Cross-Field Transport

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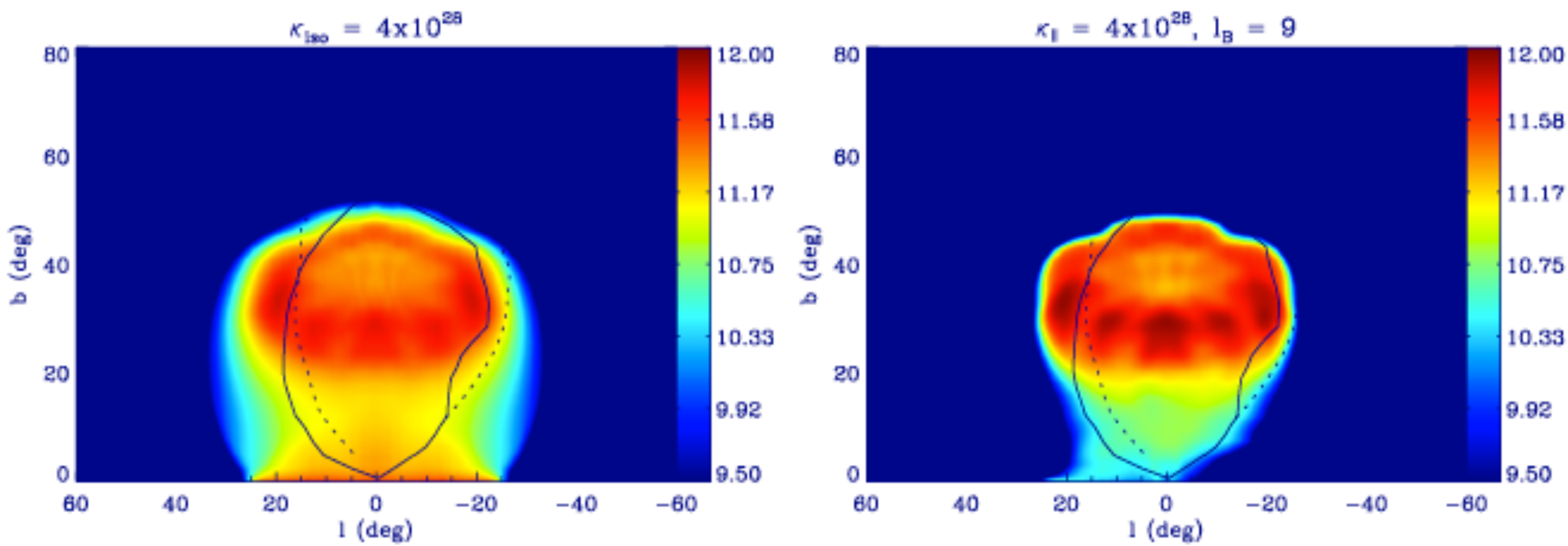
How Do Cosmic Rays Cross Magnetic Fieldlines?

- The exact field, not the mean field
  - Propagation in the Galaxy
  - Confinement in discrete structures



Yang et al. 2012

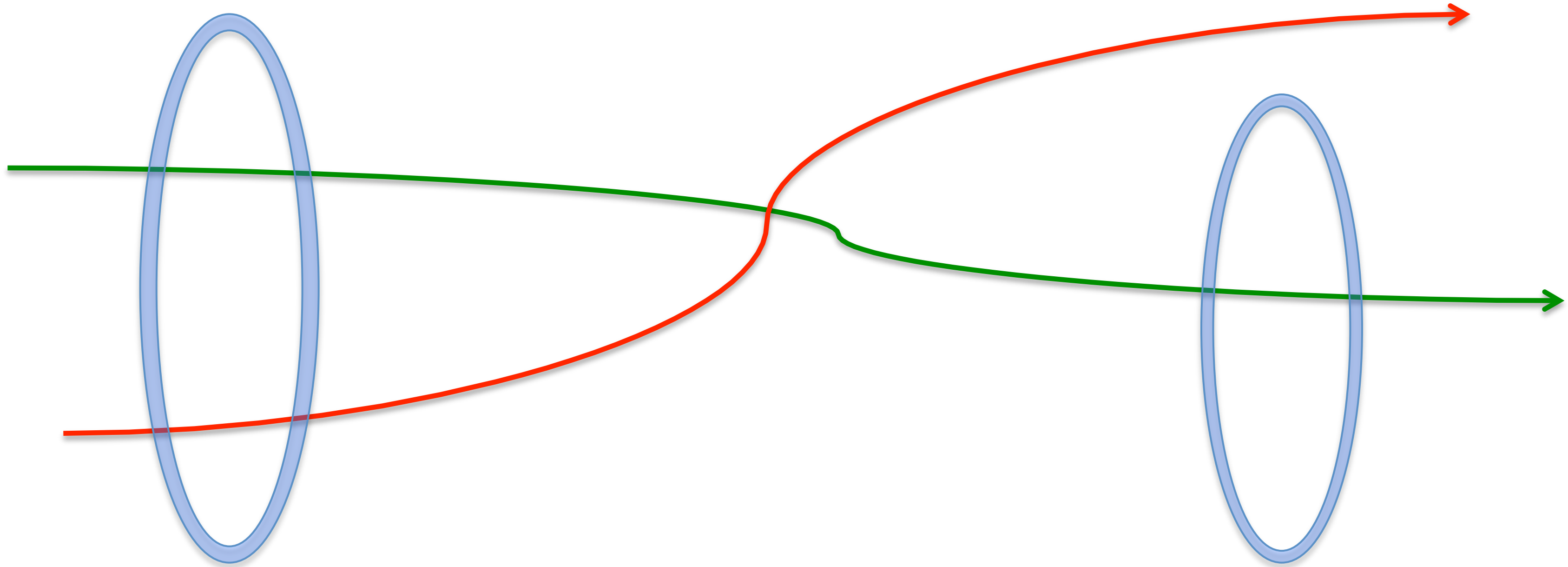
Mechanisms for Crossing Fieldlines

- Drifts due to curvature, gradients: $v_D \sim v \frac{r_g}{L_B}$
  - *Slow for large scale structure in B.*
- Gyroresonant pitch angle scattering:

