The Perpendicular Diffusion Coefficient in Cosmic Ray Transport Theories

Alexander Dosch and Gary P Zank

Center for Space Plasmas and Aeronomic Research
University of Alabama in Huntsville

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Field line random walk

Pressure-balanced structures in MHD

- *Pressure-balanced structures, observed frequently in the solar wind [e.g., Burlaga, 1995], are equilibrium solutions to the ideal MHD equations.*
- Provided $\mathbf{U} \cdot \nabla \mathbf{U} = 0$, they satisfy

$$\nabla P = \frac{1}{c} \mathbf{J} \times \mathbf{B} = 0,$$

from which $\mathbf{B} \cdot \nabla \mathbf{B} = 0$ we obtain the condition for pressure balance

$$P + \frac{B^2}{8\pi} = const.$$

- This implies that $\mathbf{B} \cdot \nabla P = 0$ and $\mathbf{J} \cdot \nabla P = 0$ and therefore both $\mathbf{B}$ and $\mathbf{J}$ lie along surfaces of constant total pressure. Thus the flux tube defined by the pressure-balanced structure represents a smooth surface that is everywhere tangent to the local magnetic field $\mathbf{B}$. By following the evolution of the pressure-balanced structure in space, we can therefore very conveniently examine magnetic field line wandering.
- Such structures emerge from a dynamical theory of nearly incompressible MHD *Zank and Matthaeus* [1992].

Field line random walk - cont.

Pressure-balanced structures in MHD - cont.

- Consider axisymmetric magnetic fluctuations **b** transverse to a uniform mean field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$ so that

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{b}(x, y, z); \qquad \mathbf{b} \cdot \mathbf{B}_0 = 0,$$

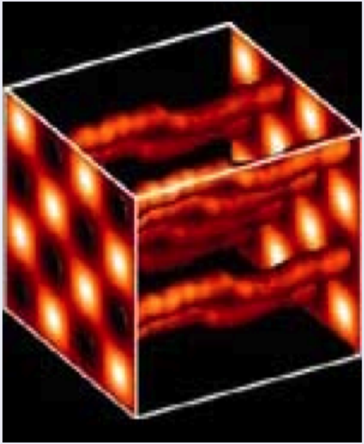
  meaning that **b** can be slab, 2d, or a superposition.
- Magnetic surfaces must therefore satisfy

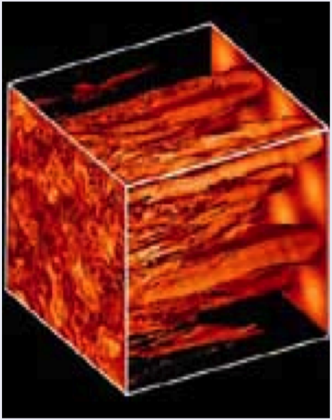
$$\frac{\partial P}{\partial z} + \frac{\mathbf{b}}{\mathbf{B}_0} \cdot \nabla_\perp P = 0.$$

- Adopting a simple slab turbulence or propagating linear waves form for the coefficient $\mathbf{b}/\mathbf{B}_0$ will not engender any complexity in the field surfaces described by the scalar P. By contrast, the inclusion of a 2-D field for $\mathbf{b}/\mathbf{B}_0$ can lead to an extraordinarily complicated magnetic field flux surfaces. Such surfaces, as expressed through the passive scalar equation will evolve in complexity with spatial displacement just as a passive scalar undergoes turbulent mixing with time when advocated in a turbulent 2-D flow field.

Field line random walk - cont.

Pressure-balanced structures in MHD - cont.





NLGC

Back to diffusion coefficients

- *Velocity of the guiding center.* The first assumption is that *the perpendicular transport is governed by the velocity of gyrocenters that follow field lines*. In this case the equation of motion is given by:

$$v_x(t) = a v_z \frac{\delta B_x}{B_0}.$$

  The parameter a is a proportionality constant and has to be determined after the fact through comparison with simulations.

