Aperiodic magnetic field fluctuations and their effect on cosmic rays

Reinhard Schlickeiser
Institut für Theoretische Physik
Lehrstuhl IV: Weltraum- und Astrophysik
Ruhr-Universität Bochum, Germany

September 27, 2013

Topics:

1. Electromagnetic acceleration and transport
2. Plasma fluctuations and kinetic description
3. Cosmic ray transport theory
4. Diffusion approximation and cosmic ray anisotropy
5. Summary and conclusions

References:

Spontaneous electromagnetic fluctuations in unmagnetized plasmas II: Relativistic form factors of aperiodic thermal modes; T. Felten, RS, P. H. Yoon, M. Lazar, 2013, Physics of Plasmas 20, 052113

Strength of the spontaneously emitted collective aperiodic magnetic field fluctuations in the reionized early intergalactic medium; RS and T. Felten, 2013, ApJ, submitted

Cosmic ray acceleration and transport in chaotic electromagnetic fields; S. Krakau and RS, 2013, in preparation

1. Electromagnetic acceleration and transport

Any **electromagnetic** CR acceleration and transport processes in turbulent electromagnetic fields $\vec{E} = \vec{0} + \delta\vec{E}$ with no ordered electric field because of huge cosmic conductivities (exceptions: pulsars, magnetic reconnection)), $\vec{B} = \vec{B}_0 + \delta\vec{B}$ orders CRs by their rigidity $R = p/q$

Lorentz force:

$$\frac{d\vec{p}}{dt} = q\left[\delta\vec{E} + \frac{\vec{v}\times(\vec{B}_0 + \delta\vec{B})}{c}\right]$$

Acceleration requires turbulent electric fields:

$$\frac{dE_{\rm kin}}{dt} = \frac{c^2}{2E_{\rm kin}}\frac{dp^2}{dt} = \frac{c^2}{E_{\rm kin}}q\vec{p}\cdot\delta\vec{E}$$

with $E_{\rm kin} = \sqrt{p^2c^2 + m^2c^4} - mc^2$.

Equal acceleration rates for charged particles at the same magnetic rigidity $R = p/q$:

$$\frac{dp^2}{dt} = 2q\vec{p}\cdot\delta\vec{E} \quad\rightarrow\quad \frac{dR^2}{dt} = 2\vec{R}\cdot\delta\vec{E}$$

The rigidity ordering successfully explains 100 times more hadrons than electrons at relativistic energies $E_{\rm kin} \gg m_pc^2 = 1$ GeV because $m_p = 1836m_e$

Assume: electrons and protons are accelerated to the same power law spectrum for the differential number density $N_e(p) = N_{0,e}p^{-s}$, $N_p(p) = N_{0,p}p^{-s}$ in momentum(=rigidity here) above the nonrelativistic kinetic energy $T_0 = 10$ keV, and equal number density

$$n_0 = \int_{T_0}^{\infty} dE_{\rm kin} N_e(E_{\rm kin}) = \int_{T_0}^{\infty} dE_{\rm kin} N_p(E_{\rm kin})$$

We find

$$N(E_{\rm kin}) = N[p(E_{\rm kin})]\frac{dp}{dE_{\rm kin}} = \frac{n_0}{c^2}(E_{\rm kin} + mc^2)\left[\frac{E_{\rm kin}^2}{c^2} + 2E_{\rm kin}m\right]^{-(s+1)/2}$$

and for the electron-proton ratio in the limit $T_0 \ll m_e c^2$

$$\frac{N_e(E_{\rm kin})}{N_p(E_{\rm kin})} = \left(\frac{m_e}{m_p}\right)^{(s-1)/2} \frac{E_{\rm kin} + m_e c^2}{E_{\rm kin} + m_p c^2}\left[\frac{E_{\rm kin} + 2m_e c^2}{E_{\rm kin} + 2m_p c^2}\right]^{-(s+1)/2},$$

which approaches at relativistic energies $E_{\rm kin} \gg m_p c^2$ =1 GeV the constant

$$\frac{N_e(E_{\rm kin} \gg m_p c^2)}{N_p(E_{\rm kin} \gg m_p c^2)} \simeq \left(\frac{m_e}{m_p}\right)^{(q-1)/2} = 0.011$$

for $s = 2.2$. Most electrons sit at $T_0 \le E_{\rm kin} \le m_e c^2$.