$$\nu \equiv \frac{\pi}{4} \omega_{cr} k \frac{\delta B_k^2}{B^2}$$

  - *Maximum $D_{perp} \sim c r_L$; typically also slow.*

Mechanisms for Fieldline Crossing:



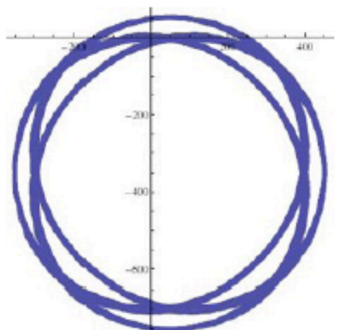
Sub-Gyroscale Magnetic Structure

Fieldline Geometry + Scattering?

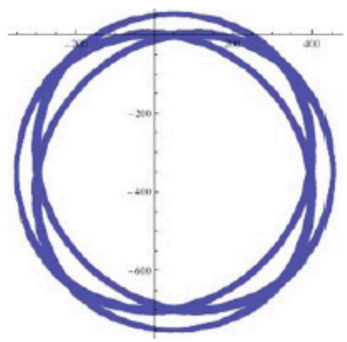
Collaboration with Paolo Desiati

- Orbits in simple model fields
  - Our numerics
  - Basic mechanisms for fieldline crossing
- Derivation of a diffusion coefficient
- Bounds on diffusivity
- Future work

