

Explanation for the Anisotropies at Small and Medium Angular Scales

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Based on :

GG & G.Sigl

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GG, M.Kachelriess, D.Semikoz

Phys. Rev. Lett. 108, 261101(2012)

Phys. Rev. D 88, 023010 (2013)



UNIVERSITY OF
OXFORD

**With thanks to Tony Bell, Brian
Reville & Klara Schure for
discussions**

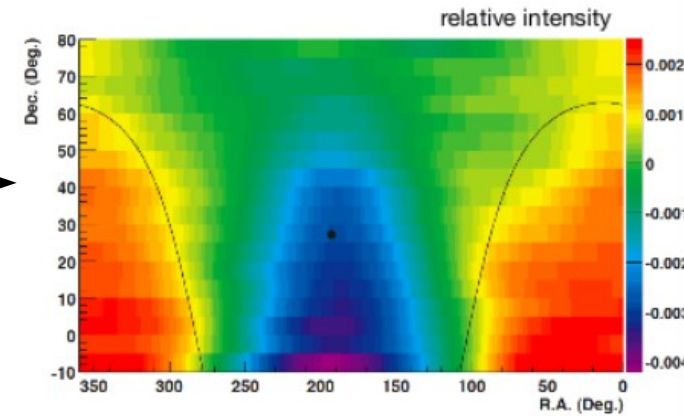
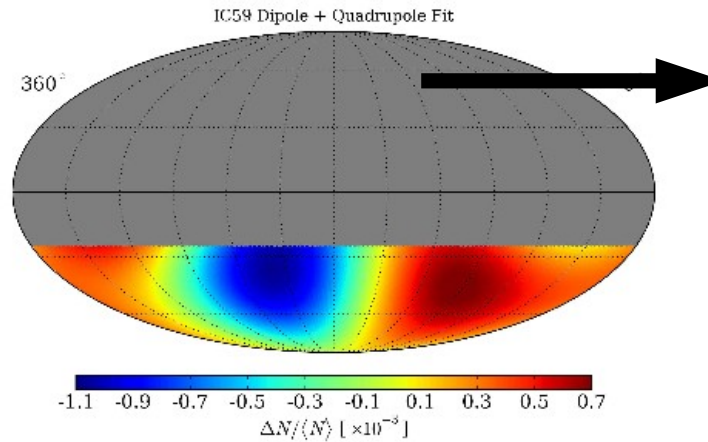


Observations

IceCube results, here

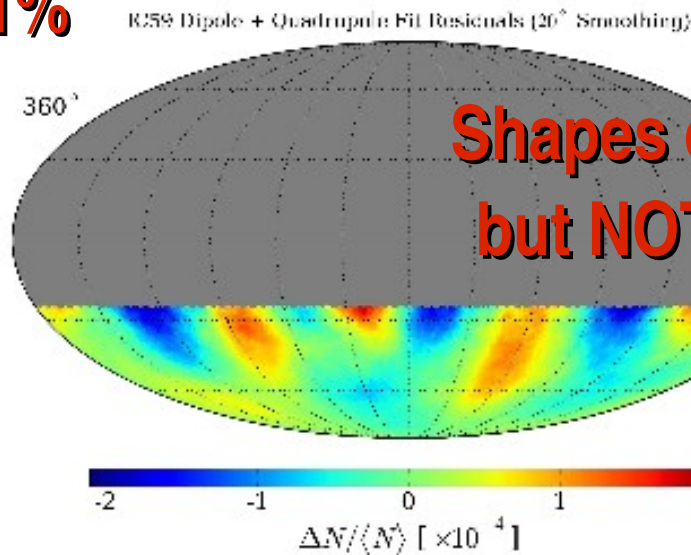
~0.1%

1105.2326

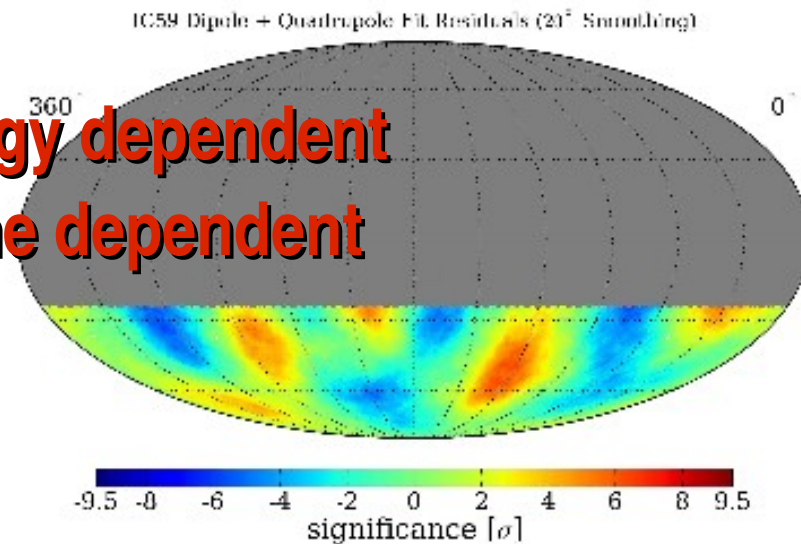


Milagro

~0.01%



**Shapes energy dependent
but NOT time dependent**

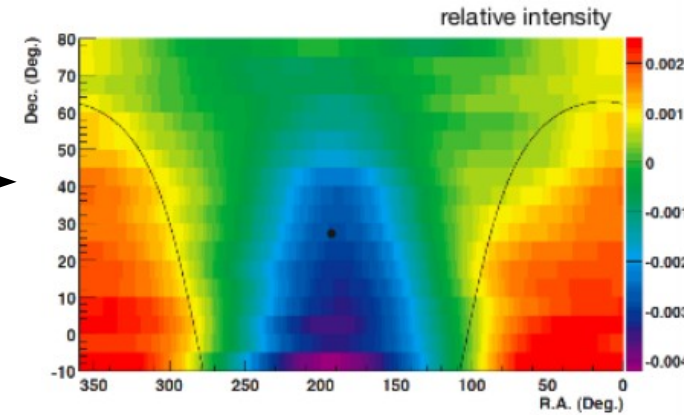
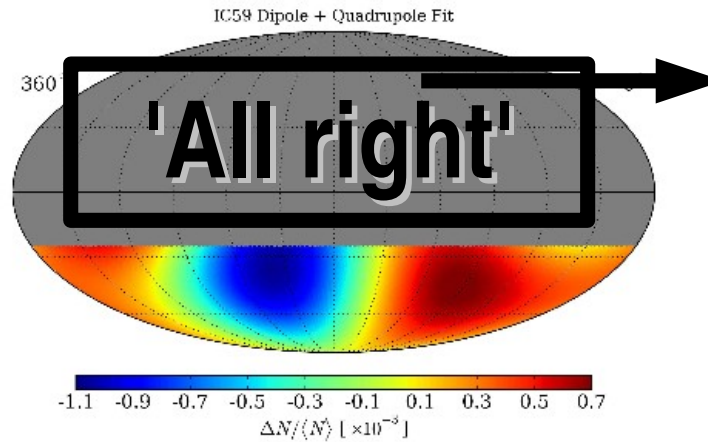


Observations

IceCube results, here

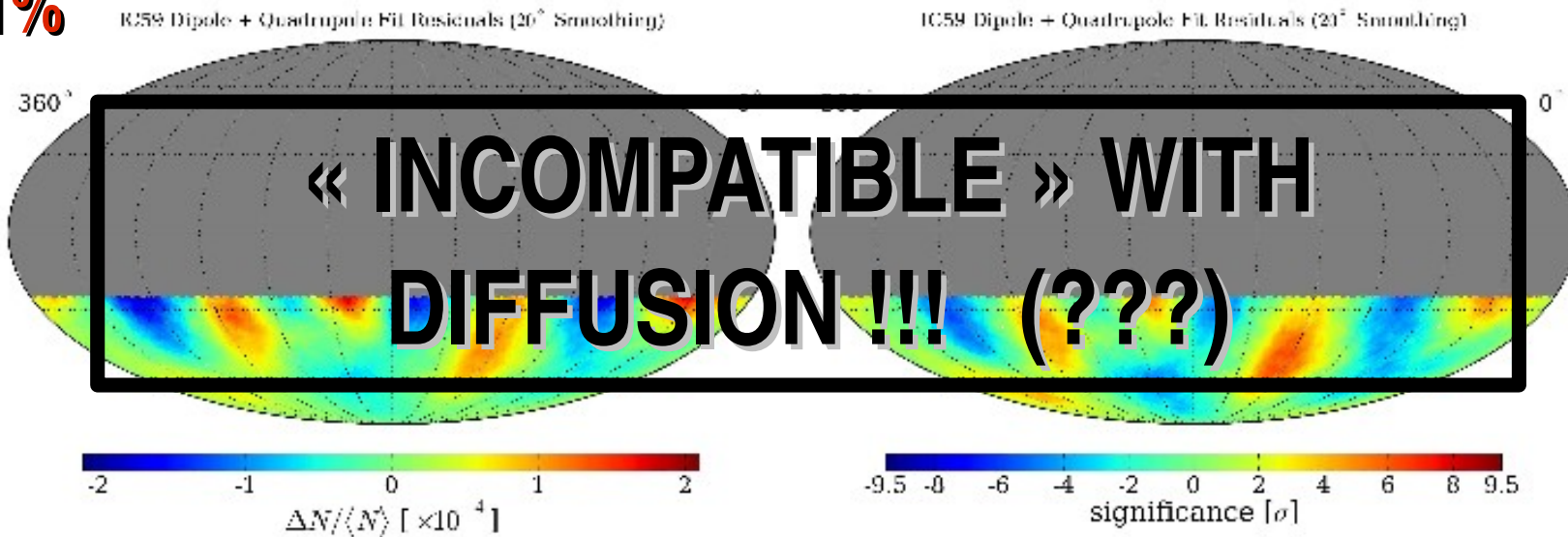
~0.1%

1105.2326



Milagro

~0.01%



The Galactic turbulent magnetic field

Satisfies : $\langle \mathbf{B}(\mathbf{x}) \rangle = \mathbf{0}$, $\langle \mathbf{B}(\mathbf{x})^2 \rangle = B_{\text{rms}}^2$, and $\text{div } \mathbf{B} = 0$

Fourier transform : $B_i(\mathbf{x}) = \int \frac{d^3 k}{(2\pi)^3} B_i(\mathbf{k}) e^{i(\mathbf{k} \cdot \mathbf{x} + \phi_i(\mathbf{k}))} \xrightarrow{+ \omega t}$

with $|\mathbf{B}(\mathbf{k})|^2 \propto k^{-\alpha-2}$

Power spectrum : $\mathcal{P}(k) \propto k^{-\alpha}$ ($\alpha = 5/3, 3/2$ plausible)

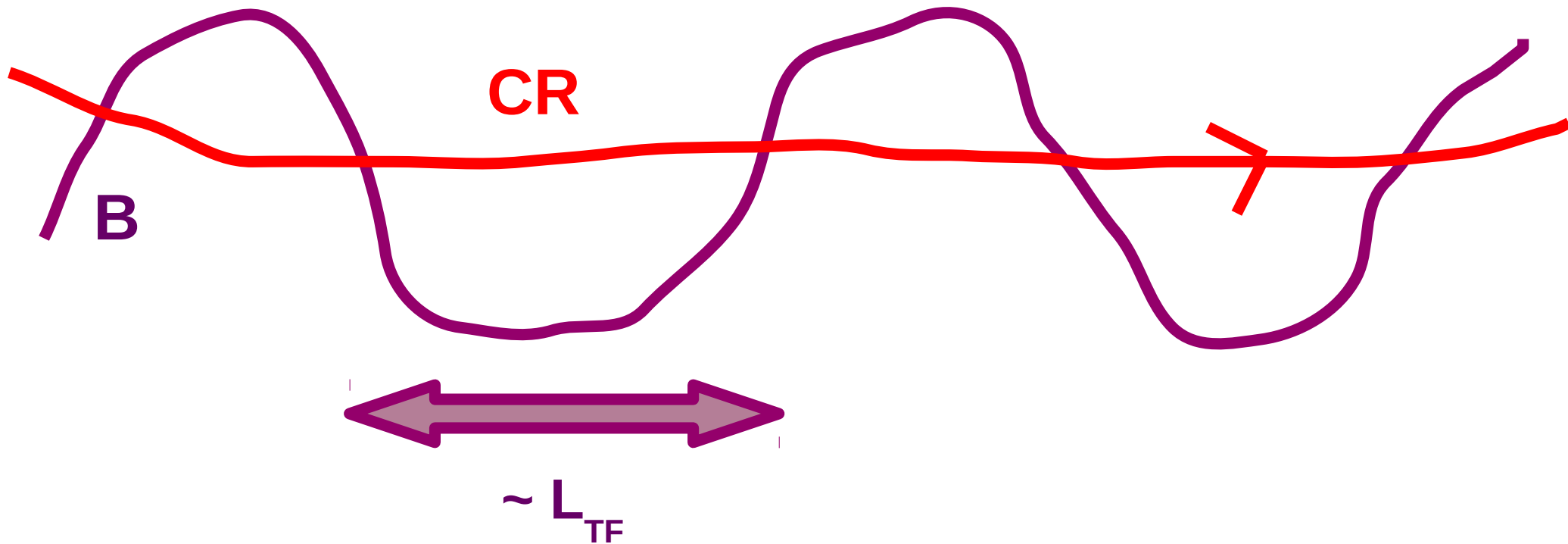
for $2\pi/L_{\text{max}} \leq k \leq 2\pi/L_{\text{min}}$ with $L_{\text{min}} < 1 \text{ AU}$, $L_{\text{max}} \sim 100 \text{ pc}$