- *TGK formalism.* The TGK formalism is given by

$$\kappa_{xx} = \int_0^\infty dt \, \langle \, v_x(t) v_x(0) \, \rangle \, .$$

  By substituting the gyro center velocity into the equation we obtain

$$\kappa_{xx} = \frac{a^2}{B_0^2} \int_0^\infty dt \, \langle \, v_z(t) \, \delta B_x(t) \, v_z(0) \, \delta B_x^*(0) \, \rangle \, .$$

NLGC

Back to diffusion coefficients

- *Fourth order correlation functions.* Matthaeus, Bieber, Qin, Zank (2003) note:

> *Next, we assume that the particle velocities are uncorrelated with the local magnetic field vector. This is exact for any distribution symmetric about* 90^o *pitch angle. Thus, the more daunting fourth-order correlation is replaced by a product of second-order correlations,*

$$\kappa_{xx} = \frac{a^2}{B_0^2} \int_0^\infty dt \, \langle \, v_z(t) \, v_z(0) \, \rangle \, \langle \, \delta B_x(t) \, \delta B_x^*(0) \, \rangle \, .$$

NLGC

Back to diffusion coefficients

- *Velocity correlation function.* We replace the velocity correlation function by

$$\langle\, v_z(t)\, v_z(0)\,\rangle = \frac{v^2}{3} e^{-\frac{vt}{\lambda_\parallel}} .$$

This assumption is consistent with the TGK formulation of the parallel diffusion coefficient

$$\kappa_{zz} = \frac{v}{3}\lambda_\parallel = \int_0^\infty dt\, \langle\, v_z(t)\, v_z(0)\,\rangle\,,$$

where we used the relation between the parallel mean free path and parallel diffusion coefficient , where $\kappa_{zz} = \kappa_\parallel$. The velocity correlation decorrelates exponentially with a characteristic decorrelation time $\tau = \lambda_\parallel / v$, which is connected to the parallel mean free path. For short times ($vt \ll \lambda_\parallel$) we find for the velocity correlation function $\langle\, v_z(t)\, v_z(0)\,\rangle = v^2/3$, i.e., isotropic initial conditions. (see generalization by Webb, Zank, 2005).

NLGC

Back to diffusion coefficients

- *Correlation function of the turbulent magnetic fields.* For the correlation function of the turbulent magnetic fields we use

$$\langle \delta B_x(t) \, \delta B_x^*(0) \rangle = \int d^3k \; P_{xx}(\boldsymbol{k}) \, \Gamma(\boldsymbol{k}, t) \, \left\langle e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \right\rangle ,$$

  where $P_{xx}(\boldsymbol{k})$ is the magnetostatic correlation tensor, $\Gamma(\boldsymbol{k}, t)$ the dynamical correlation function, and $\left\langle e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \right\rangle$ the characteristic function. The magnetostatic correlation tensor depends on the geometry (slab, 2D).

- *Dynamical correlation function.* The magnetic fields decorrelate exponentially with time, so that

$$\Gamma(\boldsymbol{k}, t) = e^{-\gamma(\boldsymbol{k})t},$$

  where γ is the inverse of a wavenumber dependent characteristic time scale.

NLGC

Back to diffusion coefficients

- *Characteristic function.* Matthaeus et al (2003) write:

> *We assume that the components of the trajectory have uncorrelated axisymmetric Gaussian distributions and, furthermore, that the distribution of displacements is diffusive for all values of time.*

This leads to

$$\left\langle e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \right\rangle = e^{-\kappa_{xx}k_{xx}^2 t-\kappa_{yy}k_{yy}^2 t-\kappa_z k_{zz}^2 t}.$$

