Limit setting

(in point source searches)

Aart Heijboer,
Nikhef

since you're here early Sunday morning, I'll assume you're interested.....

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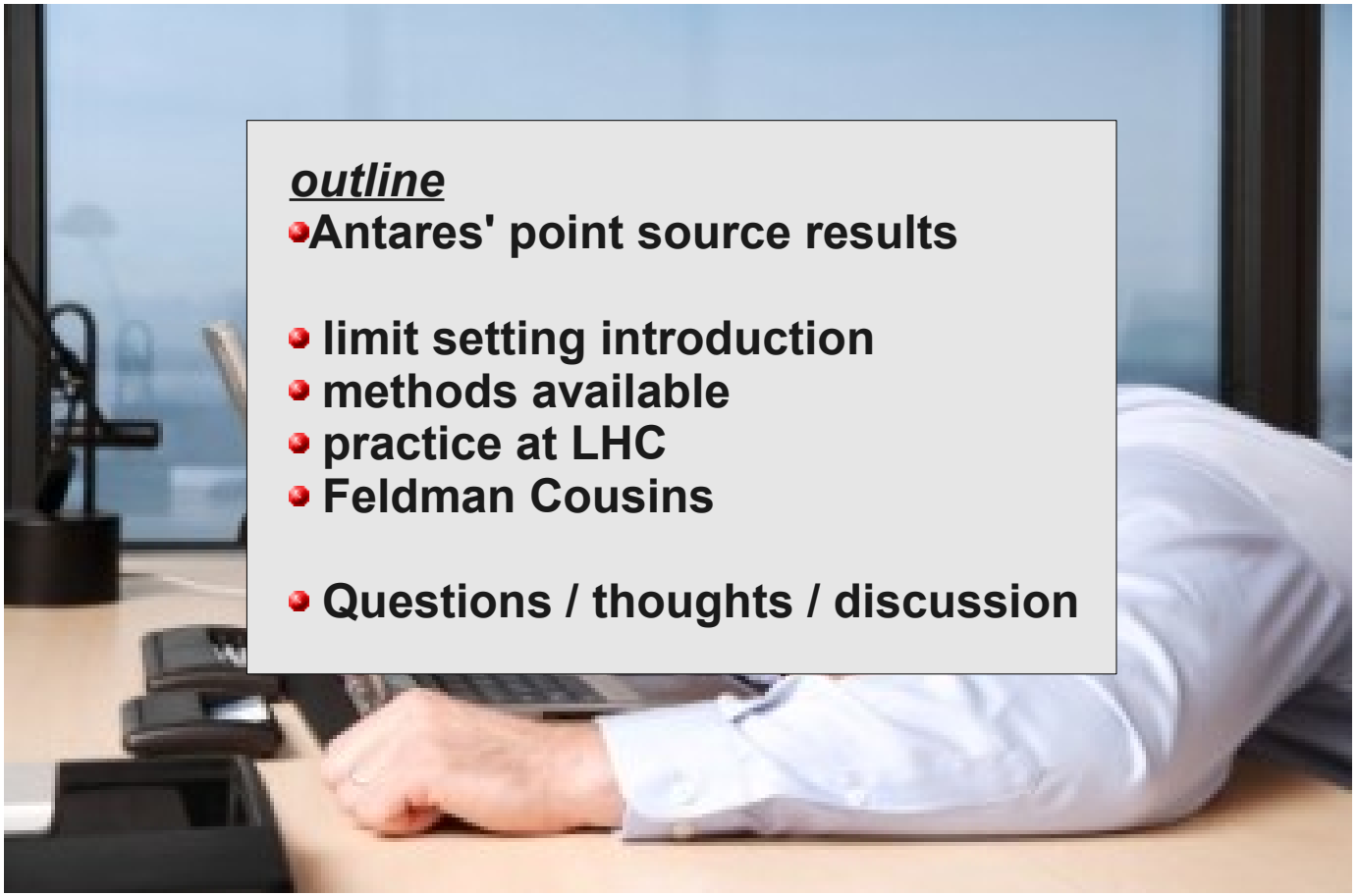
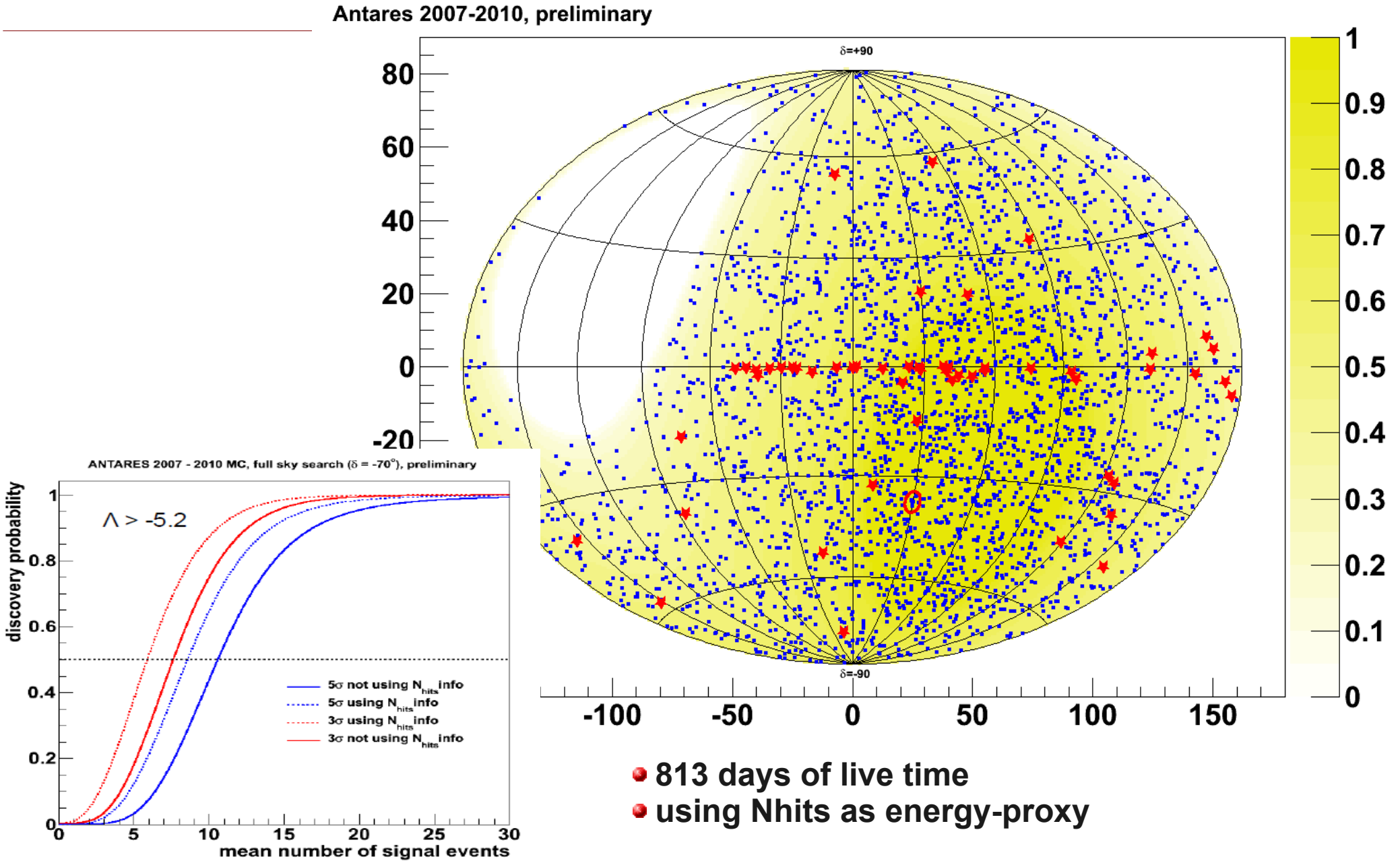