Coherence length : $L_c \sim \text{few } 10\text{s pc.}$

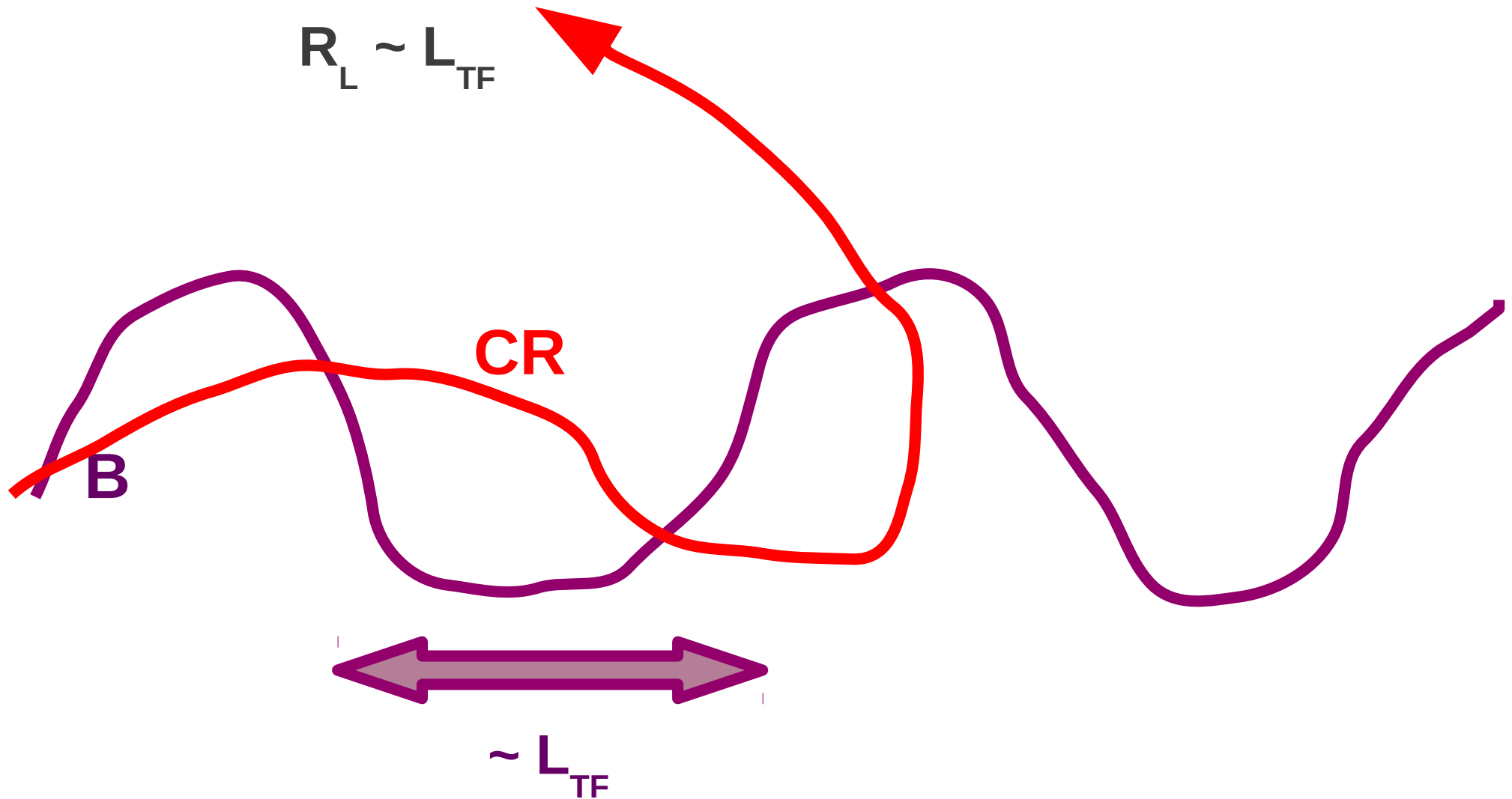
Locally : See works of P. Frisch

CR propagation in magnetic turbulence

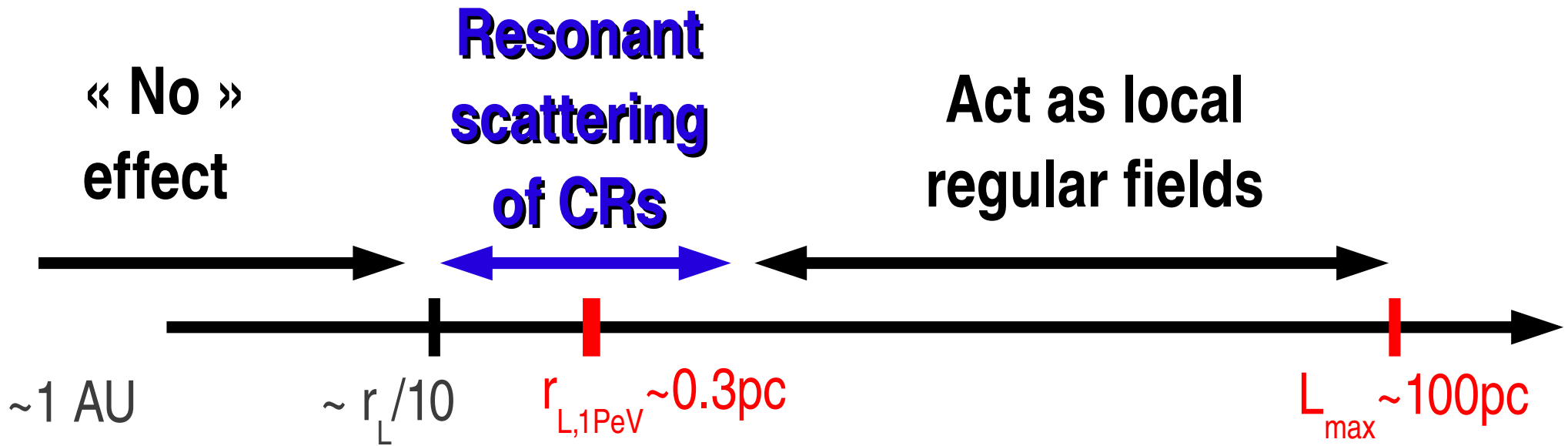
$$R_L \gg L_{TF}$$



CR propagation in magnetic turbulence



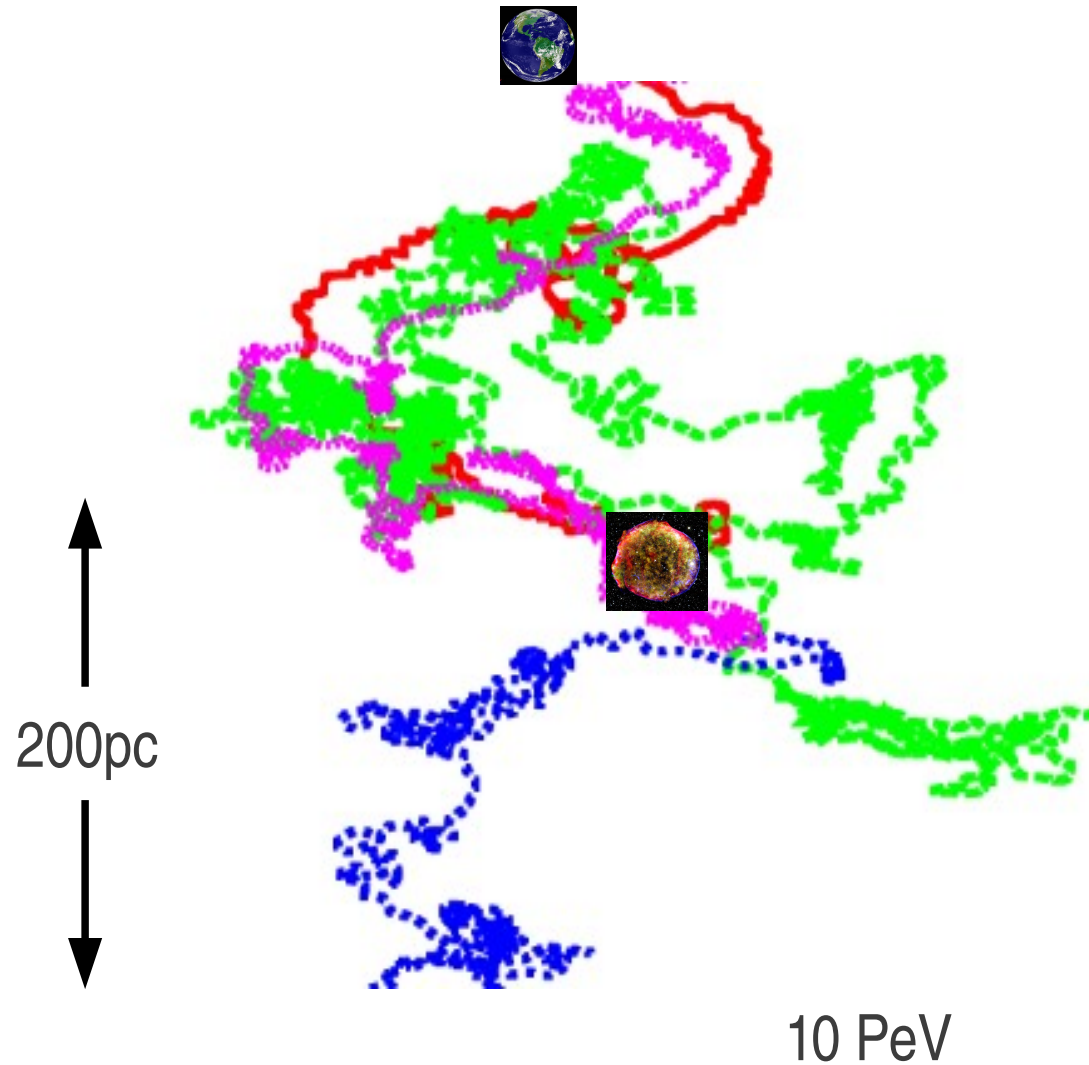
Diffusion approximation



CR propagation in magnetic turbulence can be regarded as a Markovian process

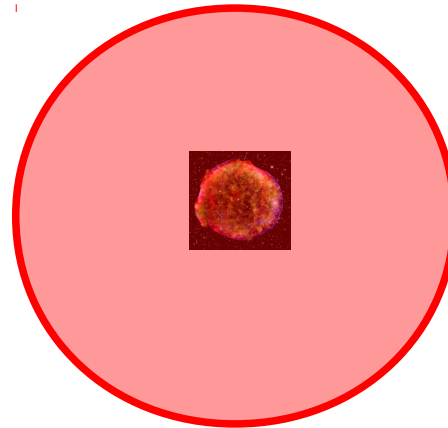
=> diffusion approximation

Diffusion approximation



Globally /
effective picture :
All right

Diffusion approximation

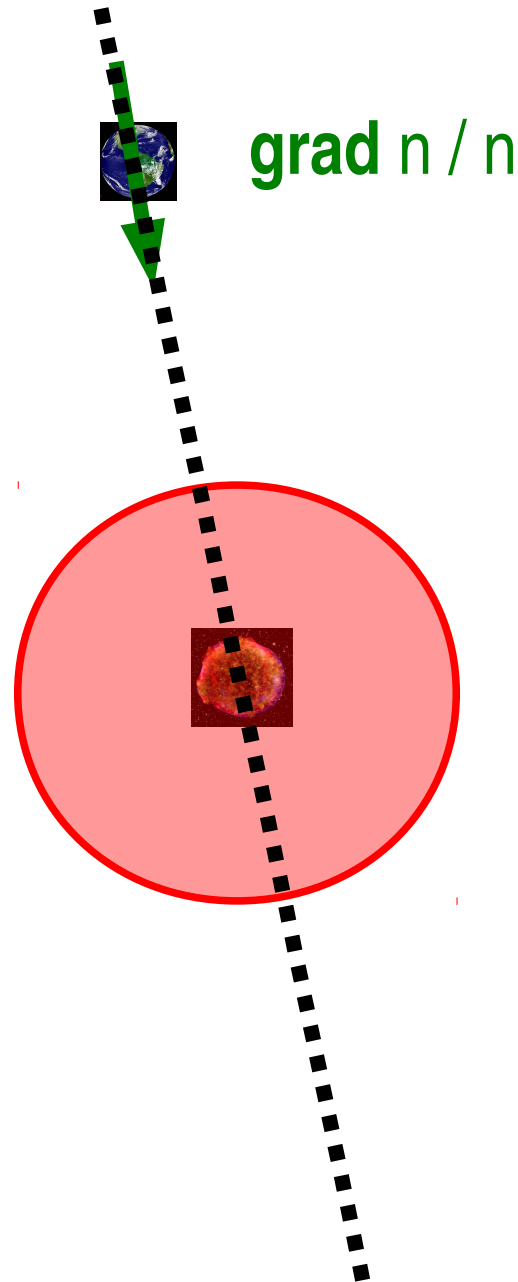


$$\delta(p) \simeq -\frac{3}{c_0} \frac{\mathbf{j}}{n} = \frac{3D(p)}{c_0} \frac{\nabla n}{n}$$

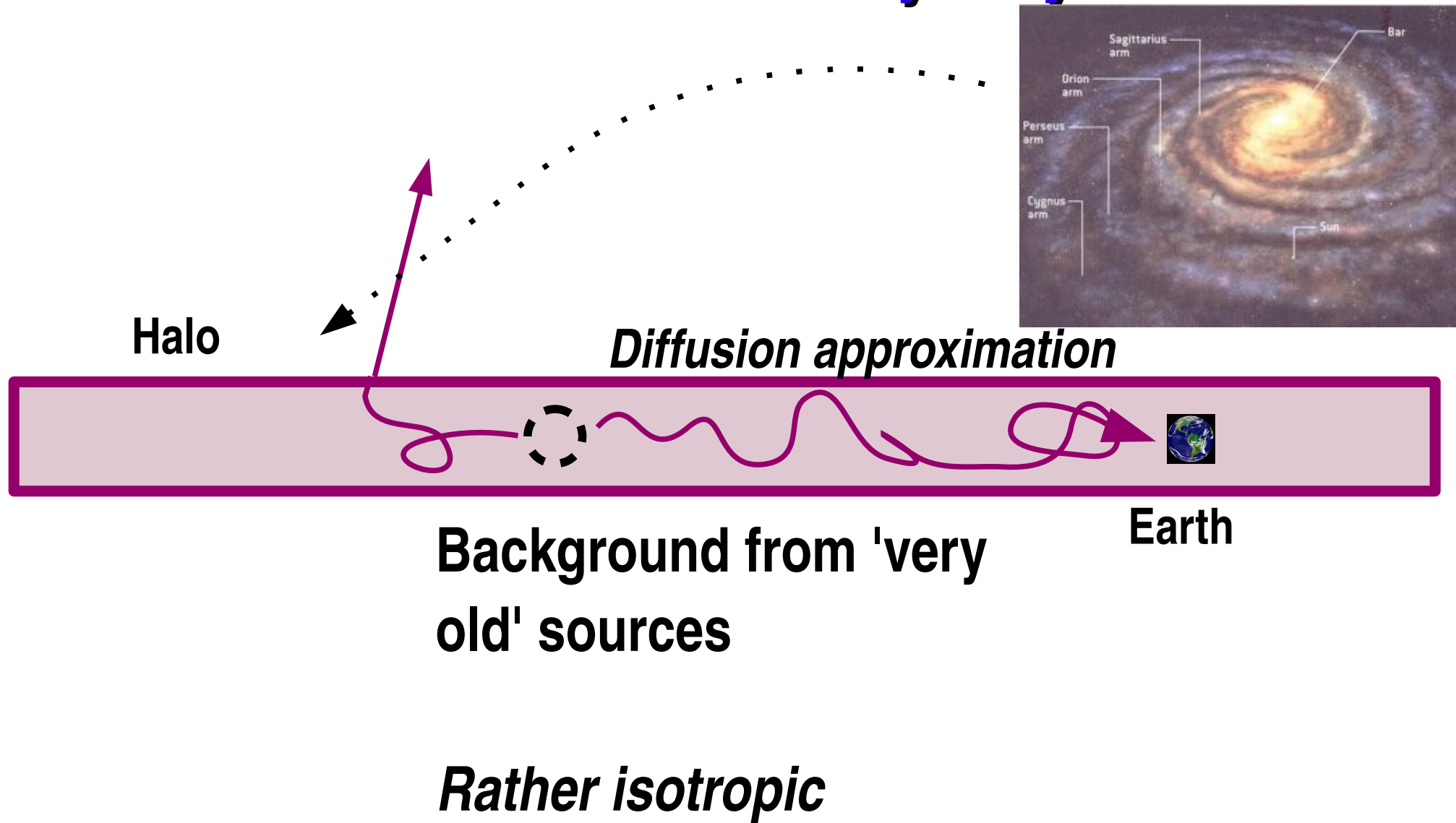
where $\mathbf{j}(\mathbf{r}, p) = -D(p) \nabla n$ is the CR current

DIPOLE $\delta \sim (E/Z)^{1/3; 1/2}$

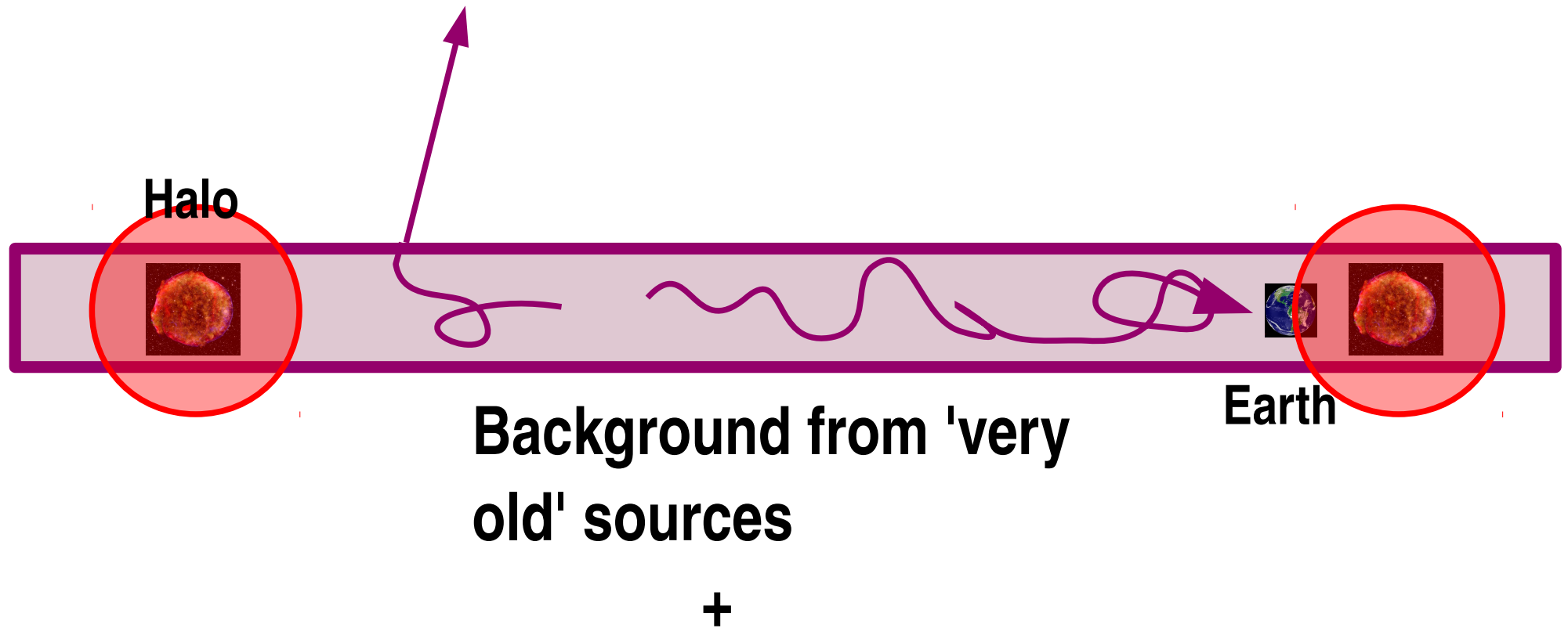
Diffusion approximation



CR diffusion in the Milky Way



CR diffusion in the Milky Way

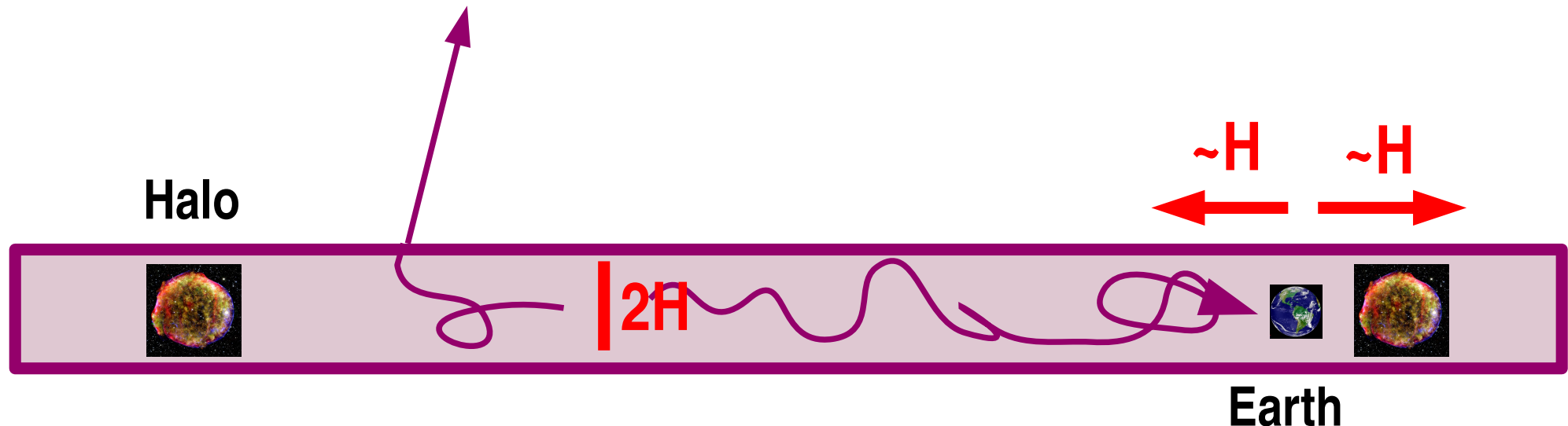


Flux from 'more recent' sources: *More anisotropic*

---> CR anisotropy at Earth due to 'most recent' sources

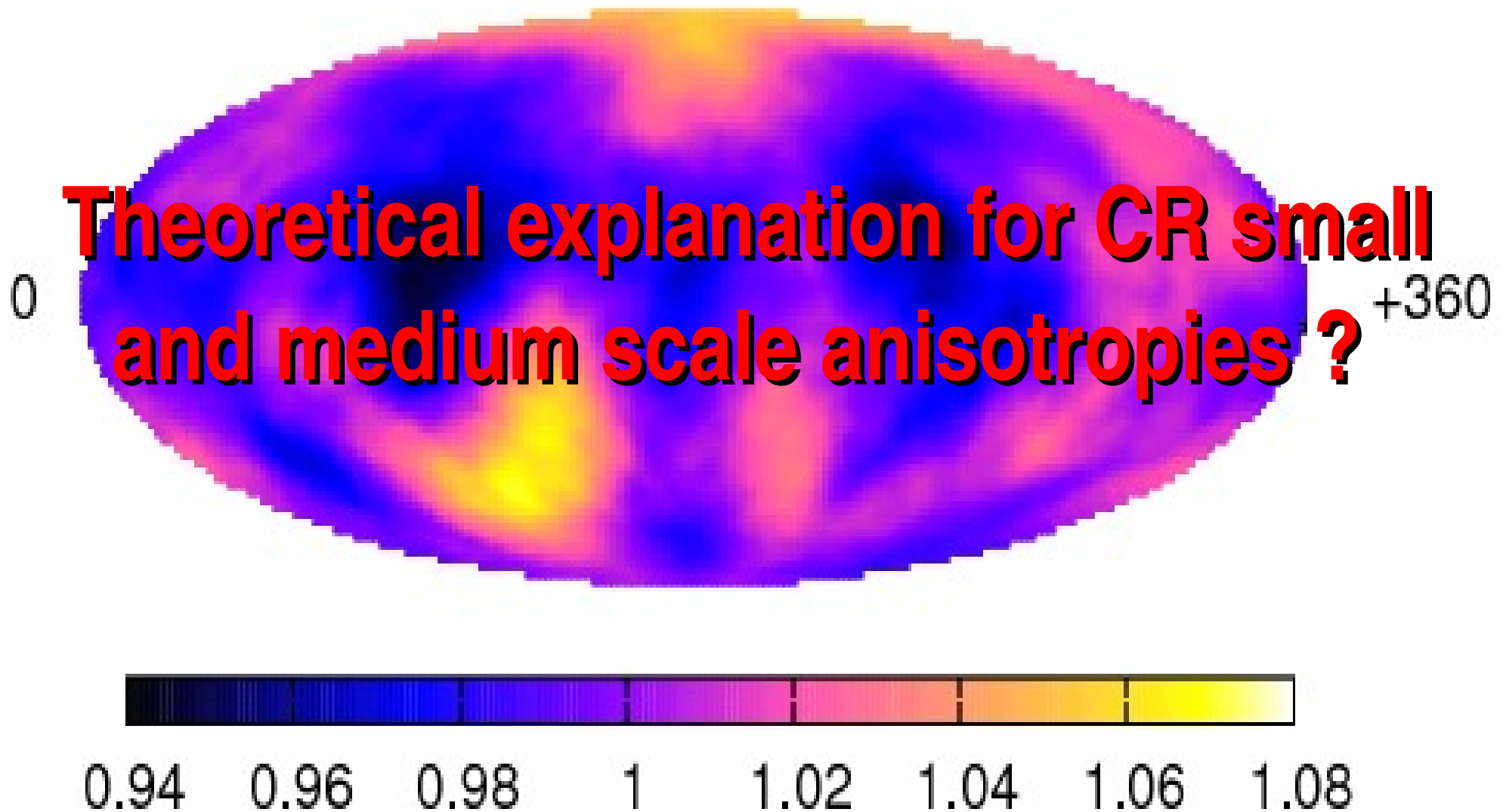
CR diffusion in the Milky Way

(Erlykin & Wolfendale '06, Blasi & Amato '11)



CR anisotropy at Earth mostly due to 'most recent'
and 'nearby' sources

$$\Rightarrow \text{DIPOLE} \quad \delta \sim (E/Z)^{2-\alpha} \sim (E/Z)^{1/3; 1/2}$$



Based on : Giacinti & Sigl Phys. Rev. Lett. 109, 071101 (2012)

Small scale anisotropies : 'mysterious'

In magnetic turbulence, only a dipolar anisotropy should be detected => Additional effects needed.

- ~~Related to sources ?~~ CRs do not point back to their sources

Small scale anisotropies : Suggestions

Magnetic funnelling (Drury & Aharonian '08)

Astroparticle Physics 29 (2008) 420–423



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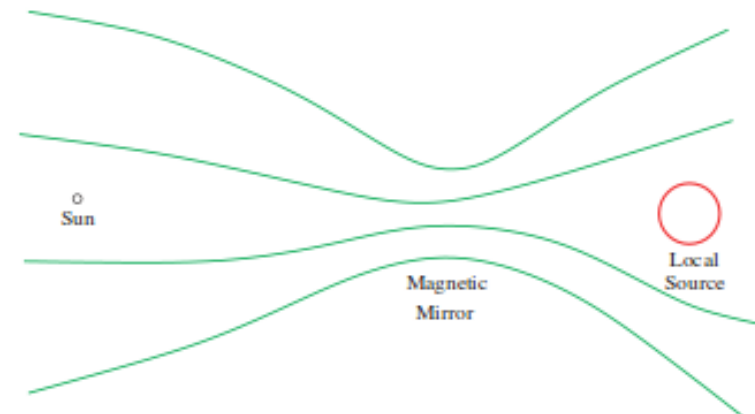


Fig. 1. A cartoon representation of the suggested model. The magnetic field near the sun is linked through a strong field region where most particles are mirrored to a region containing a slightly higher amplitude and harder spectrum of cosmic rays, presumably as a result of a local source event such as a supernova explosion. Only particles with pitch angles closely aligned to the field can penetrate through the mirror and emerge as a field-aligned beam on the other side (another possibility would be to locate the source inside the strong field region itself rather than behind the trap).

The puzzling MILAGRO hot spots

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ABSTRACT

We discuss the reported detection by the MILAGRO experiment of localised hot spots in the cosmic ray arrival distribution and the difficulty of interpreting these observations. A model based on secondary neutron production in the heliotail is shown to fail. An alternative model based on loss-cone leakage through a magnetic trap from a local source region is proposed.

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Small scale anisotropies : Suggestions

Anisotropic turbulence (Malkov *et al.* '10)

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PROBING NEARBY COSMIC-RAY ACCELERATORS AND INTERSTELLAR MEDIUM TURBULENCE WITH MILAGRO HOT SPOTS

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ABSTRACT

Both the acceleration of cosmic rays (CRs) in supernova remnant shocks and their subsequent propagation through the random magnetic field of the Galaxy are deemed to result in an almost isotropic CR spectrum. However, the MILAGRO TeV observatory discovered sharp ($\sim 10^\circ$) arrival anisotropies of CR nuclei. We suggest a mechanism for producing a weak and narrow CR beam which operates en route to the observer. The key assumption is that CRs are scattered by a strongly anisotropic Alfvén wave spectrum formed by the turbulent cascade across the local field direction. The strongest pitch-angle scattering occurs for particles moving almost precisely along the field line. Partly because this direction is also the direction of the minimum of the large-scale CR angular distribution, the enhanced scattering results in a weak but narrow particle excess. The width, the fractional excess, and the maximum momentum of the beam are calculated from a systematic transport theory depending on a single scale l which can be associated with the longest Alfvén wave, which efficiently scatters the beam. The best match to all three characteristics of the beam is achieved at $l \sim 1$ pc. The distance to a possible source of the beam is estimated to be within a few 100 pc. Possible approaches to the determination of the scale l from the characteristics of the source are discussed. Alternative scenarios of drawing the beam from the galactic CR background are considered. The beam-related large-scale anisotropic CR component is found to be energy independent, which is also consistent with the observations.