The diffusion coefficient is then given by

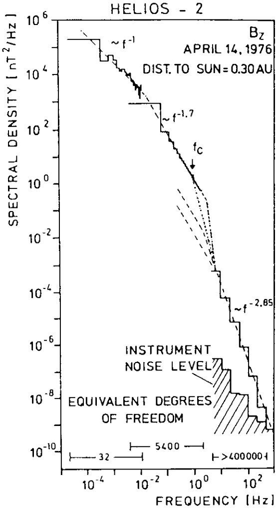
$$\kappa_{xx} = \frac{a^2v^2}{3B_0^2}\int d^3k\; P_{xx}(\boldsymbol{k})\int_0^\infty dt\; e^{-\frac{vt}{\lambda_\parallel}-\gamma(\boldsymbol{k})t-\kappa_{xx}k_{xx}^2 t-\kappa_{yy}k_{yy}^2 t-\kappa_z k_{zz}^2 t}.$$

After an elementary time integration we obtain

$$\boxed{\kappa_{xx} = \frac{a^2v^2}{3B_0^2}\int d^3k\; \frac{P_{xx}(\boldsymbol{k})}{\frac{v}{\lambda_\parallel}+\gamma(\boldsymbol{k})+\kappa_{xx}k_{xx}^2+\kappa_{yy}k_{yy}^2+\kappa_z k_{zz}^2}}.$$

This is a nonlinear integral equation and is referred to as the NLGC theory.

Turbulence Properties: Energy Spectrum



Energy Range

- small wave numbers $\rightarrow$ large scales
- Turbulence gains energy
- $E \sim k^{-1}$

Inertial Range

- medium wave numbers $\rightarrow$ medium scales
- Energy is transferred from large to small scales
- *Kolmogorov theory of turbulence*
- $E \sim k^{-5/3}$

Dissipation Range

- high wave numbers $\rightarrow$ small scales
- Turbulence loses energy through dissipation
- $E \sim k^{-3}$

Turbulence Properties: Geometry

Isotropic turbulence

- $\delta \boldsymbol{B} = \delta \boldsymbol{B}(r)$

Slab turbulence

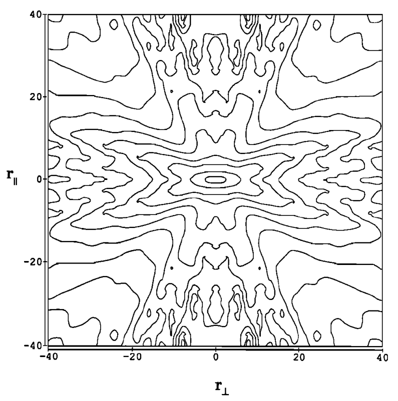
- $\delta \boldsymbol{B}^{slab} = \delta \boldsymbol{B}^{slab}(z)$ (change parallel to $\boldsymbol{B}_0$)
- $\delta B_z^{slab}(z) = 0$ (solenoidal constraint)
- $\boldsymbol{k} \parallel \boldsymbol{B}_0$ (Fourier transformation)

2D turbulence

- $\delta \boldsymbol{B}^{2D} = \delta \boldsymbol{B}^{2D}(x, y)$ (change perpendicular to $\boldsymbol{B}_0$)
- in general $\delta \boldsymbol{B}^{2D}(x, y) \neq 0$
- $\delta \boldsymbol{B}^{2D}(x, y) = 0$ (full 2D model, $\boldsymbol{k} \perp \boldsymbol{B}_0$)

