Cosmic ray sources

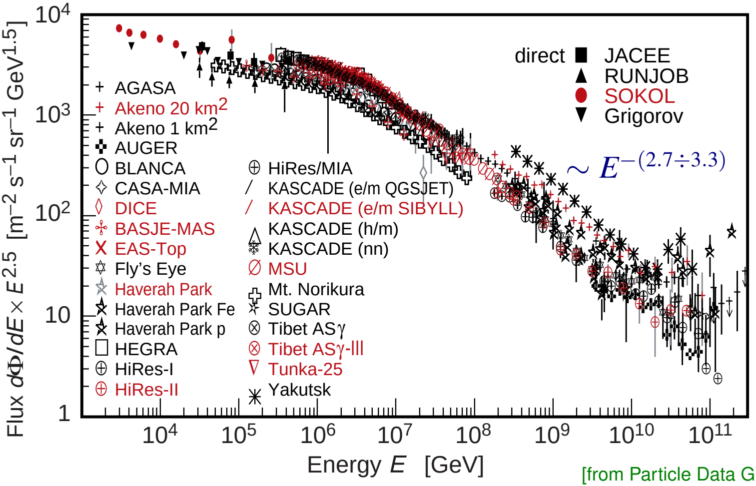
Markus Ahlers

University of Wisconsin-Madison & WIPAC

Bootcamp 2012
June 13, 2012, Madison

The cosmic leg

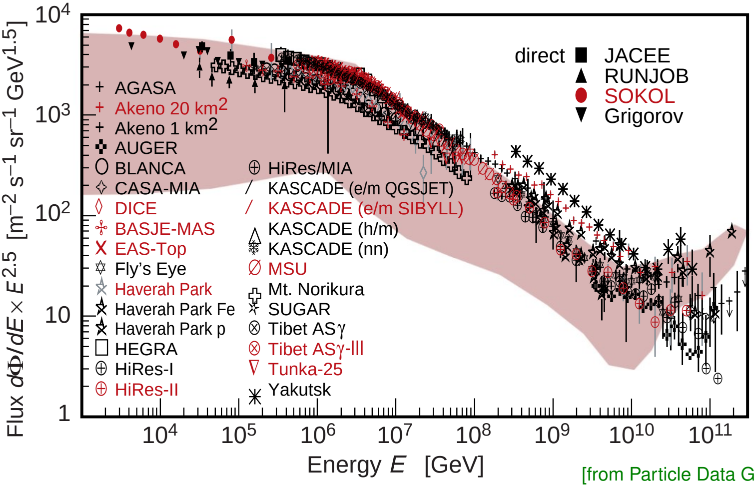
The all-particle spectrum (as $E^{2.5} \times J$) of cosmic rays.



[from Particle Data Group '05]

The cosmic leg

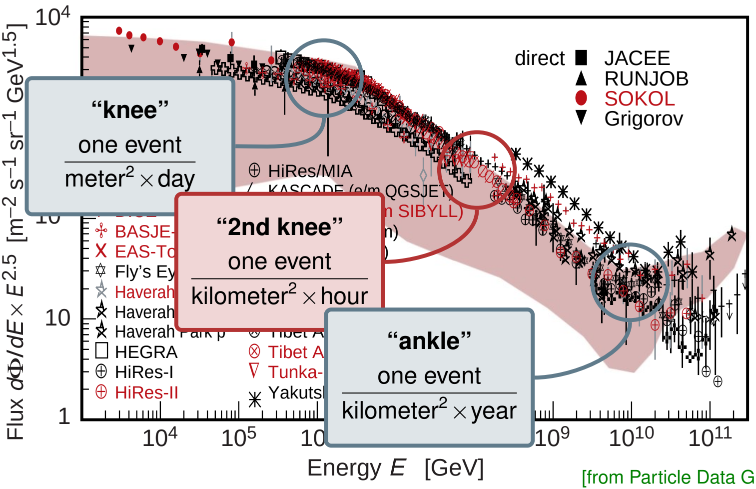
The all-particle spectrum (as $E^{2.5} \times J$) of cosmic rays.



[from Particle Data Group '05]

The cosmic leg

The all-particle spectrum (as $E^{2.5} \times J$) of cosmic rays.



[from Particle Data Group ’05]

A word on scale…

10^6 eV $= 1$ MeV	$m_e \simeq 0.5$ MeV
10^9 eV $= 1$ GeV	$m_p \simeq 1$ GeV
10^{12} eV $= 1$ TeV	$\sqrt{s_{\rm LHC}} \simeq 7$ TeV
10^{15} eV $= 1$ PeV	$E_{\rm max,Earth} \simeq 2$ PeV
10^{18} eV $= 1$ EeV	?
10^{21} eV $= 1$ ZeV	?????

Zetta-electronvolt?

$$m_{\text{ball}} \simeq 0.41\text{kg}$$

$$v \simeq 120\text{km/h}$$

$$E_{\text{kin}} = \frac{1}{2} m_{\text{ball}} v^2 = 1 \text{ ZeV}$$

A word on units...

- **natural units**: $c = \hbar = k_B = \epsilon_0 = \mu_0 = 1$
- conversion factors:

$$\hbar c \simeq 2 \times 10^{-7}\mathrm{eVm} \qquad c \simeq 3 \times 10^8 \frac{\mathrm{m}}{\mathrm{s}}$$

$$k_B \simeq 8.6 \times 10^{-5} \frac{\mathrm{eV}}{\mathrm{K}} \qquad \alpha_{\mathrm{EM}} \simeq \frac{1}{137} = \frac{\mathrm{e}^2}{4\pi}$$

- example

$$1\mathrm{Tesla} = 1\frac{\mathrm{Vs}}{\mathrm{m}^2} = \frac{c}{\mathrm{m/s}} \frac{\hbar c}{\mathrm{eVm}} \frac{1}{\sqrt{4\pi\alpha_{\mathrm{EM}}}} (\mathrm{eV})^2 \simeq 195(\mathrm{eV})^2$$

- other important relations/definitions:

$$1\mathrm{erg} \simeq 624\ \mathrm{GeV} \qquad 1\mathrm{eV} \simeq 1.8 \times 10^{-36}\ \mathrm{kg} \qquad 1\mathrm{pc} \simeq 3.26\mathrm{ly} \simeq 3.09 \times 10^{16}\ \mathrm{m}$$

Particle acceleration in the Universe

- Acceleration is a continuous process.

➜ Accelerators need to confine the particle by magnetic fields.

- Larmor radius:

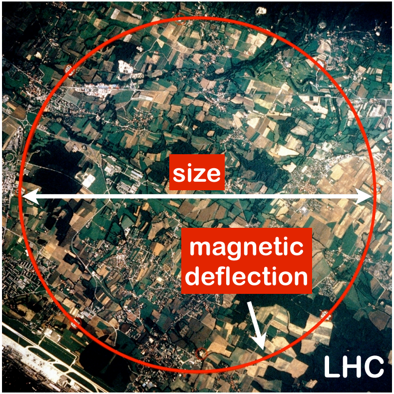
$$R_{\rm L} = \frac{E}{ZeB} \simeq \frac{1.1}{Z}\left(\frac{E}{\rm EeV}\right)\left(\frac{B}{\mu{\rm G}}\right)^{-1}{\rm kpc}\,.$$

- maximal energy from $R_{\rm L} = R_{\rm acc}$:

$$E_{\rm max} \simeq 0.9Z\left(\frac{B_{\rm acc}}{\mu{\rm G}}\right)\left(\frac{R_{\rm acc}}{\rm kpc}\right){\rm EeV}\,.$$

- for example, the LHC:

$$E_{\rm max} \simeq 9\left(\frac{B_{\rm acc}}{\rm 8T}\right)\left(\frac{R_{\rm acc}}{\rm 4km}\right){\rm TeV}\,.$$



Acceleration mechanism?

- There is a problem with this analogy.
- ✘ Universe is a "perfect conductor"
- ➜ It is unlikely to build up large potentials on long time-scales that accelerate charged particles.
- astrophysical environments are described (to leading order) as an ideal magneto-hydrodynamical (MHD) system:

$$\partial_t \rho = -\nabla(\rho \mathbf{v}) \qquad \text{(continuity)}$$

$$\rho(\partial_t + \mathbf{v}\nabla)\mathbf{v} = (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p \qquad \text{(momentum)}$$

$$\partial \mathbf{B} = -\nabla \times \mathbf{E} \qquad \text{(Faraday's law)}$$

$$\nabla \mathbf{B} = 0 \qquad \text{(no divergence)}$$

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} \qquad \text{(Ohm's law)}$$

- in particular, Ohm's law gives $\mathbf{E} \perp \mathbf{v}$
- ➜ **no acceleration** along electric fields
- ➜ exceptions (NLO effects): magnetic reconnections, double layers, relativistic motion,. . .

Fermi's idea

PHYSICAL REVIEW VOLUME 75, NUMBER 8 APRIL 15, 1949

On the Origin of the Cosmic Radiation

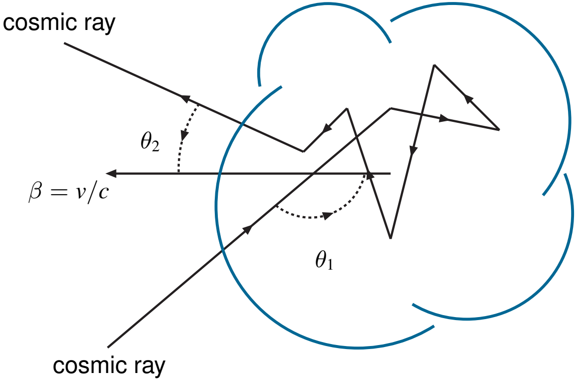
ENRICO FERMI
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magnetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

- **exercise** (my only one for today!): *Try to get this paper on the web!*
- hints:
    - `http://inspirehep.net/` (type in "`f a fermi and t cosmic`")
    - `http://adsabs.harvard.edu/abstract_service.html`
    - `http://arxiv.org/`

Fermi's original idea

"collisionless" scattering of charged particles with "magnetic clouds"



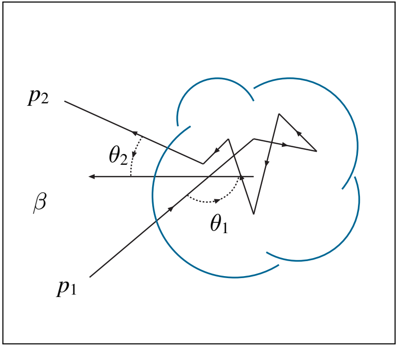
Fermi acceleration (second order)

- "magnetic cloud" with velocity β.
- momentum in rest frame

$$E_1' = \gamma E_1(1 - \beta\cos\theta_1)$$

- elastic scattering within cloud conserves energy ($E_2' = E_1'$) but isotropizes the emission direction θ_2'
- emitted energy

$$E_2 = \gamma E_2'(1 + \beta\cos\theta_2')$$



- energy gain per scatter:

$$\frac{\Delta E}{E_1} = \frac{E_2 - E_1}{E_1} = \gamma^2(1 + \beta\cos\theta_2')(1 - \beta\cos\theta_1) - 1$$

➜ can be **positive or negative** depending on scattering angle

Fermi acceleration (second order)