Keywords: acceleration of particles – cosmic rays – diffusion – ISM: supernova remnants – magnetohydrodynamics (MHD) – scattering – shock waves – turbulence

Small scale anisotropies : Suggestions

A local source in the heliotail (Lazarian & Desiati '10)

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doi:10.1088/0004-637X/722/1/188

MAGNETIC RECONNECTION AS THE CAUSE OF COSMIC RAY EXCESS FROM THE HELIOSPHERIC TAIL

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Received 2010 May 10; accepted 2010 August 1; published 2010 September 17

ABSTRACT

The observation of a broad excess of sub-TeV cosmic rays compatible with the direction of the heliospheric tail and the discovery of two significant localized excess regions of multi-TeV cosmic rays by the MILAGRO Collaboration, also from the same region of the sky, have raised questions on their origin. In particular, the coincidence of the most significant localized region with the direction of the heliospheric tail and the small angular scale of the observed anisotropy ($\sim 10^\circ$) is suggestive of a local origin and of a possible connection to the low-energy broad excess. Cosmic ray acceleration from magnetic reconnection in the magnetotail is proposed as a possible source of the energetic particles.

Key words: acceleration of particles – cosmic rays – magnetic fields – magnetosphere – solar wind

Online-only material: color figures

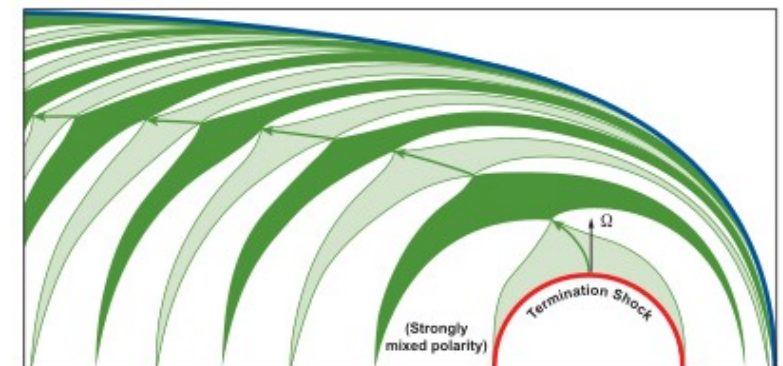
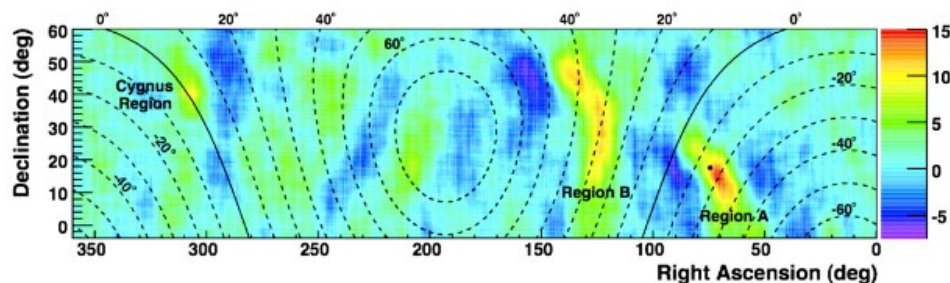


Figure 4. Meridional view of the boundary sectors of the heliospheric current sheet and how the opposite sectors get tighter closer to the heliopause and into the heliotail. The thickness of the outflow regions in the reconnection region depends on the level of turbulence. The length of the outflow regions L depends on the mean geometry of magnetic field and turbulence. Adapted from Nerney et al. (1995) and also Lazarian & Opher (2009).

Small scale anisotropies : Suggestions

Strangelets – Molecular Clouds (Kotera *et al.* '13)

Strangelets and the TeV-PeV cosmic-ray anisotropies

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¹*Institut d'Astrophysique de Paris, UMR 7095 - CNRS,
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(Dated: March 7, 2013)

Several experiments (e.g., Milagro and IceCube) have reported the presence in the sky of regions with significant excess in the arrival direction distributions of Galactic cosmic rays in the TeV to PeV energy range. Here we study the possibility that these hotspots are a manifestation of the peculiar nature of these cosmic rays, and of the presence of molecular clouds near the sources. We propose that stable quark matter lumps or so-called *strangelets* can be emitted in the course of the transition of a neutron star to a more compact astrophysical object. A fraction of these massive particles would lose their charge by spallation or electron capture in molecular clouds located in the immediate neighborhood of their source, and propagate rectilinearly without decaying further, hence inducing anisotropies of the order of the cloud size. With reasonable astrophysical assumptions regarding the neutron star transition rate, strangelet injection and neutralization rates, we can reproduce successfully the observed hotspot characteristics and their distribution in the sky.

Several experiments have reported strong anisotropy measurements in the arrival direction distributions of Galactic cosmic rays (CRs) in the TeV to PeV energy range (Super-Kamiokande, Tibet III, Milagro, ARGO-YBJ, and IceCube [1, 2]). The data reveal the presence of large scale anisotropies of amplitude $\sim 0.1\%$. Smaller scale anisotropies of size $\sim 10^\circ - 30^\circ$ are also detected with amplitude a factor of a few lower. Milagro has reported the detection at significance $> 12\sigma$ of two *hotspots* (regions with enhanced CR intensity) with amplitude $\approx 10^{-4}$, at a median energy of 1 TeV. ARGO-YBJ report similar excesses. IceCube observes localized regions of angular scale $\sim 15^\circ$ of excess and deficit in CR flux with significance $\sim 5\sigma$ around a median energy of 20 TeV [2].

Various phenomena, such as heliospheric tion, neutron sources, nearby pulsars, peculiar s of the local Galactic magnetic fields have been but none seem to give an obvious explanation [

In this work we study the possibility that the hotspots in the skymap are a manifestation of the peculiar nature of CRs, and of the presence of molecular clouds (MCs) near the sources. We propose that quark matter lumps or so-called *strangelets* could be produced and accelerated while a neutron star (NS) transitions to a quark star (QS). A fraction of these heavy particles would suffer spallation or electron capture in molecular clouds located in the immediate neighborhood of their source, and produce neutral fragments that would propagate rectilinearly without decaying further, hence inducing

COMPARISON WITH DATA AND SIGNATURES

Most observed hotspots could be produced by MCs in the Gould Belt (a star forming region concentrating many MCs, that forms a ring at a distance from the Sun of $\sim 0.7 - 2$ kpc), at the location where NS-QS transitions may have occurred. Interestingly, the Milagro hotspot labelled “Region A” [1] lies in the direction of the Taurus Molecular Cloud, the nearest star formation region located at 140 pc, and that covers $\sim 100 \text{ deg}^2$ in the sky [19]. “Region 1” of IceCube [2] is also in the direction of a remarkable MC: the Vela Molecular Ridge, located at $0.7 - 2$ kpc distance, of size $\sim 15^\circ$ in sky [19].

Small scale anisotropies : Suggestions

Dark Matter (Harding '13)

The TeV Cosmic-Ray Anisotropy from Local Dark Matter Annihilation

J. Patrick Harding*

Physics Division, Los Alamos National Laboratory, Los Alamos, NM, USA

(Dated: July 25, 2013)

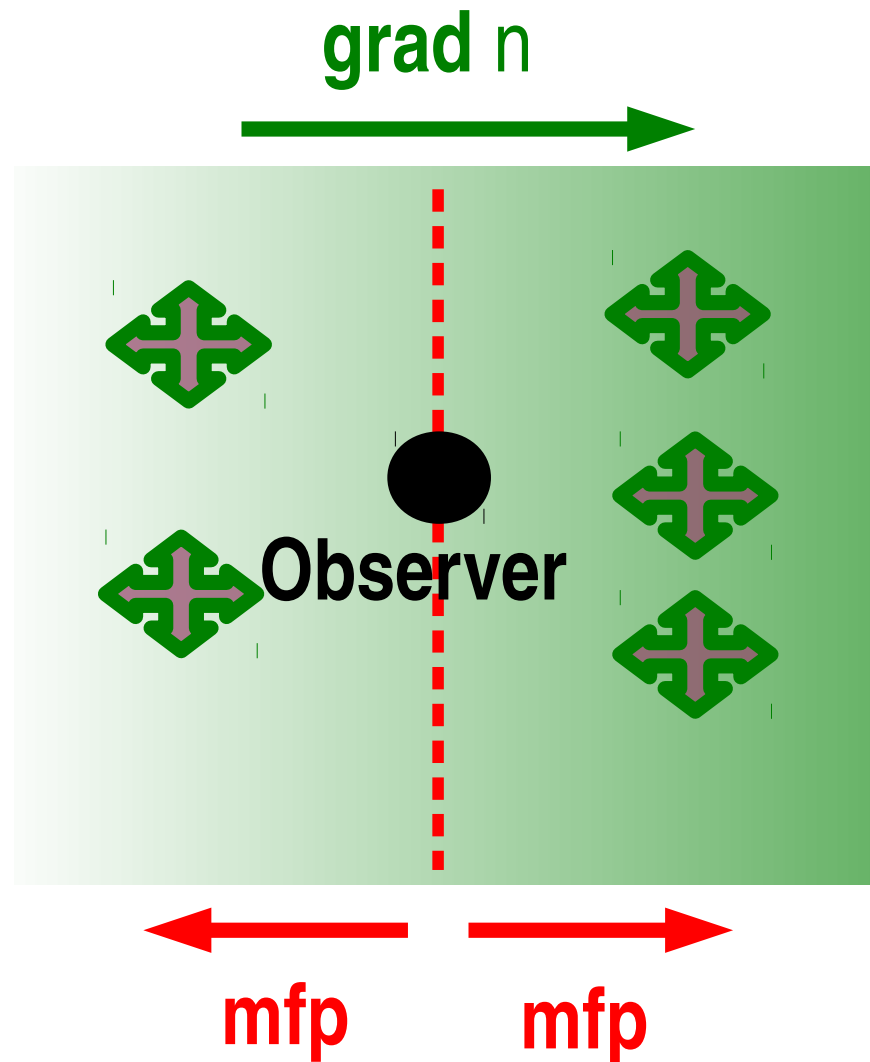
Several experiments have reported regions in the TeV sky with an excess of cosmic rays. Here we consider the possibility that the excess cosmic rays are coming from dark matter annihilation in nearby subhalos. We provide motivation for dark matter as the source of the excess and show that dark matter annihilations can naturally produce the cosmic ray anisotropy without the need for exotic dark matter models. We show that viable dark matter explanations of the cosmic ray excess are consistent with current measurements of antiprotons, positrons, and gamma-rays. Additionally, we show that the dark matter interpretation of the observed anisotropy predicts detections in several cosmic ray channels by the next generation of experiments.

Solution we propose:

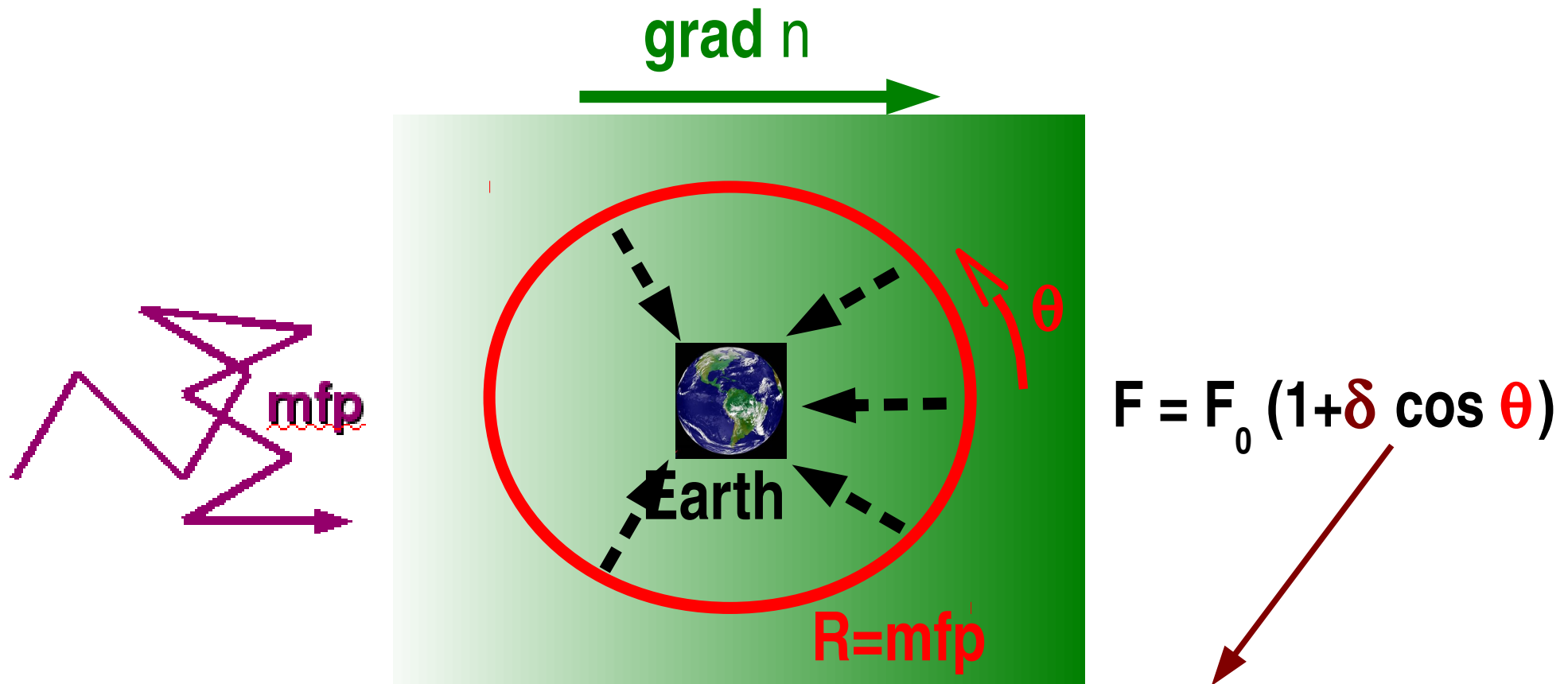
~~In magnetic turbulence, only a dipolar anisotropy should be detected $\Rightarrow$ Additional effects needed.~~

- Energy-dependent smaller scales must ***automatically*** appear at multiple angular scales, provided a large scale anisotropy exists... *even* in pure *isotropic turbulence*
- ***No additional assumption***

Diffusion approximation (1D)



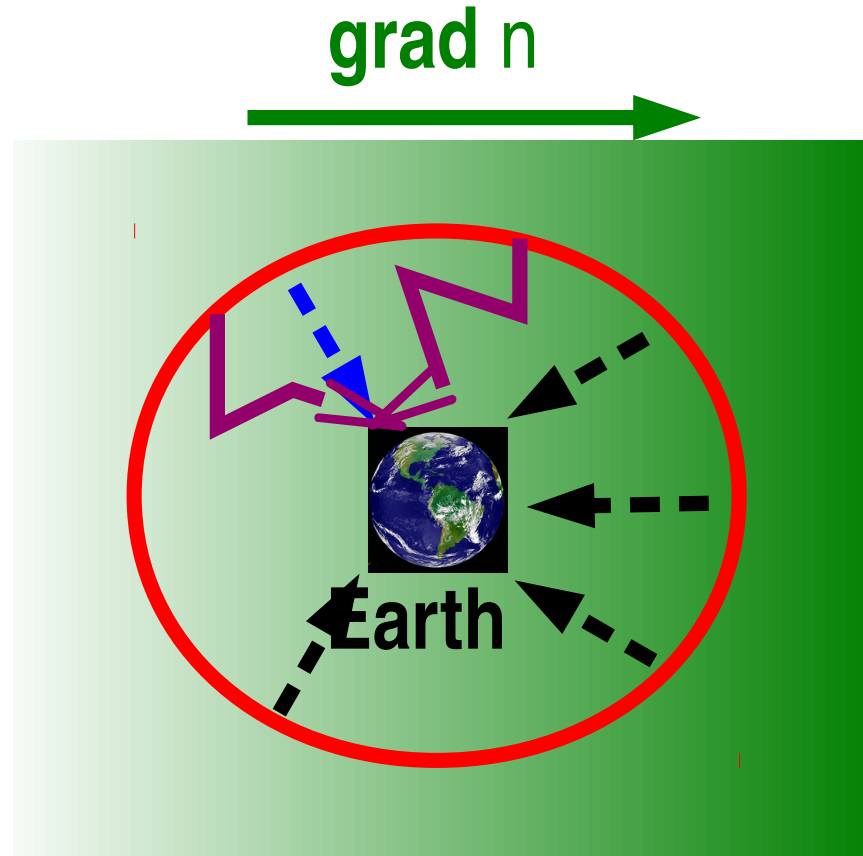
Diffusion approximation - Dipole



$$\delta(p) \simeq -\frac{3}{c_0} \frac{\mathbf{j}}{n} = \frac{3D(p)}{c_0} \frac{\nabla n}{n}$$

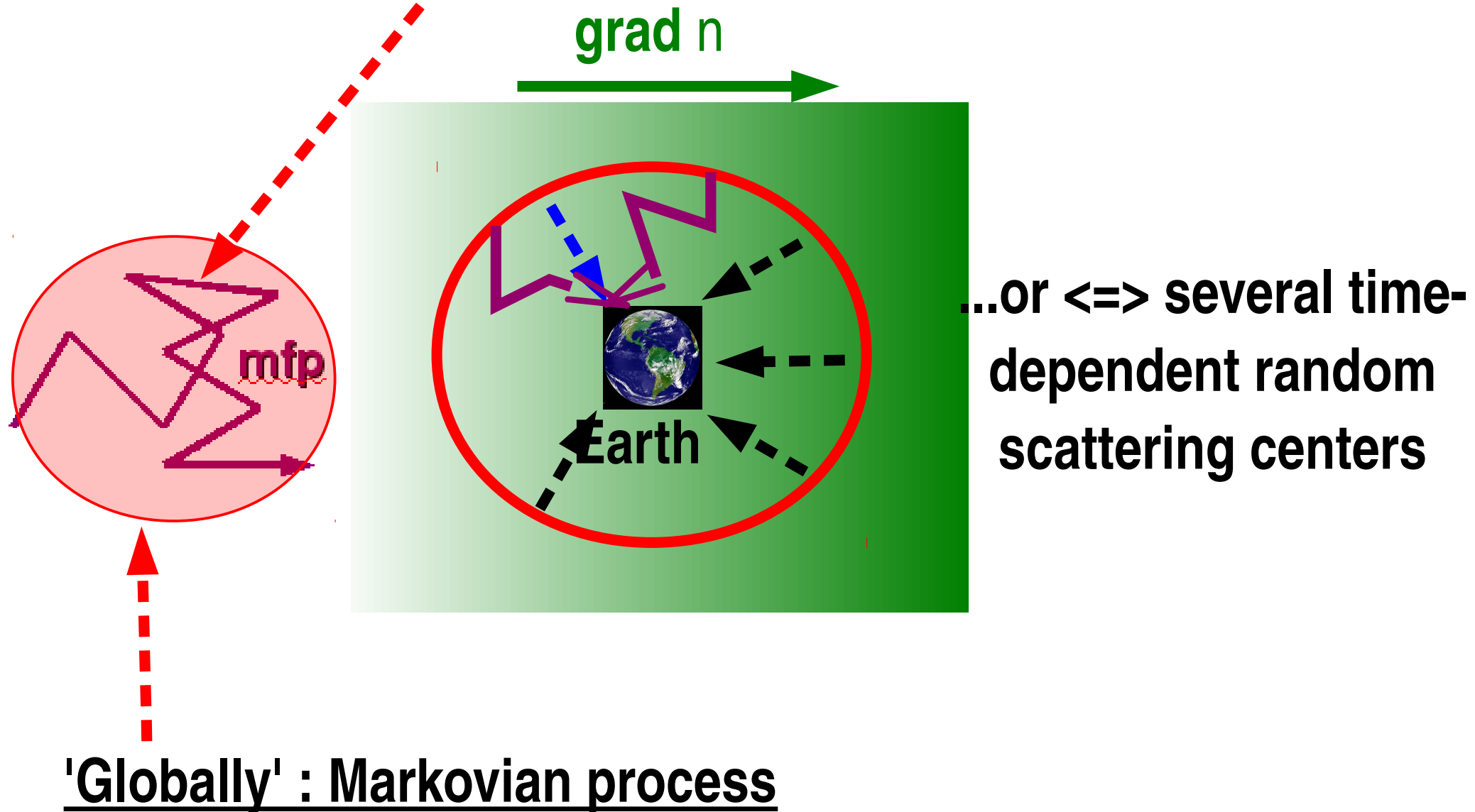
where $\mathbf{j}(\mathbf{r}, p) = -D(p) \nabla n$ is the CR current