Turbulence Properties: Geometry

Turbulence Geometry form Measurements



- Maltese cross
- approximated by superposition:
  15 - 20 % slab
  80 - 85 % 2D

NLGC models

Slab plus 2D

- Turbulence in the solar wind is thought to comprise a superimposed slab and 2-D component; this based on theory [*Zank and Matthaeus*, 1992] and observations [*Matthaeus et al.*, 1990; *Bieber et al.*, 1996]. The two-component slab 2-D model ignores the usually smaller parallel variance and includes only fluctuations with wave vectors either purely parallel (k_z) to or perpendicular ($k_\perp$) to the mean magnetic field $\mathbf{B}_0$. Thus we may express

$$S_{xx}(\mathbf{k}) = S_{xx}^{2D}(k_\perp)\delta(k_z) + S_{xx}^{slab}(k_z)\delta(k_\perp)$$

where

$$\begin{aligned} S_{xx}^{slab}(k_z) &= C\left\langle b_{slab}^2\right\rangle \lambda_{slab}\left(1 + k_z^2\lambda_{slab}^2\right)^{-\nu}; \\ S_{xx}^{2D}(k_\perp) &= \frac{C}{\pi}\left\langle b_{2D}^2\right\rangle \lambda_{2D}\frac{\left(1 + k_z^2\lambda_{slab}^2\right)^{-\nu}}{k_\perp}. \end{aligned}$$

We assume that the spectrum has an inertial range that is characteristic of fully developed Kolmogorov turbulence and thus $\nu = 5/6$.

NLGC models

Slab plus 2D

- *Zank et al.*, 2004 and *Shalchi et al.*, 2004 develop attractive approximate solution to the integral equation.

$$\lambda_{xx} \simeq \left(\sqrt{3}\pi a^2 C\right)^{2/3} \left(\frac{\langle b_{2D}^2 \rangle}{B_0^2}\right)^{2/3} \lambda_{2-D}^{2/3} \lambda_{\parallel}^{1/3} \cdot \left[1 + \frac{(a^2 C)^{1/3}}{(\sqrt{3}\pi)^{2/3}} \frac{\langle b_{slab}^2 \rangle}{\langle b_{2D}^2 \rangle^{2/3} (B_0^2)^{1/3}} \times \frac{\min(\lambda_{slab}, \lambda_{\parallel}/\sqrt{3})}{\lambda_{2D}^{2/3} \lambda_{\parallel}^{1/3}} \cdot \left(4.33 H\left(\lambda_{slab} - \left(\lambda_{\parallel}/\sqrt{3}\right)\right) + 3.091 H\left(\lambda_{\parallel}/\sqrt{3} - \lambda_{slab}\right)\right)\right]^{2/3}. \quad (12)$$

NLGC models

Slab plus 2D

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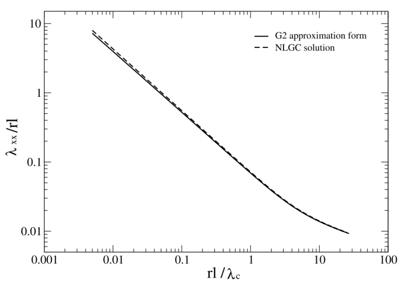

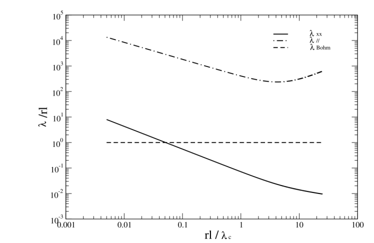

NLGC models

Slab plus 2D

- Numerical simulations compared to NLGC theory:

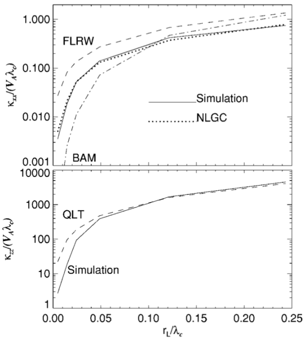

FIG. 2.—Perpendicular (*upper panel*) and parallel (*lower panel*) diffusion coefficient as a function of r_L/λ_c, with $b/B_0 = 0.2$ and $E^{slab} : E^{2D} = 20 : 80$. The particle velocity varies by a factor of 50. *Upper panel, solid line*: κ_{xx} from numerical simulation; *dotted line*: κ_{xx} from present NLGC theory (eq. [7]); *dashed line*: κ_{xx} from FLRW limit; *dash-dotted line*: κ_{xx} from BAM theory. *Lower panel, solid line*: κ_{zz} from numerical simulation; *dashed line*: κ_{zz} from QLT.