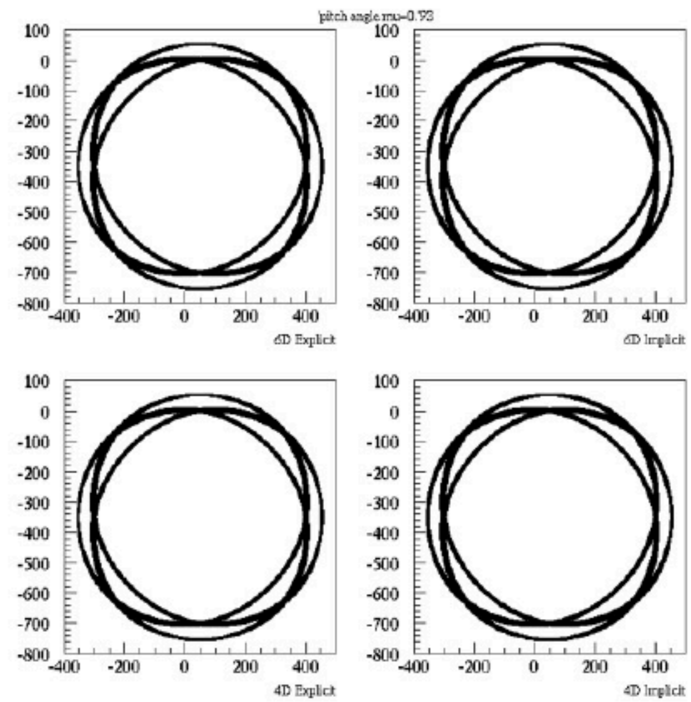
Numerical Tests



6D - mathematica



4D - mathematica



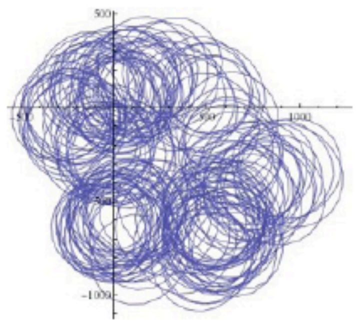
$p = 1000$ $\mu = 0.93$

$\varepsilon = 0.1$ $L = 200$

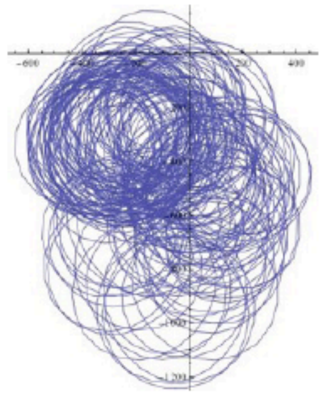
$t = 10^6$

Six schemes, one result

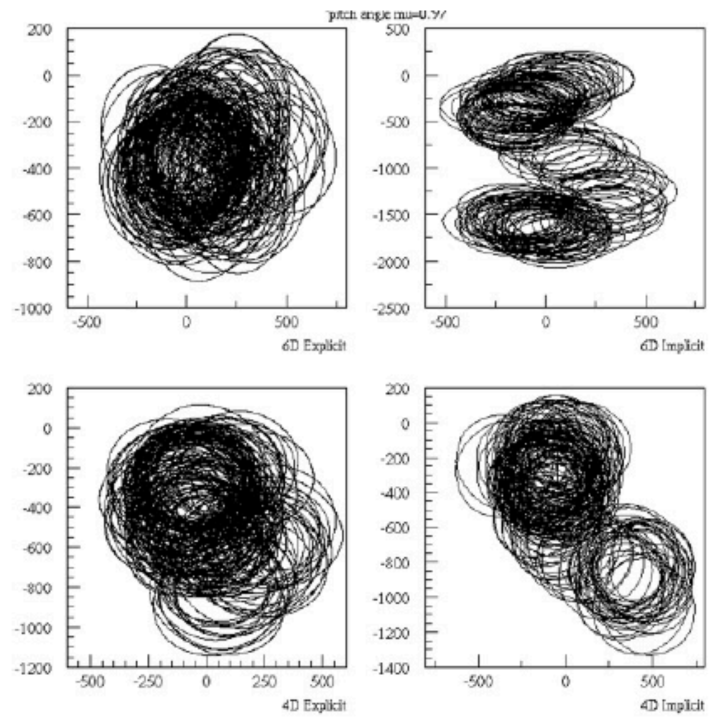
Numerical Tests



6D - mathematica



4D - mathematica



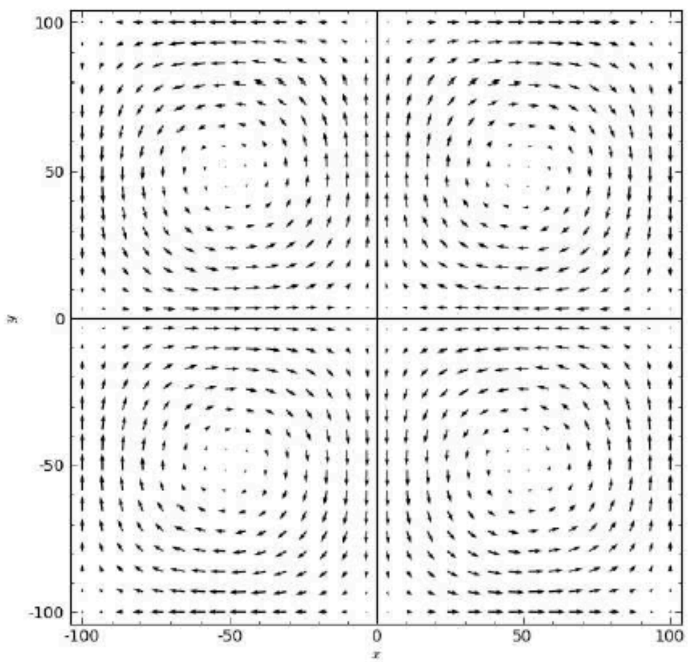
$p = 1000$ $\mu = 0.97$

$\varepsilon = 0.1$ $L = 200$

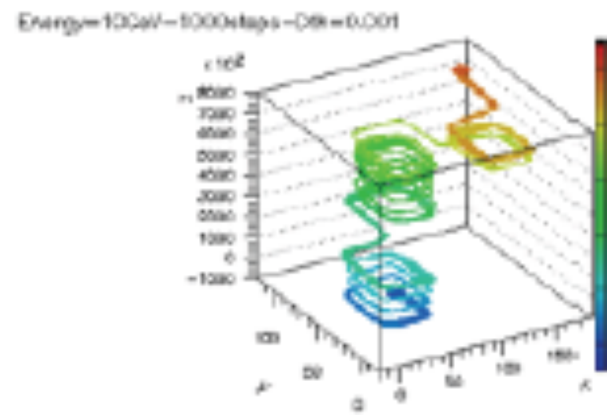
$t = 10^6$

6 schemes agree on whether orbits are chaotic

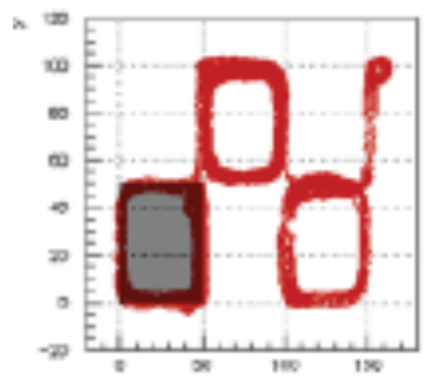
Pitch Angle Scattering -> Hopping Between Cells



Projection of an array of helices; dominant component out of the plane.