To a large extent, our progress in understanding CR dynamics in cosmic plasmas depends on our understanding of the magnetic and electric field fluctuations

Here we address two important issues:

- Nature of cosmic electromagnetic fluctuations in magnetized (e.g. ISM) and nonmagnetized (intergalactic medium (IGM)) cosmic plasmas
- CR anisotropy and parallel mean free path in magnetized and nonmagnetized plasmas

2. Plasma fluctuations and kinetic description

All plasmas, including unmagnetized and those in thermal equilibrium, have fluctuations. Because of the large sizes of astrophysical systems compared to the plasma Debye length, the fluctuations are descibed by real wave vectors ($\vec{k}$) and complex frequencies $\omega(\vec{k}) = \omega_R(\vec{k}) + \imath\gamma(\vec{k})$, implying for the space- and time-dependence of e.g. magnetic fluctuations the superposition of

$$\delta\vec{B}(\vec{x}, t) \propto \exp[\imath(\vec{k} \cdot \vec{x} - \omega_R t) + \gamma t] \quad (1)$$

One distinguishes between

- **collective** modes with a fixed dispersion relation $\omega = \omega(\vec{k})$, e.g. electromagnetic waves in vacuum $\omega_R^2 = c^2k^2$ and $\gamma = 0$,
- **non-collective** modes with no dispersion relation $\omega = \omega(\vec{k})$,

and, regarding the real (ω_R) and imaginary (γ) part of the frequency,

- **weakly damped/amplified** wave-like modes with $|\gamma| \ll |\omega_R|$, e.g. collective Alfven and magnetosonic waves,
- **weakly propagating** modes with $|\omega_R| \ll |\gamma|$, e.g. collective mirror und firehose fluctuations,
- **aperiodic** modes with $\omega_R = 0$ fluctuate only in space, do not propagate as $\omega_R = 0$, but permanently grow or decrease in time depending on the sign of γ, e.g. collective Weibel fluctuations. And $|\delta B|^2 \gg |\delta E|^2$!

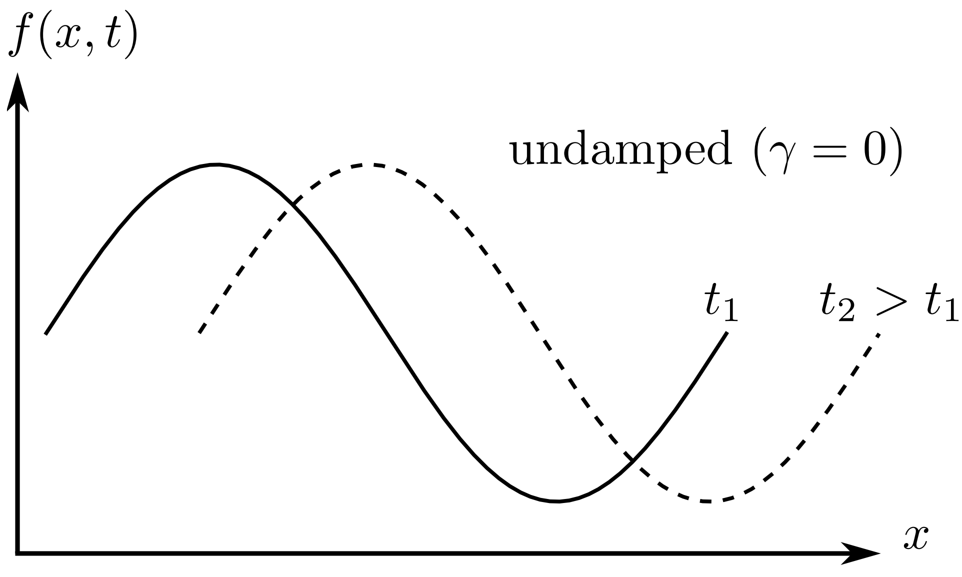

Figure 1: Sketch of undamped ($\gamma = 0$) plasma wave ($\omega_R \neq 0$).

2.1. Spontaneous emission

Because of their comparably low gas densities, all cosmic fully and partially ionized non-stellar plasmas are collision-poor, as indicated by the very small values of the plasma parameter

$$g = \nu_{ee}/\omega_{p,e} = 7.3 \cdot 10^{-4} \left(n_e/\mathrm{cm}^{-3}\right)^{1/2} \left(T_e/\mathrm{K}\right)^{-3/2} \leq \mathcal{O}\left(10^{-10}\right),$$

given by the ratio of the electron-electron Coulomb collision frequency ν_{ee} to the electron plasma frequency $\omega_{p,e}$, characterizing interactions with electromagnetic fields, so that fully kinetic plasma descriptions are necessary.

Unlike for weakly amplified/damped modes (see Salpeter 1960, Sitenko 1967, Ichimaru 1973, Kegel 1998), however, for aperiodic and weakly propagating fluctuations the expected fluctuation level of spontaneously emitted fluctuations has never been calculated quantitatively. Only recently general expressions for the electromagnetic fluctuation spectra (electric and magnetic field, charge and current densities) from uncorrelated plasma particles in unmagnetized plasmas for arbitrary frequencies have been derived (RS and Yoon 2012, Felten et al. 2013) using the system of the Klimontovich and Maxwell equations, which are appropriate for fluctuations wavelengths longer than the mean distance between plasma particles, i.e. $k \leq k_{\max} = 2\pi n_e^{1/3}$.