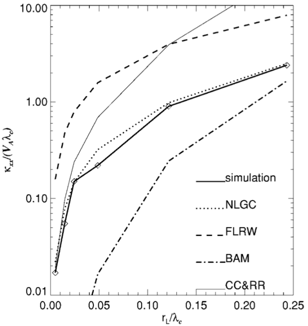

FIG. 3.—Perpendicular diffusion coefficients as a function of r_L/λ_c, with $b/B_0 = 1$ and $E^{slab} : E^{2D} = 20 : 80$. *Thick solid line*: κ_{xx} from numerical simulation; *dotted line*: κ_{xx} from NLGC theory (eq. [7]); *thin solid line*: CC&RR theory; *dashed line*: FLRW limit; *dash-dotted line*: BAM theory. The turbulence amplitude is larger than in Fig. 2, and parallel diffusion (not shown) is no longer accurately given by QLT for these parameters. The NLGC theory is more accurate than the other theories shown.

Magnetic correlation tensor

Fourier Transformation

The perturbed equations of motion depend on the turbulent magnetic fields $\delta B(\boldsymbol{r}, t)$ and the components of the correlation tensor are

$$R_{lm}(\boldsymbol{r}_1, t_1, \mathbf{r}_2, t_2) = \langle \, \delta B_l(\boldsymbol{r}_1, t_1) \; \delta B_m(\boldsymbol{r}_2, t_2) \, \rangle \, ,$$

with $l, m = x, y, z$. The position vectors are given by $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ and the time by t_1 and t_2. Usually the turbulent magnetic fields are represented by a Fourier transformation,

$$\delta B_l(\boldsymbol{r}, t) = \int d^3k \; \delta B_l(\boldsymbol{k}, t) e^{i\boldsymbol{k}\cdot\boldsymbol{r}},$$

where $\boldsymbol{k}$ is a wave vector. The correlation tensor becomes

$$\begin{aligned} & R_{lm}(\boldsymbol{r}_1, t_1, \boldsymbol{r}_2, t_2) \\ & = \left\langle \int d^3k_1 \; \delta B_l(\boldsymbol{k}_1, t_1) e^{i\boldsymbol{k}_1\cdot\boldsymbol{r}_1} \int d^3k_2 \; \delta B_m(\boldsymbol{k}_2, t) e^{i\boldsymbol{k}_2\cdot\boldsymbol{r}_2} \right\rangle \\ & = \int d^3k_1 \int d^3k_2 \; \langle \, \delta B_l(\boldsymbol{k}_1, t_1) \; \delta B_m(\boldsymbol{k}_2, t_2) \; e^{i(\boldsymbol{k}_1\cdot\boldsymbol{r}_1 + \boldsymbol{k}_2\cdot\boldsymbol{r}_2)} \, \rangle \, . \end{aligned}$$

Magnetic correlation tensor

Corrsin's (independence) hypothesis - a random phase approximation

Corrsin's hypothesis states that

$$\left\langle \delta B_l(\boldsymbol{k}_1, t_1)\, \delta B_m(\boldsymbol{k}_2, t_2)\, e^{i(\boldsymbol{k}_1\cdot\boldsymbol{r}_1+\boldsymbol{k}_2\cdot\boldsymbol{r}_2)} \right\rangle$$
$$\approx \langle \delta B_l(\boldsymbol{k}_1, t_1)\, \delta B_m(\boldsymbol{k}_2, t_2) \rangle \left\langle e^{i(\boldsymbol{k}_1\cdot\boldsymbol{r}_1+\boldsymbol{k}_2\cdot\boldsymbol{r}_2)} \right\rangle .$$

Corrsin's (independence) hypothesis can be formulated in different ways:

- Corrsin suggested that at long diffusion times the probability distribution of particle displacements and the probability distribution of the Eulerian velocity field would become statistically independent of each other. At large values of the diffusion time, the independence hypothesis asserts that the joint average can be expressed as the product of two separate averages.
- The statistics of the magnetic fluctuations can be separated from those of the individual trajectories, Matthaeus et al 1995.