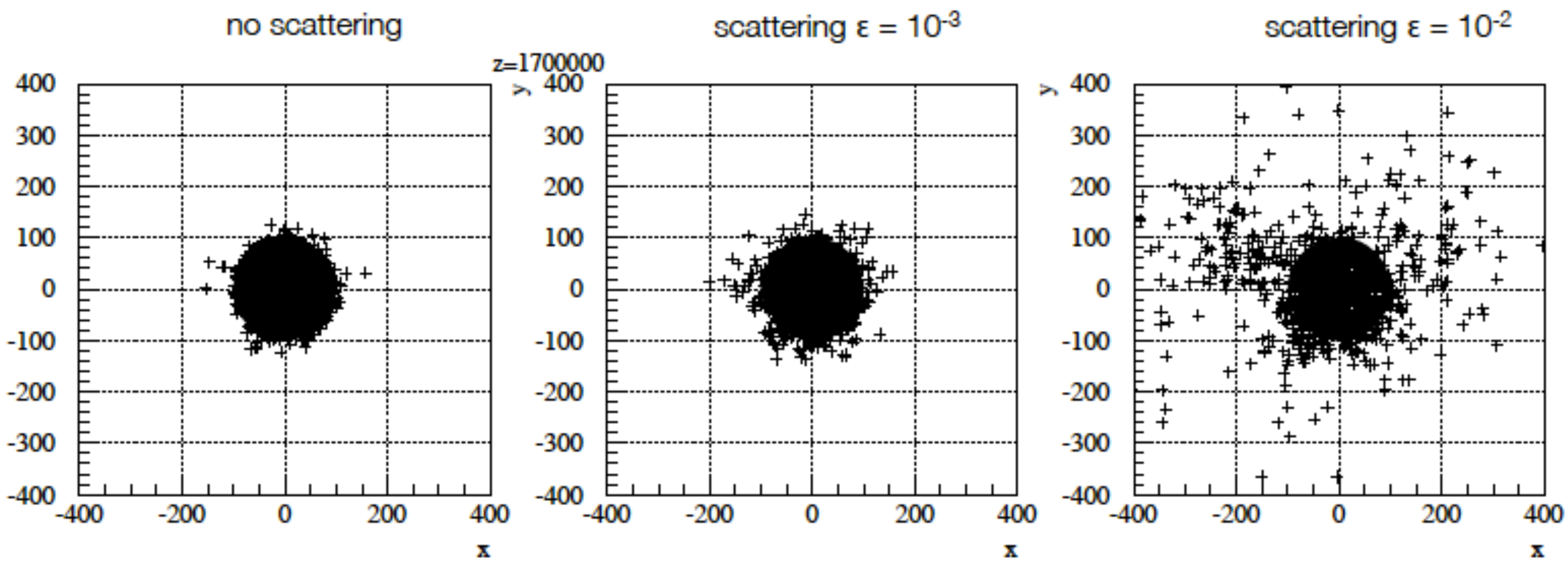


3D View



2D View

Spreading of an Upward-Going Patch



ε is the relative strength of the transverse field

Cross-Field Diffusion Coefficient

Measure diffusion for N_p particles:

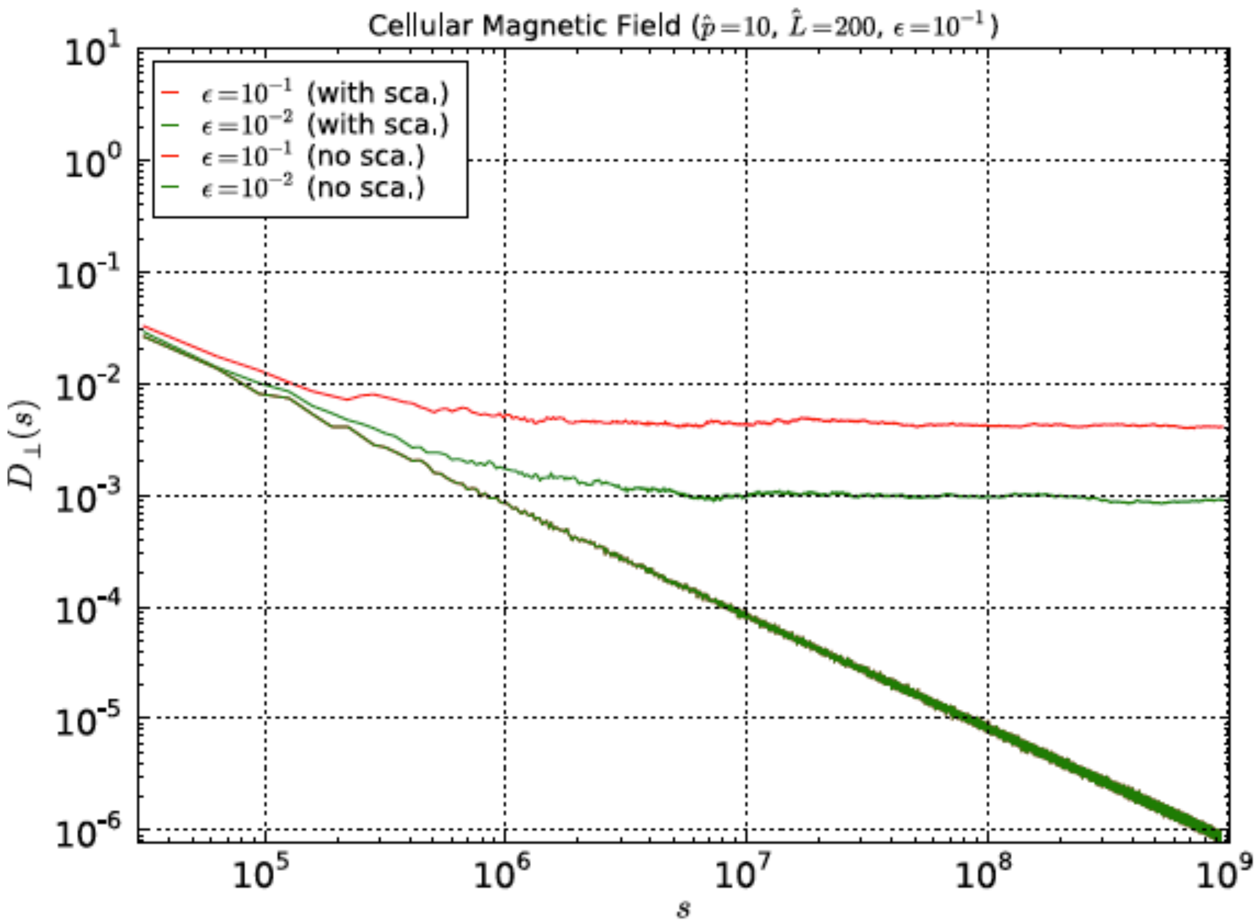
running diffusion coefficient

$$D_{ij}(t) = \frac{1}{N_p} \sum_{n=0}^{N_p} \frac{(x_i(t) - x_i(t_0)) \cdot (x_j(t) - x_j(t_0))}{t - t_0}$$