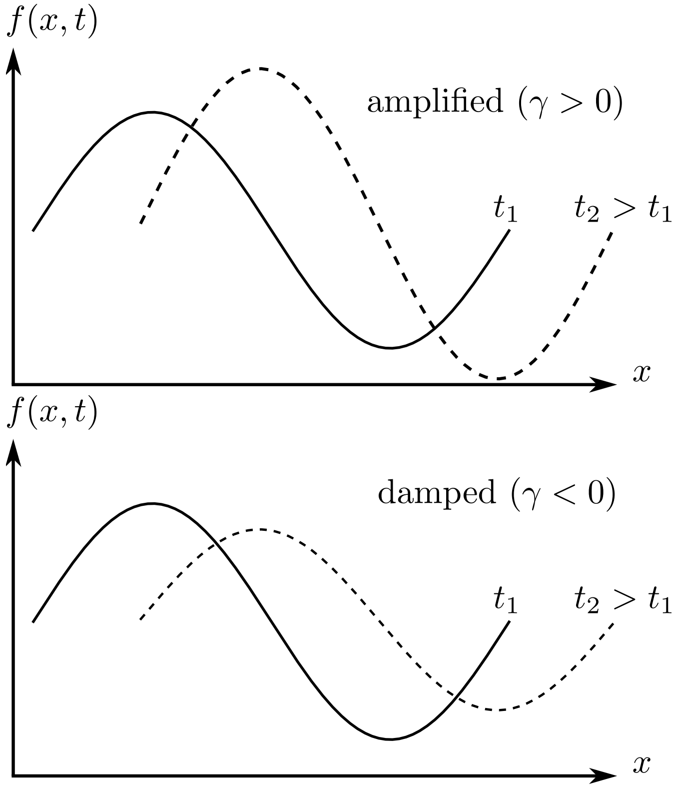

Figure 2: Sketch of weakly amplified ($\gamma > 0$, top) and weakly damped ($\gamma < 0$, bottom) plasma wave ($\omega_R \gg |\gamma|$).

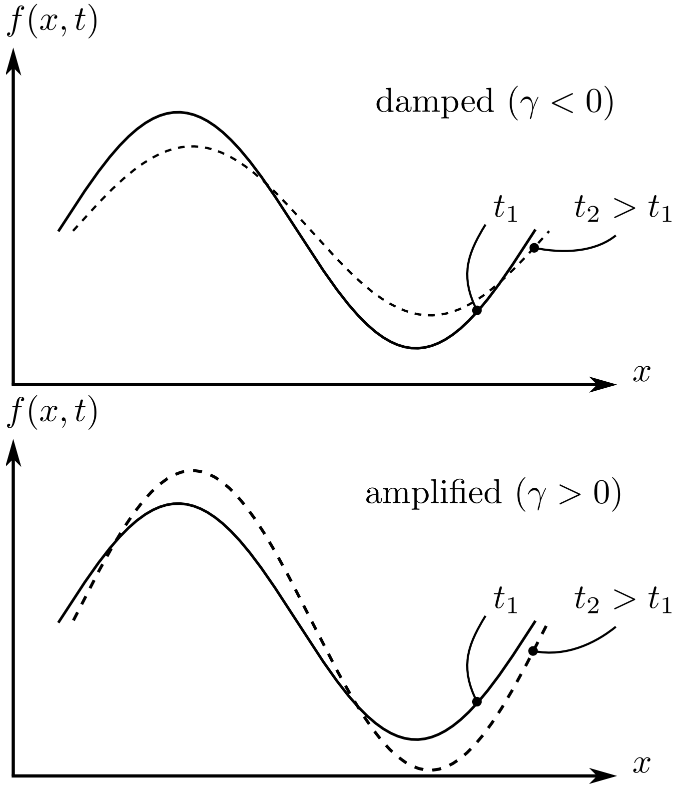

Figure 3: Sketch of weakly propagating damped ($\gamma < 0$, top) and weakly propagating amplified ($\gamma < 0$, bottom) fluctuation ($\omega_R \ll |\gamma|$).