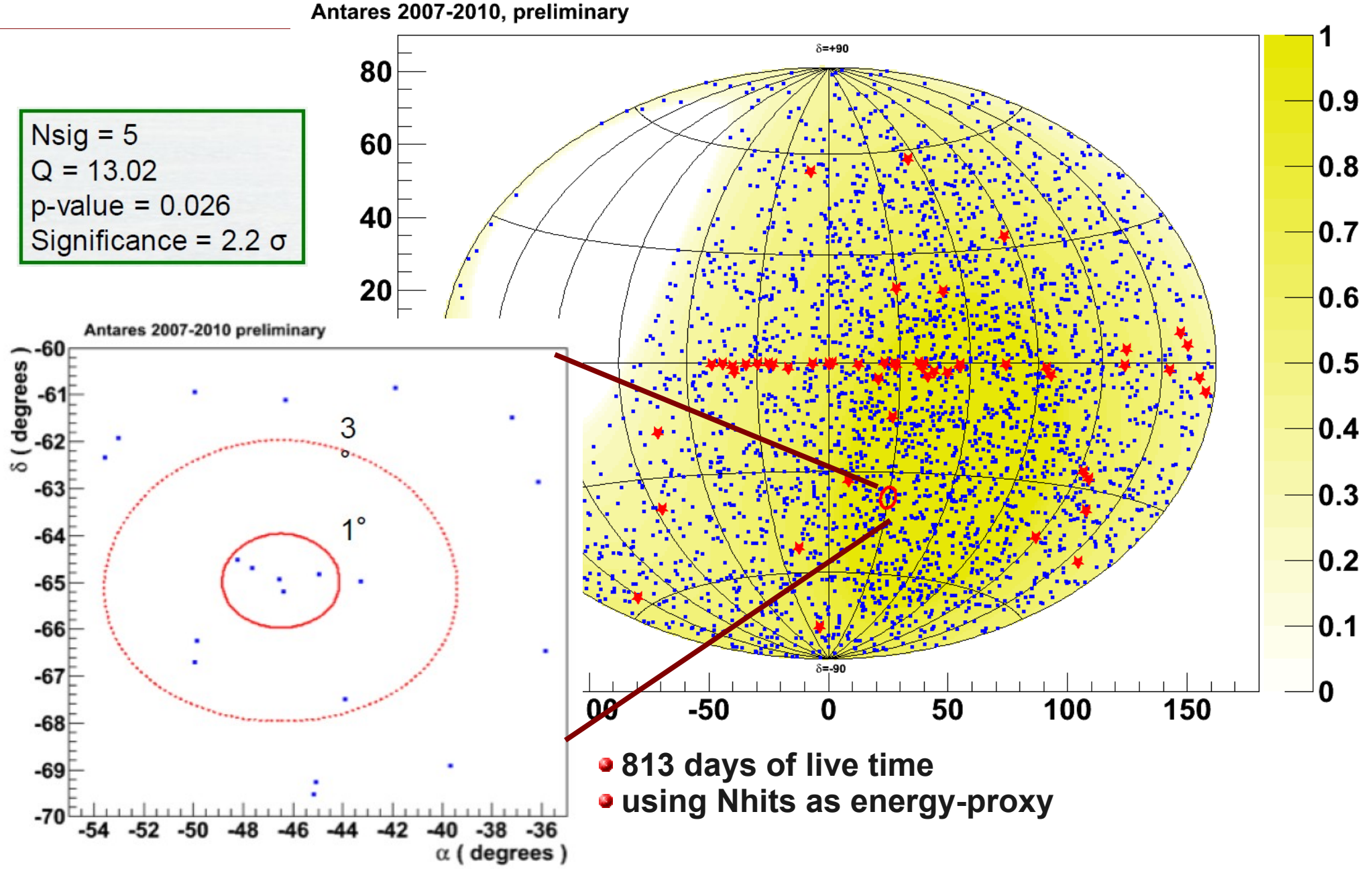
Antares' latest point source results



- 813 days of live time
- using Nhits as energy-proxy

Antares' latest point source results

Nsig = 5
Q = 13.02
p-value = 0.026
Significance = 2.2 σ



- 813 days of live time
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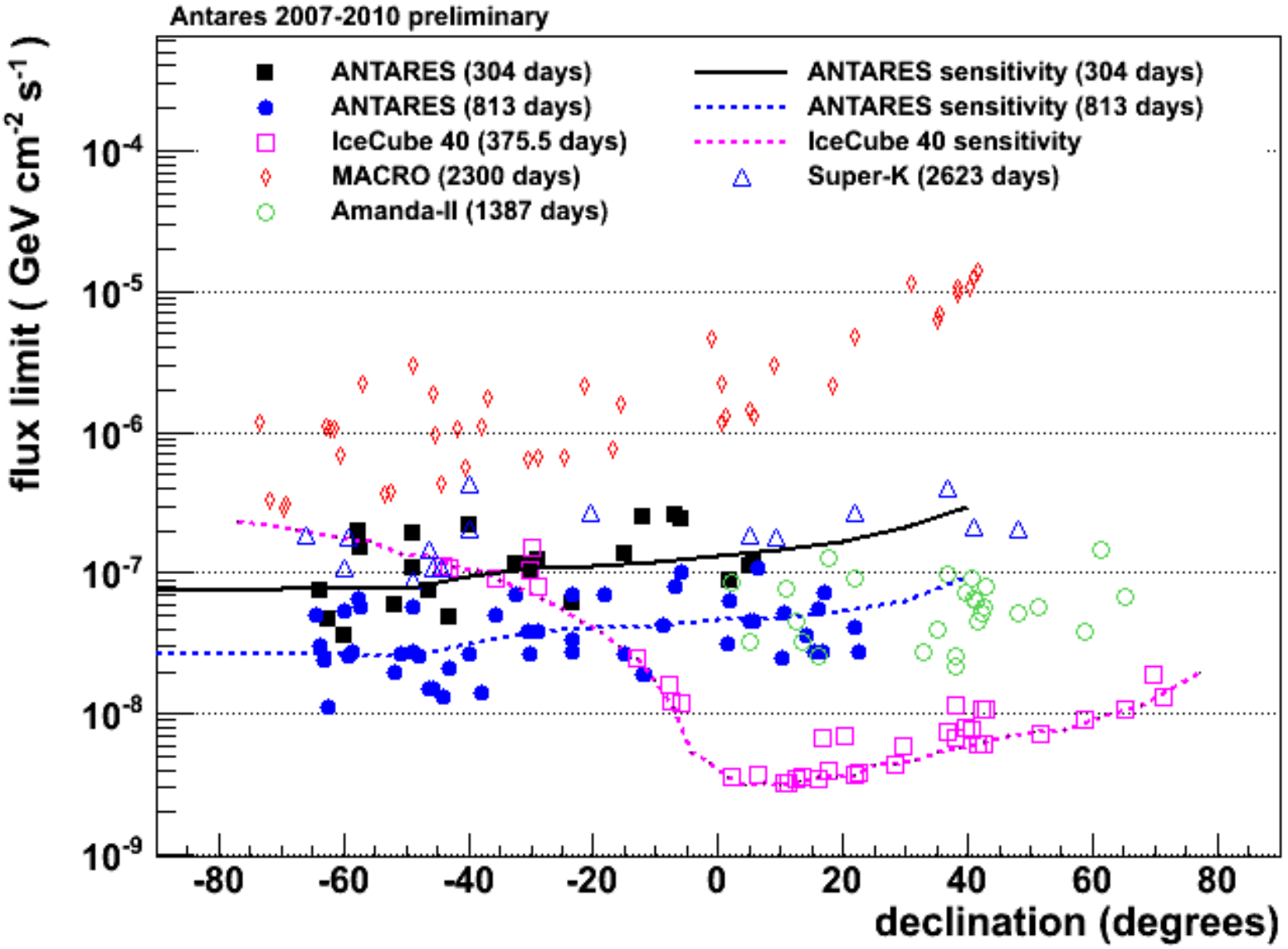
Limits (F&C)


Antares 2007-2010 preliminary
ANTARES (304 days)
ANTARES (813 days)
IceCube 40 (375.5 days)
MACRO (2300 days)
Amanda-II (1387 days)
ANTARES sensitivity (304 days)
ANTARES sensitivity (813 days)
IceCube 40 sensitivity
Super-K (2623 days)
flux limit (GeV cm^{-2} s^{-1})
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
-80
-60
-40
-20
0
20
40
60
80
declination (degrees)