- distribution of θ'_2 is (appr.) isotropic $\frac{\mathrm{d}n}{\mathrm{d}\cos\theta'_2} \propto 1$
- averaging over θ'_2:

$$\frac{\langle\Delta E\rangle_{\theta'_2}}{E_1} = \int_{-1}^{1} \mathrm{d}\cos\theta'_2 \frac{\mathrm{d}n}{\mathrm{d}\cos\theta'_2}\frac{\Delta E}{E_1} = \frac{1}{2}\int_{-1}^{1}\mathrm{d}\cos\theta'_2\frac{\Delta E}{E_1} = \gamma^2(1-\beta\cos\theta_1) - 1$$

- distribution of θ_1 follows number of particles per second in direction θ_1

$$\frac{\mathrm{d}n}{\mathrm{d}\cos\theta_1} \propto (1-\beta\cos\theta_1)$$

- further averaging over θ_1

$$\frac{\langle\Delta E\rangle_{\theta_1\&\theta'_2}}{E_1} = \int_{-1}^{1}\mathrm{d}\cos\theta_1\frac{\mathrm{d}n}{\mathrm{d}\cos\theta_1}\frac{\langle\Delta E\rangle_{\theta'_2}}{E_1}$$

$$= \frac{1}{2}\int_{-1}^{1}\mathrm{d}\cos\theta_1(1-\beta\cos\theta_1)[\gamma^2(1-\beta\cos\theta_1)-1]$$

$$= \gamma^2\left(1+\frac{\beta^2}{3}\right) - 1 = \frac{1+\frac{\beta^2}{3}}{1-\beta^2} - 1 \simeq 1 + \frac{\beta^2}{3} + \beta^2 - 1 = \frac{4}{3}\beta^2$$

➜ **on average energy gain** with $\Delta E/E \propto \beta^2$

➜ slow for $\beta \ll 1$; these days called *second order* Fermi acceleration

Macroscopic treatment

- second order Fermi acceleration can be treated by a diffusion equation in momentum space:

$$\partial_t n = \nabla_p \mathbf{D} \nabla_p n - \frac{1}{\tau} n + Q$$

- diffusion tensor $\mathbf{D}$ can be anisotropic, for instance if scattering centers have preferred direction

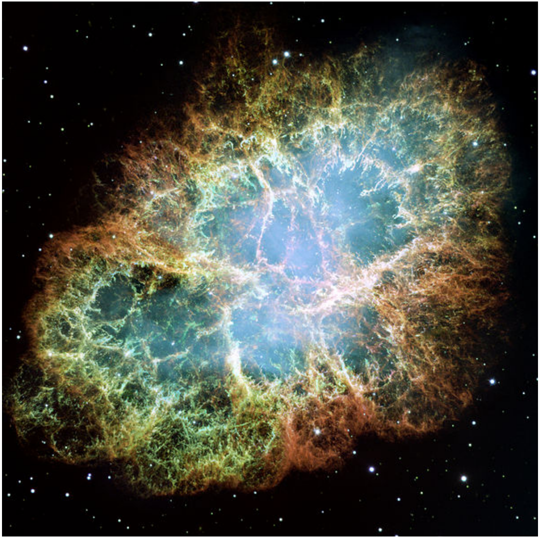
$$\mathbf{D}_{ij} = \frac{\langle \Delta p_i \Delta p_j \rangle}{2\lambda}$$

- diffusion coefficient is momentum dependent $\mathbf{D} = D_p \mathbf{1}$ and $D_p \propto D_0 (p/p_0)^{2-\delta}$
- Bohm diffusion $\delta = 1$, Kolmogorov diffusion $\delta = 5/3$, ...
- steady-state solution for $\delta = 2$ [*e.g.* Mertsch'11]

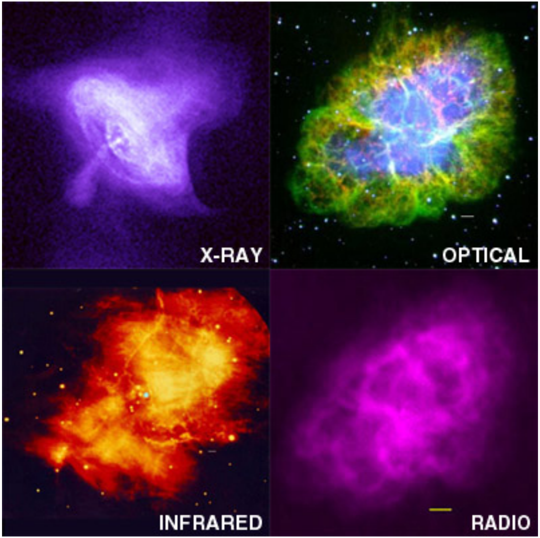
$$n \propto E^{-\gamma} \qquad \gamma \simeq \frac{1}{2} + \frac{1}{3} \frac{p_0^2}{D_0 \tau}$$

✘ no **universal** power law

Diffuse shock acceleration

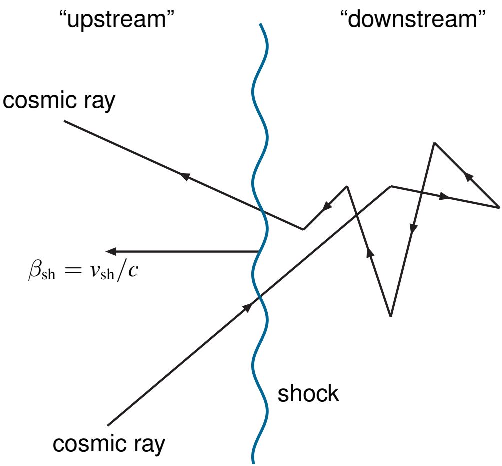


Diffuse shock acceleration



Diffuse shock acceleration

"collisionless" scattering of charged particles across shocks



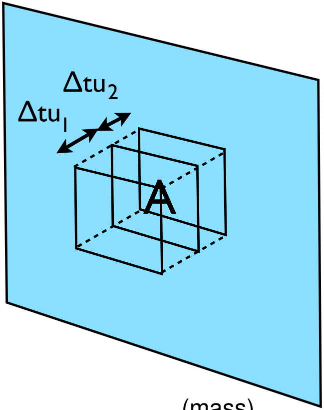
Rankine-Hugoniot relations

- consider the particle flow through an area A on the shock front during a time Δt in the rest-frame of the shock

$$u_2^* = u_{\rm sh} \qquad\qquad u_1^* = \frac{u_{\rm sh} - u_2}{1 - u_{\rm sh}u_2}$$

➜ differential volume $\Delta V_{1,2} = \Delta x_{1,2}A = \Delta t u_{1,2}^* A$

➜ relation between mass density $\rho_{1,2}$, pressure $p_{1,2}$ and energy density $\epsilon_{1,2}$ from mass, momentum and energy conservation across shock



$$\Delta V_1 \rho_1 = \Delta V_2 \rho_2 \qquad \text{(mass)}$$

$$\Delta V_1 \rho_1 u_1^* + \Delta t A p_1 = \Delta V_2 \rho_2 u_2^* + \Delta t A p_2 \qquad \text{(momentum)}$$

$$\frac{1}{2}\Delta V_1 \rho_1 (u_1^*)^2 + \Delta V_1 \epsilon_1 + \Delta t u_1^* A p_1 = \frac{1}{2}\Delta V_2 \rho_2 {u_2^*}^2 + \Delta V_2 \epsilon_2 + \Delta t A u_2^* p_2 \qquad \text{(energy)}$$

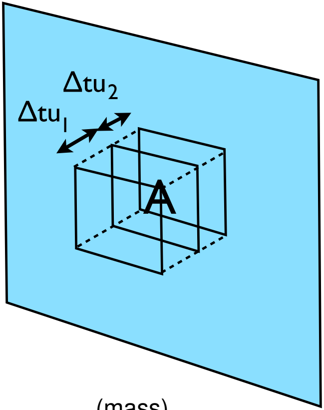
Rankine-Hugoniot relations

- consider the particle flow through an area A on the shock front during a time Δt in the rest-frame of the shock

$$u_2^* = u_{\rm sh} \qquad\qquad u_1^* = \frac{u_{\rm sh} - u_2}{1 - u_{\rm sh} u_2}$$

➜ differential volume $\Delta V_{1,2} = \Delta x_{1,2} A = \Delta t u_{1,2}^* A$

➜ relation between mass density $\rho_{1,2}$, pressure $p_{1,2}$ and energy density $\epsilon_{1,2}$ from mass, momentum and energy conservation across shock



$$u_1^* \rho_1 = u_2^* \rho_2 \qquad \text{(mass)}$$

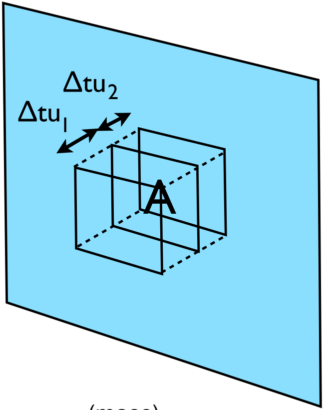
$$\rho_1 u_1^2 + p_1 = \rho_2 u_2^{*\,2} + p_2 \qquad \text{(momentum)}$$

$$\frac{1}{2}\rho_1 u_1^{*\,3} + u_1^*(\epsilon_1 + p_1) = \frac{1}{2}\rho_2 u_2^{*\,3} + u_2^*(\epsilon_2 + p_2) \qquad \text{(energy)}$$

Rankine-Hugoniot relations

- consider the particle flow through an area A on the shock front during a time Δt in the rest-frame of the shock

$$u_2^* = u_{\rm sh} \qquad\qquad u_1^* = \frac{u_{\rm sh} - u_2}{1 - u_{\rm sh}u_2}$$



➜ differential volume $\Delta V_{1,2} = \Delta x_{1,2}A = \Delta t u^*_{1,2}A$

➜ relation between mass density $\rho_{1,2}$, pressure $p_{1,2}$ and energy density $\epsilon_{1,2}$ from mass, momentum and energy conservation across shock

$$u_1^*\rho_1 = u_2^*\rho_2 = \Phi \qquad \text{(mass)}$$

$$\Phi(u_1^* - u_2^*) = p_2 - p_1 \qquad \text{(momentum)}$$

$$\frac{1}{2}\Phi(u_1^{*2} - u_2^{*2}) = u_2^*(\epsilon_2 + p_2) - u_1^*(\epsilon_1 + p_1) \qquad \text{(energy)}$$

Rankine-Hugoniot relations

- finally...

$$\frac{1}{2}(p_2 - p_1)(u_1^* + u_2^*) = u_2^*(\epsilon_2 + p_2) - u_1^*(\epsilon_1 + p_1)$$

- compression ratio:

$$r = \frac{u_1^*}{u_2^*} = \frac{\rho_2}{\rho_1} = \frac{2(\epsilon_2 + p_2) - (p_2 - p_1)}{2(\epsilon_1 + p_1) + (p_2 - p_1)}$$

- equation of state: $p = \omega\epsilon$

$$\omega = \frac{1}{3} \text{ (relativistic)} \qquad \omega = \frac{2}{3} \text{ (non-relativistic)}$$