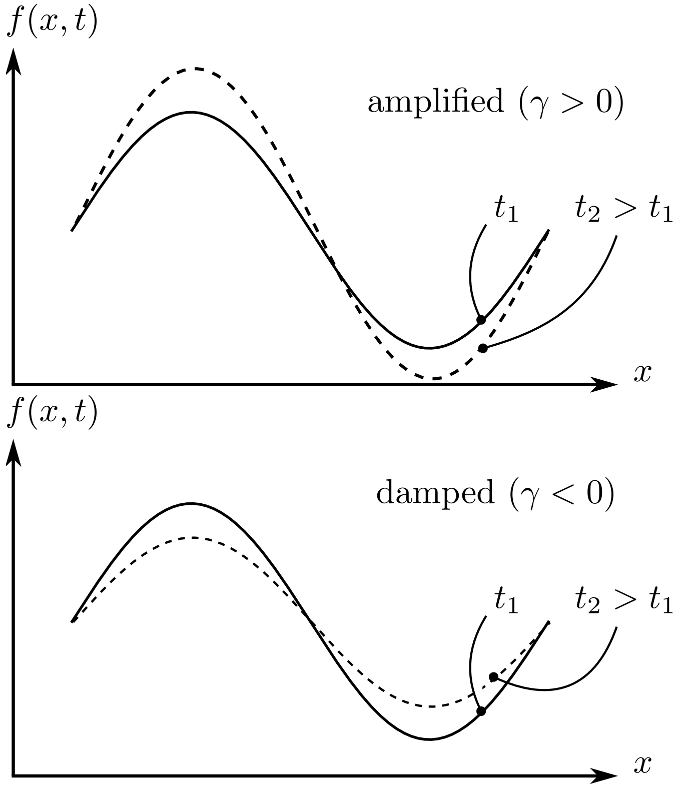

Figure 4: Sketch of aperiodic ($\omega_R = 0$) growing ($\gamma > 0$, top) and aperiodic damped ($\gamma < 0$, bottom)fluctuation.

In Fig. 5 we show the resulting magnetic fluctuation spectrum for the unmagnetized, fully-ionized IGM immediately after the reionization onset. The bright red ribbon in this figure at negative imaginary frequencies $\gamma < 0$ clearly indicates the existence of a new collective, transverse, damped aperiodic mode $\gamma_0(k) < 0$, resulting from the solution of the dispersion relation $\Lambda_T(k, \gamma_0) = 0$. The highest fluctuation intensities occur near this collective mode.

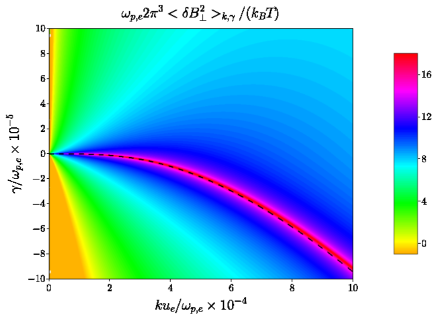

Figure 5: Contour plot of the spontaneously emitted aperiodic magnetic field fluctuation spectrum in the thermal nonrelativistic electron-proton IGM plasma distribution in units of $k_B T_e/(2\pi^3 \omega_{p,e})$. Equal electron and proton temperatures ($T_i = T_e = 10^4$ K) are adopted. The colour scale is logarithmic in powers of e. From Schlickeiser and Felten (2013)

By integrating over all values of γ, RS and Felten (2013) derived the thermal wavenumber spectra of spontaneously emitted aperiodic fluctuations shown in Fig. 6.

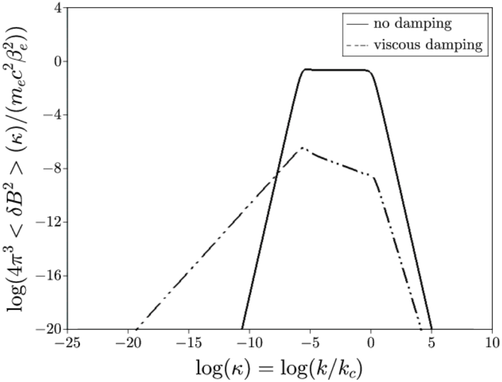

Figure 6: Thermal wavenumber spectra in the case of no damping (full curve) and in the case of viscous damping (dot-dashed curve) calculated for IGM cosmic void parameters $T_4 = 1$ and $n_{-7} = 1$. From Schlickeiser and Felten (2013)

For the case of competing viscous damping, the wavenumber spectrum at large turbulence spatial scales rises only linearly $< \delta B^2 >_{\rm eq} (\kappa < \kappa_1) \propto \kappa$. The dominant contribution arises from small turbulence scales $\lambda < \lambda_2$, corresponding to $\kappa > \kappa_2 = 1$. In this wavenumber range the wavenumber spectrum $< \delta B^2 >_{\rm eq} (\kappa > \kappa_2) \propto \kappa^{-3}$. Maximum spatial scales of 10^{15} cm of the emitted aperiodic fluctuations in cosmic voids are possible.

By integrating over all wavenumbers the total magnetic field strength of spontaneously emitted aperiodic fluctuations is

$$|\delta B| = \left(< \delta B^2 >\right)^{1/2} = 2305 g (n_e m_e c^2)^{1/2} = 1.5 \cdot 10^{-16} n_{-7} T_4^{-3/2} \text{ G}$$

These guaranteed magnetic fields in the form of randomly distributed fluctuations, produced by the spontaneous emission of the isotropic thermal IGM or ISM plasma, serve as seed fields for possible amplification by later possible plasma instabilities from anisotropic plasma particle distribution functions (RS, Ibscher ans Supsar 2012), MHD instabilities and/or the MHD dynamo process. Neither the dynamo process nor plasma instabilities generate magnetic fields without such seed fields.