Limit setting : overview of methods and issues

Introduction

All searches use some likelihood ratio test statistic. We call it Q:

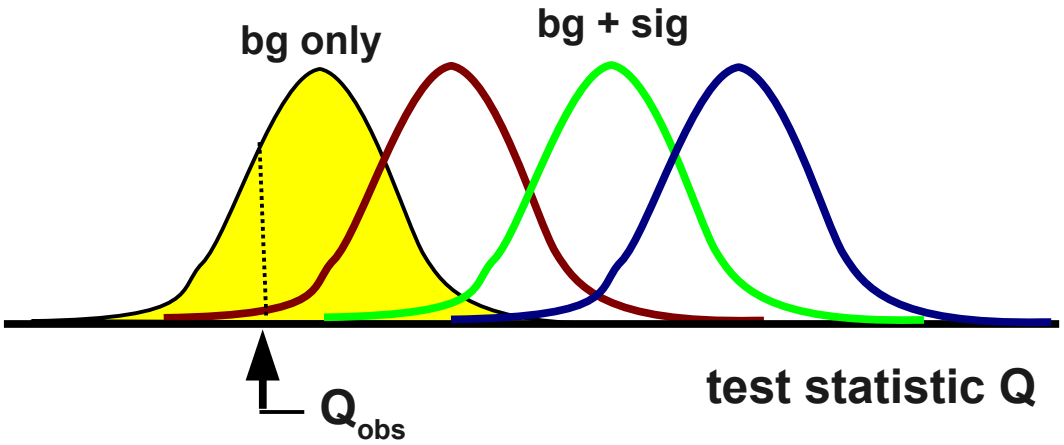
$$\log \mathcal{L}_{\mathrm{s+b}} = \sum \log[\mu_{\mathrm{sig}} \times \mathcal{F}(\beta_i(\delta_s, \alpha_s)) \times \mathcal{N}(N_{hits}^{i,sig}) + \mathcal{B}_i \times \mathcal{N}(N_{hits}^{i,bkg})] + \mu_{\mathrm{tot}}$$

$$Q = \log \mathcal{L}_{\mathrm{s+b}}^{\max} - \log \mathcal{L}_{\mathrm{b}}$$

- making discoveries
  - easy!
  - p-values easy to compute
  - ~no systematics

- setting limits
  - surprisingly hard:
  - choices involved that matter for the numbers
    different limit setting method can change result by 40%
  - possibility of nonsense-results
  - statisticians do not agree

end up with distributions of the test statistic (from MC) and one observed value (from data)