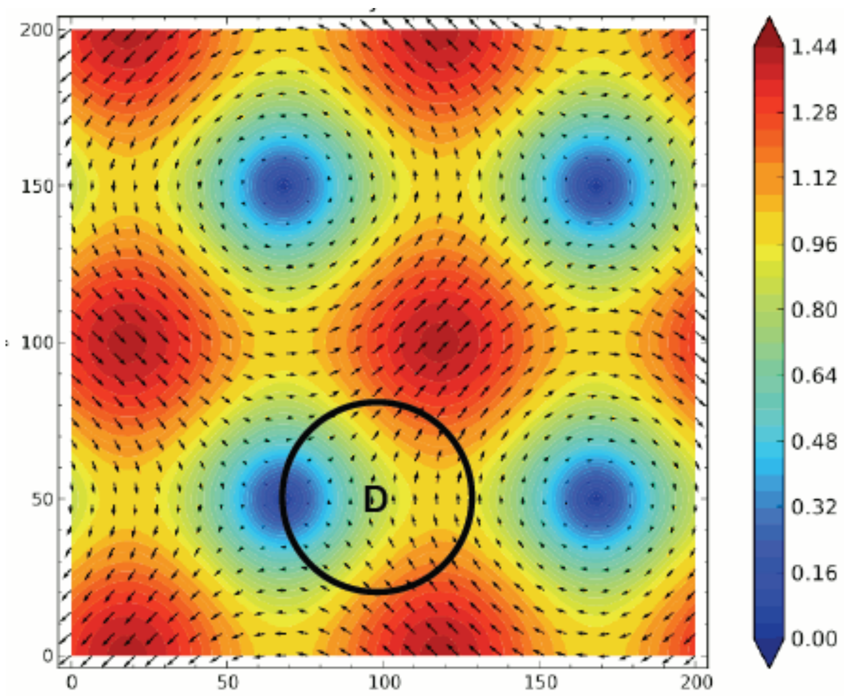
Without transport, D ~ t^{-1}

For standard (random walk type) diffusion D levels off.

Diffusion Across the Cellular Field

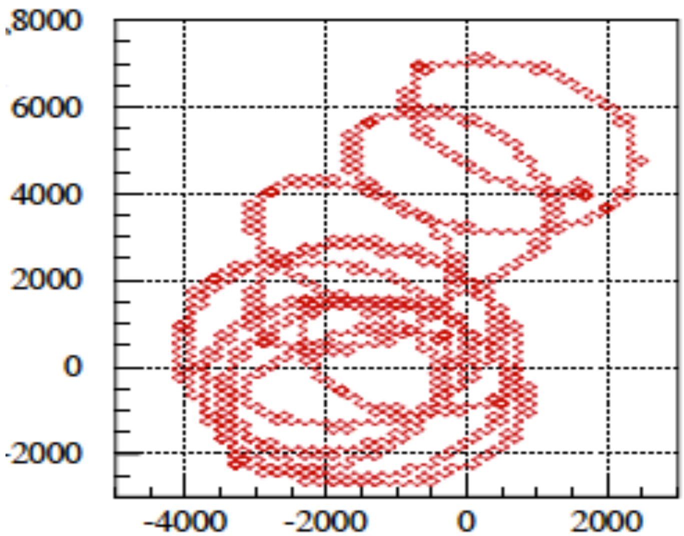


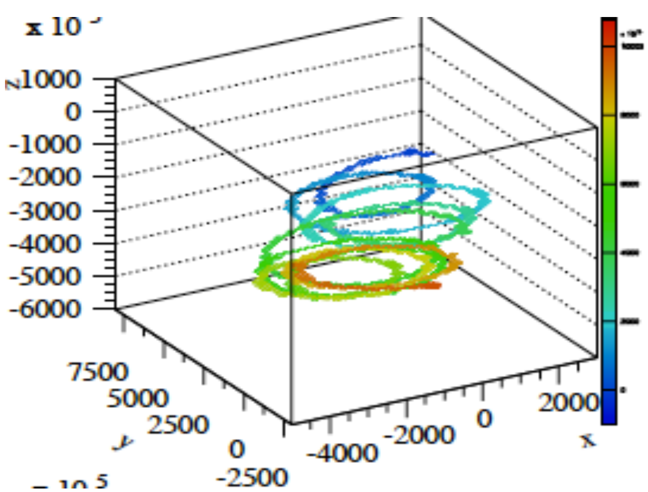
A Time Dependent Field



- Magnetic analog of Galloway-Proctor flow in dynamo theory, plus a large out of plane field.
- Cells oscillate back and forth with amplitude shown in circle & frequency $\omega=2\pi/\tau$.