Using Corrsin's independent hypothesis we obtain

$$R_{lm}(\boldsymbol{r}_1, t_1, \boldsymbol{r}_2, t_2)$$
$$= \int d^3k_1 \int d^3k_2\ \langle \delta B_l(\boldsymbol{k}_1, t_1)\, \delta B_m(\boldsymbol{k}_2, t_2) \rangle \left\langle e^{i(\boldsymbol{k}_1\cdot\boldsymbol{r}_1+\boldsymbol{k}_2\cdot\boldsymbol{r}_2)} \right\rangle .$$

Magnetic correlation tensor

Homogeneity in space

Tautz and Lerche (2011) write:

> *In a homogeneous (but not necessarily isotropic) medium both the left-hand and the right-hand sides of the last equation must depend on $|\boldsymbol{r}_1 - \boldsymbol{r}_2|$ only, hence a factor $\delta(\boldsymbol{k}_1 + \boldsymbol{k}_2)$ is invoked.*

This means we have to multiply with $\delta(\boldsymbol{k}_1 + \boldsymbol{k}_2)$. The integration with respect to wave vector $\boldsymbol{k}_2$ can easily be solved (the integration contributes only for $\boldsymbol{k}_2 = -\boldsymbol{k}_1$) and we obtain

$$R_{lm}(\boldsymbol{r}_1, t_1, \boldsymbol{r}_2, t_2) = \int d^3k_1 \left\langle \delta B_l(\boldsymbol{k}_1, t_1) \delta B_m(-\boldsymbol{k}_1, t_2) \right\rangle \left\langle e^{i(\boldsymbol{k}_1 \cdot \boldsymbol{r}_1 - \boldsymbol{k}_1 \cdot \boldsymbol{r}_2)} \right\rangle .$$

From the definition of the Fourier transform of the magnetic fluctuation (or better, the definition of the back transformation), it is clear that $\delta B_m(-\boldsymbol{k}_1, t_2) = \delta B_m^*(\boldsymbol{k}_1, t_2)$, where the asterisk $(*)$ denotes a complex conjugate quantity. We obtain

$$R_{lm}(\boldsymbol{r}_1, t_1, \boldsymbol{r}_2, t_2) = \int d^3k_1 \; \left\langle \delta B_l(\boldsymbol{k}_1, t_1) \, \delta B_m^*(\boldsymbol{k}_1, t_2) \right\rangle \left\langle e^{i\boldsymbol{k}_1 \cdot (\boldsymbol{r}_1 - \boldsymbol{r}_2)} \right\rangle .$$

Magnetic correlation tensor

Homogeneity in space

By defining $\tilde{P}_{lm}(\boldsymbol{k}_1, t_1, t_2) = \langle\, \delta B_l(\boldsymbol{k}_1, t_1)\, \delta B^*_m(\boldsymbol{k}_1, t_2)\, \rangle$ as the magnetic correlation tensor in wave vector space, we obtain

$$R_{lm}(\boldsymbol{r}_1, t_1, \boldsymbol{r}_2, t_2) = \int d^3k_1\; \tilde{P}_{lm}(\boldsymbol{k}_1, t_1, t_2) \left\langle\, e^{i\boldsymbol{k}_1\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)} \,\right\rangle .$$