➜ for $\epsilon_1 \ll \epsilon_2$ we have $r \simeq (2 + \omega)/\omega$ and

$$r = 7 \text{ (relativistic)} \qquad r = 4 \text{ (non-relativistic)}$$

➜ cosmic frame velocity:

$$u_2 = \frac{(r - 1)\beta_{\rm sh}}{r - \beta_{\rm sh}^2}$$

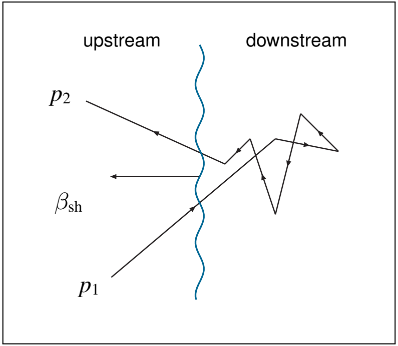
Fermi acceleration (first order)

- shock with velocity $\beta_{\rm sh}$ ➜ $\beta = \frac{(r-1)\beta_{\rm sh}}{r-\beta_{\rm sh}^2}$.
- momentum in rest frame

$$E'_1 = \gamma E_1 (1 - \beta \cos\theta_1)$$

- elastic scattering within cloud conserves energy ($E'_2 = E'_1$) but isotropizes direction
- emitted energy

$$E_2 = \gamma E'_2 (1 + \beta \cos\theta'_2)$$



- energy gain per scatter:

$$\frac{\Delta E}{E_1} = \frac{E_2 - E_1}{E_1} = \gamma^2 (1 + \beta\cos\theta'_2)(1 - \beta\cos\theta_1) - 1$$

➜ **always positive** since $\cos\theta_1 < 0$ and $\cos\theta'_2 > 0$

Fermi acceleration (first order)

- distributions of θ_1 and θ_2' follow projection onto the shock:

$$\frac{\mathrm{d}n}{\mathrm{d}\cos\theta_1} \propto \cos\theta_1 \;\; (\cos\theta_1 < 0) \qquad\qquad \frac{\mathrm{d}n}{\mathrm{d}\cos\theta_2'} \propto \cos\theta_2' \;\; (\cos\theta_2' > 0)$$

- averaging over θ_2':

$$\frac{\langle\Delta E\rangle_{\theta_2'}}{E_1} = \int_{-1}^{1}\mathrm{d}\cos\theta_2' \frac{\mathrm{d}n}{\mathrm{d}\cos\theta_2'}\frac{\Delta E}{E_1} = 2\int_{0}^{1}\mathrm{d}\cos\theta_2' \cos\theta_2' \frac{\Delta E}{E_1}$$
$$= \gamma^2(1 - \beta\cos\theta_1 + \frac{2}{3}\beta - \frac{2}{3}\beta^2\cos\theta_1) - 1$$

- also averaging over θ_1

$$\frac{\langle\Delta E\rangle_{\theta_1\&\theta_2'}}{E_1} = \int_{-1}^{1}\mathrm{d}\cos\theta_1 \frac{\mathrm{d}n}{\mathrm{d}\cos\theta_1}\frac{\langle\Delta E\rangle_{\theta_2'}}{E_1}$$
$$= -2\int_{-1}^{0}\mathrm{d}\cos\theta_1\cos\theta_1[\gamma^2(1 - \beta\cos\theta_1 + \frac{2}{3}\beta - \frac{2}{3}\beta^2\cos\theta_1) - 1]$$
$$= \gamma^2\left(1 + \frac{2}{3}\beta\right)^2 - 1 \simeq 1 + \frac{4}{3}\beta - 1 = \frac{4}{3}\beta$$

➜ **on average energy gain** with $\Delta E/E \propto \beta$

➜ *first order* Fermi acceleration more efficient

Spectrum

- particle acceleration per crossing $\Delta E/E = \xi$
- relative rate of particles crossing the shock from upstream to downstream:

$$R_{\rm cross} = \frac{1}{4\pi}2\pi\int_0^1 {\rm d}\cos\theta_1\cos\theta_1 = \frac{1}{4}$$

- relative rate of particles escaping the shock region

$$R_{\rm esc} = \frac{u_2^*}{c}$$

- probability that particle crosses the shock and escapes downstream:

$$P_{\rm esc} = \frac{R_{\rm esc}}{R_{\rm cross}} = \frac{4u_2^*}{c}$$

- evolution of energy and particle number

$$\partial_t E = \frac{\xi}{t_{\rm cycle}}E \qquad \partial_t N = -\frac{P_{\rm esc}}{t_{\rm cycle}}N$$

Spectrum

➜ dividing

$$\partial_E N = -\frac{P_{\rm esc}}{\xi}\frac{N}{E}$$

➜ re-arranging

$$\frac{{\rm d}N}{N} = -\frac{P_{\rm esc}}{\xi}\frac{{\rm d}E}{E}$$

➜ integrating

$$\int_{N_0}^{N(E)}\frac{{\rm d}N'}{N'} = -\int_{E_0}^{E}\frac{P_{\rm esc}}{\xi}\frac{{\rm d}E'}{E'}$$

➜ final spectrum

$$N(E) = N_0(E/E_0)^{-\gamma}$$

- power index for non-relativistic plasma ($r = 4$) and strong shock

$$\gamma = \frac{P_{\rm esc}}{\xi} \simeq \frac{4u_2^*}{(4/3)(u_1^* - u_2^*)} = \frac{3}{r-1} \simeq 1$$

➜ differential spectrum

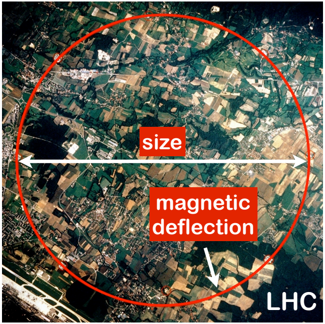
$$\frac{{\rm d}N}{{\rm d}E} \propto E^{-2}$$

Candidate sources

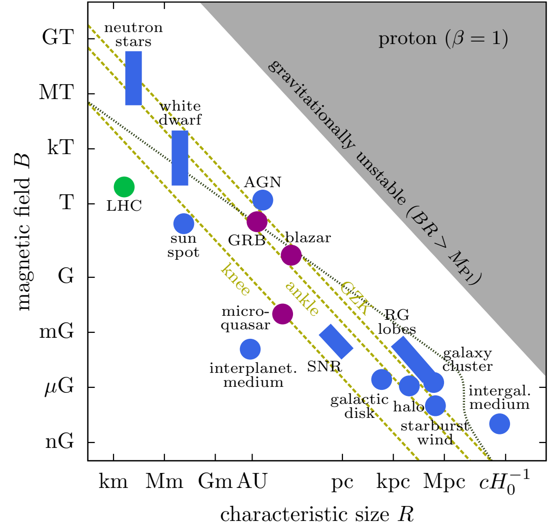
- CR acceleration is (most likely) a continuous process.

➜ Accelerators need to confine the particle by magnetic fields.

- $E_{\max} \sim$ size $\times$ field strength



Hillas plot



Neutrino flux predictions

- pion production in CR interactions with ambient radiation

$$\pi^+ \to \mu^+\nu_\mu \to e^+\nu_e\bar{\nu}_\mu\nu_\mu$$
$$\pi^0 \to \gamma\gamma$$

- inelasticity:

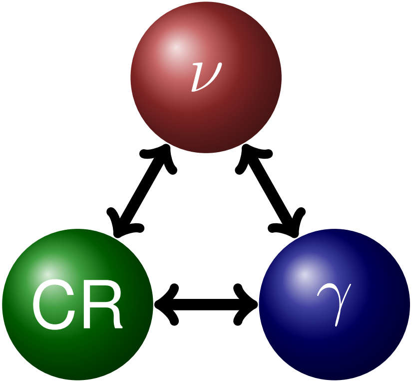
$$E_\nu \simeq E_\gamma/2 \simeq \kappa E_p/4$$

- relative multiplicity:

$$K = N_{\pi^\pm}/N_{\pi^0}$$

- pion fraction:

$$f_\pi \simeq 1 - e^{-\kappa\tau}$$



Neutrino flux predictions

- pion production in CR interactions with ambient radiation

$$\pi^+ \to \mu^+\nu_\mu \to e^+\nu_e\bar{\nu}_\mu\nu_\mu$$
$$\pi^0 \to \gamma\gamma$$

- inelasticity:

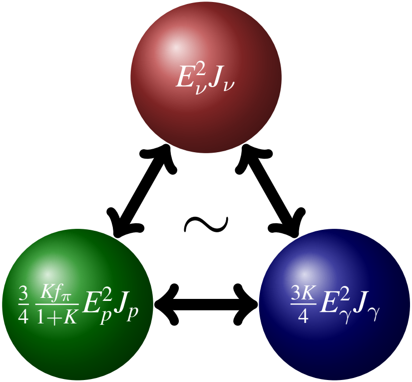
$$E_\nu \simeq E_\gamma/2 \simeq \kappa E_p/4$$

- relative multiplicity:

$$K = N_{\pi^\pm}/N_{\pi^0}$$

- pion fraction:

$$f_\pi \simeq 1 - e^{-\kappa\tau}$$



($E_\nu^2 J_\nu \sim$ energy density ω)

Average pion fraction

- 3 neutrinos per pion; equally distributed *after* oscillation
- average energy loss in a single $p\gamma$ interaction via Δ-resonance:

$$\frac{E_\nu}{E_\pi} \simeq \frac{1}{4} \qquad \text{and} \qquad K \equiv \frac{N_{\pi^+} + N_{\pi^-}}{N_{\pi^0}} \simeq \frac{1}{2}$$

- f_π depends on optical depth $\tau_{p\gamma}(E_p)$ and mean inelasticity $\langle x \rangle \simeq 0.2$

➜ particle number conservation (**no magnetic field** at the moment):

$$E_\nu \frac{\mathrm{d}N_\nu}{\mathrm{d}E_\nu}(E_\nu) \simeq \frac{3K}{1+K} E_\pi \frac{\mathrm{d}N_\pi}{\mathrm{d}E_\pi}(E_\pi) \simeq \frac{3K}{1+K} E_p \left(\frac{1 - e^{-\langle x \rangle \tau_{p\gamma}(E_p)}}{\langle x \rangle}\right) \frac{\mathrm{d}N_p}{\mathrm{d}E_p}(E_p)$$

- can be rewritten as an (approximate) energy relation with $E_\nu \simeq \langle x \rangle E_p / 4$:

$$E_\nu^2 \frac{\mathrm{d}N_\nu}{\mathrm{d}E_\nu}(E_\nu) \simeq \frac{3K}{4(1+K)} \underbrace{\left(1 - e^{-\langle x \rangle \tau_{p\gamma}(E_p)}\right)}_{\text{“}f_\pi\text{”}} E_p^2 \frac{\mathrm{d}N_p}{\mathrm{d}E_p}(E_p)$$

- final neutrino spectra after meson/muon **cooling in magnetic fields**

Optical depth for $p\gamma$

- interaction rate averaged over isotropic spectrum ($E'_\gamma = (E_p/m_p)E_\gamma(1-\cos\theta)$)

$$\Gamma_{p\gamma}(E) \equiv \frac{1}{2}\int\limits_{-1}^{1} \mathrm{d}\cos\theta \int \mathrm{d}E_\gamma\,(1-\cos\theta)\frac{\mathrm{d}N_\gamma}{\mathrm{d}E_\gamma}(E_\gamma)\sigma_{p\gamma}(E_\gamma{}')$$