Transport Without Scattering



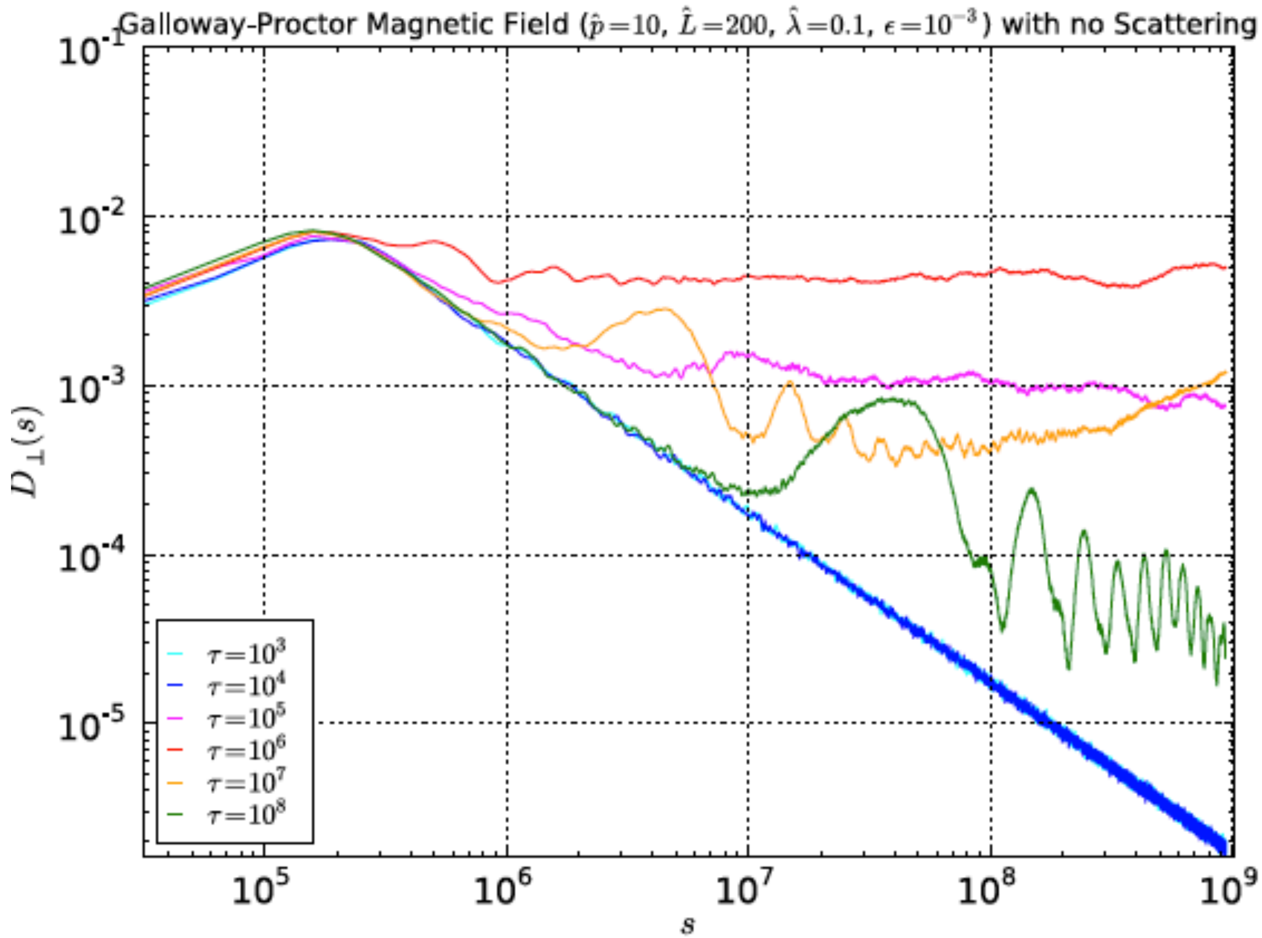


Even without scattering, there is cross-field transport.