Diffusion approximation - Dipole



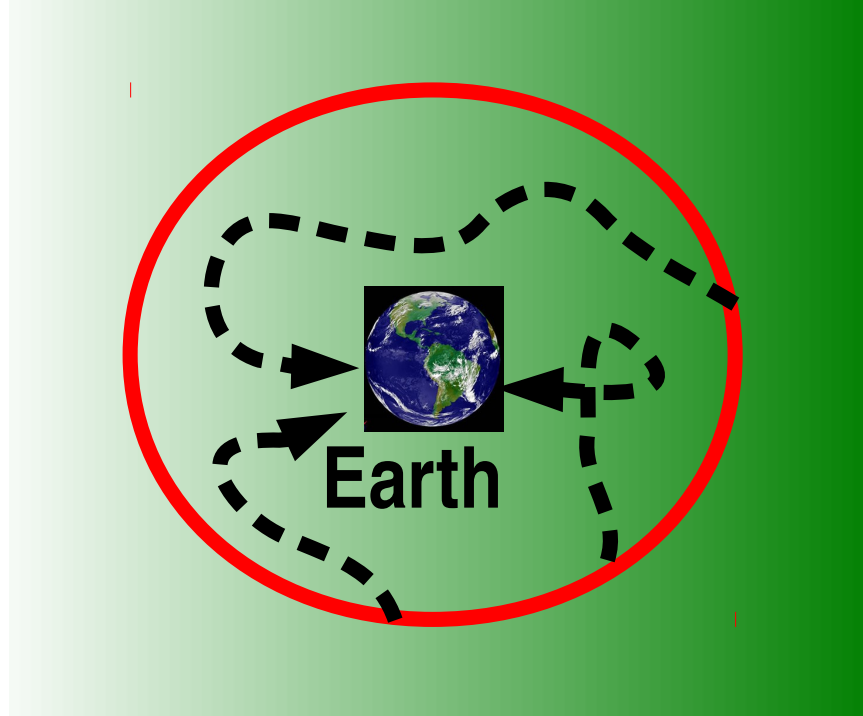
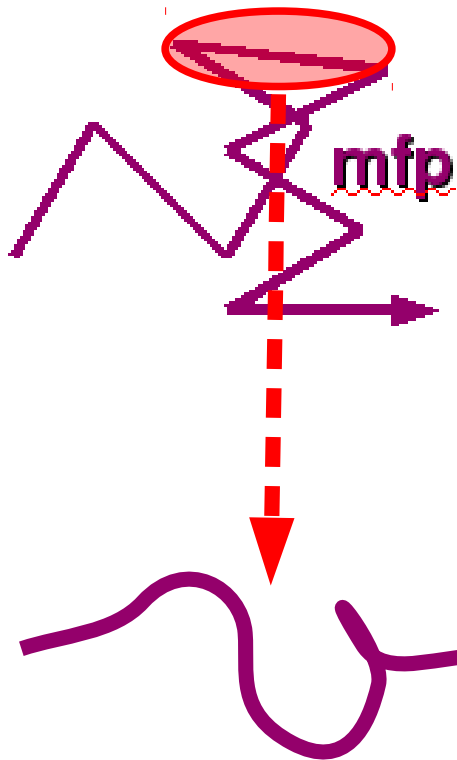
...or $\Leftrightarrow$ several time-dependent random scattering centers

Diffusion approximation - Dipole



In reality :

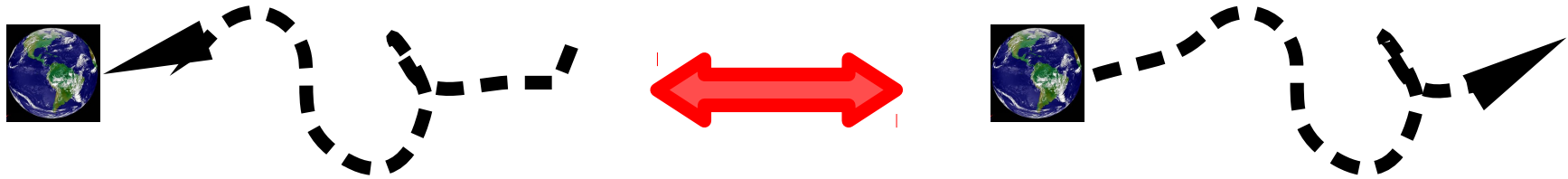
grad n



**Not diffusion -> Need
individual trajectories**

**1) The local field
does not vary
signif. within λ/c
(10 km/s $\ll$ c)**

In reality :

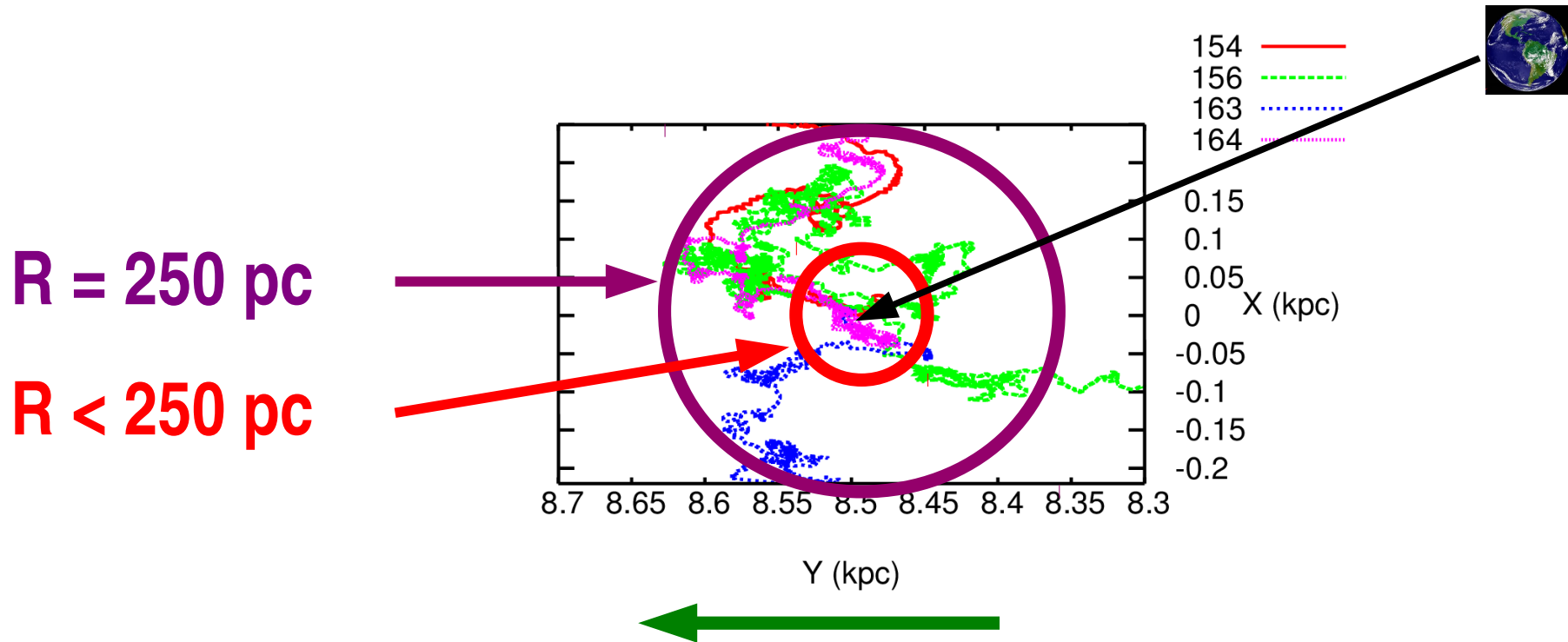


Equivalence between forward- and back-tracking of individual cosmic rays

... the process is locally non-Markovian

2) The Earth is point-like ($/r_L$)

Numerical simulations



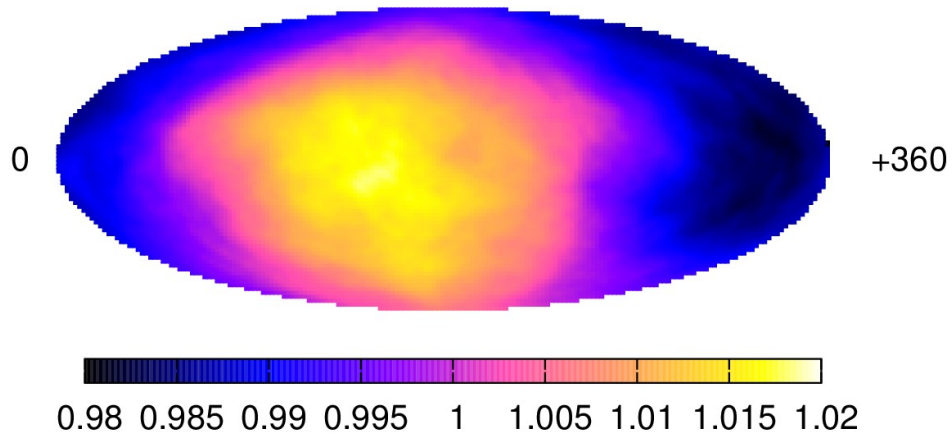
$$|(\text{grad } n)/n| \sim (290 \text{ pc})^{-1} \rightarrow (1 \text{ kpc})^{-1}$$

$$\text{and } E/Z = 10^{16} \text{ eV} \rightarrow 10^{13} \text{ eV}$$

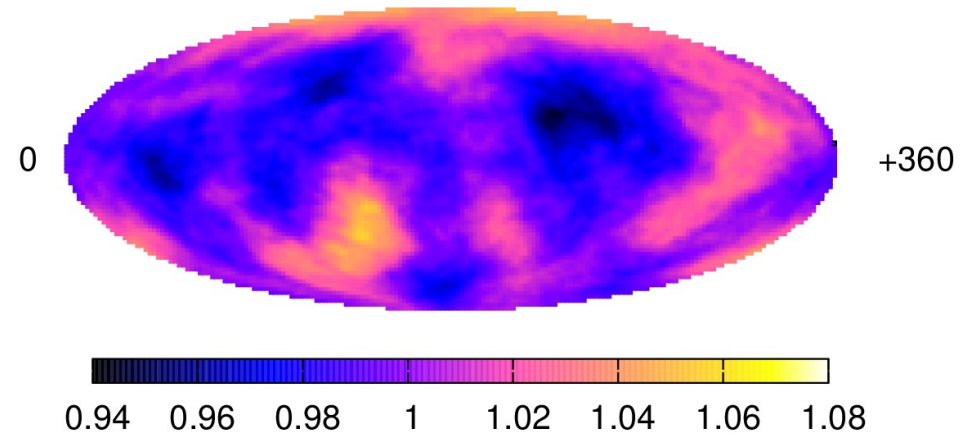
Can be extrapolated to lower $(\text{grad } n)/n$ and E/Z

Dipole / Smaller scales

90°smoothing



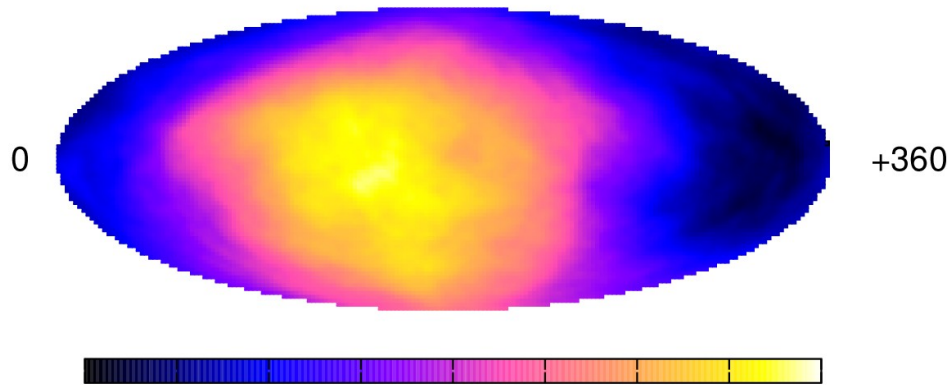
20°smoothing - Dipole



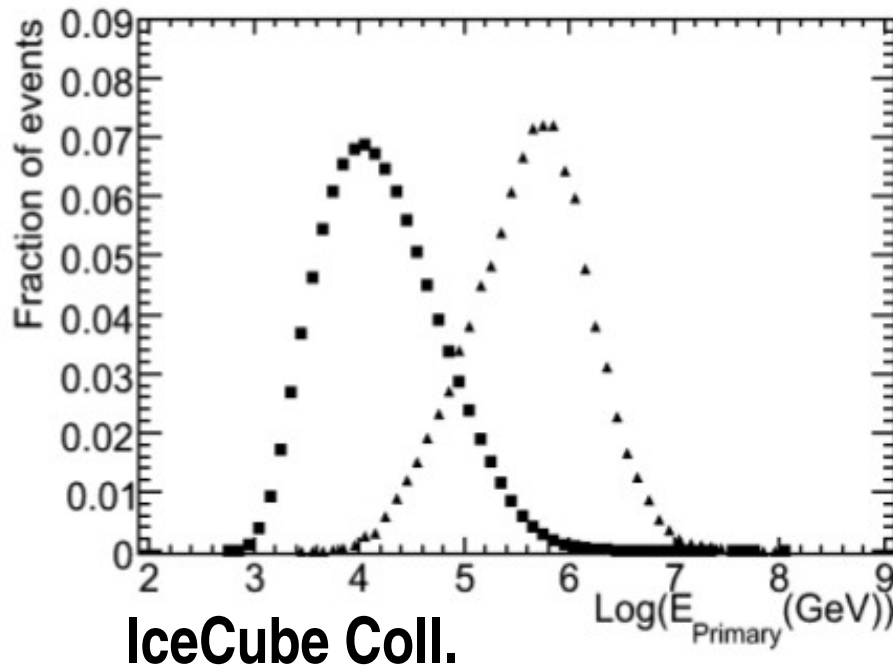
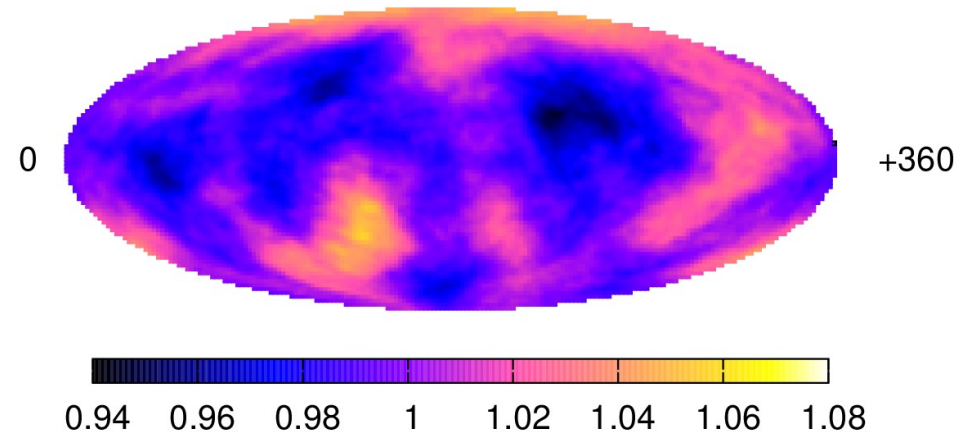
Amplitude ~ 6 % here

Dipole / Smaller scales

90°smoothing



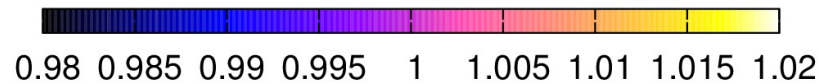
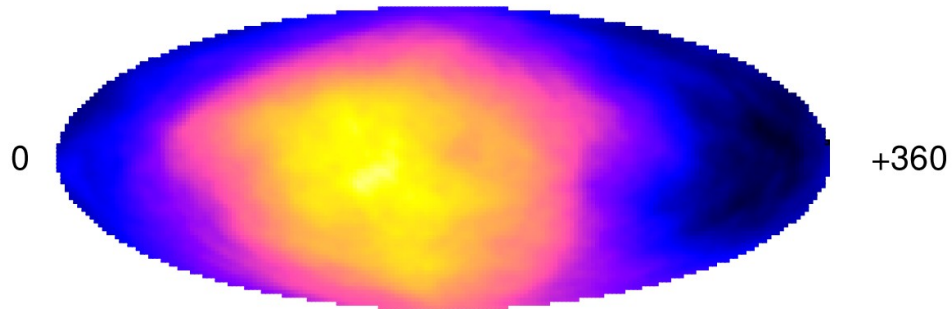
20°smoothing - Dipole



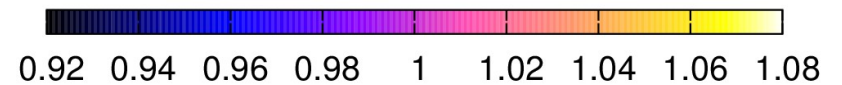
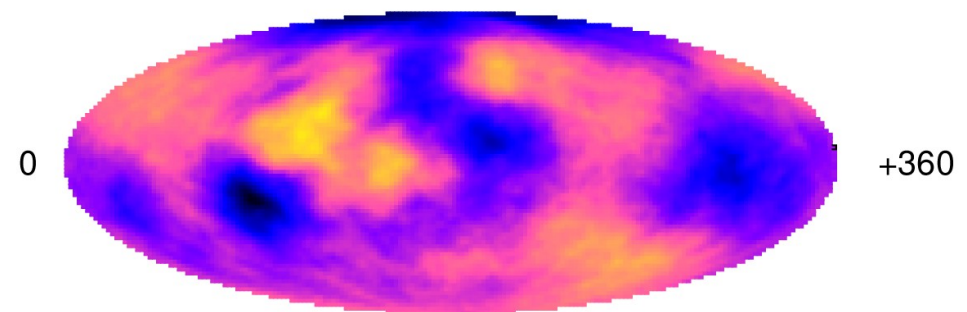
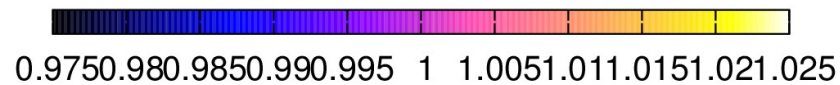
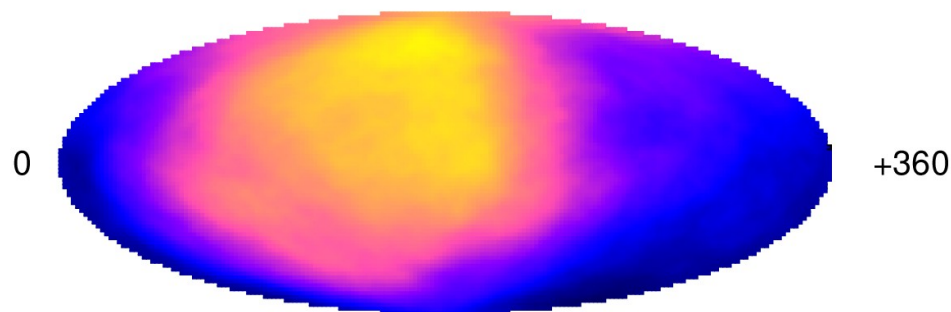
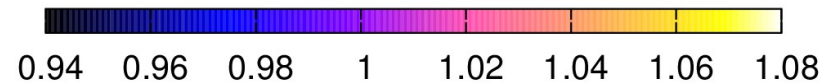
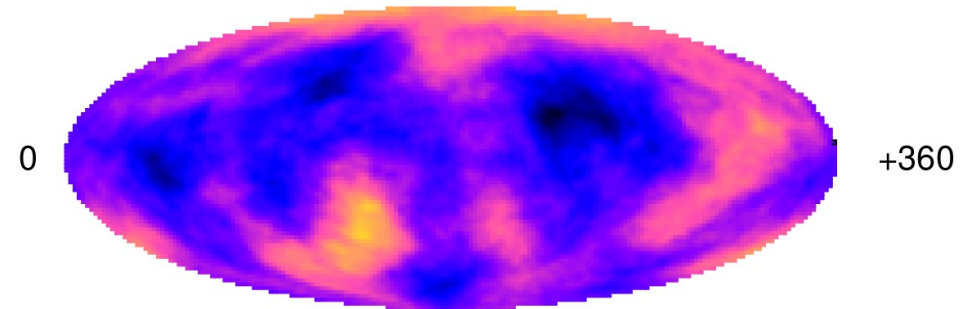
**Different E/Z add
non-constructively
→ Smaller (~10x)
(Giacinti, PPCF '13)**