- Breit-Wigner approximation (width $\Gamma_\Delta \simeq 0.11$ GeV and $\sigma_0 \simeq 34\ \mu$b)

$$\sigma_{p\gamma}(E_\gamma{}') \simeq \underbrace{\frac{s}{E'_\gamma{}^2}\frac{\sigma_0\Gamma_\Delta^2 s}{(s-m_\Delta^2)^2+\Gamma_\Delta^2 s}}_{\text{Breit-Wigner}} \simeq \underbrace{\frac{s}{E'_\gamma{}^2}\Gamma_\Delta\sqrt{s}\sigma_0\pi\delta(s-m_\Delta^2)}_{\text{narrow-width approximation}}$$

- opacity of $p\gamma$ collision ($\epsilon_{\min} = (m_\Delta^2 - m_p^2)/4E_p$)

$$\tau_{p\gamma}(E_p) = R_{\rm size}\Gamma_{p\gamma}(E_p) \simeq R_{\rm size}\underbrace{\left(\frac{\pi}{2}\frac{\Gamma_\Delta\sigma_0 m_\Delta^3}{m_\Delta^2-m_p^2}\right)}_{0.04}\frac{m_p^2}{E_p{}^2}\int\limits_{\epsilon_{\min}}\frac{\mathrm{d}\epsilon}{\epsilon^2}n_\gamma(\epsilon)$$

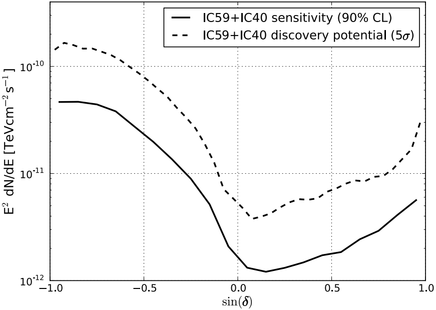
Galactic γ-ray sources

- hadronic interaction relate neutrinos and γ-rays **on production** ($E_\gamma \simeq 2E_\nu$)

$$Q_{\mathrm{all}\,\nu}(E_\nu) \simeq 3KQ_\gamma(E_\gamma)\,,$$

- for close-by (galactic) sources this translates into a direct relation between the observed point-source spectra

$$J_{\mathrm{all}\,\nu}(E_\nu) \simeq 3KJ_\gamma(E_\gamma)\,,$$



- typical IceCube sensitivity for TeV-PeV neutrino sources in the northern sky:

$$E^2J_{\nu_\mu} \simeq 10^{-11}\mathrm{TeVcm}^{-2}\mathrm{s}^{-1}$$

Neutrino point sources

- in general, flux F (erg/cm^2/s) and luminosity $\mathcal{L}$ (erg/s) of a source (γ-ray, neutrino, ...) are related via the luminosity distance d_L

$$F = \int \mathrm{d}E\, E\, J(E) = \frac{\mathcal{L}}{4\pi d_L^2}$$

- for close-by sources d_L is just the Euclidian distance
- ✗ not so for cosmic sources at redshift $z \gg 0$:

$$d_L = (1+z)\int_0^z \frac{\mathrm{d}z'}{H(z')}\,.$$

- Hubble parameter accounts for the expansion of the universe; for ΛCDM model:

$$H(z) = H_0\sqrt{\Omega_\Lambda + \Omega_m(1+z)^3}$$

$$\Omega_\Lambda \simeq 0.74, \qquad \Omega_m \simeq 0.26, \qquad H_0 \simeq 72\frac{\mathrm{km}}{\mathrm{s}}\mathrm{Mpc}^{-1}$$

- neutrino spectrum:

$$J_\nu(z,E) = \frac{(1+z)^2}{4\pi d_L^2}Q_\nu((1+z)E)$$

Jetted sources

- many γ-ray sources show a jet-like outflow, *e.g.* quasars, micro-quasars, γ-ray bursts
- neutrino in co-moving and observatory frame satisfy $\Delta t = \Delta x$ and $\Delta t' = \Delta x'$ and are related by a Lorentz tranformation

$$\Delta t' = \Gamma \Delta t - \Gamma \Delta \vec{x} \cdot \vec{\beta} \qquad \Delta \vec{x}' \cdot \vec{\beta} = \Gamma \Delta \vec{x} \cdot \vec{\beta} - \beta^2 \Gamma \Delta t$$

- observation angle relative to the velocity

$$\cos\theta' = \frac{\cos\theta - \beta}{1 - \beta\cos\theta}$$

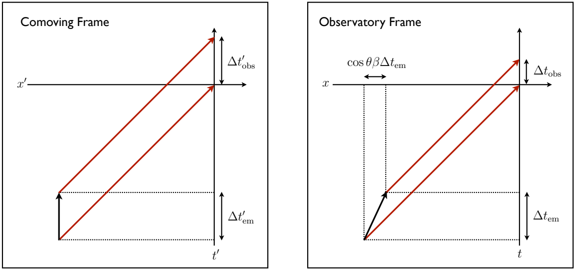
- convenient to define the Doppler factor

$$\delta = \frac{1}{\Gamma(1 - \beta\cos\theta)}$$

➜ with this we can define

$$\sin\theta' = \delta \sin\theta \qquad E' = E/\delta$$

Jetted sources



- two neutrinos emitted in a time-interval $\Delta t'_{\rm em}$ and $\Delta x'_{\rm em} = 0$ are observed in the co-moving frame within $\Delta t'_{\rm obs} = \Delta t'_{\rm em}$
- in the observer frame $\Delta t_{\rm em} = \Gamma \Delta t'_{\rm em}$ and $\Delta t_{\rm obs} = \Delta t_{\rm em} - \beta\cos\theta\Delta t_{\rm em} = \Delta t'_{\rm obs}/\delta$
- *apparent* displacement of the source projected onto the night-sky is $\Delta x_{\rm obs} = \sin\theta\beta\Delta t_{\rm em}$ after the emission time intervall $\Delta t_{\rm em}$

➜ **super-luminal** motion:

$$\beta_{\rm app} = \frac{\Delta x_{\rm obs}}{\Delta t_{\rm obs}} = \frac{\sin\theta\beta}{1-\beta\cos\theta}$$

Jetted sources

- transformation of spherical elements

$$\frac{\mathrm{d}\Omega'}{\mathrm{d}\Omega} = \frac{\mathrm{d}\cos\theta'}{\mathrm{d}\cos\theta} = \delta^2$$

- number of particles emitted into a volume element $\Delta\omega$ per energy ΔE_γ and observation time $\Delta t_{\rm obs}$ is independent of the frame of reference

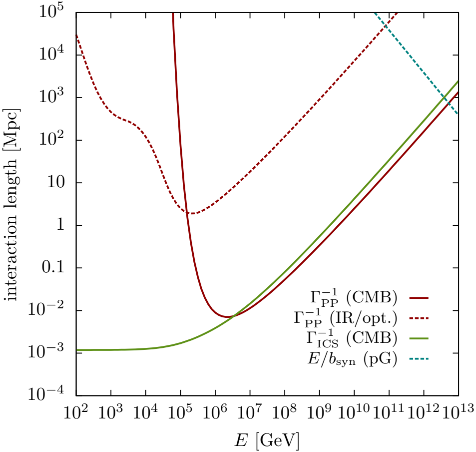
$$\frac{\mathrm{d}F_\nu}{\mathrm{d}E}(E) = \frac{\mathrm{d}\Omega'}{\mathrm{d}\Omega}\frac{\Delta t'_{\rm obs}}{\Delta t_{\rm obs}}\frac{\mathrm{d}E'}{\mathrm{d}E}\frac{\mathrm{d}F'_\nu}{\mathrm{d}E'}(E') = \delta^2\frac{\mathrm{d}F'_\nu}{\mathrm{d}E'}(E/\delta)$$

- for comoving emissivity $Q'_\nu = Q_0(E/\mathrm{TeV})^{-\alpha}$ (GeV^{-1} s^{-1}) and also including red-shift scaling we have

$$J(z,E) = \frac{(1+z)^{2-\alpha}\delta^{2+\alpha}}{4\pi d_L^2}Q_0\left(\frac{E}{\mathrm{TeV}}\right)^{-\alpha}$$

Extra-galactic γ-ray sources?

- CMB interactions **(solid lines)** dominate in casade:
  - inverse Compton scattering (ICS)
    $e^{\pm} + \gamma_{\rm CMB} \to e^{\pm} + \gamma$
  - pair production (PP)
    $\gamma + \gamma_{\rm CMB} \to e^{+} + e^{-}$
- PP in IR/optical background **(red dashed line)** determines the "edge" of the spectrum.
- this calculation:
  Franceschini *et al.* '08

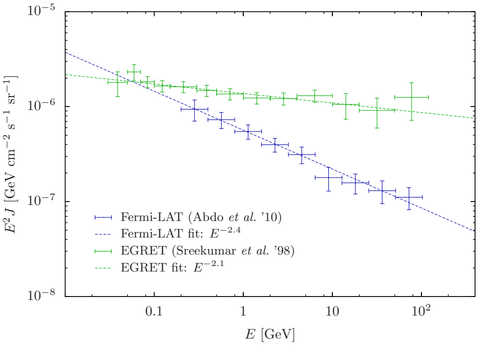


Rapid cascade interactions produce universal GeV-TeV emission (almost) independent of injection spectrum and source distribution.
➜ **"cascade bound" for neutrinos** [Berezinsky&Smirnov'75]

Extra-galactic γ-ray sources?

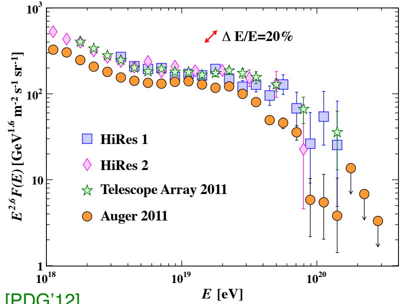
- CMB interactions **(solid lines)** dominate in casade:
  - inverse Compton scattering (ICS)
    $e^{\pm} + \gamma_{\rm CMB} \to e^{\pm} + \gamma$
  - pair production (PP)
    $\gamma + \gamma_{\rm CMB} \to e^{+} + e^{-}$
- PP in IR/optical background **(red dashed line)** determines the "edge" of the spectrum.
- this calculation:
  Franceschini *et al.* '08

diffuse γ-ray background

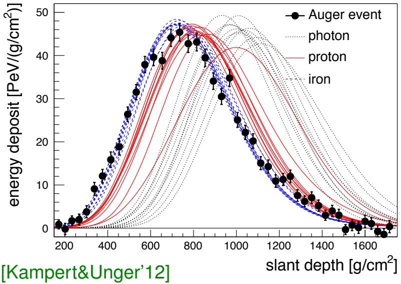


Rapid cascade interactions produce universal GeV-TeV emission (almost) independent of injection spectrum and source distribution.
➜ **"cascade bound" for neutrinos** [Berezinsky&Smirnov'75]

