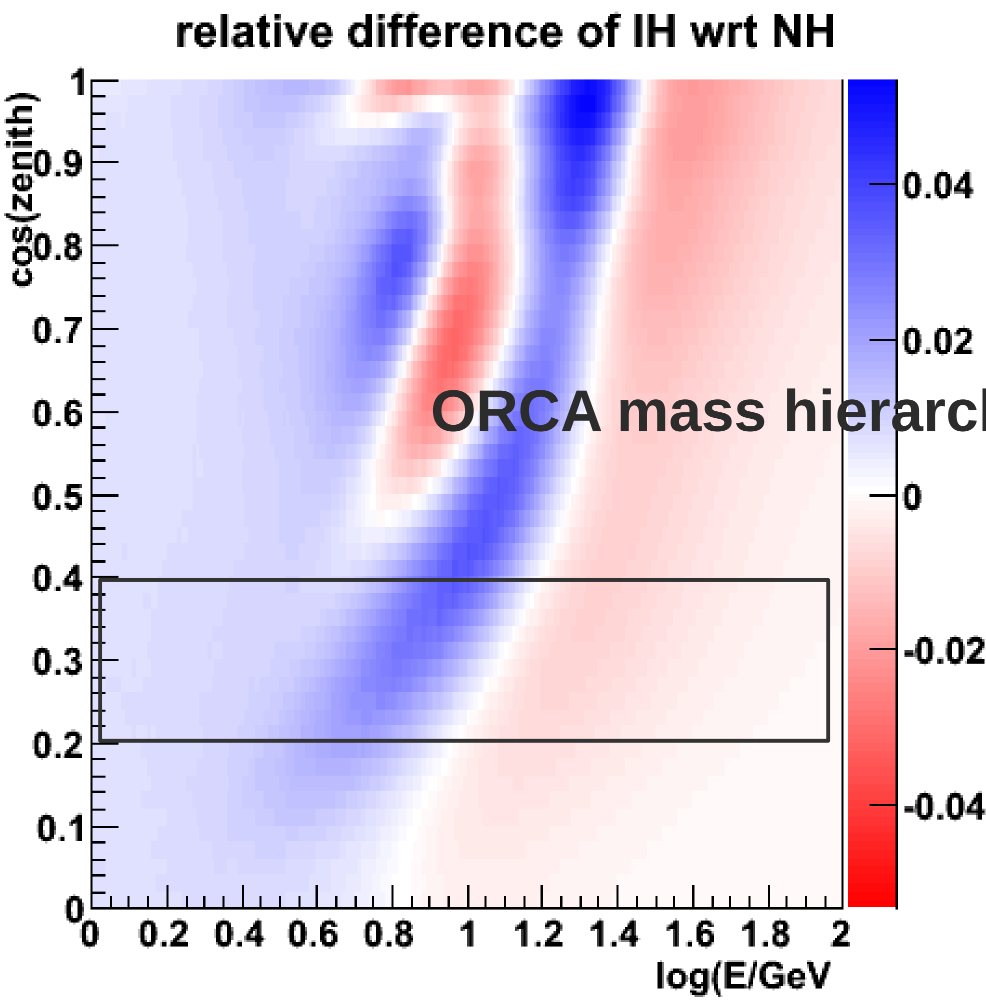
Reconstruction algorithm: bbfit + analysis