Dipole / Smaller scales

90°smoothing

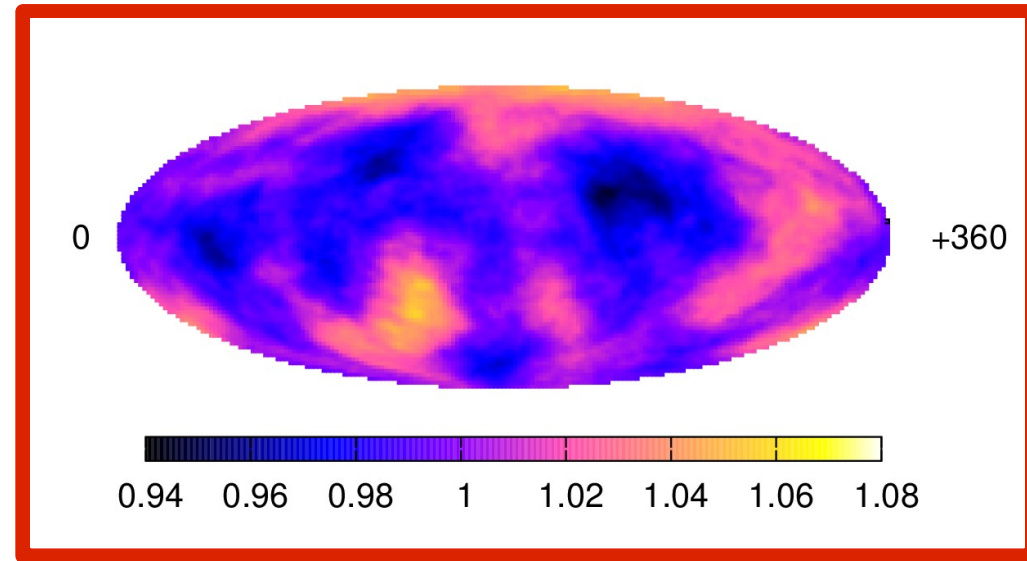


20°smoothing - Dipole

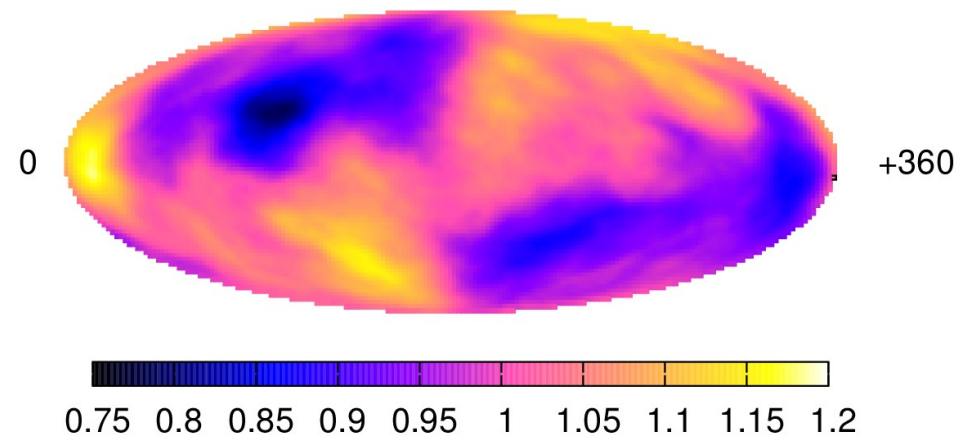


Dependence on the CR rigidity E/Z

$E/Z = 10^{16}$ eV

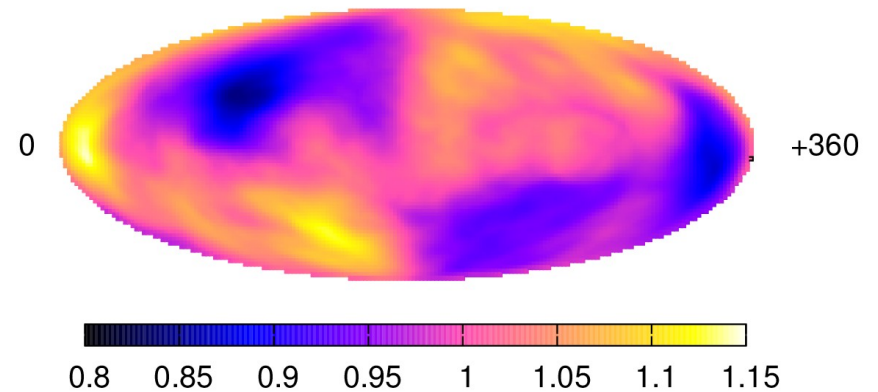
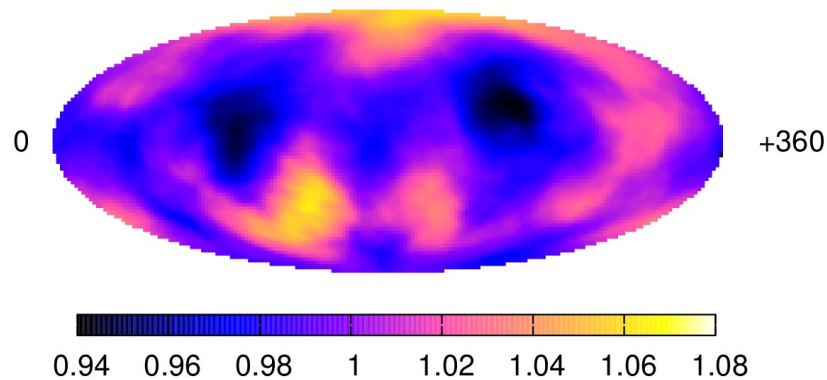
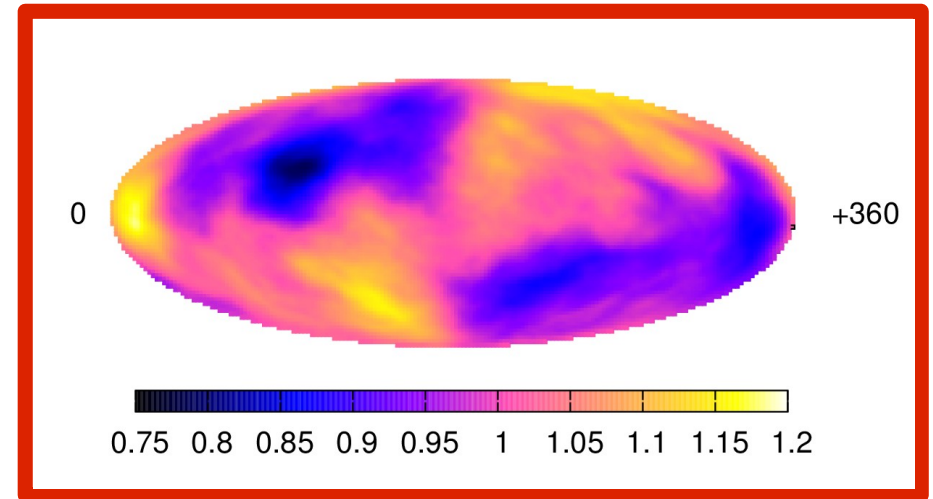
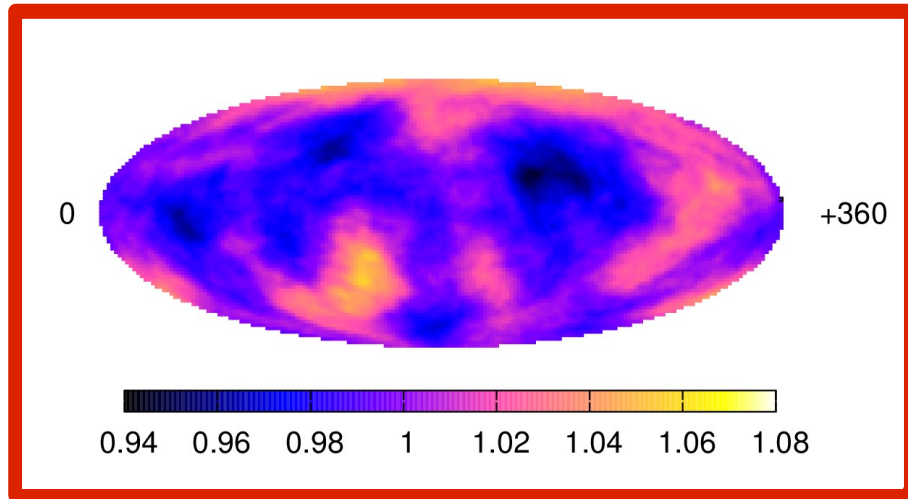


$E/Z = 5 \times 10^{16}$ eV



Dependence on the 'domain' size

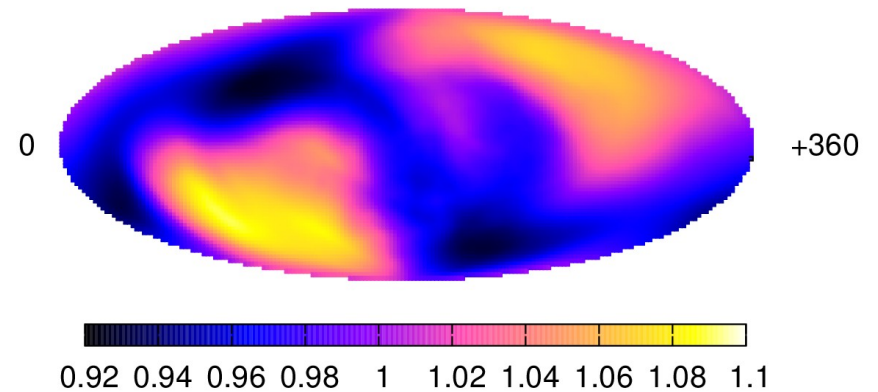
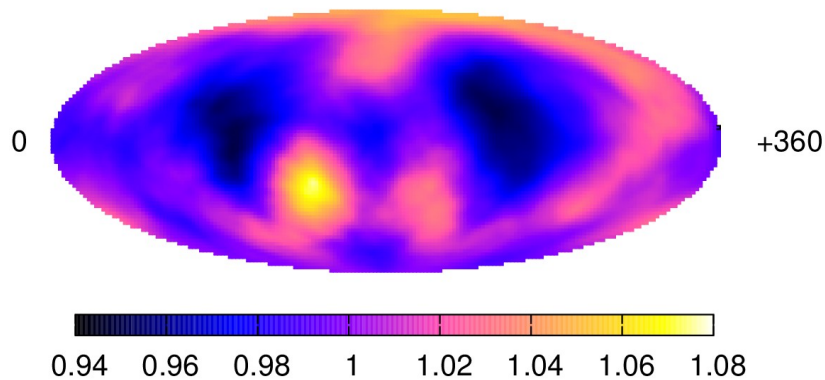
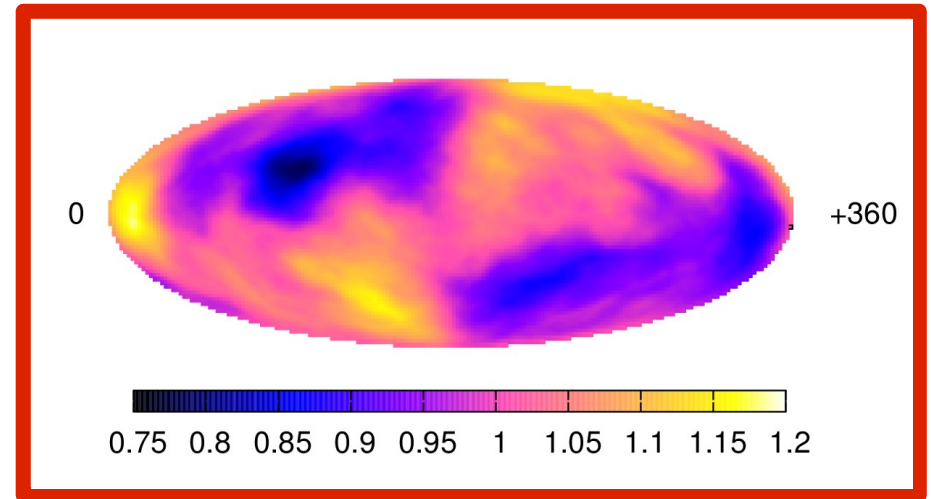
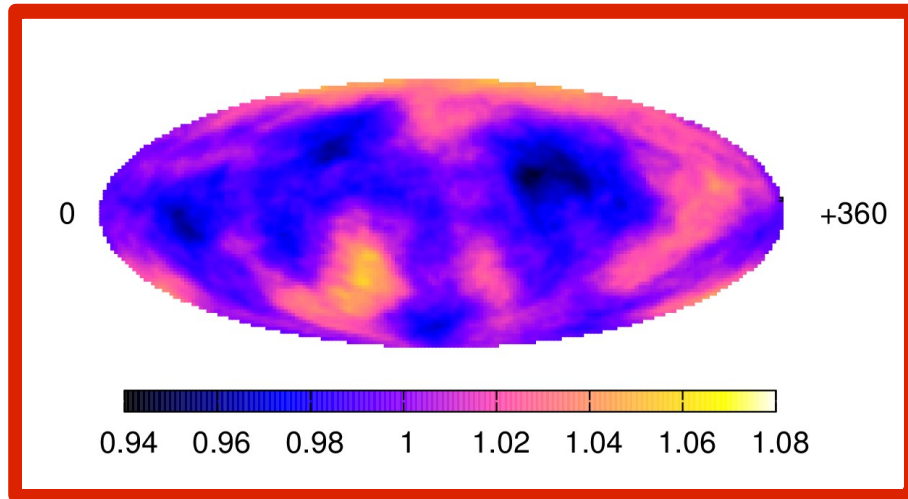
250 pc



100 pc

Dependence on the 'domain' size

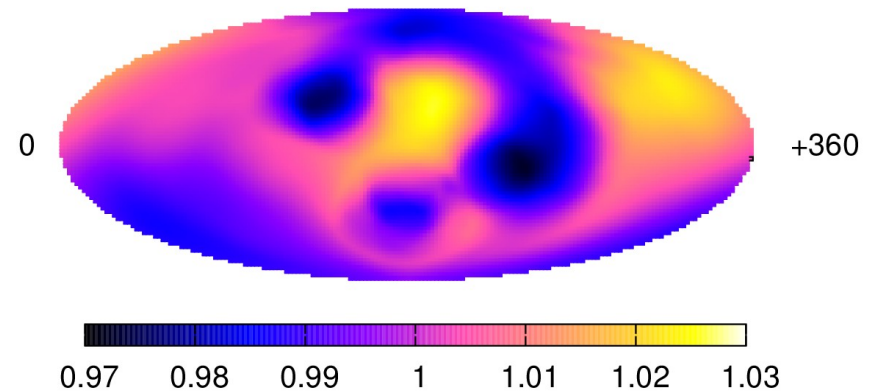
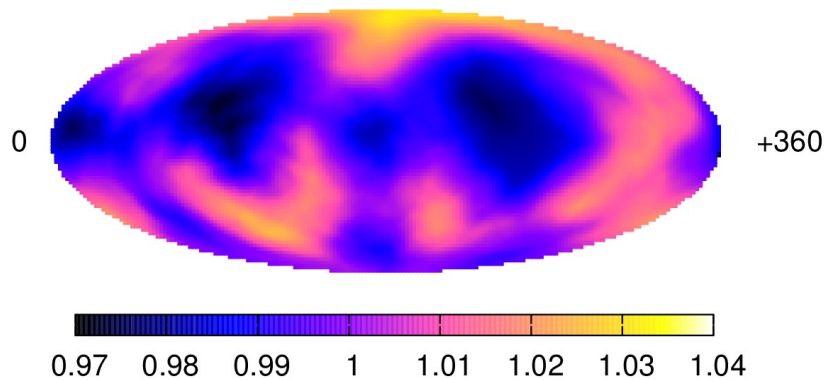
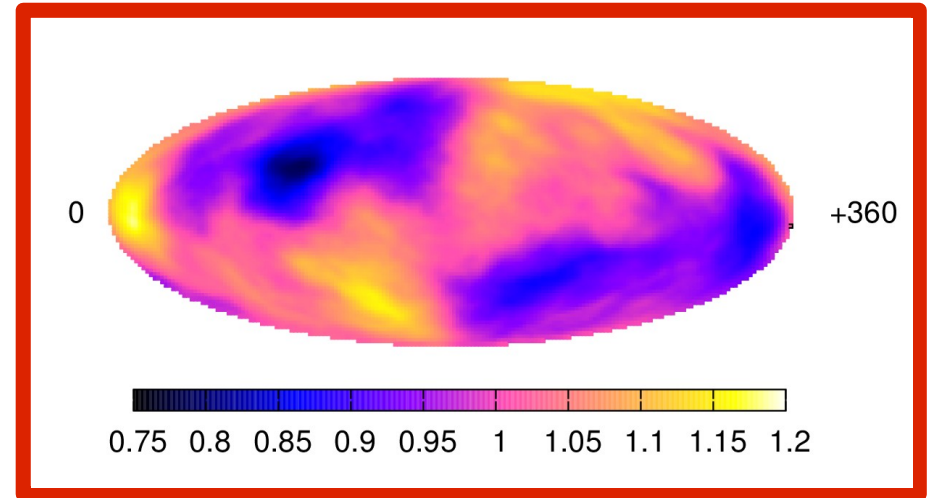
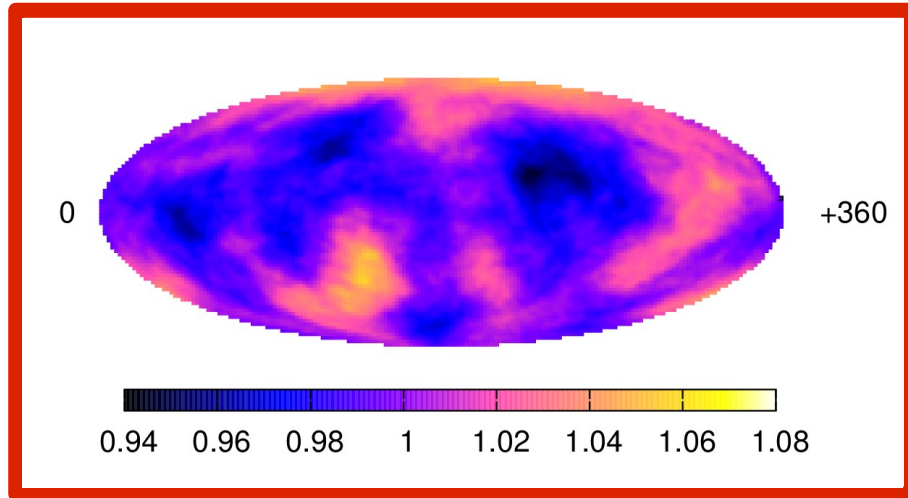
250 pc



50 pc

Dependence on the 'domain' size

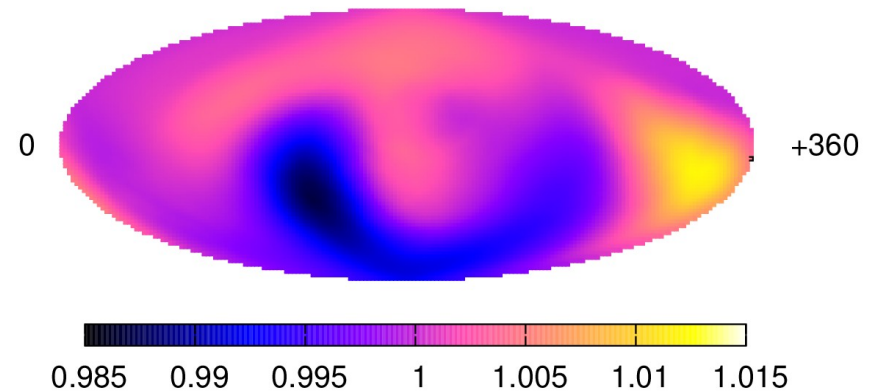
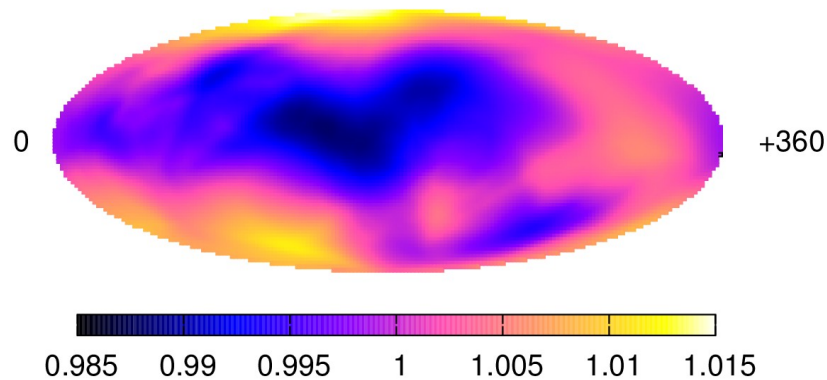
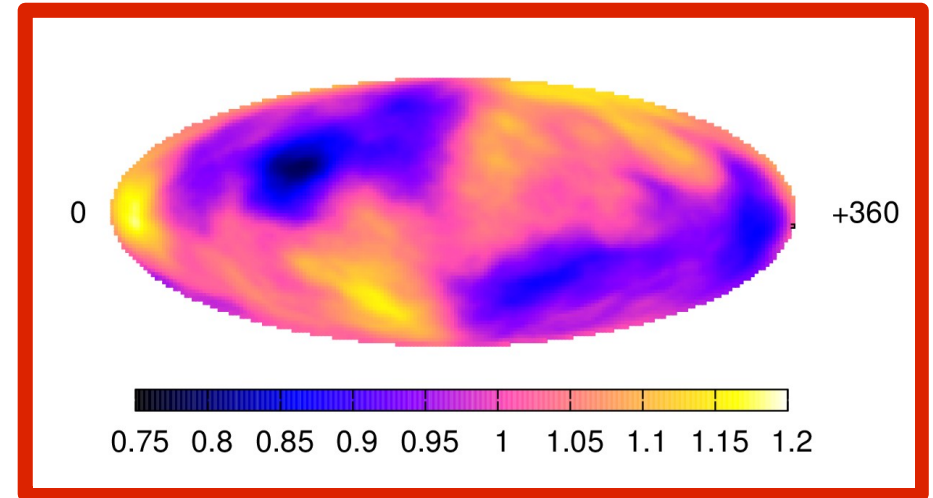
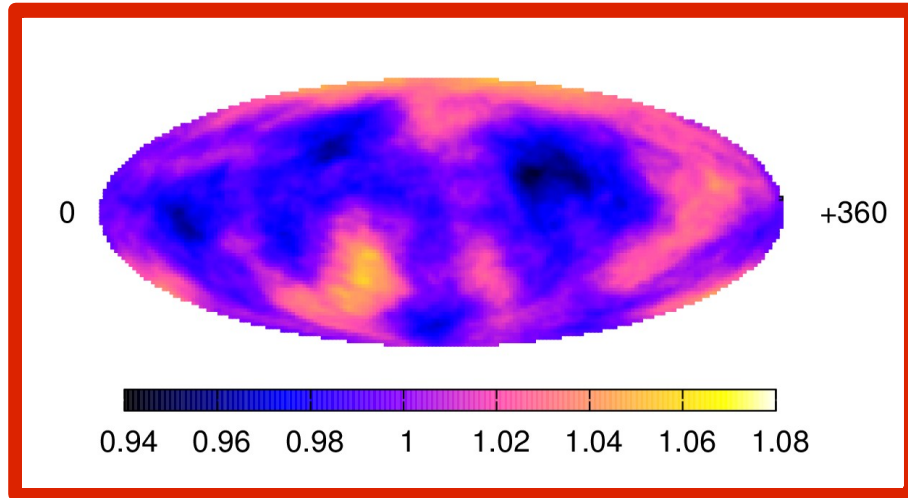
250 pc