UHE CR observation

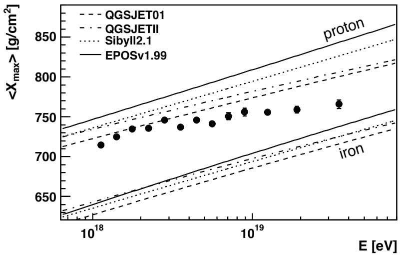


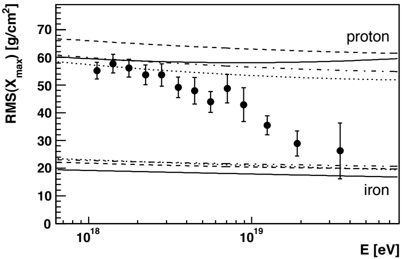
[PDG'12]



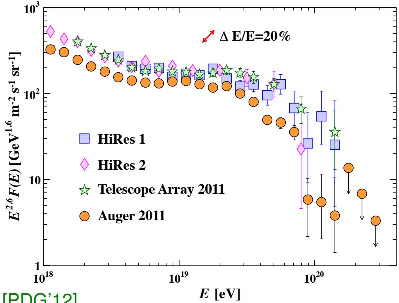
[Kampert&Unger'12]

Auger composition



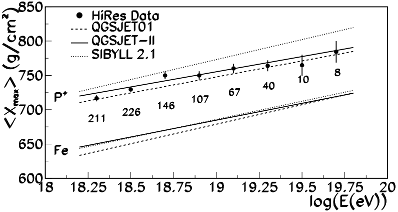


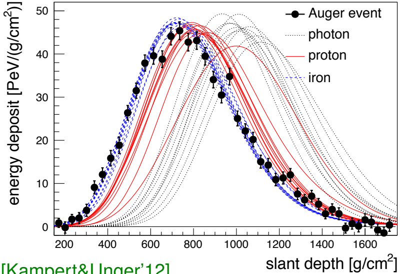
UHE CR observation



[PDG'12]

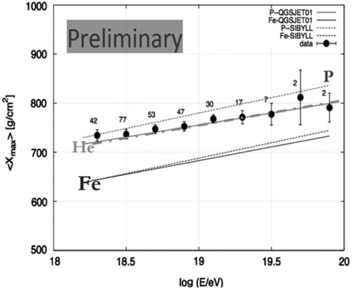
HiRes composition





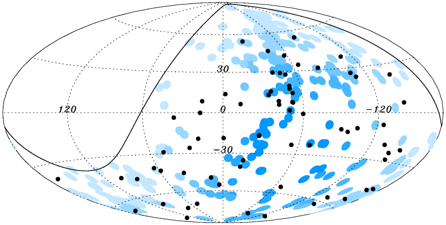
[Kampert&Unger'12]

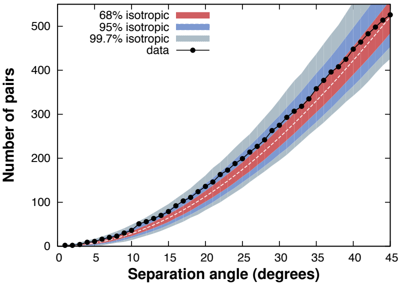
TA composition



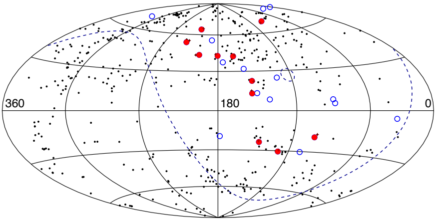
UHE CR observation

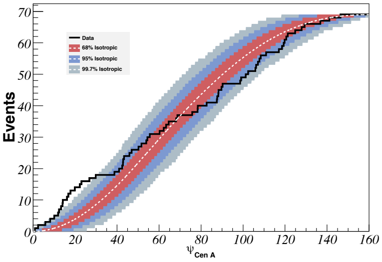
Auger distribution ($E > 55$ EeV)





TA distribution ($E > 57$ EeV)





Diffuse fluxes

- **spatially homogeneous and isotropic** distribution of sources
- Boltzmann equation of comoving number density ($Y = n/(1+z)^3$):

$$\dot{Y}_i = \partial_E(HEY_i) + \partial_E(b_i Y_i) - \Gamma_i\, Y_i + \sum_j \int \mathrm{d}E_j\, \gamma_{ji} Y_j + \mathcal{L}_i\,,$$

H : Hubble rate
b_i : continuous energy loss
γ_{ji} (Γ_i) : differential (total) interaction rate

- **power-law** proton emission rate:

$$\mathcal{L}_p(0,E) \propto (E/E_0)^{-\gamma} \exp(-E/E_{\max}) \exp(-E_{\min}/E)$$

- **redshift evolution** of source emission or distribution:

$$\mathcal{L}_p(z,E) = \mathcal{L}_p(0,E)(1+z)^n \Theta(z_{\max} - z)\Theta(z - z_{\min})$$

Diffuse neutrino fluxes

- homogenous distribution of neutrino sources $\mathcal{L}_\nu$

$$J_\nu(E) = \frac{1}{4\pi}\int_0^\infty \frac{\mathrm{d}z}{H(z)}\mathcal{L}_\nu(z,(1+z)E)\,.$$

- cosmogenic neutrinos from CR propagation

$$J_\nu(E_\nu) \simeq \frac{1}{4\pi}\int_0^\infty \frac{\mathrm{d}z'}{H(z')}\int \mathrm{d}\mathcal{E}_p\,\gamma_{p\nu}(z',\mathcal{E}_p,(1+z')E_\nu)\,Y_p(z',\mathcal{E}_p)$$

- proton spectrum

$$Y_p(z,\mathcal{E}_p(z,E_p)) \simeq \frac{1}{1+z}\int_z^\infty \frac{\mathrm{d}z'}{H(z')}\mathcal{L}_{p,\mathrm{eff}}(z',\mathcal{E}_p(z',E_p)) \times \exp\left[\int_z^{z'} \mathrm{d}z''\,\frac{\partial_E b_{\mathrm{BH}}(z'',\mathcal{E}_p(z'',E_p))}{(1+z'')H(z'')}\right]$$

- effective source term is defined as

$$\mathcal{L}_{p,\mathrm{eff}}(z,E_p) = \mathcal{L}_p(z,E_p) + \int \mathrm{d}\mathcal{E}_p\,\gamma_{pp}(z,\mathcal{E}_p,E_p)Y_p(z,\mathcal{E}_p)$$

UHE CRs and neutrinos

- observed UHE CR spectrum can be used to give an upper limit on diffuse neutrino fluxes [Waxman&Bahcall'97]

$$E_\nu^2 J_\nu(E_\nu) \simeq \frac{3K}{4(1+K)} \underbrace{\left(1 - e^{-\langle x \rangle \tau_{p\gamma}(E_p)}\right)}_{\text{“}f_\pi\text{”}} E_p^2 \frac{\mathrm{d}N_p}{\mathrm{d}E_p}(E_p) \leq \frac{3K}{4(1+K)} E_p^2 J_p(E_p)$$

- we can estimate the proton flux as

$$E^2 J_p(E_p) \simeq \frac{t_H}{4\pi} \zeta_z \mathcal{Q}_{\rm CR} \qquad t_H \simeq H_0^{-1} \simeq 14\,\mathrm{Gyr} \qquad \mathcal{Q}_{\rm CR} \simeq 10^{44} \mathrm{erg\,Mpc}^{-3}\,\mathrm{yr}^{-1}$$

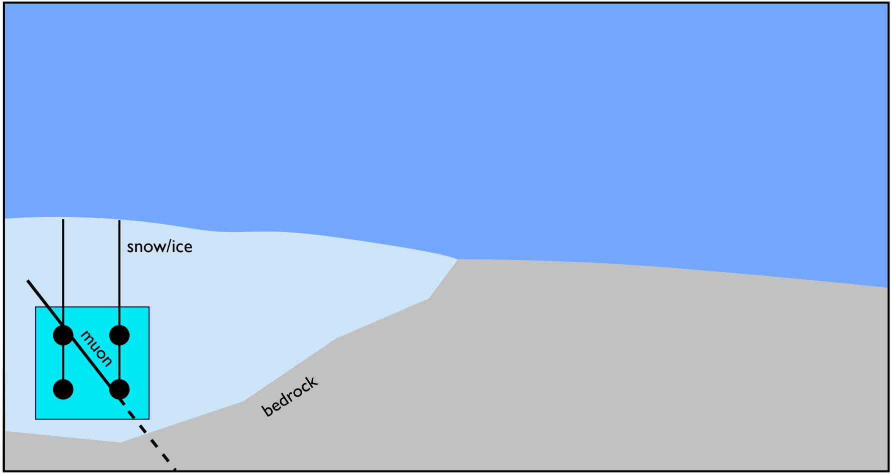
- evolution factor

$$\zeta_z = H_0 \int_0^{z_{\rm max}} \mathrm{d}z \frac{(1+z)^{n-\gamma}}{H(x)}$$

- with $K = 1$ and $\zeta_z = 0.6 - 3$ (no to strong evolution) we get

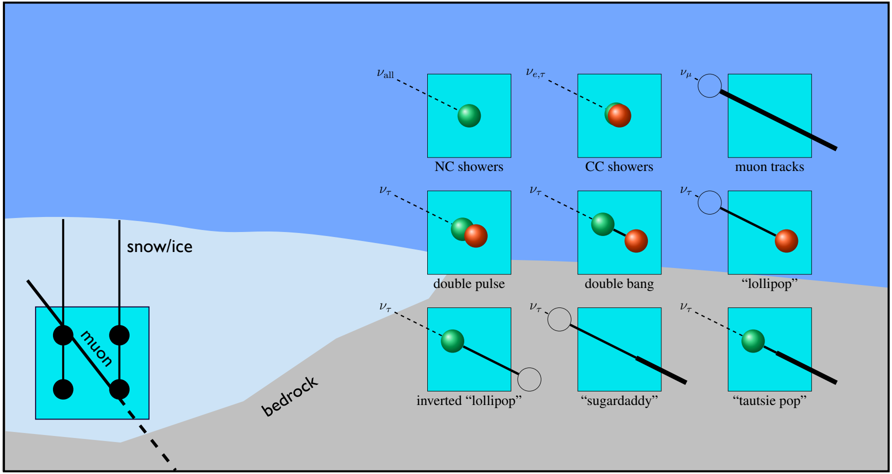
$$E_\nu^2 J_{\mathrm{all}\,\nu}^{\rm WB} \simeq (1.6 - 8.0) \times 10^{-8}\,\mathrm{GeV\,cm^{-2}\,s^{-1}\,sr^{-1}}$$

Neutrino observation at very high energies



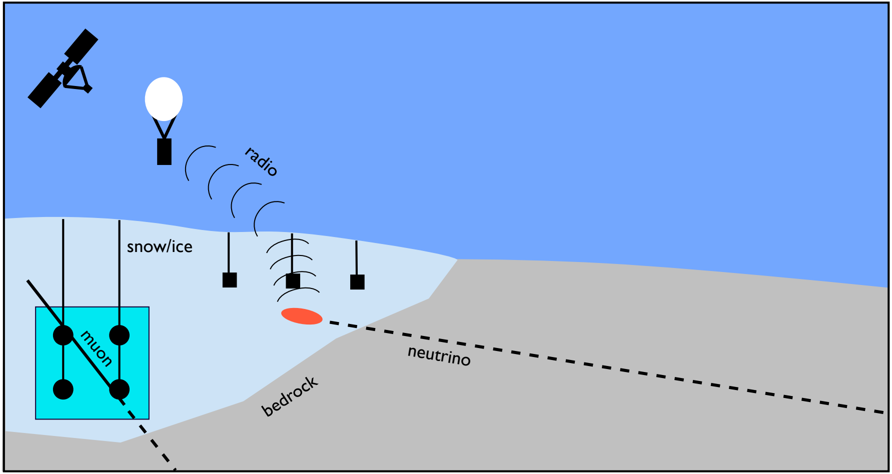
Cherenkov radiation in transparent media (glaciers, lakes, oceans,. . .).