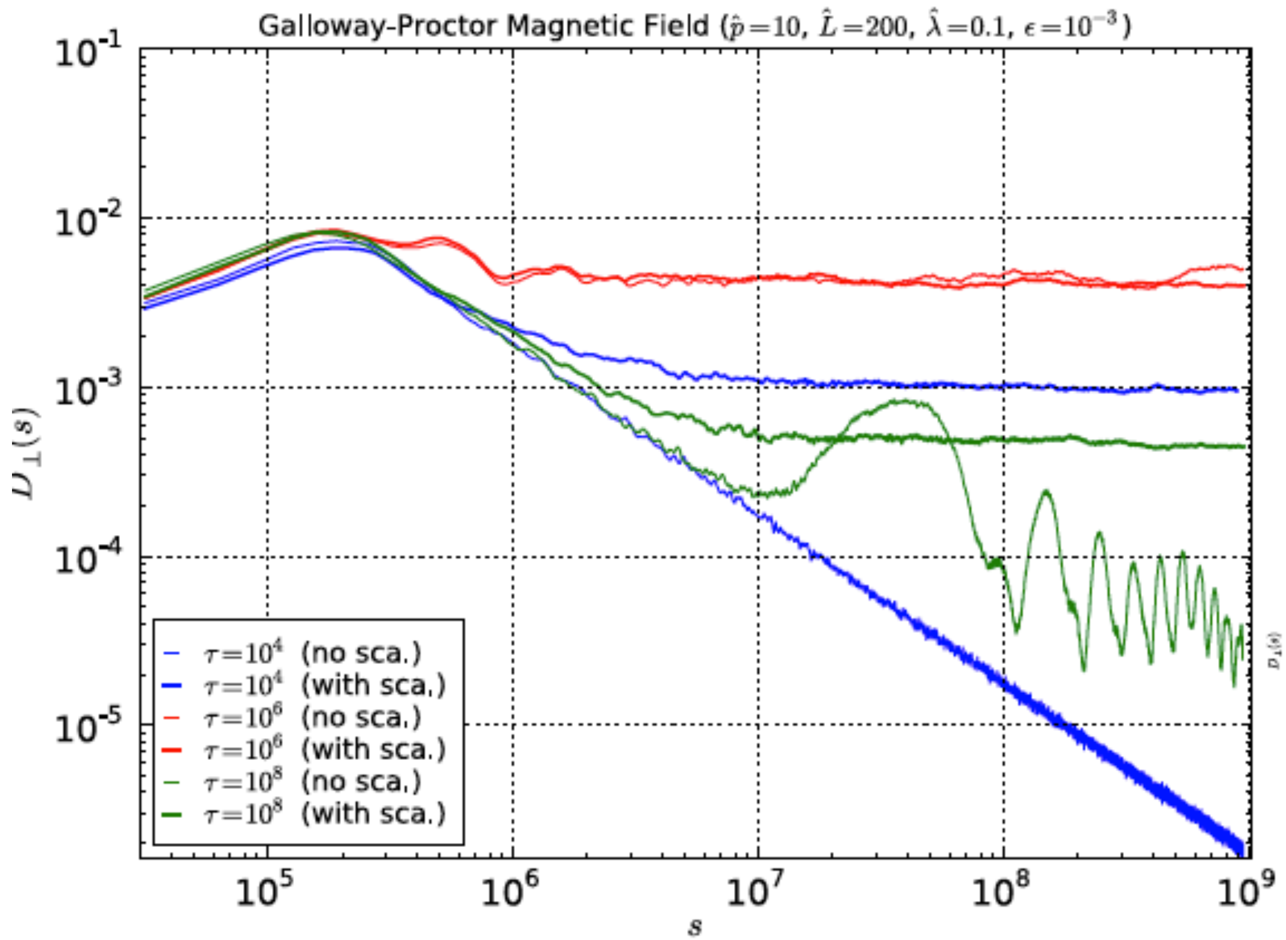
We attribute this to chaos in the field.

Let's compute the diffusion.

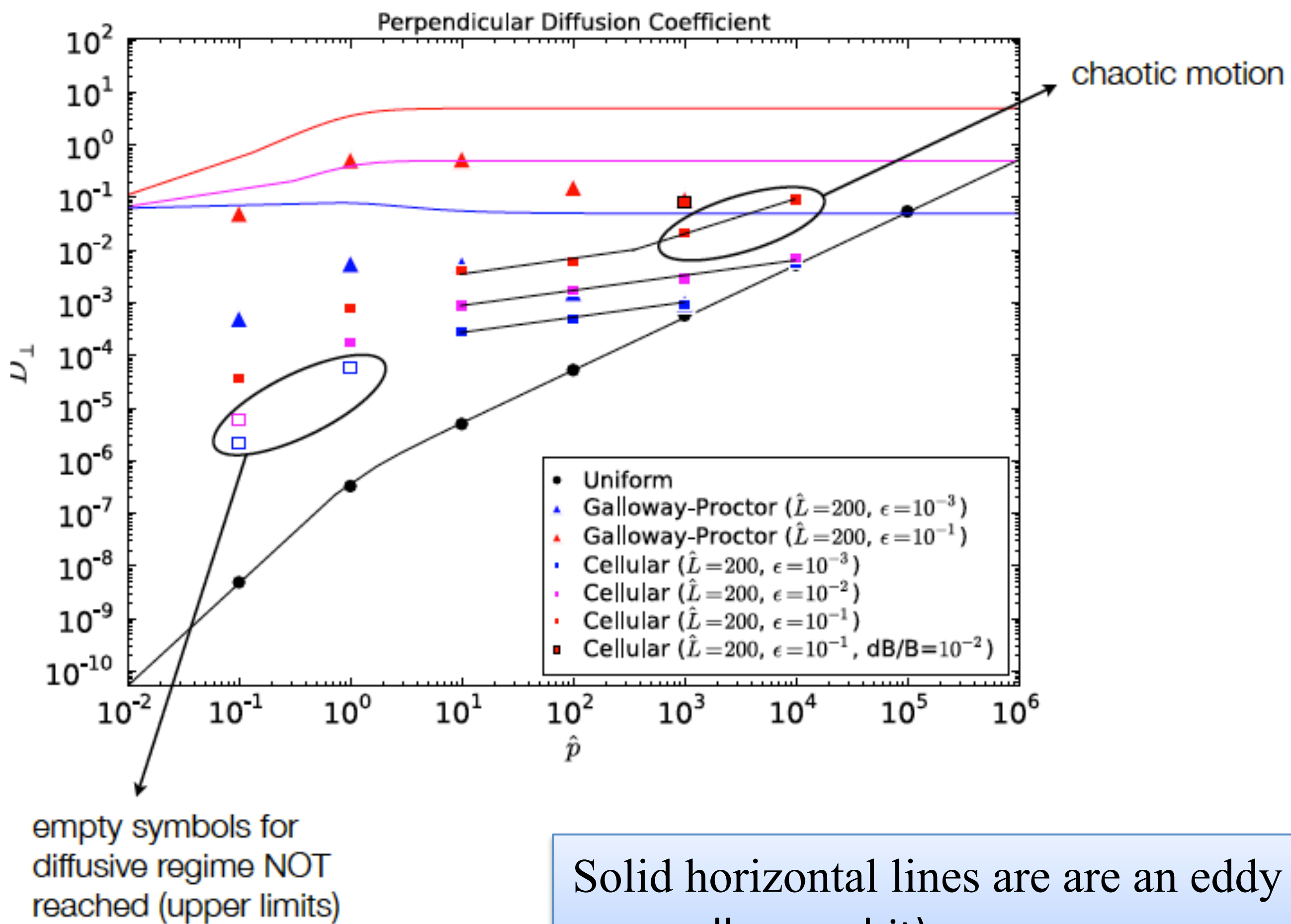
Diffusion is Maximized for $t_{orb} \sim \tau$