Introduction

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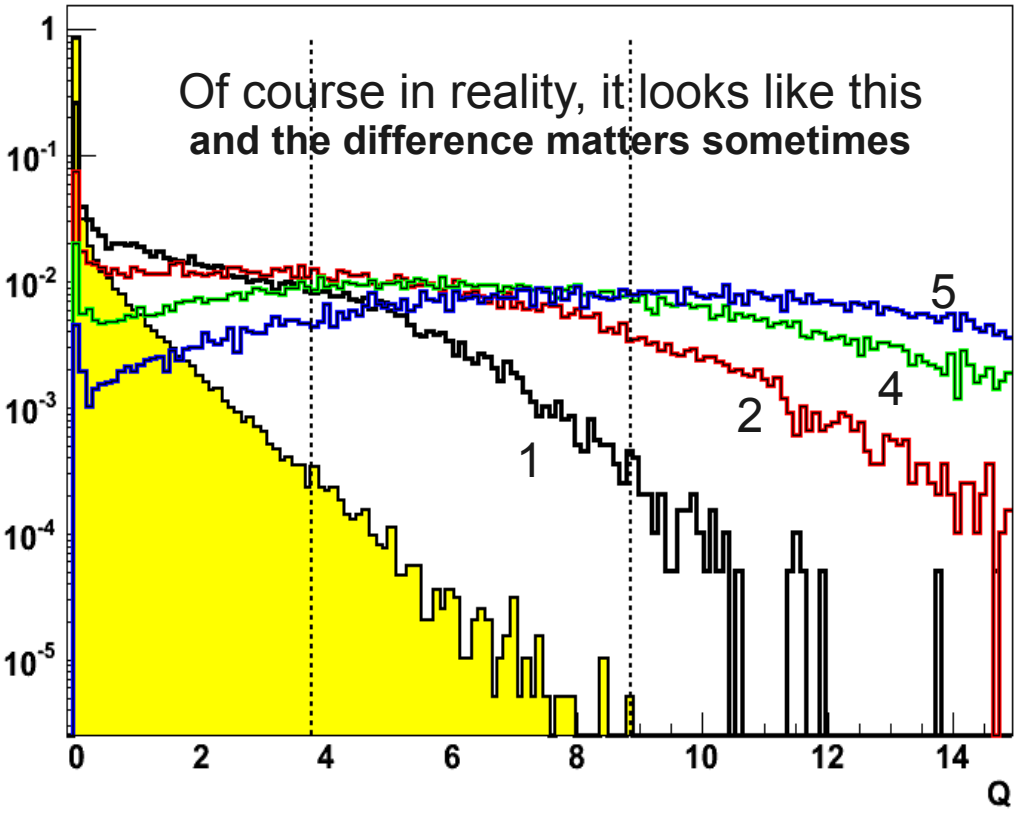
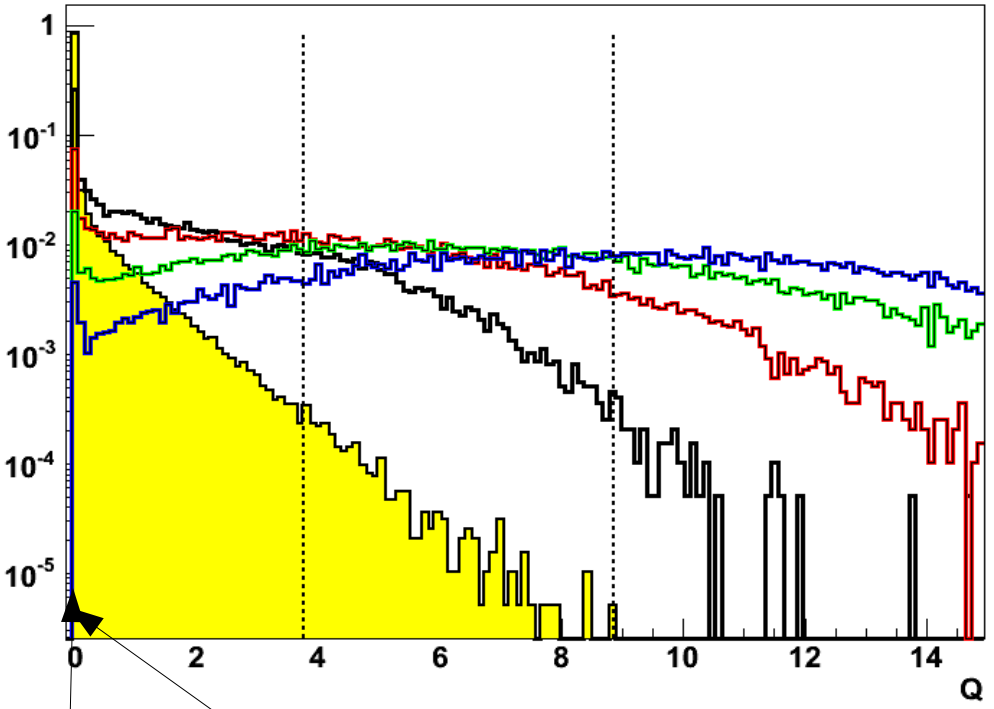
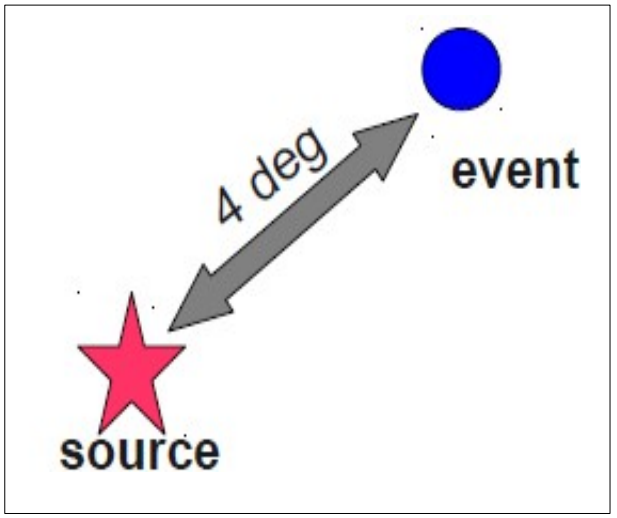