Neutrino observation at very high energies



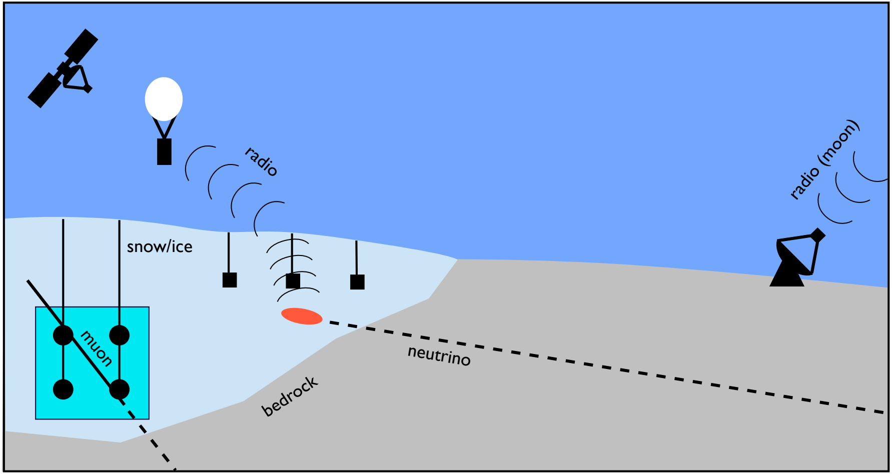
Cherenkov radiation in transparent media (glaciers, lakes, oceans,. . .).

Neutrino observation at very high energies



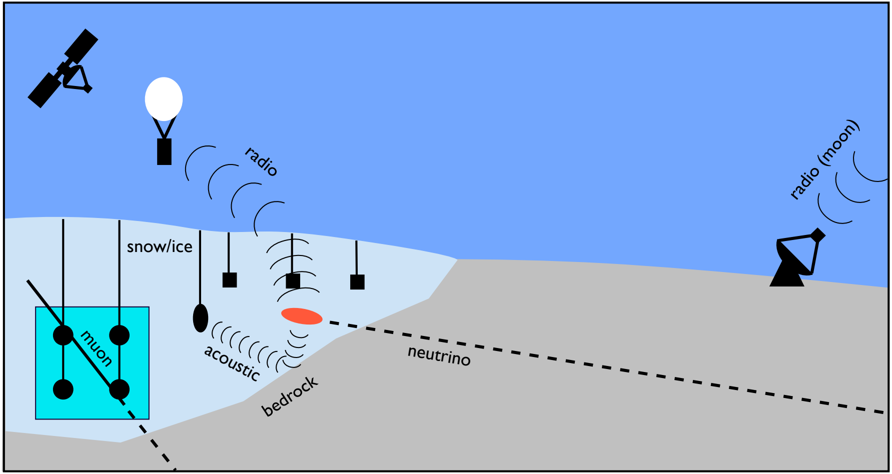
Coherent radio Cherenkov emission (Askaryan effect).
Observation in-situ, balloons or satellites.

Neutrino observation at very high energies



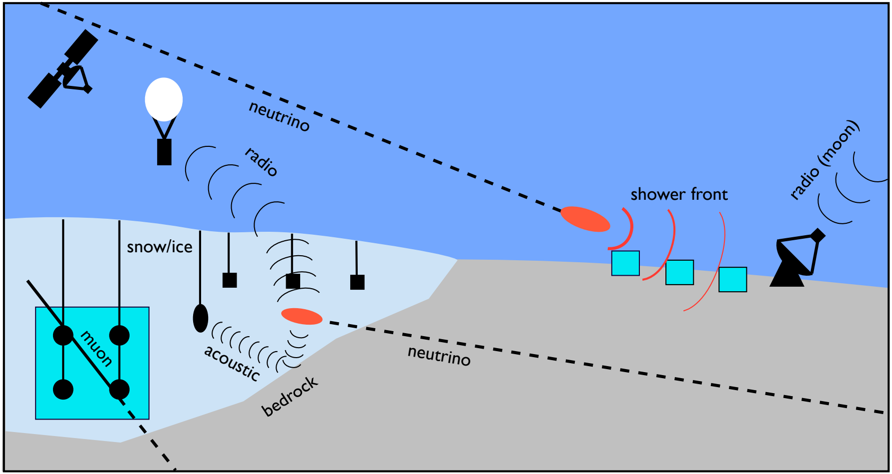
Coherent radio Cherenkov emission (Askaryan effect).
Observation from lunar regolith.

Neutrino observation at very high energies



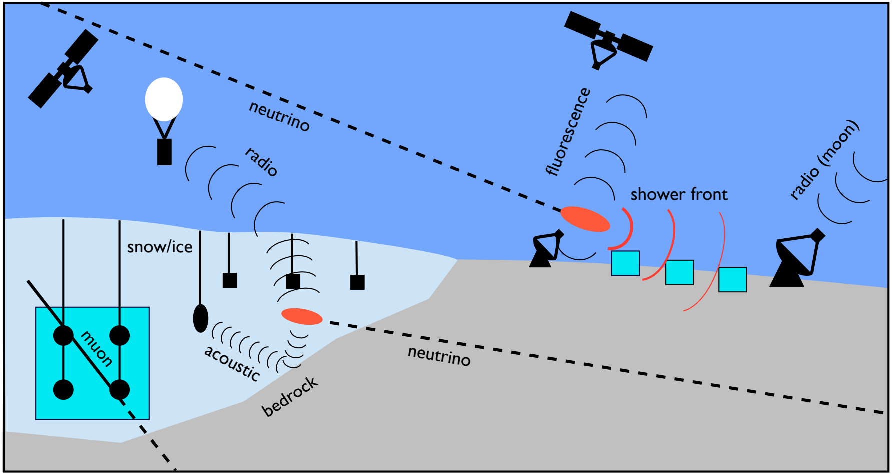
Acoustic detection?

Neutrino observation at very high energies



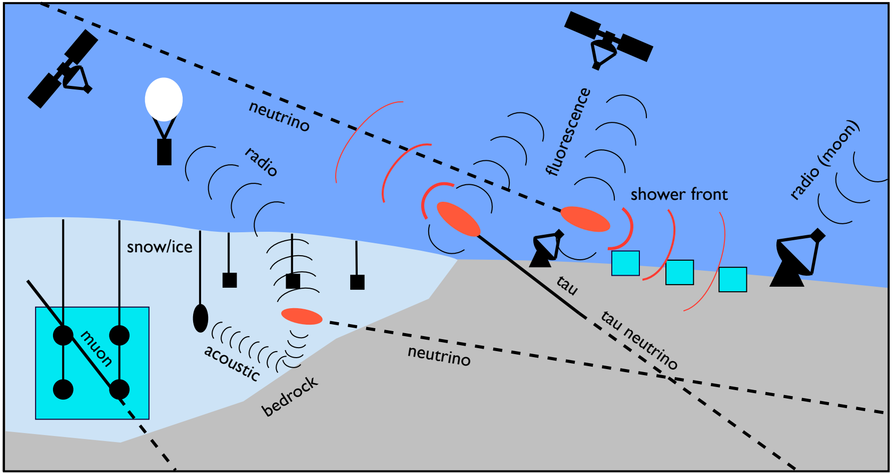
Deeply penetrating quasi-horizontal showers.
Observation by CR surface arrays.

Neutrino observation at very high energies



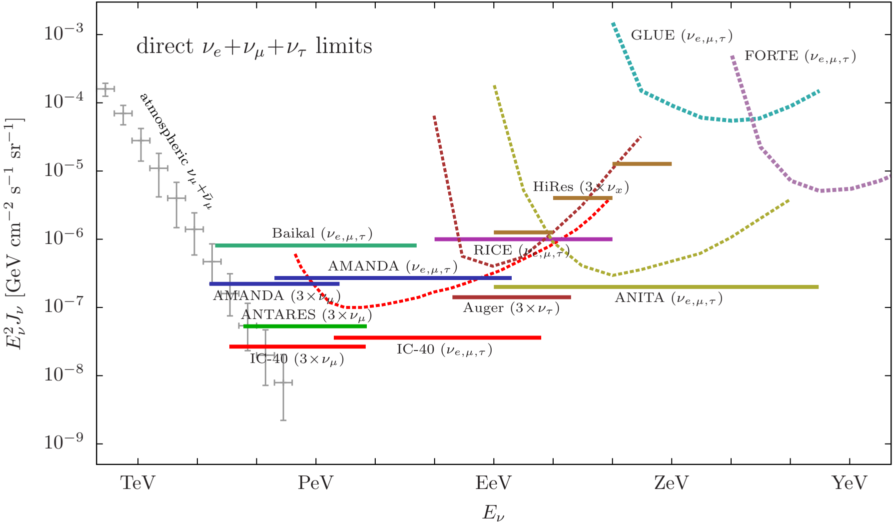
Observation by CR surface arrays and/or fluorescence detectors/satellites.

Neutrino observation at very high energies

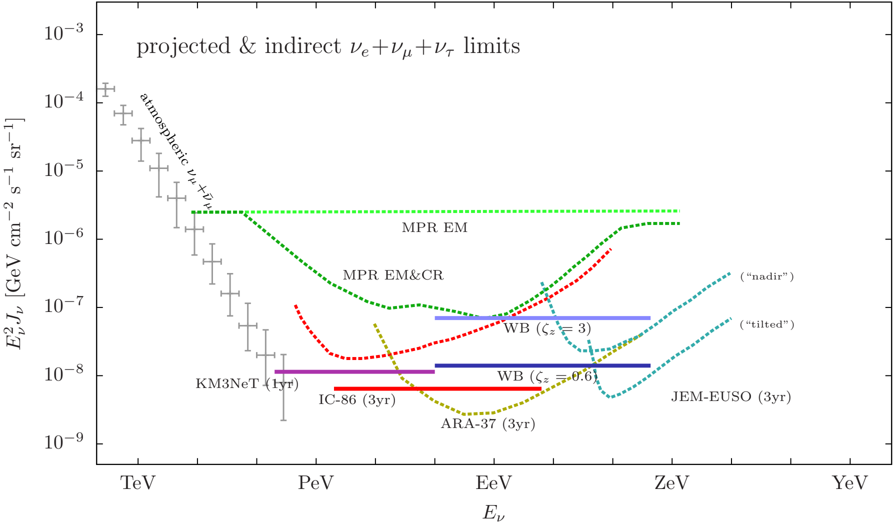


Earth-skimming tau neutrinos.