Homogeneity in time

Under the assumption that the turbulence is also homogeneous in time, i.e., only the time difference $|t_1 - t_2|$ is important, we may set $t_2 = 0$ und $\boldsymbol{r}_2(t_2 = 0) = 0$ and obtain

$$R_{lm}(\boldsymbol{r}_1, t_1) = \int d^3k_1\; \tilde{P}_{lm}(\boldsymbol{k}_1, t_1) \left\langle\, e^{i\boldsymbol{k}_1\cdot\boldsymbol{r}_1} \,\right\rangle .$$

Assuming that all components of the correlation tensor possess the same temporal behavior, we may define

$$\tilde{P}_{lm}(\boldsymbol{k}_1, t_1) = P_{lm}(\boldsymbol{k}_1)\, \Gamma(\boldsymbol{k}_1, t_1)$$

so that

Magnetic correlation tensor

Homogeneity in space

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Magnetic correlation tensor

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so that

$$R_{lm}(\boldsymbol{r}_1, t_1) = \int d^3k_1\; P_{lm}(\boldsymbol{k}_1)\, \Gamma(\boldsymbol{k}_1, t_1) \left\langle e^{i\boldsymbol{k}_1 \cdot \boldsymbol{r}_1} \right\rangle .$$

Here, $P_{lm}(\boldsymbol{k}_1)$ denotes the components of the magnetostatic correlation tensor, $\Gamma(\boldsymbol{k}_1, t_1)$ the dynamical correlation function and $\left\langle e^{i\boldsymbol{k}_1 \cdot \boldsymbol{r}_1} \right\rangle$ the so called characteristic function. As an example, for magnetostatic turbulence, the dynamical correlation function is $\Gamma(\boldsymbol{k}_1, t_1) = 1$.

Equations of motion

- gyrocenters of charged particles follow magnetic field lines.

$$\tilde{v}_x(t) = v_z(t)\frac{\delta B_x(t)}{B_0} \qquad \tilde{v}_y(t) = v_z(t)\frac{\delta B_y(t)}{B_0}$$

Equations of motion

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$$\tilde{v}_x(t) = v_z(t)\frac{\delta B_x(t)}{B_0} \qquad \tilde{v}_y(t) = v_z(t)\frac{\delta B_y(t)}{B_0}$$

- Newton-Lorentz equation

$$\begin{aligned} v_x(\xi) =& v_\perp \cos(\phi_0 - \Omega\xi) \\ &+ \frac{\Omega}{B_0}\int_0^\xi dt\ v_z(t)\delta B_x(t)\sin\left[\Omega(\xi - t)\right] \\ &- \frac{\Omega}{B_0}\int_0^\xi dt\ v_z(t)\delta B_y(t)\cos\left[\Omega(\xi - t)\right] \\ v_y(\xi) =& v_\perp \sin(\phi_0 - \Omega\xi) \\ &+ \frac{\Omega}{B_0}\int_0^\xi dt\ v_z(t)\delta B_x(t)\cos\left[\Omega(\xi - t)\right] \\ &+ \frac{\Omega}{B_0}\int_0^\xi dt\ v_z(t)\delta B_y(t)\sin\left[\Omega(\xi - t)\right] \end{aligned}$$

Higher order correlation functions

4th order correlation function

- general expression

$$C_{ij}(t_1, t_2) = \langle\, v_z(t_1)\delta B_i(t_1) v_z(t_2)\delta B_j(t_2)\,\rangle$$

- assumption that the particle velocities are uncorrelated with the local magnetic field vector

$$C_{ij}(t_1, t_2) \approx \langle\, v_z(t_1) v_z(t_2)\,\rangle \langle\, \delta B_i(t_1)\delta B_j(t_2)\,\rangle$$

- Fourier transformation of turbulent fields

$$C_{ij}(t_1, t_2) = \langle\, v_z(t_1) v_z(t_2)\,\rangle \int d^3k \; P_{ij}(\vec{k}) \left\langle\, e^{i\vec{k}\cdot[\vec{r}(t_1)-\vec{r}(t_2)]}\,\right\rangle$$

where $P_{ij}(\vec{k}) = < \delta B_i(\vec{k})\delta B_j^*(\vec{k}) >$ magnetostatic correlation tensor