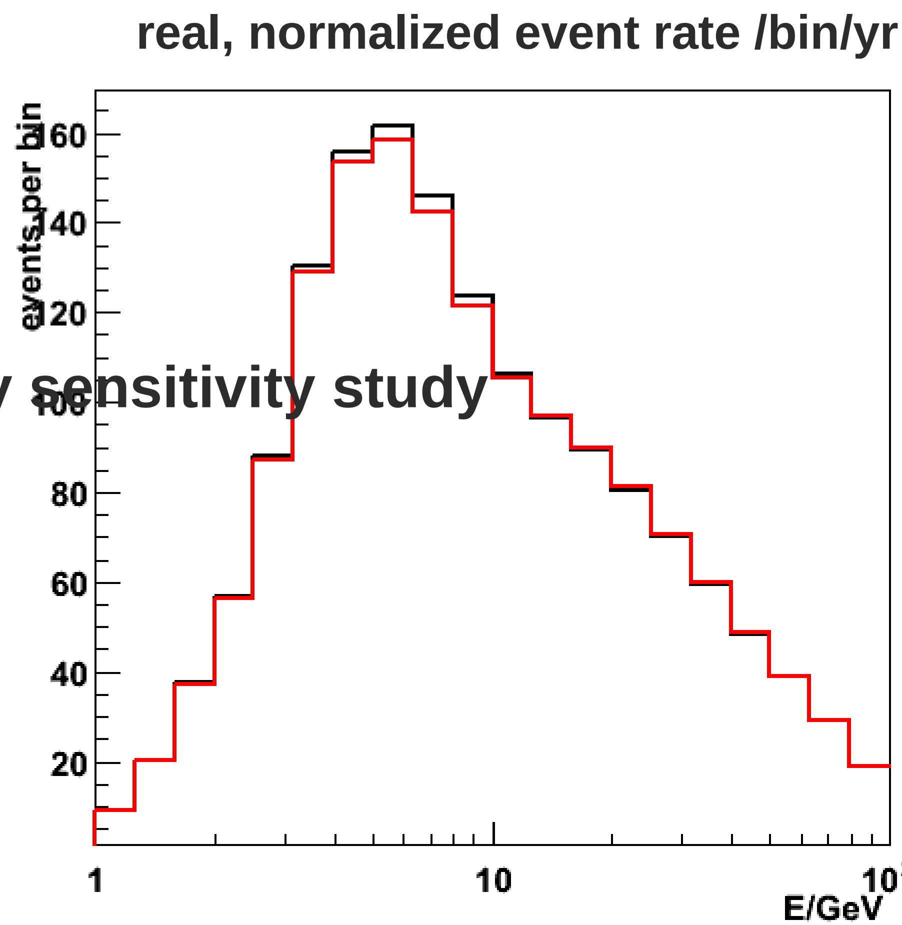
- Analysis along the line of Antares Oscillation analysis : E/cos(θ) histogram
- Works will using mc-truth, but resolutions quickly reduce significance
- Also: using only muon energy is not good enough: need combined track+shower reconstruction.