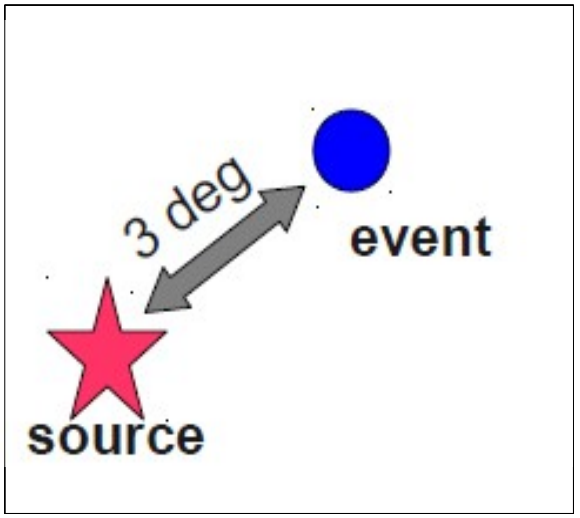
Q distributions from running analysis on pseudo-experiments. PE generation can include all the systematics.

BG-like experiments

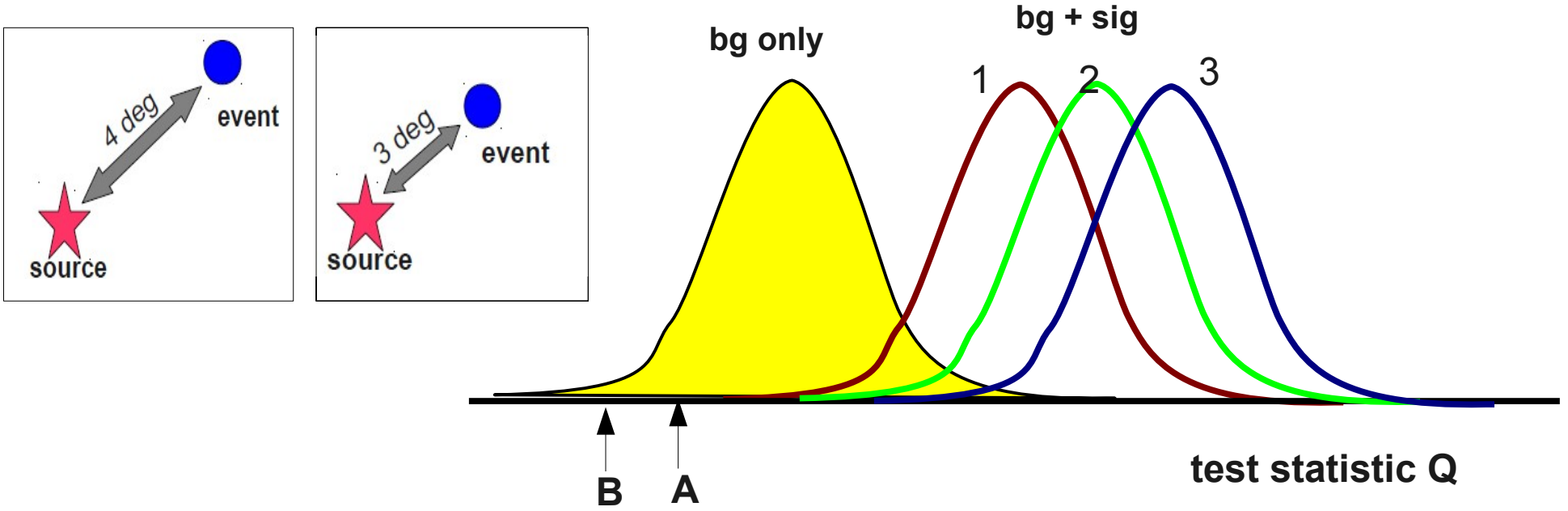
How to treat this peak?