3. CR transport theory

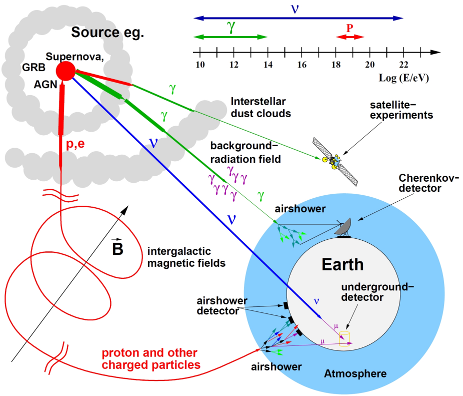

Figure 7: Sketch of cosmic ray life. Courtesy R. Wagner.

Fig. 7 sketches the typical life of a cosmic ray particle: after being accelerated in individual sources such like supernova remnants, active galactic nuclei or gamma-ray bursts, it stochastically propagates in the partially turbulent magnetic field and interacts with the ambient photon and matter fields, generating nonthermal photon and neutrino radiation. In the case of galactic CRs this takes about 10^7 years before detection by near-Earth or ground based detectors.

3.1. Fokker-Planck transport equation

The Fokker-Planck particle transport equation for the CR particle phase space density holds if the following seven physical assumptions about the fluctuating electromagnetic field turbulence are made:

- (1) Gaussian statistics of fluctuations,
- (2) adiabatic approximation,
- (3) homogeneous and quasi-stationary turbulence,
- (4) existence of a small enough finite decorrelation time of second-order correlation functions,
- (5) random phase (between particles and fluctuations),
- (6) in magnetized systems $B_0 \gg \delta B$; in nonmagnetized systems aperiodic fluctuations $\delta B \gg \delta E$
- (7) in magnetized systems parallel flows with respect to $\vec{B}_0$.

3.2. Magnetized systems (ISM)

In a moving medium of arbitrary speed $U \parallel \vec{B}_0 \parallel \vec{e}_Z$ the Fokker-Planck equation reads with $\Gamma = [1-(U/c)^2]^{-1/2}$:

$$
\begin{aligned}
&\Gamma\left[1+\frac{Uv\mu}{c^2}\right]\left[\frac{\partial f_0}{\partial t}-\frac{1}{v}\frac{\partial U}{\partial t}\Gamma^2\left(\mu p\frac{\partial f_0}{\partial p}+(1-\mu^2)\frac{\partial f_0}{\partial \mu}\right)\right] \\
&+\Gamma\left[U+v\mu\right]\left[\frac{\partial f_0}{\partial Z}-\frac{1}{v}\frac{\partial U}{\partial z}\Gamma^2\left(\mu p\frac{\partial f_0}{\partial p}+(1-\mu^2)\frac{\partial f_0}{\partial \mu}\right)\right] \\
&+\mathcal{N}f_0+\mathcal{R}f_0-S(\vec{X},p,\mu,t)=p^{-2}\frac{\partial}{\partial x_\alpha}p^2 D_{\alpha\sigma}\frac{\partial f_0}{\partial x_\sigma}
\end{aligned}
\tag{2}
$$

The phase space coordinates have to be taken in the mixed comoving coordinate system (time and space coordinates $\vec{x}$ in the laboratory (=observer) system and particle's momentum coordinates p and $\mu = p_{\|}/p$ in the rest frame of the streaming plasma). In Eq. (2) we use the Einstein sum convention for indices, and $x_\alpha \in [\mu, p, X, Y]$ represent the four phase space variables with non-vanishing stochastic fields $\delta\vec{E}$ and $\delta\vec{B}$. Consequently, the term on the right-hand side represents 16 different Fokker-Planck coefficients: but, depending on the turbulent fields considered not all of them are non-zero and some are much larger than others. $S(\vec{X},p,\mu,t)$ represents additional sources and sinks of particles.

$\mathcal{N} f_0$ (RS and Jenko 2010) represents effects to the mirror force in large scale inhomogenous magnetic fields:

$$\mathcal{N} f_0 = \frac{v(1-\mu^2)}{2}\left[\frac{1}{L_3}\frac{\partial f_0}{\partial \mu} + \frac{sign(q_a)R_L}{L_2}\frac{\partial f_0}{\partial X} - \frac{sign(q_a)R_L}{L_1}\frac{\partial f_0}{\partial Y}\right]$$
$$= \frac{v(1-\mu^2)}{2}\left[\frac{1}{L_3}\frac{\partial f_0}{\partial \mu} + \partial_\perp f_0\right] \qquad (3)$$

with

$$\partial_\perp f_0 = sign(q_a)R_L\left(\frac{1}{L_2}\frac{\partial f_0}{\partial X} - \frac{1}{L_1}\frac{\partial f_0}{\partial Y}\right) = sign(q_a)\left(\frac{\partial R_L}{\partial Y}\frac{\partial f_0}{\partial X} - \frac{\partial R_L}{\partial X}\frac{\partial f_0}{\partial Y}\right),$$

where $L_1^{-1} = -\partial_x \ln B_0$, $L_2^{-1} = -\partial_y \ln B_0$, $L_3^{-1} = -\partial_z \ln B_0$ denote the large spatial gradients of the guide magnetic field B_0.