Antares OMs 1.5 Mton instrumented + semi-contained events in these plots



difference between IH and NH
example slice 1



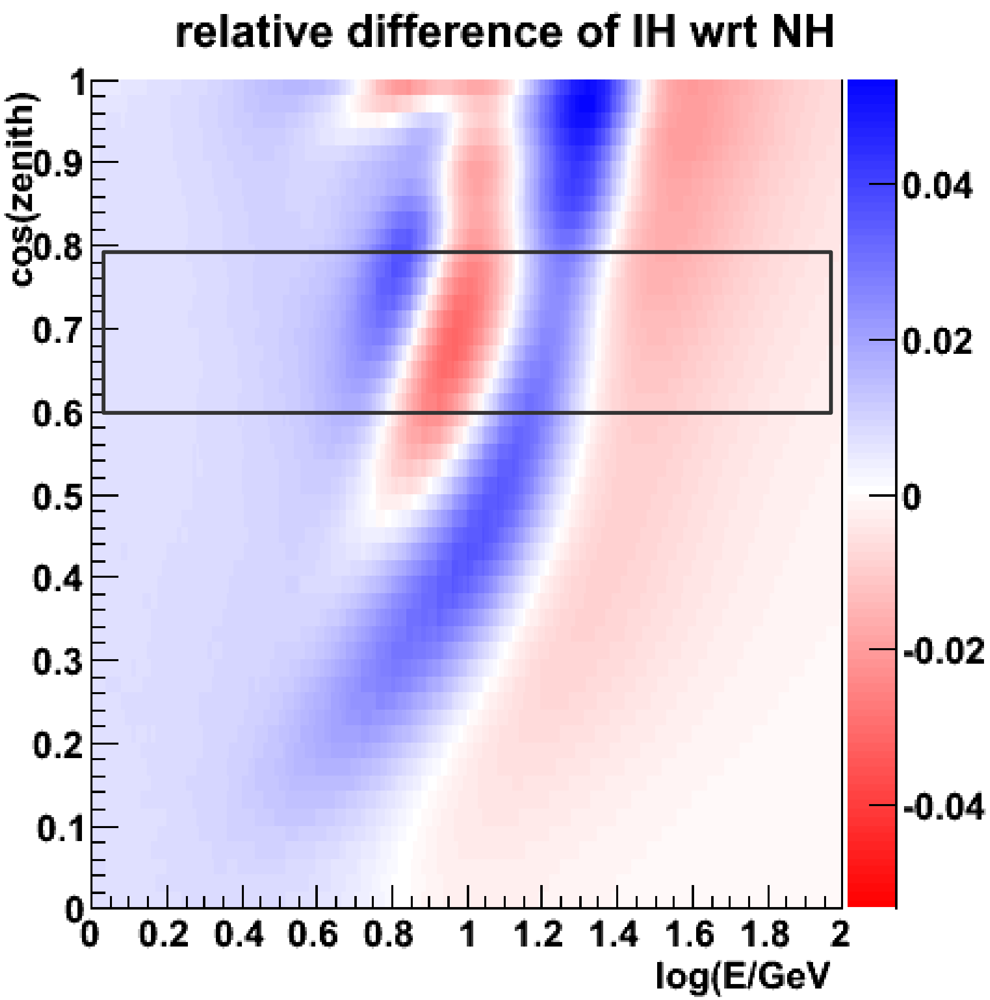


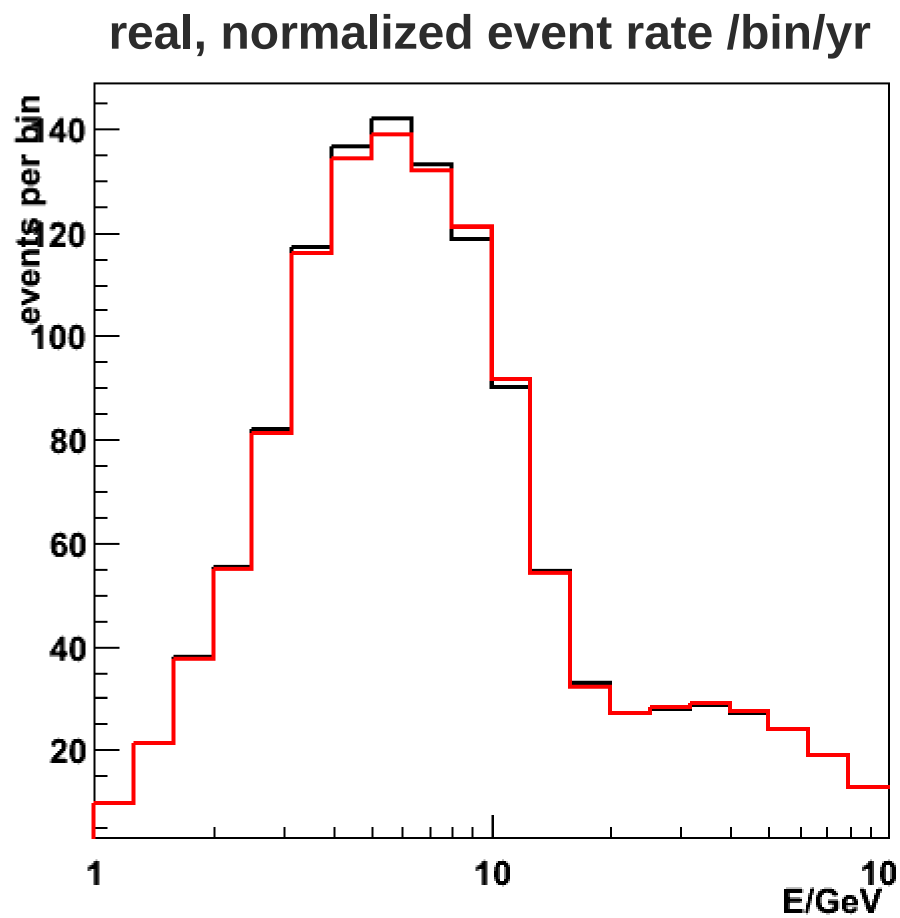
ORCA mass hierarchy sensitivity study

maximal ~5% effect

difference between IH and NH

example slice 1





difference between IH and NH

example slice 1