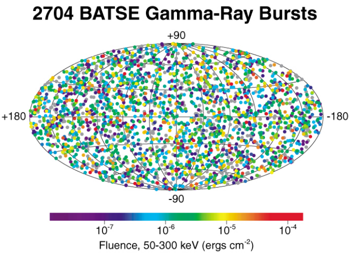
Diffuse neutrino limits

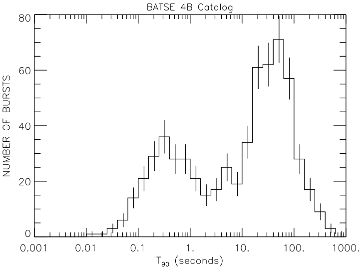


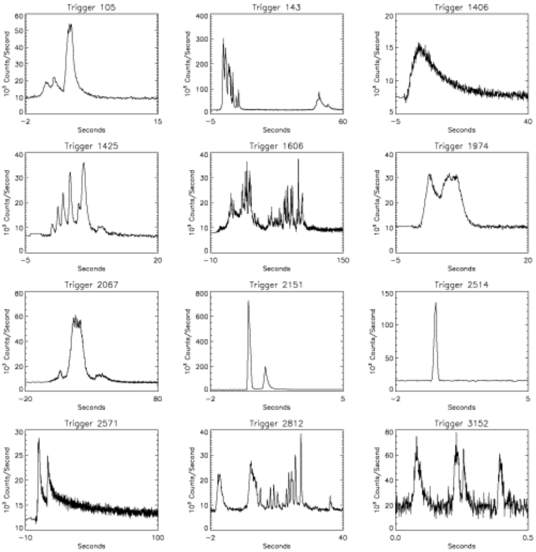
Diffuse neutrino limits



Gamma-ray bursts (GRBs)







Gamma-ray bursts & UHE CRs

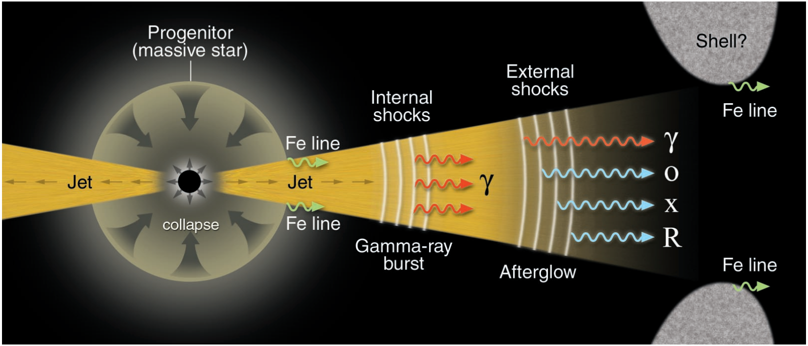
- possible sources of UHE CRs:
  - ✔ comparable **energy density**: $10^{53}\,\mathrm{erg}\,\mathrm{t_{Hubble}}^{-3}\,\mathrm{day}^{-1} \simeq 10^{44}\,\mathrm{erg}\,\mathrm{Mpc}^{-3}\,\mathrm{yr}^{-1}$
  - ✔ fulfill necessary conditions on time-scales (dynamical, cooling, acceleration) to reach **ultra-high energies** [Hillas'84]
  - ✔ acceleration of UHE CRs possible, *e.g.*, in **internal or external reverse shocks** [Vietri'95;Waxman'95]

➜ *smoking gun signal*: **neutrino production**

- Neutrino emission of GRBs is one of the best-tested models: [IceCube, Nature'12]
  - ✔ **cosmological sources** ("one per day and 4π")
  - ✔ **wealth of data** from Swift and Fermi
  - ✔ good information on **timing and location** (➜ background reduction)

GRB neutrino emission

- Neutrino production at various stages of GRB, *e.g.*
  - ➜ **precursor** pp and $p\gamma$ interactions in stellar envelope; also possible for "failed" GRBs [Razzaque,Meszaros&Waxman'03]
  - ➜ **burst** $p\gamma$ interactions in internal shocks [Waxman&Bahcall'97]
  - ➜ **afterglow** $p\gamma$ interactions in reverse external shocks [Waxman&Bahcall'00;Murase&Nagataki'06;Murase'07]



[Meszaros'01]

Burst neutrino emission

- neutrinos from meson production, *e.g.*

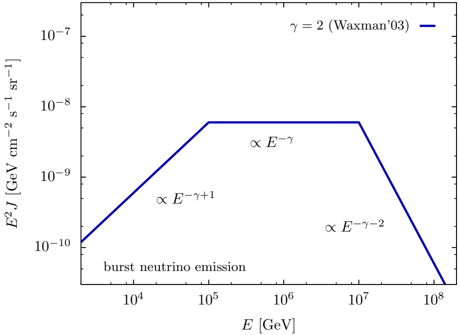
$$\pi^+ \to \mu^+\nu_\mu \to e^+\nu_e\bar{\nu}_\mu\nu_\mu$$

- spectra shaped by burst and proton spectrum and synchrotron loss of pions and muons before decay

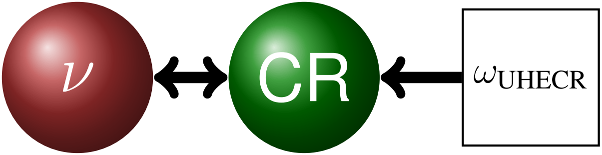
[Waxman & Bahcall'97]

- for typical burst spectra this c s a "plateau" of neutrinos

$$100\,\text{TeV} \lesssim E_\nu \lesssim 10\,\text{PeV}$$



➜ **Different models for absolut normalization:**



Burst neutrino emission

- neutrinos from meson production, *e.g.*

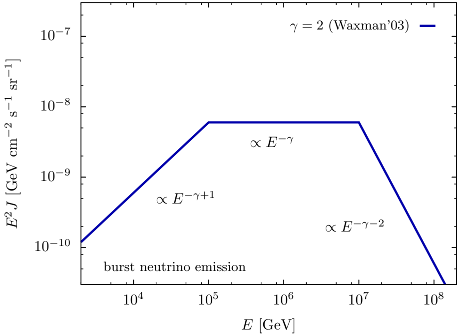
$$\pi^+ \to \mu^+\nu_\mu \to e^+\nu_e\bar{\nu}_\mu\nu_\mu$$

- spectra shaped by burst and proton spectrum and synchrotron loss of pions and muons before decay

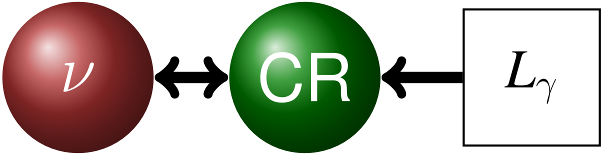
[Waxman & Bahcall'97]

- for typical burst spectra this c s a "plateau" of neutrinos

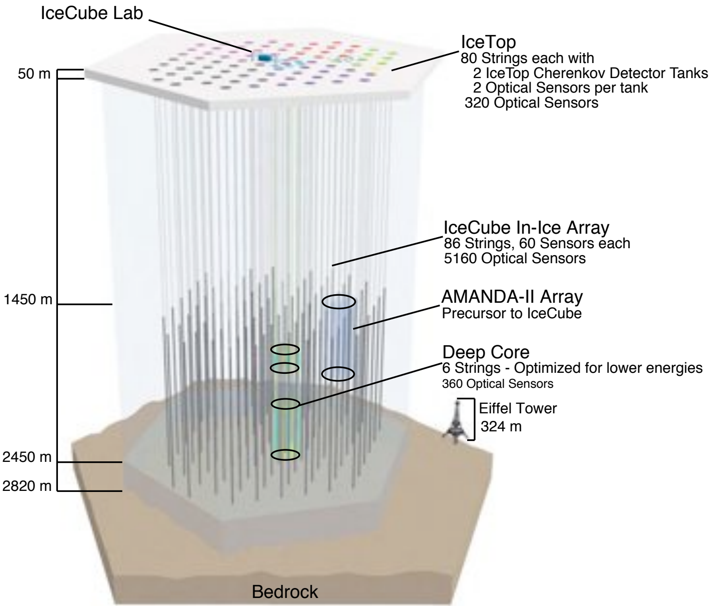
$$100\,\mathrm{TeV} \lesssim E_\nu \lesssim 10\,\mathrm{PeV}$$



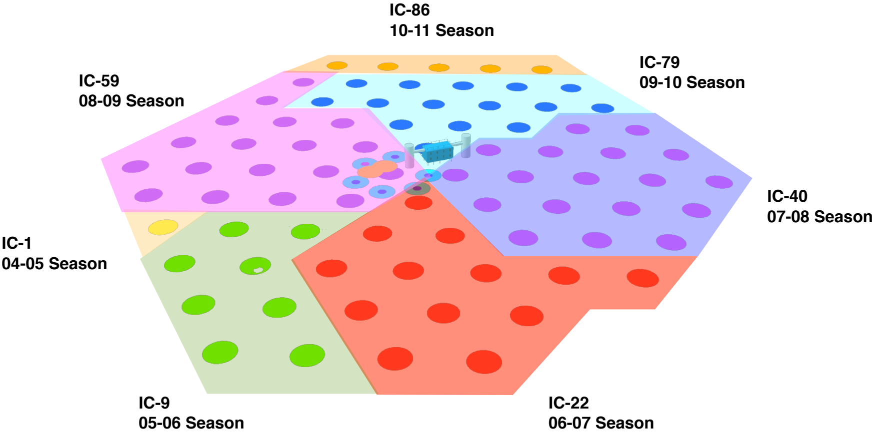
➜ **Different models for absolut normalization:**



IceCube search for burst neutrinos



IceCube search for burst neutrinos



[courtesy of M. Santander]

IC40+59 results

- Limits on neutrino emission coincident with 215 (85) northern (southern) sky GRBs between April 2008 and May 2010 ("IC40+59"). [Abbasi *et al.*'11;'12]

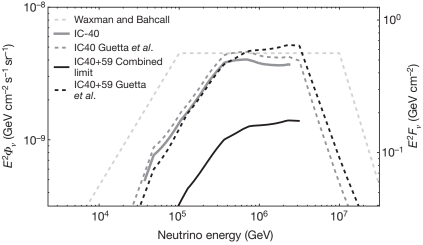
→ **Model-dependent** limit for prompt emission model.

→ **Model-independent** limit for general neutrino coincidences (no spectrum assumed) with sliding time window $\pm\Delta t$ from burst.

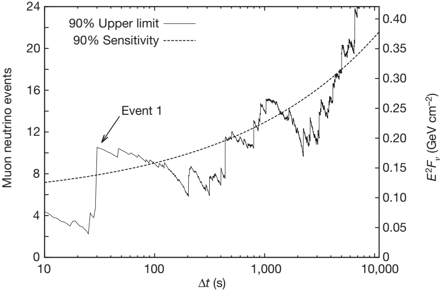
- **Stacked flux** below "benchmark" prediction of burst neutrino emission by a factor 3-4. [Guetta *et al.*'04]

→ **conversion to diffuse flux** via cosmic GRB rate.