$\mathcal{R} f_0$ accounts for continuous ($\dot{p}_{\rm loss}$) and catastrophic (T_c) momentum loss processes of CR particles:

$$\mathcal{R} f = p^{-2}\partial_p\left[p^2 \dot{p}_{\rm loss} f\right] + \frac{f}{T_c} \qquad (4)$$

In a medium at rest the Fokker-Planck transport equation (2) reduces to

$$\frac{\partial f_0}{\partial t} + v\mu\frac{\partial f_0}{\partial Z} + \mathcal{N}f_0 + \mathcal{R}f_0 - S(\vec{X}, p, \mu, t) = p^{-2}\frac{\partial}{\partial x_\alpha}p^2 D_{\alpha\sigma}\frac{\partial f_0}{\partial x_\sigma}, \qquad (5)$$

3.2.1. Importance of low-frequency MHD waves

Magnetized space plasmas contain low-frequency linear ($\delta B \ll B_0$) transverse MHD waves (such as shear Alfven and magnetosonic plasma waves) with dispersion relations $\omega_R^2 = V_A^2 k_\parallel^2$ and $\omega_R^2 = V_A^2 k^2$, respectively. The induction law then indicates for MHD waves $\delta E = (V_A/c)\delta B \ll \delta B$

Then a perturbation scheme based on $B_0 \gg \delta B \gg \delta E$ corresponds to the reduction

$$< f > (\vec{X}, p, \mu, \phi, t) \to f_0(\vec{X}, p, \mu, t) \to F(\vec{X}, p, t) \qquad (6)$$

to gyrotropic $f_0(\vec{X}, p, \mu, t)$ and to isotropic, gyrotropic distributions functions $F(\vec{X}, p, t)$, respectively, in excellent agreement with the observed near isotropy of CRs.

3.2.2. Order of magnitude estimate of Fokker-Planck coefficients

Before proceeding, we estimate the relative strength of the different Fokker-Planck coefficients. With $\epsilon = V_A/v \ll 1$ these scale as

$$D_{\mu\mu} \, (\simeq D_{\phi\phi}) \simeq D_0 = a_1 \Omega_p \frac{\delta B^2}{B_0^2} \ll a_1 \Omega_p,$$

$$D_{pp} \simeq D_0 \epsilon^2 p^2, \; D_{X,Y} \simeq R_L^2 D_0,$$

$$D_{\mu p} (\simeq D_{\phi p}) \simeq D_0 \epsilon p, \; D_{\mu X} \simeq D_{\phi X} = R_L D_0 \tag{7}$$

Consequently, the associated times scales for pitch-angle scattering ($T_\mu \simeq D_{\mu\mu}^{-1}$), momentum diffusion ($T_p \simeq p^2/D_{pp}$) and perpendicular spatial gyro-center diffusion ($T_X \simeq X^2/D_{XX}$) scale as

$$T_\mu \simeq T_\phi = T_0 \simeq D_0^{-1}, \;\; T_p \simeq \frac{T_0}{\epsilon^2} \gg T_0, \;\; T_X \simeq \frac{X^2}{R_L^2} T_0 \gg T_0 \tag{8}$$

Therefore, in the presence of low-frequency MHD fluctuations the particles will relax most quickly on the time scale $\min[\Omega_p^{-1}, T_0]$ to an isotropic, gyrotropic distribution function, which then on considerably longer time scales T_X and T_p undergoes diffusion in position space and momentum space, respectively.

3.3. Nonmagnetized systems (IGM)

With the regular force operator

$$\mathcal{L}_0 = \partial_t + \ v\mu\partial_z + \ v\sqrt{1-\mu^2}[\cos\phi\partial_x + \sin\phi\partial_y], \tag{9}$$

here the Fokker-Planck equation reads (Krakau and RS 2013)

$$\mathcal{L}_0 < f > (t) - Q_a(z,X,Y,p,\mu,\phi,t) = -p^{-2}\frac{\partial}{\partial y_\alpha}\left[p^2\mathcal{P}_{\alpha\sigma} < f > (t)\right] \tag{10}$$

with the Fokker-Planck coefficients

$$P_{\alpha\sigma} = \Re\int d^3k\int_0^\infty d\tau\, C_{\alpha,\sigma}(\tau)e^{\imath k_\parallel v\mu\tau}J_0\left(k_\perp v\tau\sqrt{1-\mu^2}\right), \tag{11}$$

where $y_\alpha \in [\mu,p,\phi]$ represent the three phase space variables with non-vanishing stochastic fields $\delta\vec{E}$ and $\delta\vec{B}$, and the respective correlation functions of the stochastic forces

$$C_{\alpha,\sigma}(\tau) = H_\alpha(\vec{k},\vec{p},0)H^*_\sigma(\vec{k},\vec{p},-\tau) \tag{12}$$