25 pc

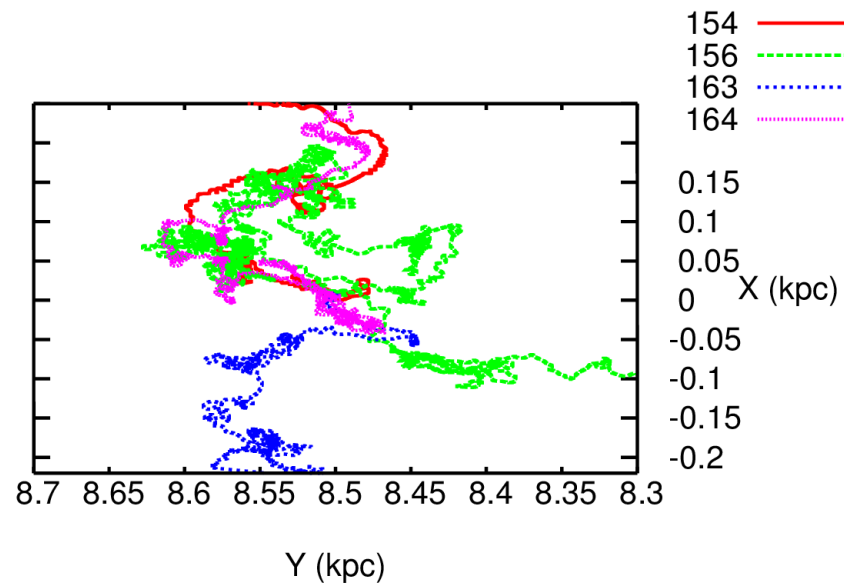
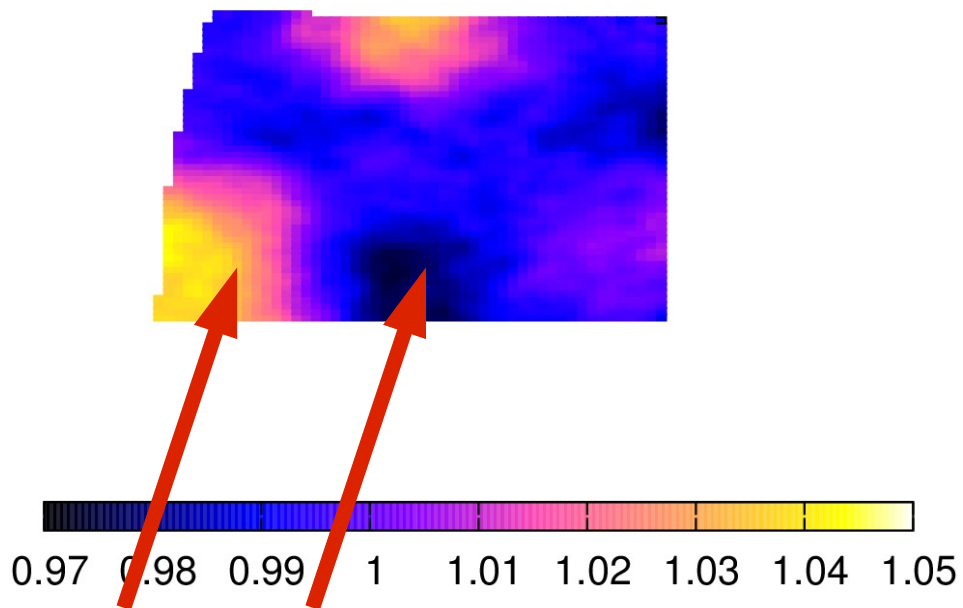
Dependence on the 'domain' size

250 pc



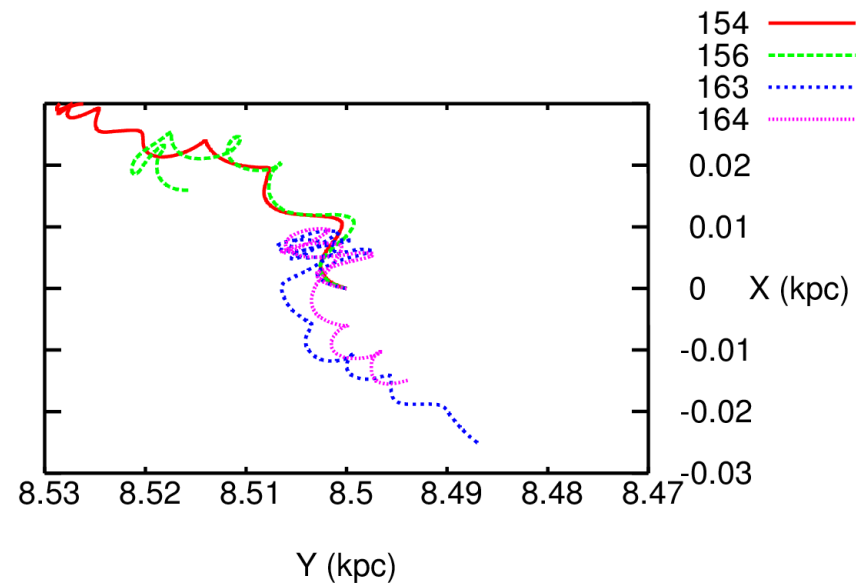
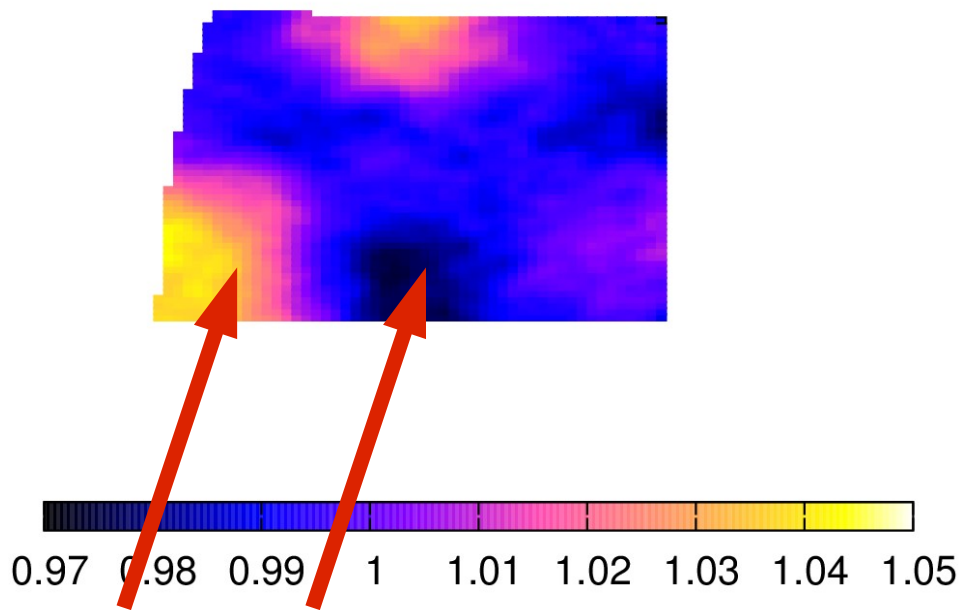
10 pc

Local trajectories



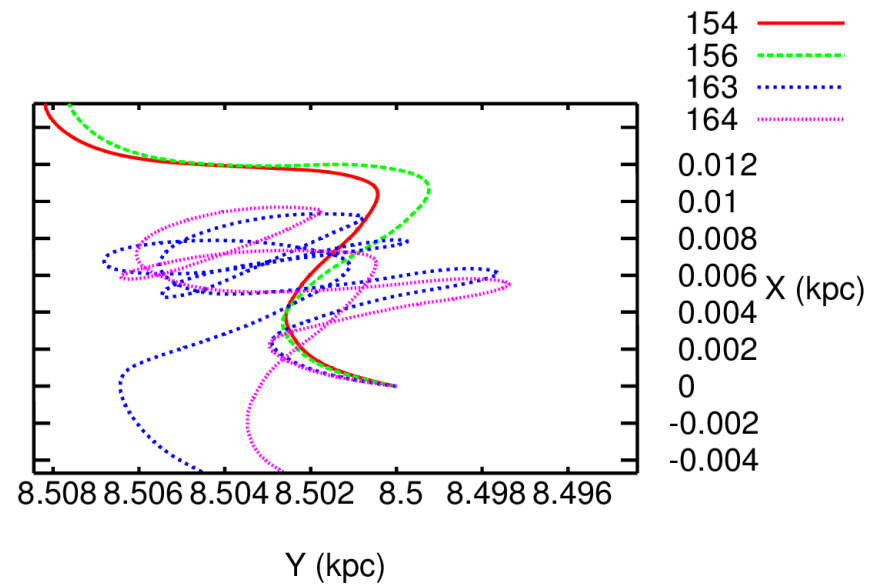
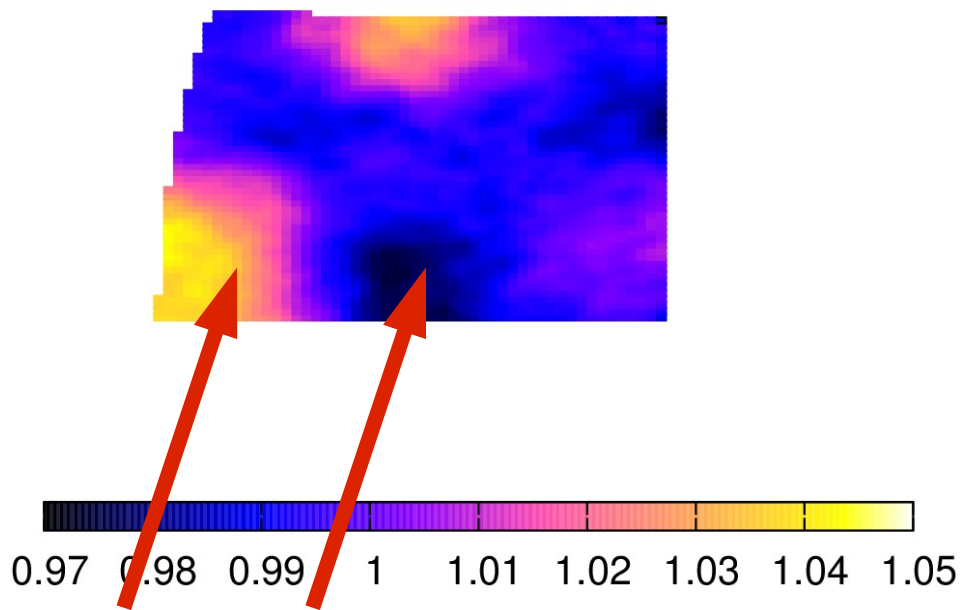
←
grad n

Local trajectories



←
grad n

Local trajectories



←
grad n

Heliospheric B fields at ~ TeV energies ?

At PeV energies : probe ISMF up to ~10s of pc

At 1 - 10 TeV energies : Same argument ; may start to probe fields in the heliosphere, see P. Desiati and A. Lazarian, Astrophys. J. 762, 44 (2013).

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doi:10.1088/0004-637X/762/1/44

ANISOTROPY OF TeV COSMIC RAYS AND OUTER HELIOSPHERIC BOUNDARIES

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ABSTRACT

in the energy range from about tens of GeV to several hundreds of TeV are o
gy-dependent anisotropy of order 0.01%–0.1% and a consistent topology that appe
her energy. The nearest and most recent galactic cosmic-ray sources might stoch
on and possibly explain a change in orientation of the anisotropy as a function of en
roximation is not able to explain its non-dipolar structure and, in particular, the signi
lar scale features. Particle propagation within the mean free path in the local interste
role in determining the properties of galactic cosmic rays, such as their arrival distrib

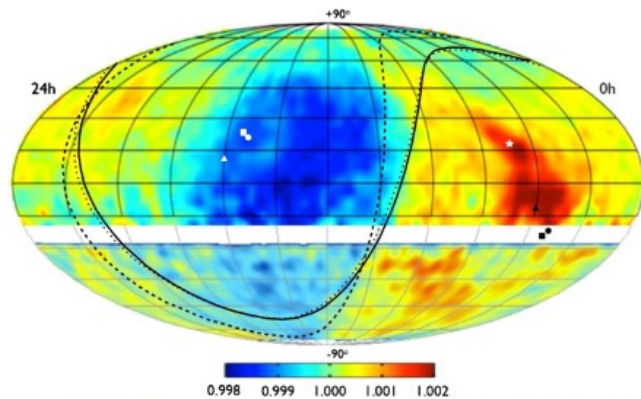


Figure 1. Map in equatorial coordinates constructed by combining relative intensity distributions of cosmic-ray counts independently normalized within Online-only material

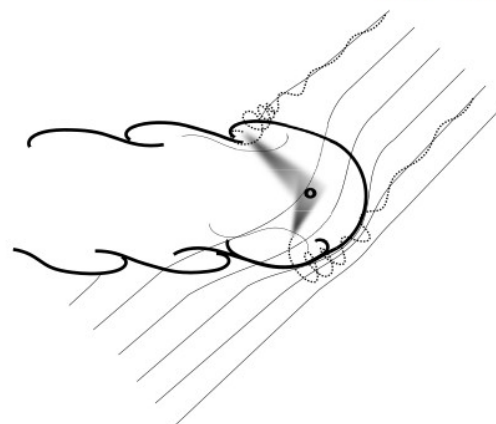


Figure 4. Illustration of the turbulent ripples at large scale on the outer

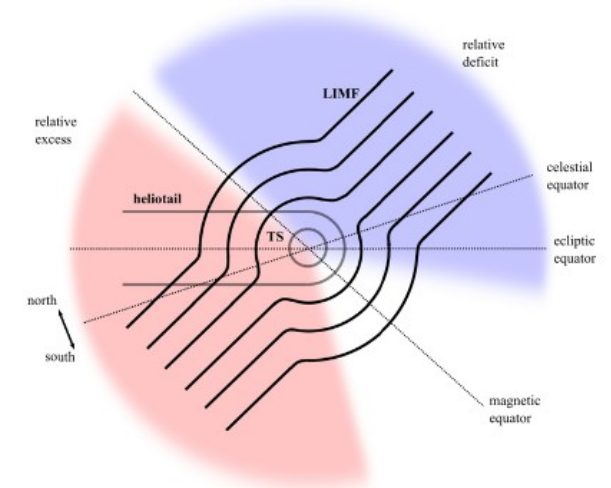


Figure 2. Schematic representation of the elongated heliosphere with respect to the LIMF drawing around it. The arrival distribution of cosmic rays with

Heliospheric B fields at ~ TeV energies ?

Alternative possibility at 1 – 10 TeV : heliospheric electric fields , see L. Drury, arXiv:1305.6752 [astro-ph.HE].

33RD INTERNATIONAL COSMIC RAY CONFERENCE, RIO DE JANEIRO 2013
THE ASTROPARTICLE PHYSICS CONFERENCE

ICRC
2013

The problem of small angular scale structure in the cosmic ray anisotropy data

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Abstract: Recent observations have revealed structure on small angular scales in the anisotropy data of cosmic rays at multi TeV energies. Even though the absolute amplitudes are very small, these effects are somewhat surprising and a wide range of possible causes have been discussed. A possible origin associated with heliospheric electric fields is proposed.

Keywords: anisotropies

1 Introduction

Our knowledge of directional variations (usually referred to as anisotropies) in the flux of cosmic rays reaching the Earth at TeV energies and above has greatly improved in recent years, largely because of the need to control the charged cosmic ray background in experiments searching

targets, have no difficulty with but fail because they require densities in close proximity to those by astronomical observations [1]. A recent fascinating but high is that the anisotropy might be due

s of a few nT. Thus above about a TeV cosmic ray particles are expected to penetrate the heliosphere with relatively little deflection. Such particles arriving at the Earth will thus have seen an effective potential shift due to induction fields in the heliosphere of

$$\int -V \times B \cdot ds \quad (2)$$

where the integral is taken along the trajectory of the incoming particle out from Earth through the heliosphere and into the local ISM. It is important that all these particles arriving in a given direction and energy band will have essentially the same trajectory through the heliosphere. If

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Refer

- [1] G.
- [2] M
- [3] A.
- [4] A.
- [5] S.
- [6] R.
- [7] R.
- [8] M

Conclusions and perspectives

Medium and small scale CRA must be present because of the MF structure within $\sim$ a CR mean free path

---> May become (one of) the best way(s) to probe the structure of local magnetic fields in the ISM in the future.

---> Once this question is solved : Potential for other interesting discoveries.

NEXT WORKS : Local MFs (ISM, Heliosphere) $\rightarrow$ CRA maps

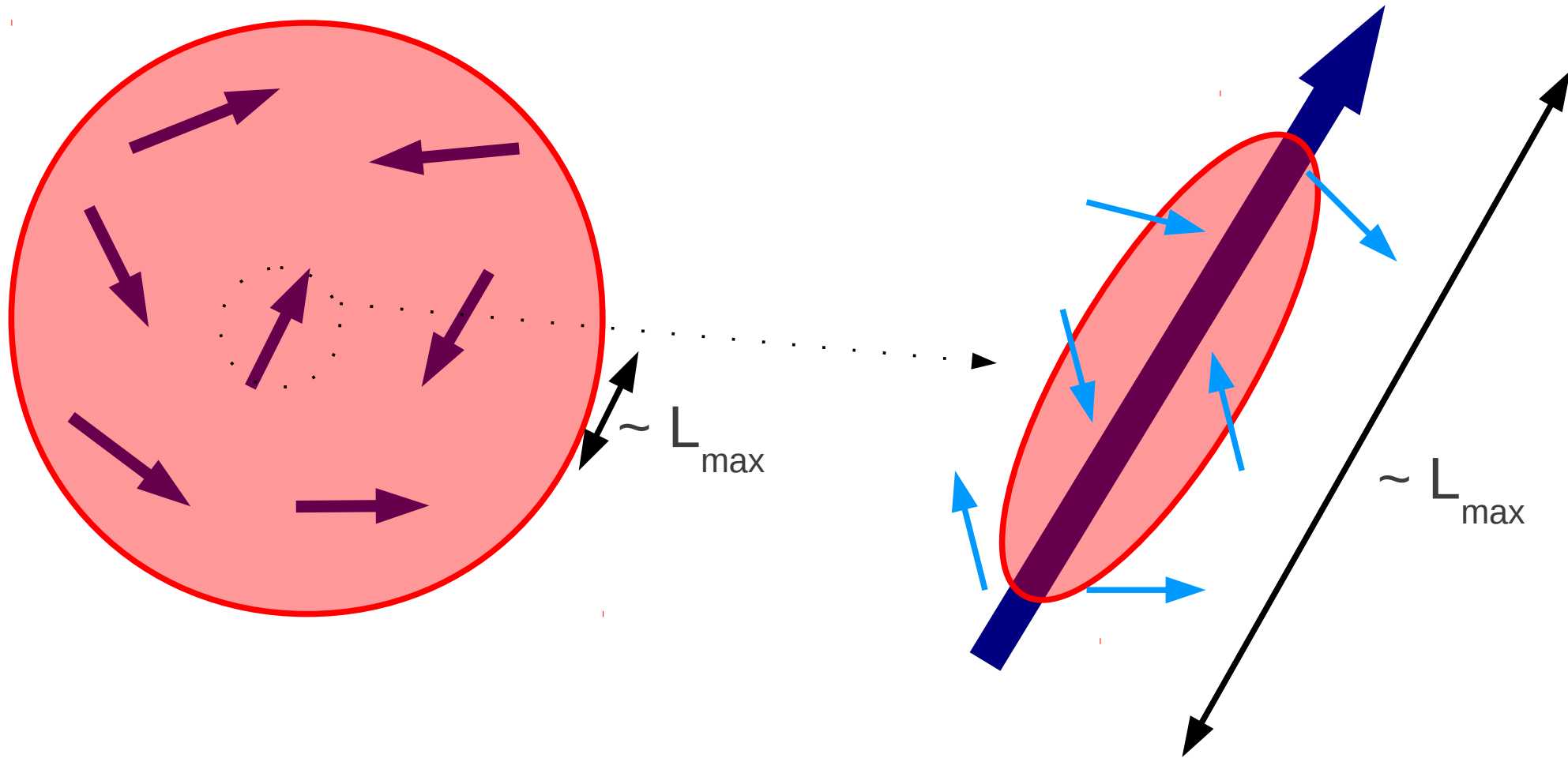
Generated magnetic turbulence

$$L_{\text{max}}, L_{\text{c}} \gg r_L \text{ (TeV - PeV)}$$

$|B| : 0 - 8 \mu\text{G}$

100 pc

***Anisotropic* diffusion in *isotropic* turbulence**



For $\Delta t \sim 1 \text{ kyr}$, $v_{10\text{km/s}} \Delta t \sim 0.01 \text{ pc} \ll (6D_{1\text{PeV}} \Delta t)^{1/2} \sim \text{few } 10\text{s pc}$