"model-dependent"



"model-independent"



IC40+59 results

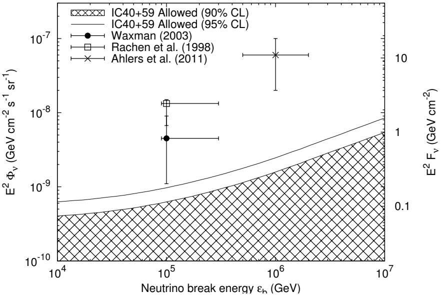
- IceCube limit below **benchmark diffuse models** normalized to UHE CR data. [Waxman&Bahcall'03; Rachen *et al.*'98]

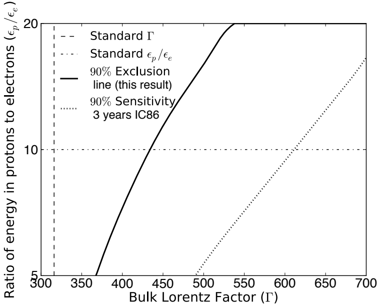
➜ **IceCube's results challenge GRBs as the sources of UHE CRs!**

- **Limit** on burst neutrino emission depends on neutrino break energy "$\varepsilon_b \propto \Gamma^2$" (break in optical depth).

- Results from model-dependent analysis translate into bounds of GRB parameters. [Guetta *et al.*'04]

➜ **Neutron emission models largely ruled out.** [MA, Gonzalez-Garcia & Halzen'11]





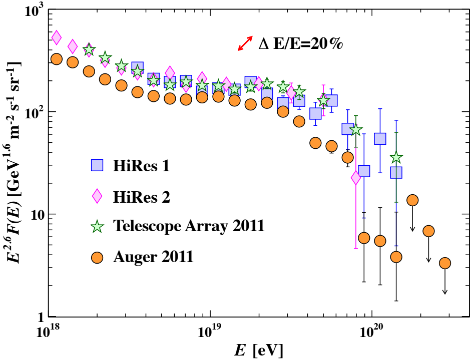
Cosmogenic neutrinos

- "Guaranteed" neutrino production from UHE CR propagation in cosmic radiation background. [Greisen&Zatsepin'66;Kuzmin'66;Berezinsky&Zatsepin'70]

➜ resonant proton interaction $p\gamma \to \Delta \to n\pi^+$ with CMB: $E_{\rm CR} < E_{\rm GZK} \simeq 40{\rm EeV}$

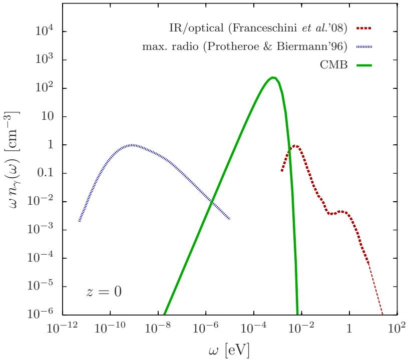
➜ peak neutrino contribution at $E_\nu \simeq 1{\rm EeV}$

UHE CR spectrum



[Particle Data Group'12]

radiation background



Cosmogenic neutrinos

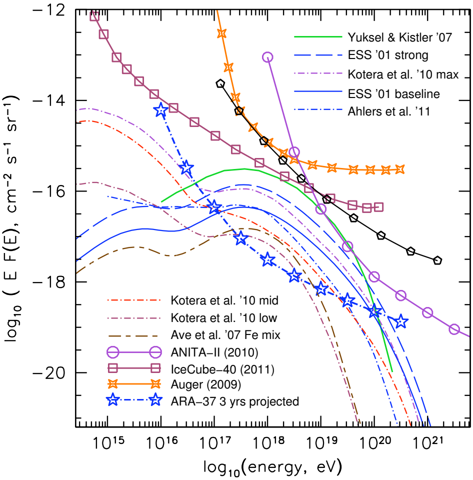


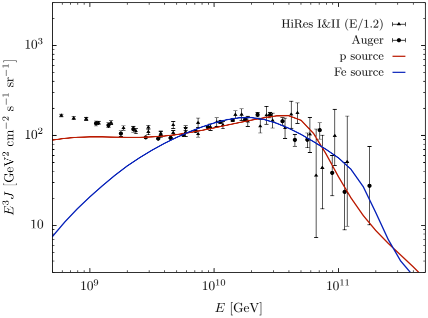
TABLE II: Expected numbers of events N_ν from several UHE neutrino models, comparing published values from the 2008 ANITA-II flight with predicted events for a three-year exposure for ARA-37.

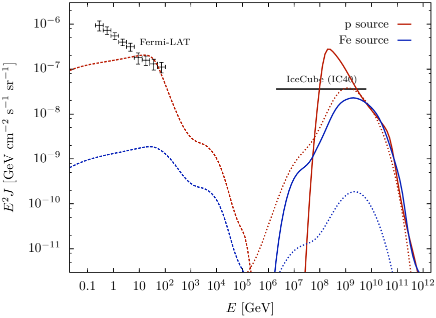
Model & references N_ν:	ANITA-II, (2008 flight)	ARA, 3 years
Baseline cosmogenic models:		
Protheroe & Johnson 1996 [27]	0.6	59
Engel, Seckel, Stanev 2001 [28]	0.33	47
Kotera,Allard, & Olinto 2010 [29]	0.5	59
Strong source evolution models:		
Engel, Seckel, Stanev 2001 [28]	1.0	148
Kalashev *et al.* 2002 [30]	5.8	146
Barger, Huber, & Marfatia 2006 [32]	3.5	154
Yuksel & Kistler 2007 [33]	1.7	221
Mixed-Iron-Composition:		
Ave *et al.* 2005 [34]	0.01	6.6
Stanev 2008 [35]	0.0002	1.5
Kotera, Allard, & Olinto 2010 [29] upper	0.08	11.3
Kotera, Allard, & Olinto 2010 [29] lower	0.005	4.1
Models constrained by Fermi cascade bound:		
Ahlers *et al.* 2010 [36]	0.09	20.7
Waxman-Bahcall (WB) fluxes:		
WB 1999, evolved sources [37]	1.5	76
WB 1999, standard [37]	0.5	27

[ARA'11]

Best-fit range of GZK neutrino predictions (∼two orders of magnitude!) cover various evolution models and source compositions.

Composition dependence of UHE CR sources





- UHE CR emission toy-model: $\mathcal{Q}(z,E) \propto E^{-\gamma} e^{-E/E_{\max}} (1+z)^n \Theta(z_{\max} - z)$
  - 100% proton: $n = 5$ & $z_{\max} = 2$ & $\gamma = 2.3$ & $E_{\max} = 10^{20.5}$ eV
  - 100% iron: $n = 0$ & $z_{\max} = 2$ & $\gamma = 2.3$ & $E_{\max} = 26 \times 10^{20.5}$ eV
- Diffuse spectra of cosmogenic γ-rays (dashed lines) and neutrinos (dotted lines) **vastly different**. [MA&Salvado'11]

Approximate* scaling law of energy densities

$$\omega_\nu \propto \underbrace{\sum_i A_i^{2-\gamma_i} \frac{E_{\rm th}^2 Q_i(E_{\rm th})}{2-\gamma_i}}_{\text{composition}} \times \underbrace{\int_0^{z_{\max}} {\rm d}z \frac{(1+z)^{n+\gamma_i-4}}{H(z)}}_{\text{evolution}}$$

*** disclaimer**:

- source composition Q_i with mass number A_i and index γ_i
- applies only to models with large rigidity cutoff $E_{\max,i} \gg A_i \times E_{\rm GZK}$

previous examples ($z_{\max} = 2$ & $\gamma = 2.3$):

- 100% proton: $n = 5$ & $E_{\max} = 10^{20.5}$ eV
  $\omega_\gamma \propto 1 \times 12$
- 100% iron: $n = 0$ & $E_{\max} = 26 \times 10^{20.5}$ eV
  $\omega_\gamma \propto 0.27 \times 0.5$

➜ **relative difference:** $\sim 82.$

Guaranteed cosmogenic neutrinos

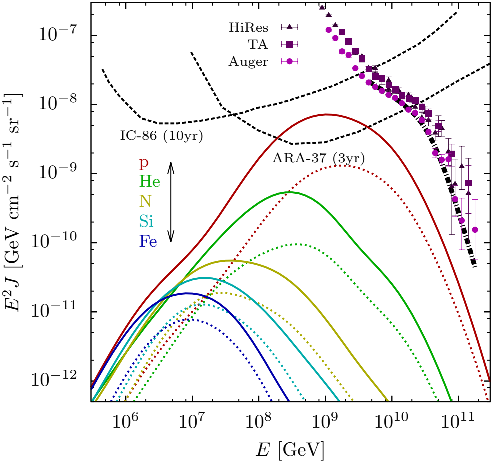
- Cascades of UHE CR nuclei in background conserve $E_N \simeq E_{CR}/A$.

➜ **minimal cosmogenic neutrinos** from nucleon spectrum:

$$J_N^{\rm min}(E_N) = A_{\rm obs}^2 J_{\rm CR}(E_{\rm CR})$$

- dependence on cosmic evolution of sources:
  - no evolution (dotted)
  - star-formation rate (solid)

➜ **ultimate test** of UHE CR proton models with **ARA-37**



[MA&Halzen'12]

Guaranteed cosmogenic neutrinos

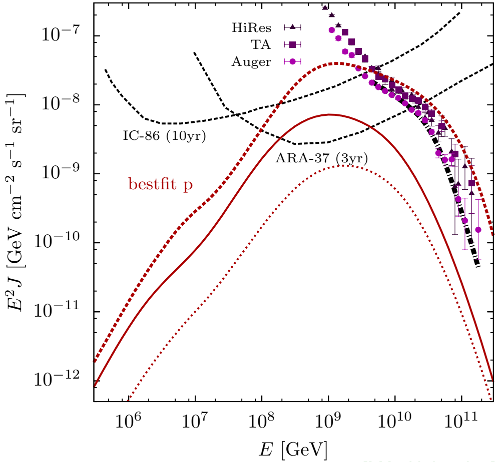
- Cascades of UHE CR nuclei in background conserve $E_N \simeq E_{CR}/A$.

➜ **minimal cosmogenic neutrinos** from nucleon spectrum:

$$J_N^{\rm min}(E_N) = A_{\rm obs}^2 J_{\rm CR}(E_{\rm CR})$$

- dependence on cosmic evolution of sources:
  - no evolution (dotted)
  - star-formation rate (solid)

➜ **ultimate test** of UHE CR proton models with **ARA-37**



[MA&Halzen'12]

Guaranteed cosmogenic neutrinos

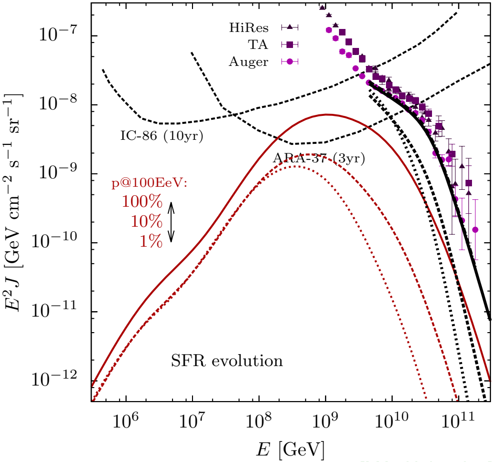
- Cascades of UHE CR nuclei in background conserve $E_N \simeq E_{CR}/A$.

→ **minimal cosmogenic neutrinos** from nucleon spectrum:

$$J_N^{min}(E_N) = A_{obs}^2 J_{CR}(E_{CR})$$

- dependence on cosmic evolution of sources:
  - no evolution (dotted)
  - star-formation rate (solid)

→ **ultimate test** of UHE CR proton models with **ARA-37**



[MA&Halzen'12]