For isotropically distributed aperiodic magnetic fluctuations we obtain

$$\begin{pmatrix} P_{\mu\mu} \\ P_{\phi\phi} \\ P_{\phi\mu} = P_{\mu\phi} \end{pmatrix}$$

$$= \frac{2\pi q_a^2}{m_a^2 c^2 \gamma^2} \int_0^1 d\eta \int_0^\infty dk\, k^2 \int_0^\infty dt\, g(k,t) \cos(kv\mu\eta t)\, J_0\left(kvt\sqrt{(1-\mu^2)(1-\eta^2)}\right)$$

$$\times \begin{pmatrix} (1-\mu^2)(\eta^2 \sin^2\phi + \cos^2\phi) \\ \frac{\mu^2\eta^2 + (1-\eta^2)[1-\mu^2\cos^2\phi]}{1-\mu^2} \\ (1-\eta^2)\mu \sin\phi \cos\phi \end{pmatrix} \quad (13)$$

where $g(k,t) =< \delta B^2 >_k (t)$ denotes the time-dependent wavenumber correlation spectrum, given by the simultaneous operating spontaneous emission at the rate $p_k(t)$ and collisional damping with the constant Coulomb damping rate (Huba 2009) $\Gamma = 10^{-11} n_{-7} T_4^{3/2}$ Hz:

$$\frac{d}{dt} g(k,t) = p_k(t) - \Gamma g(k,t) \quad (14)$$

Results are extremely sensitive to the adopted collisional damping process of aperiodic fluctuations!

According to RS and Felten (2013)

$$p_k(t) = p_0(k) e^{-s_0(k)t} \left[1 + \frac{a_2(k)}{a_0(k)s_0(k)} - \frac{a_2(k)}{a_0(k)} t\right] \quad (15)$$

so that

$$g(k,t) = \frac{p_0}{\Gamma - s_0}\left(\left(1 + \frac{a_2\Gamma}{a_0 s_0(\Gamma - s_0)}\right)\left[e^{-s_0(k)t} - e^{-\Gamma t}\right] - \frac{a_2 t}{a_0} e^{-s_0 t}\right) \quad (16)$$

and consequently

$$\int_0^\infty dt\, g(k,t) = \frac{p_0(k)}{s_0(k)\Gamma} \quad (17)$$

3.3.1. Upper limit for Fokker-Planck coefficients

We calculate upper limits to the Fokker-Planck coefficients by noting that $J_0(Z) \leq 1$ and $\cos(Z) \leq 1$ for all arguments Z. Then

$$\begin{pmatrix} P_{\mu\mu} \\ P_{\phi\phi} \\ P_{\phi\mu} = P_{\mu\phi} \end{pmatrix} < \frac{4\pi q_a^2}{3 m_a^2 c^2 \gamma^2} \begin{pmatrix} (1-\mu^2)(1+2\cos^2\phi) \\ \frac{2+\mu^2-2\mu^2\cos^2\phi}{1-\mu^2} \\ \mu \sin\phi \cos\phi \end{pmatrix} \int_0^\infty dk\, \frac{p_0(k)k^2}{s_0(k)\Gamma} \quad (18)$$

For the remaining integral we obtain

$$\int_0^\infty dk \frac{p_0(k)k^2}{s_0(k)\Gamma} = 10.87\left(\frac{\omega_{p,e}}{c}\right)^3 \frac{m_e c^2}{\beta_e^{5/2}\Gamma} = 1.3\cdot 10^{-26}\frac{n_{-7}^{3/2} T_4^{-5/4}}{\Gamma}\ \text{erg cm}^{-3}\,\text{Hz}^{-1}, \quad (19)$$

so that

$$\begin{pmatrix} P_{\mu\mu} \\ P_{\phi\phi} \\ P_{\phi\mu} = P_{\mu\phi} \end{pmatrix} < \frac{Z^2 \Omega_q^2}{A^2} \frac{\gamma^2}{\Gamma} \begin{pmatrix} (1-\mu^2)(1+2\cos^2\phi) \\ \frac{2+\mu^2-2\mu^2\cos^2\phi}{1-\mu^2} \\ \mu \sin\phi \cos\phi \end{pmatrix} \tag{20}$$

with the proton quiver frequency (=effective gyrofrequeny)

$$\Omega_q = 2.2 \cdot 10^{-9} n_{-7}^{3/4} T_4^{-5/8} \text{ Hz} \tag{21}$$

For the associated CR hadron parallel mean free path we obtain the lower limit

$$\lambda_{\parallel,\mathrm{hadron}} = \frac{v}{2P_{\mu\mu}} > \lambda^L_{\parallel,\mathrm{hadron}} = 3.1 \cdot 10^{16} \frac{A^2\gamma^2}{Z^2} T_4^{-1/4} n_{-7}^{-1/2} \text{ cm} \tag{22}$$

For CR electrons factor

$$\lambda_{\parallel,\mathrm{electron}} > \lambda^L_{\parallel,\mathrm{electron}} = 3.1 \cdot 10^{10} \gamma^2 T_4^{-1/4} n_{-7}^{-1/2} \text{ cm} \tag{23}$$

The spontaneously emitted aperiodic fluctuations seriously affect the propagation of CR protons with Lorentz factors below 10^6 in the IGM.
Only at proton and electron energies above 10^{15} eV, the lower limit of the mean free path is greater than 10^4 Mpc!