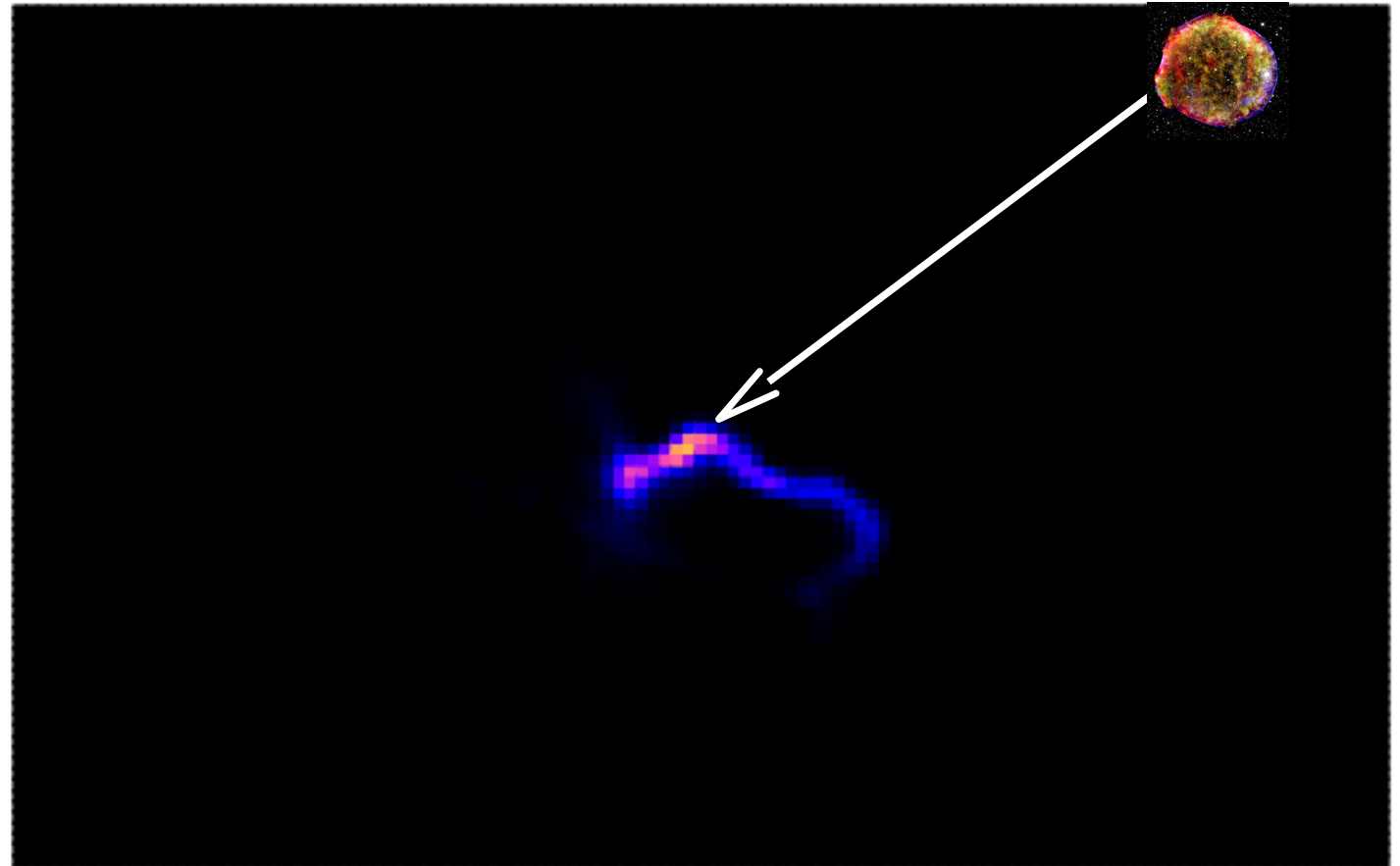
Results of numerical simulations

$E/Z=1\text{PeV}$, Kolmogorov spectrum, $L_{\text{max}}=150\text{pc}$

$t = 1\text{kyr}$

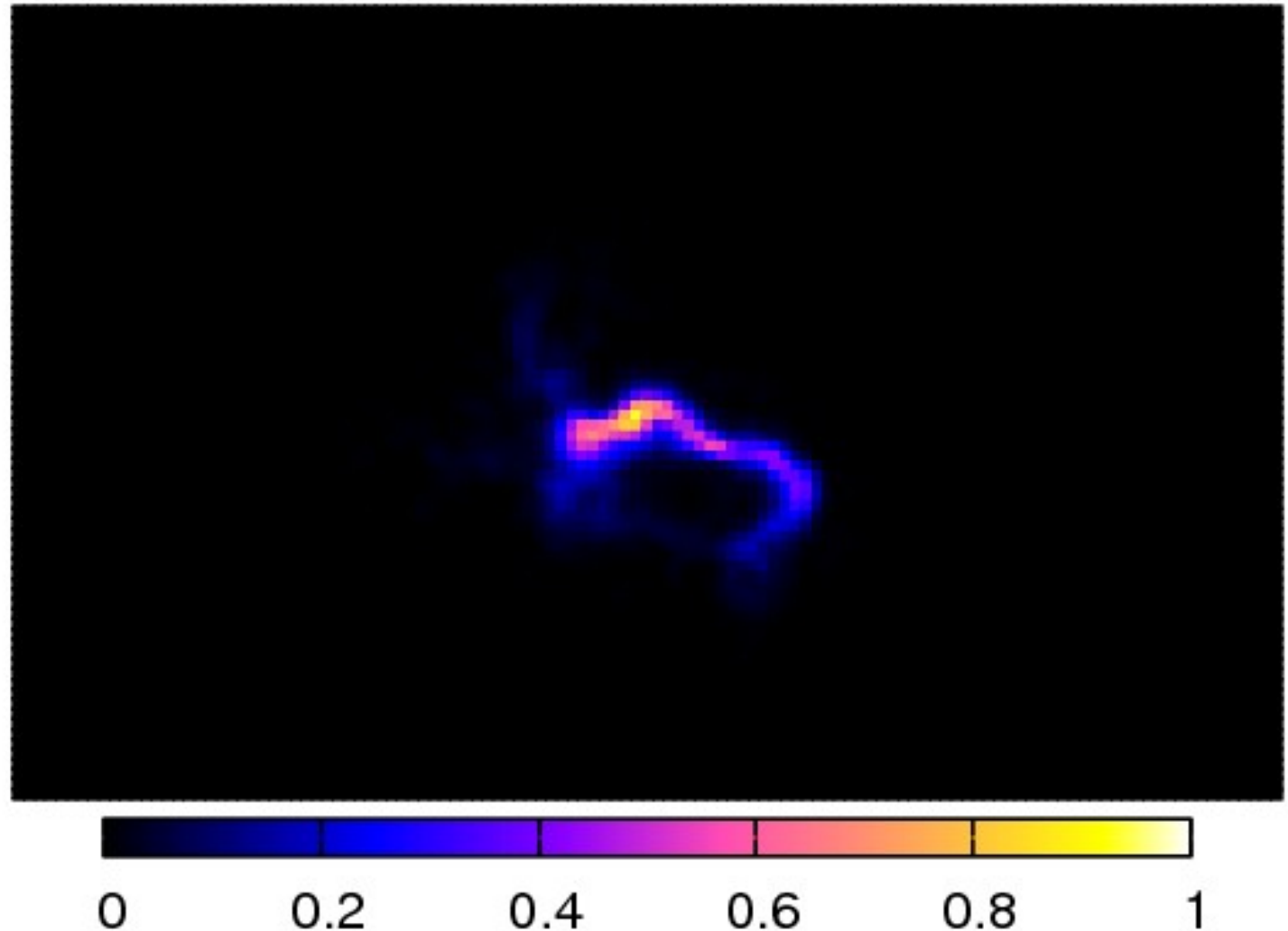
200 pc

$n_{\text{CR}}(X,Y)$



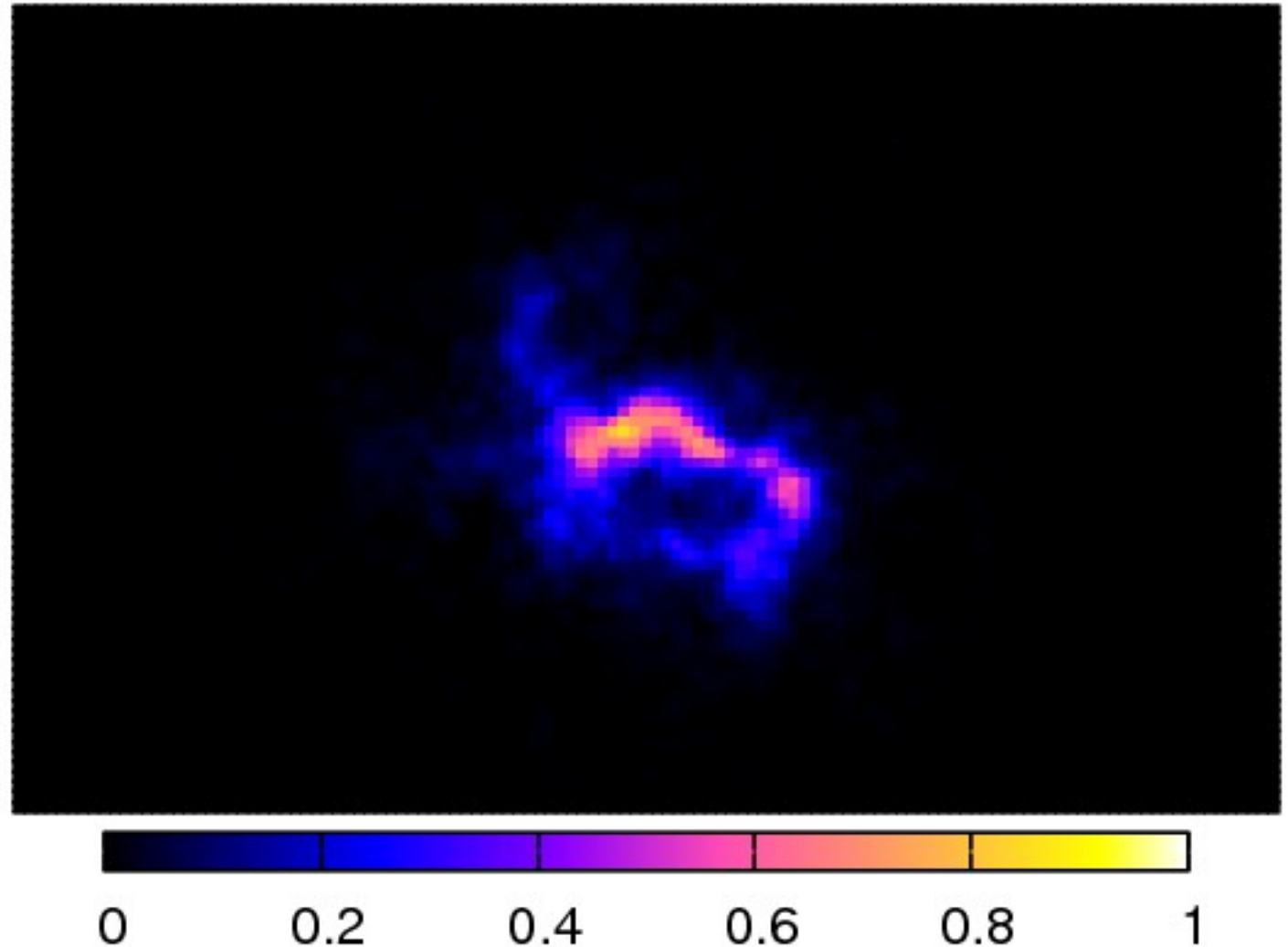
Results of numerical simulations

$t = 2\text{kyr}$



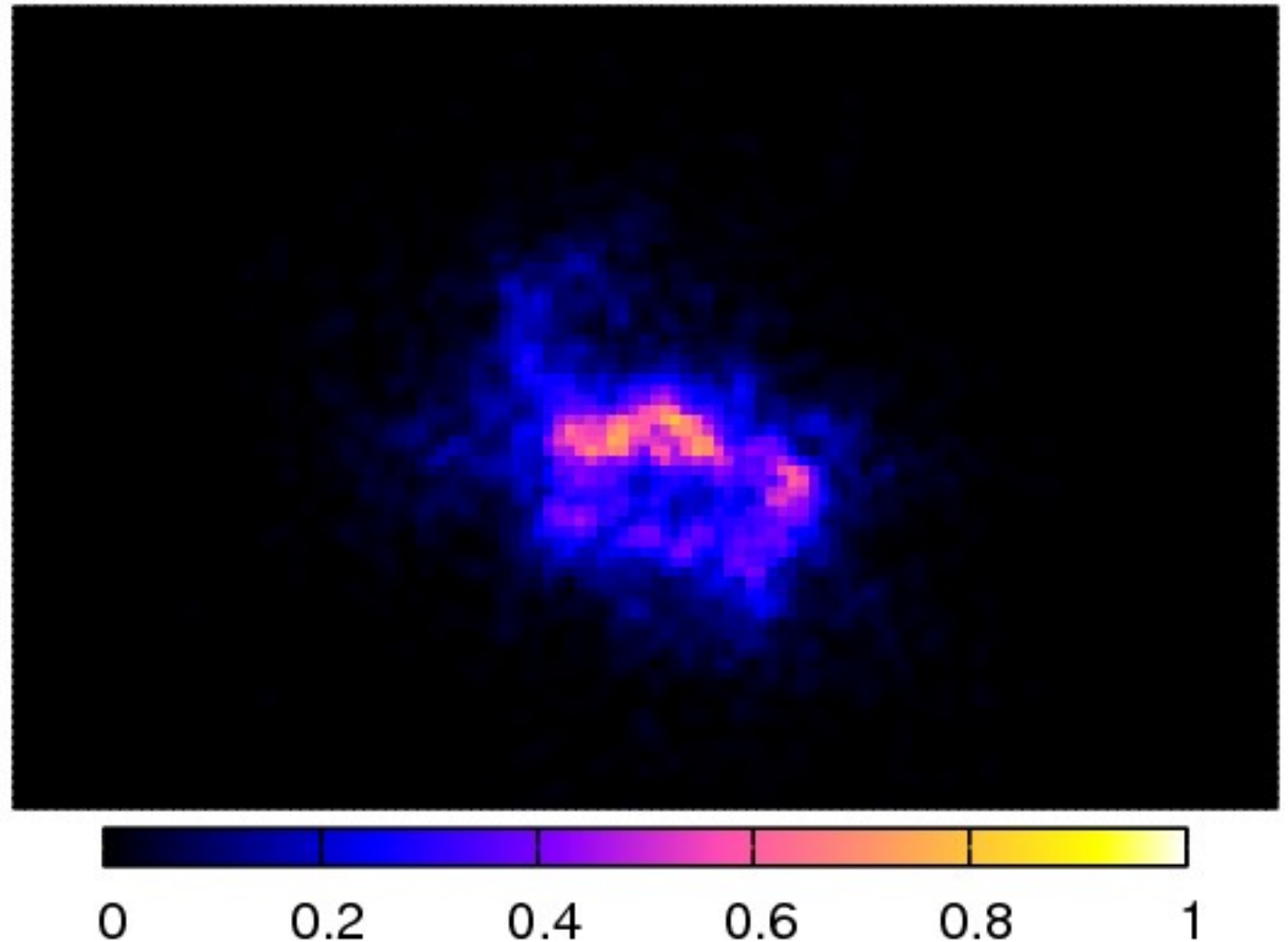
Results of numerical simulations

$t = 4\text{kyr}$



Results of numerical simulations

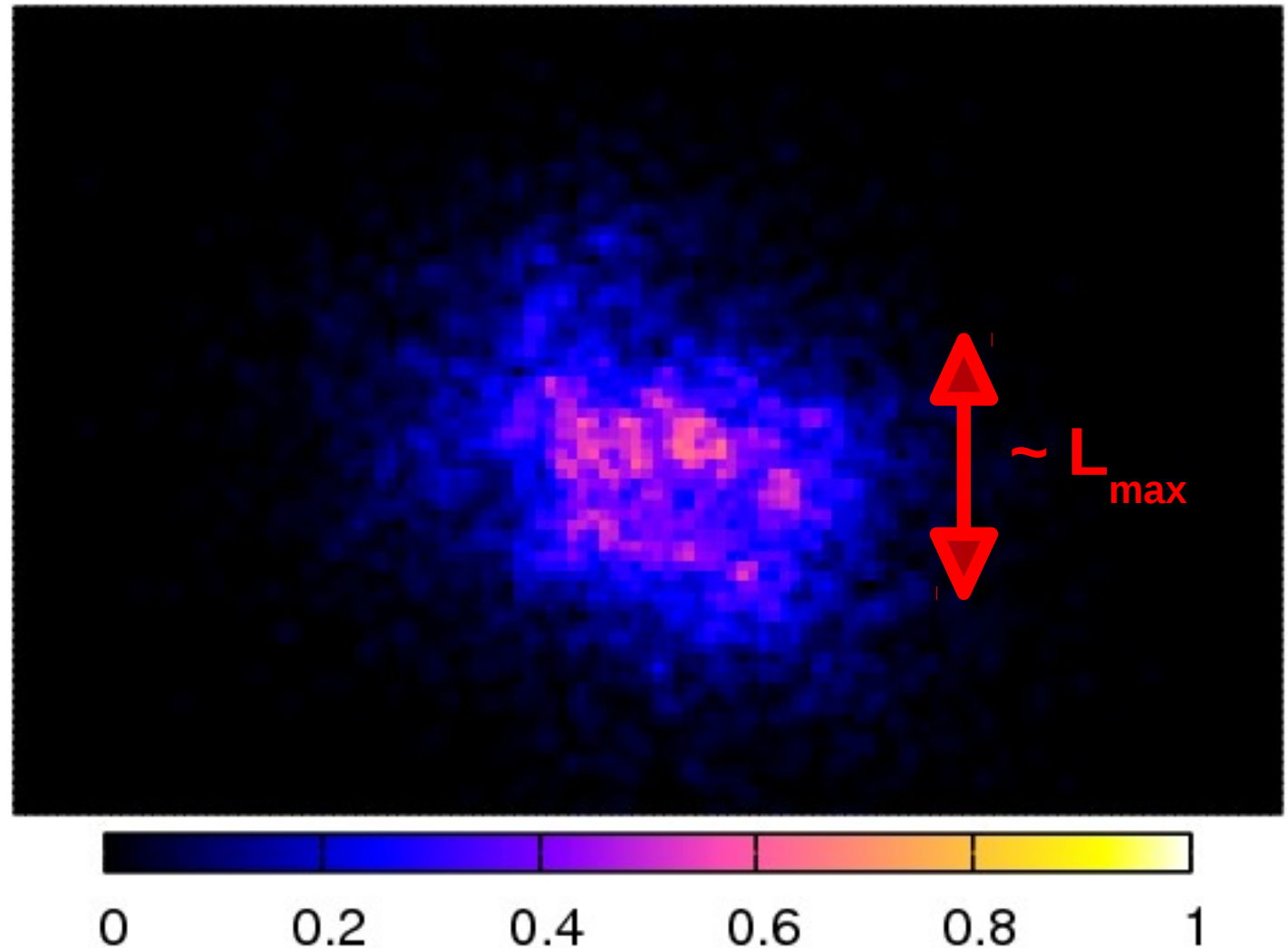
$t = 6\text{kyr}$



Results of numerical simulations

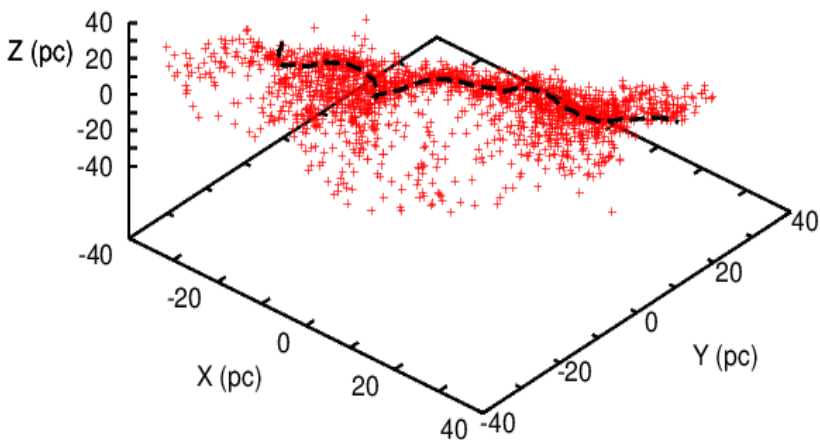
... then starts to tend towards the $r \propto t^{1/2}$ behaviour

$t = 10\text{kyr}$



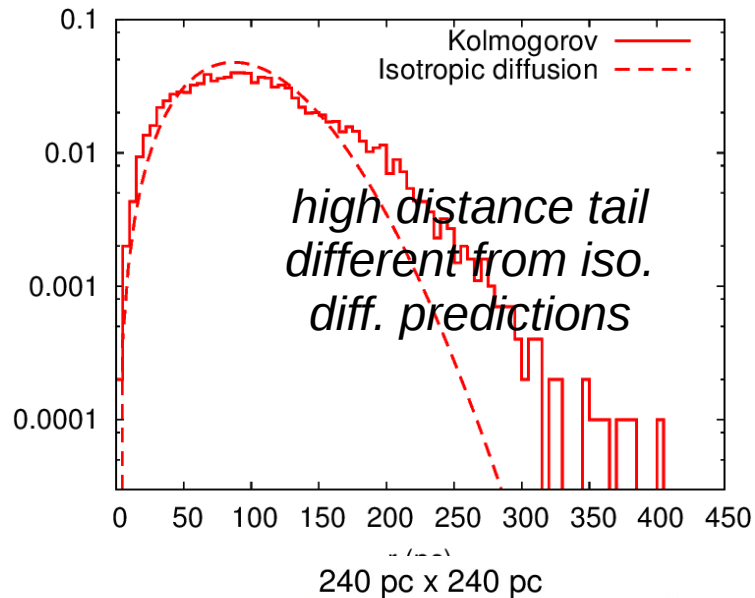
CRs and local configuration of the magnetic field