Scattering Makes Little Difference

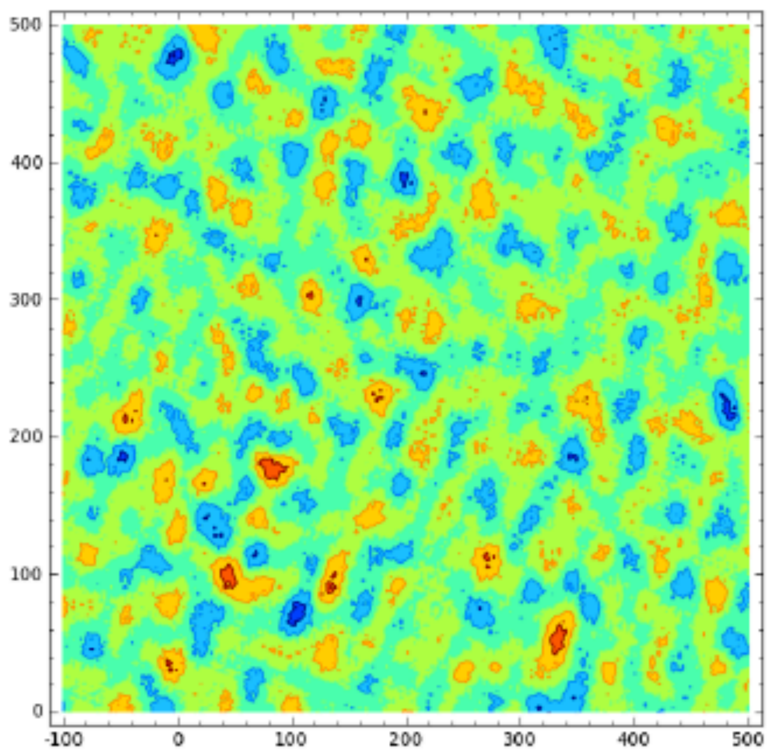


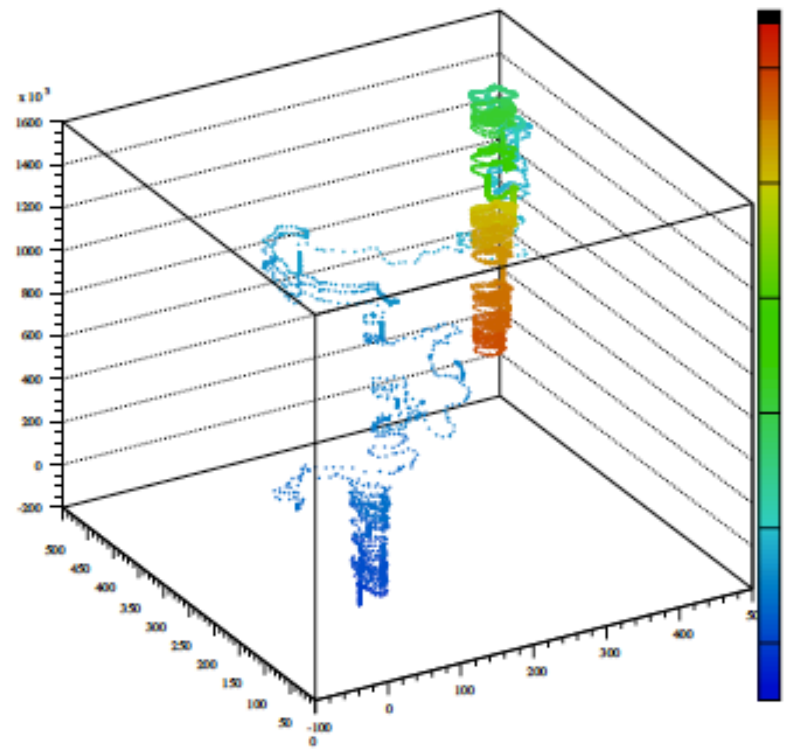
Diffusion Summary



Solid horizontal lines are are an eddy model (hop one cell per orbit).

Next – More Complex Structure





Summary

- Cross-field transport is a key issue in cosmic ray propagation.
  - Drifts driven by magnetic structure
  - Pitch angle scattering in simple magnetic structure

    Slow, not diffusive
  - Pitch angle scattering plus magnetic structure can be diffusive.
  - "Orbital" resonance in a time dependent field can produce diffusion even when structure is large scale.
- Seeking scaling laws & generalizations.