4. Diffusion approximation and cosmic ray anisotropy

Our earlier qualitative estimate of Fokker-Planck coefficients for energetic particles with $v \gg V_A$ indicated that the pitch angle Fokker-Planck coefficient $D_{\mu\mu}$ is the largest one. We therefore make the basic assumption of diffusion theory that the gyrotropic particle distribution function $f_0(\vec{X}, p, \mu, t)$ under the action of low-frequency magnetohydrodynamic waves adjusts very quickly to a distribution function through pitch-angle diffusion which is close to the isotropic distribution in the rest frame of the moving background plasma. Defining the isotropic part of the phase space density $F(\vec{X}, z, p, t)$ as the μ-averaged phase space density

$$F(\vec{X}, p, t) \equiv \frac{1}{2} \int_{-1}^{1} d\mu \; f_0(\vec{X}, p, \mu, t), \tag{24}$$

we follow the analysis of Jokipii (1966) and Hasselmann and Wibberenz (1968) to split the total density f_0 into the isotropic part F and an anisotropic part g,

$$f_0(\vec{X}, p, \mu, t) = F(\vec{X}, p, t) + \; g(\vec{X}, p, \mu, t) \tag{25}$$

where because of Eq. (24)

$$\int_{-1}^{1} d\mu \, g(\vec{X}, p, \mu, t) = 0 \tag{26}$$

The diffusion approximation in the weak focusing limit provides three contributions to the CR anisotropy

$$g(\vec{X},p,\mu,t) = g_1(\vec{X},p,\mu,t) + g_2(\vec{X},p,\mu,t) + g_3(\vec{X},p,\mu,t) \tag{27}$$

with the streaming contribution

$$g_1(\mu) = \frac{v\Gamma}{4}\frac{\partial F}{\partial Z}\left[\int_{-1}^{1} d\mu \frac{(1-\mu)(1-\mu^2)}{D_{\mu\mu}(\mu)} - 2\int_{-1}^{\mu} ds \frac{1-s^2}{D_{\mu\mu}(s)}\right], \tag{28}$$

the Compton-Getting contribution

$$g_2(\mu) = \frac{1}{2}\frac{\partial F}{\partial p}\left[\int_{-1}^{1} d\mu \frac{(1-\mu)D_{\mu p}(\mu)}{D_{\mu\mu}(\mu)} - 2\int_{-1}^{\mu} ds \frac{D_{\mu p}(\mu)}{D_{\mu\mu}(s)}\right] \tag{29}$$

and the perpendicular contribution

$$g_3(\mu) = \frac{v}{12}\partial_\perp F\left[2\int_{-1}^{\mu} ds \frac{s(1-s^2)}{D_{\mu\mu}(s)} - \int_{-1}^{1} d\mu \frac{\mu(1-\mu)(1-\mu^2)}{D_{\mu\mu}(\mu)}\right] \tag{30}$$

stemming from the gradients of F with respect to Z, p, X, Y, respectively. The pitch-angle cosine $\mu = p_{\|}/p$ is defined with respect to guide magnetic field direction.

5. Summary and conclusions

- Understanding cosmic $(\delta B, \delta E)$-fluctuations in magnetized (ISM) and nonmagnetized (IGM) plasmas is of crucial importance e.g. the role of collective and noncollective modes and wave-like, weakly-propagating and aperiodic fluctuations.
- The ordering $B_0 \gg \delta B \gg \delta E$ in magnetized systems, necessary for explaining the observed nearly isotropic CR momentum distribution function, is the basis for a perturbation scheme leading to the modified diffusion-convection CR transport equation and expressions for the CR anisotropy.
- The nonmagnetized IGM medium containes aperiodic magnetic fluctuations which are spontaneously emitted by the fully-ionized thermal electron-proton IGM plasma at a level of $|\delta B| = 1.5 \cdot 10^{-16} n_{-7} T_4^{-3/2}$ G.
- The spontaneously emitted fluctuations affect the propagation of CR protons and electrons in the IGM at energies below 10^{15} eV.

- I thank my collaborators Tim Felten, Dominik Ibscher, Steffen Krakau, Marian Lazar, Markus Supsar, Peter Yoon and the HESS-Team.

- Financial support by the Deutsche Forschungsgemeinschaft, Alexander-von-Humboldt-Stiftung, Projektförderung Mercator Research Center Ruhr (MERCUR) and Verbundforschung Astroteilchenphysik is gratefully acknowledged.