- $t < \sim t^* / 10$:



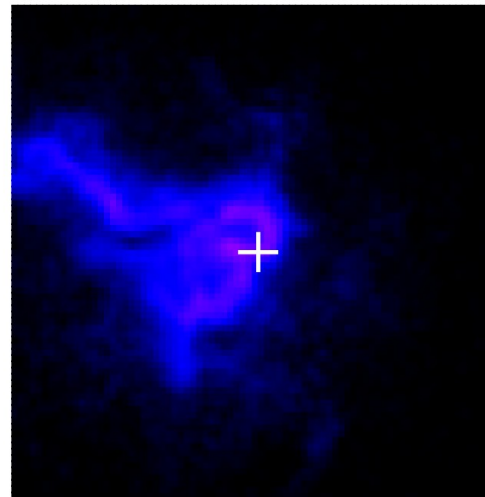
**Nearby field lines
coherent over $\sim L_c$:
CRs in a flux tube**

- $t \sim t^*$:

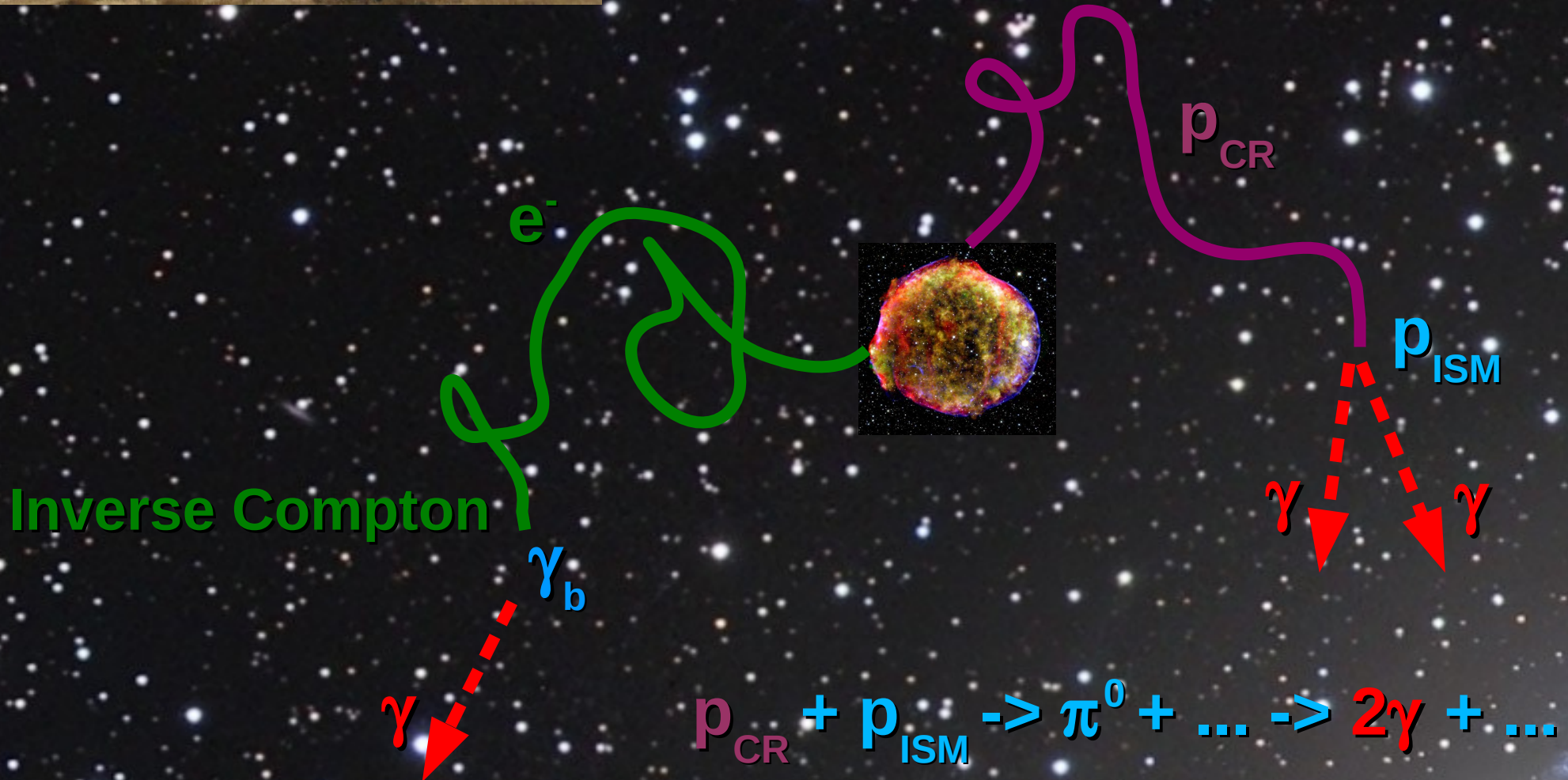
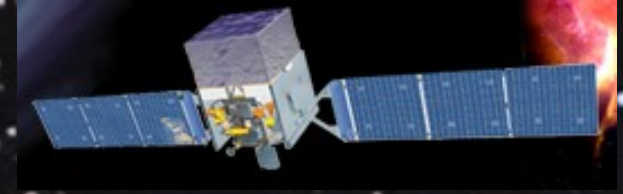


- $t > \sim 10 t^*$:

**No visible
difference**

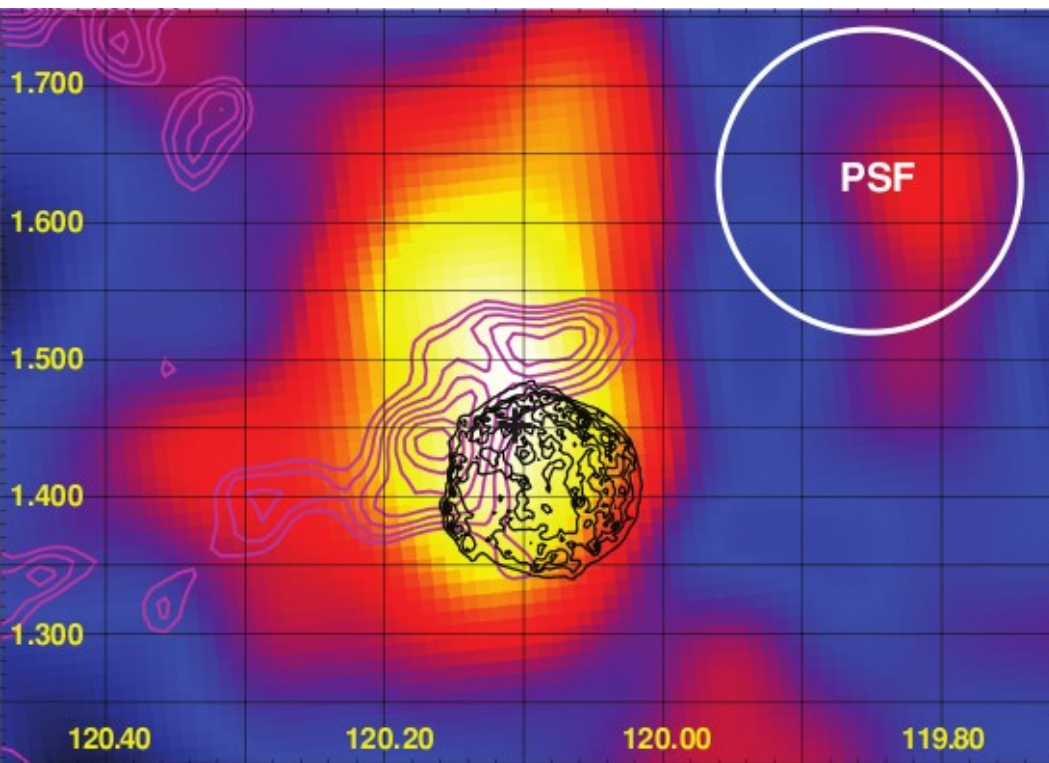


γ ray emission around sources



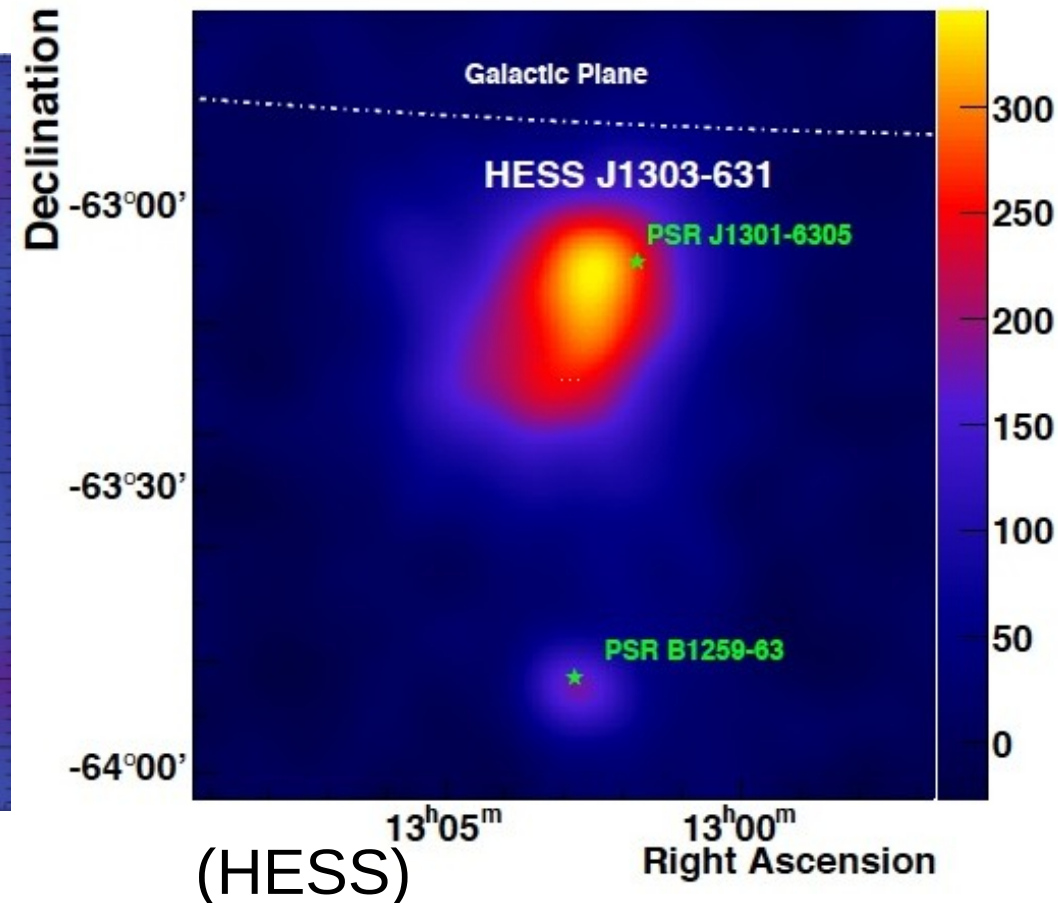
Observations : First hints at our findings ?

- Irregular images :



(Tycho - VERITAS)

-Offset from the source :



(HESS)

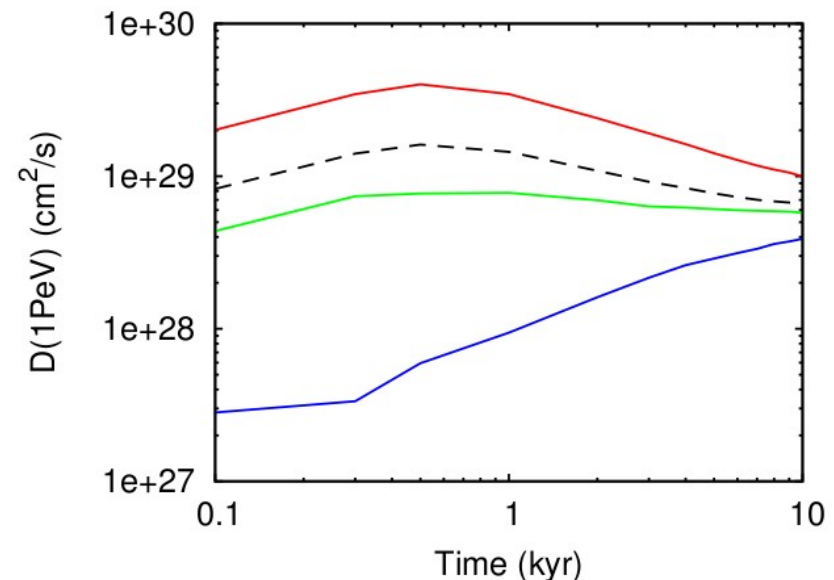
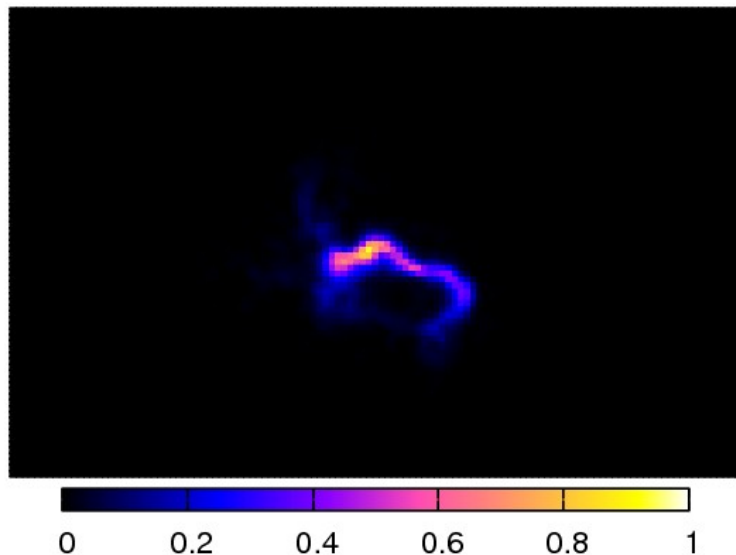
- Anisotropic diffusion around SNR W28 ? :
Nava & Gabici MNRAS '12

Eigenvalues of the diffusion tensor

Inject N particles at $\mathbf{x} = \mathbf{0}$ in **one single** B field realization ' b '

Calculate
$$D_{ij}^{(b)} = \frac{1}{N} \sum_{a=1}^N \frac{x_i^{(a)} x_j^{(a)}}{2t} \quad (i, j = X, Y, Z)$$

Compute its eigenvalues : $d_1^{(b)} < d_2^{(b)} < d_3^{(b)}$

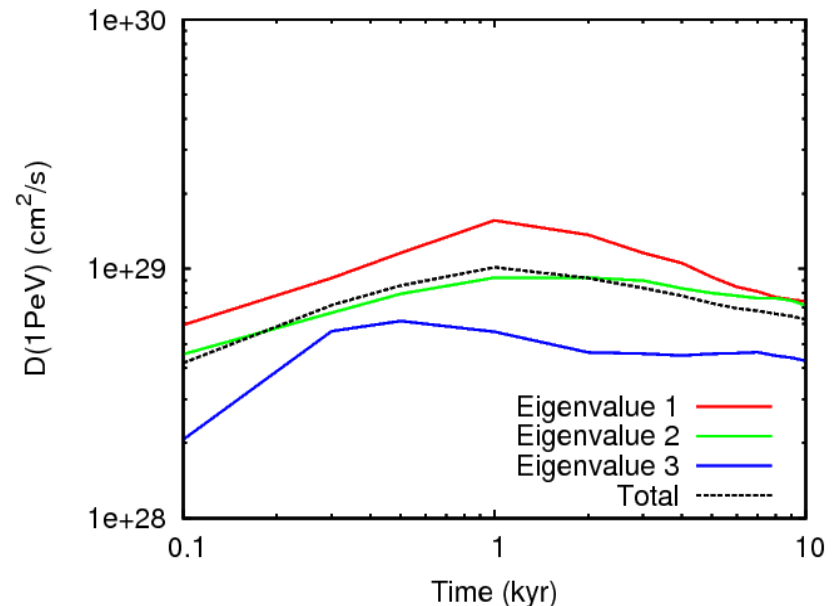
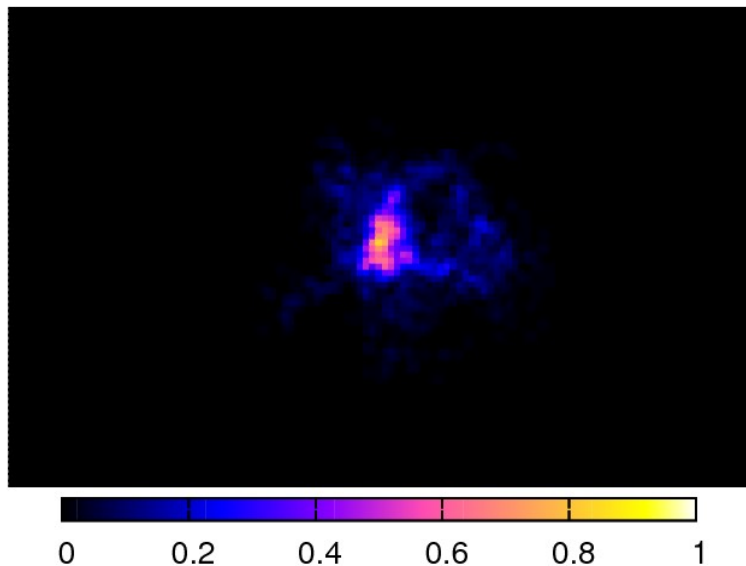


Eigenvalues of the diffusion tensor

Inject N particles at $\mathbf{x} = \mathbf{0}$ in **one single** B field realization 'b'

Calculate
$$D_{ij}^{(b)} = \frac{1}{N} \sum_{a=1}^N \frac{x_i^{(a)} x_j^{(a)}}{2t} \quad (i, j = X, Y, Z)$$

Compute its eigenvalues : $d_1^{(b)} < d_2^{(b)} < d_3^{(b)}$



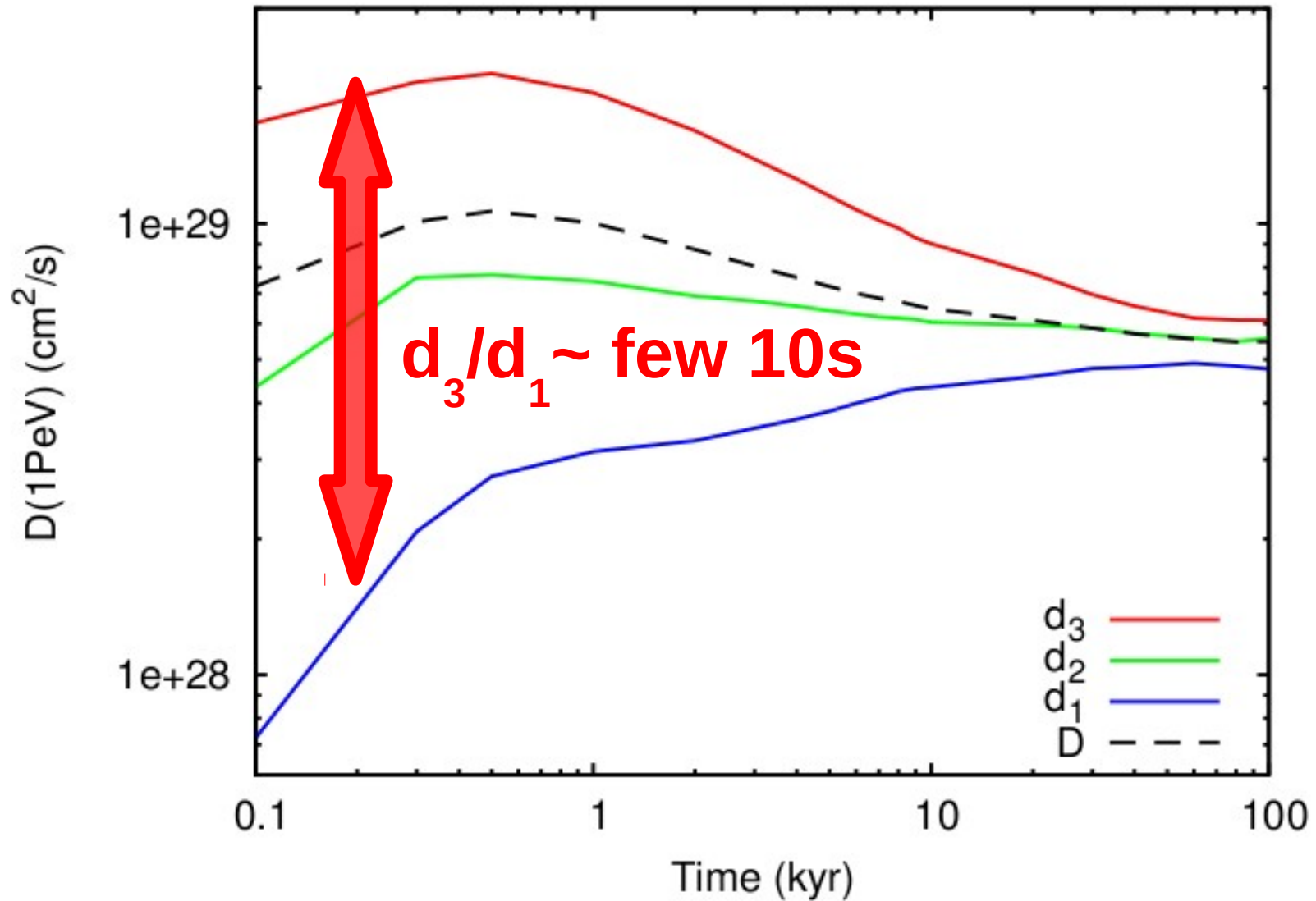
Eigenvalues of the diffusion tensor

Inject

Calcul

Comp

Average



n 'b'

)