Higher order correlation functions

Transport Theories - a selection

- Quasilinear Theory

$$C_{ij}^{QLT}(t_1, t_2) = v^2\mu^2 \int d^3k \; P_{ij}^s(k_\parallel) \cos\left[k_\parallel v \mu (t_1 - t_2)\right]$$

Higher order correlation functions

Transport Theories - a selection

- Quasilinear Theory

$$C_{ij}^{QLT}(t_1, t_2) = v^2\mu^2 \int d^3k \; P_{ij}^s(k_\parallel) \cos\left[k_\parallel v\mu(t_1 - t_2)\right]$$

- nonlinear guiding center theory with $\alpha(\vec{k}) = \gamma(\vec{k}) + v/\lambda_\parallel + \sum_{n,m} \kappa_{nm} k_n k_m$

$$C_{ij}^{NL}(t_1, t_2) = \frac{v^2}{3} \int d^3k \; P_{ij}(\vec{k}) e^{-\alpha(\vec{k})|t_1 - t_2|}$$

Higher order correlation functions

Transport Theories - a selection

- Quasilinear Theory

$$C_{ij}^{QLT}(t_1, t_2) = v^2\mu^2 \int d^3k \; P_{ij}^s(k_\parallel) \cos\left[k_\parallel v\mu(t_1 - t_2)\right]$$

- nonlinear guiding center theory with $\alpha(\vec{k}) = \gamma(\vec{k}) + v/\lambda_\parallel + \sum_{n,m} \kappa_{nm} k_n k_m$

$$C_{ij}^{NL}(t_1, t_2) = \frac{v^2}{3} \int d^3k \; P_{ij}(\vec{k}) e^{-\alpha(\vec{k})|t_1 - t_2|}$$

- distinguish between particle and field properties

$$C_{ij}^{FT}(t_1, t_2) = \frac{v^2}{3} \int d^3k \; P_{ij}(\vec{k}) \left[\frac{\omega_+}{\omega_+ - \omega_-} e^{(\omega_+ - \rho)\tau} - \frac{\omega_-}{\omega_+ - \omega_-} e^{(\omega_- - \rho)\tau}\right]$$

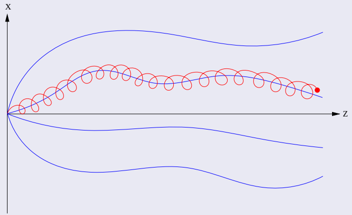
with $\tau = |t_1 - t_2|$, $\omega_\pm = -D \pm \sqrt{D^2 - (vk_\parallel)^2/3}$, D is a constant and $\rho = \sum_{i,j=x,y} k_i k_j \kappa_{ij}$.

Higher order correlation functions

Guiding center motion

Field Line Random Walk Limit

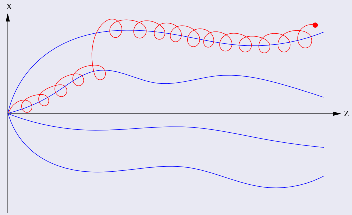
$$\lambda_\perp = \lambda_\perp^{FLRW}$$



Newton-Lorentz equation

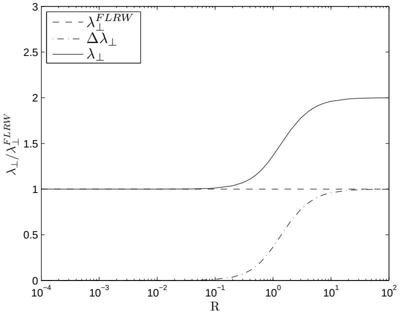
Contribution for higher energies

$$\lambda_\perp = \lambda_\perp^{FLRW} + \Delta\lambda_\perp(R)$$



Higher order correlation functions

QLT



NLGC - theory