BG-like experiments

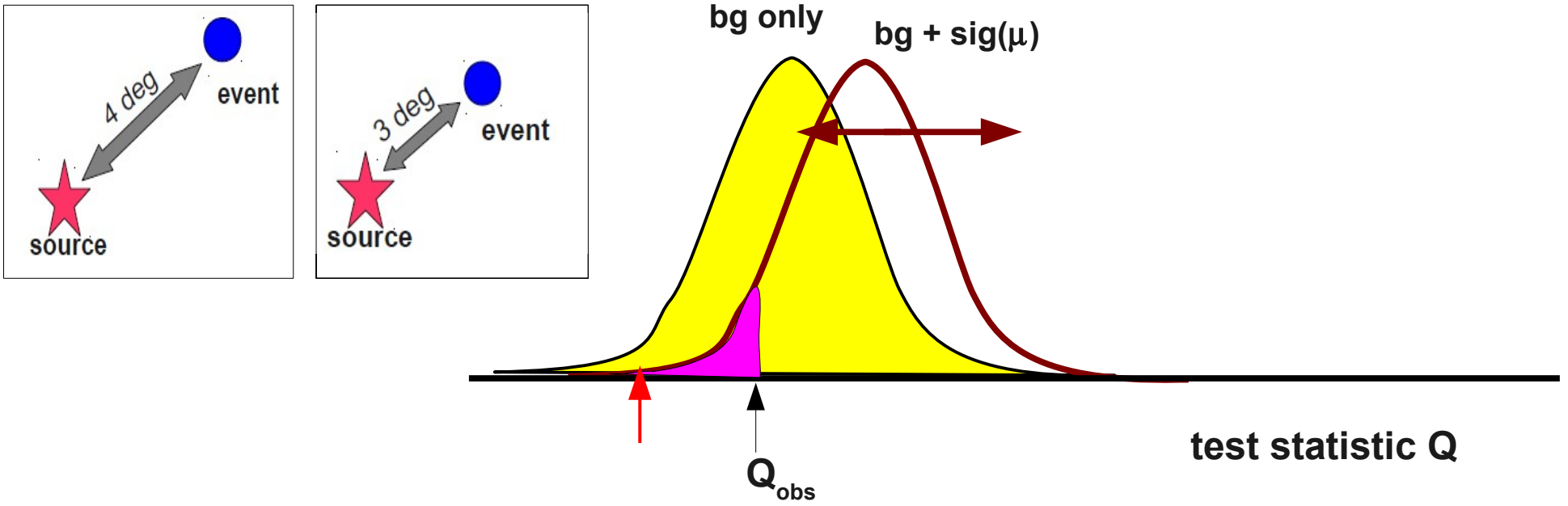


two schools of thought:

- experiment A is more signal-like that experiment
  → B should have a more stringent limit
- both experiments are ~equally compatible with any signal being present and the difference is just due to background fluctuation
  → They should yield the same limit

'Neyman' limits

(a.k.a CL_{s+b} limits)



'neyman limits' or CL_{s+b} : find the signal strength m so that

$P(Q<Q_{obs} \mid \mu) = 10\ \%$

- produces very different limits for different background fluctuations typically in the region <~1 signal event.
- If Q_{obs} is very bg-like (in the 10% tail) → exclude even $\mu=0$

Excluding a flux of zero

from CLs paper

bounded. When an experimental result appears consistent with little or no signal together with a downward fluctuation of the background, the exclusion may be so strong that even zero signal is excluded at confidence levels higher than 95%. Although a perfectly valid result from a statistical point of view, it tends to say more about the probability of observing a similar or stronger exclusion in future experiments with the same expected signal and background than about the non-existence of the signal itself, and it is the latter which is of more interest to the physicist. Presumably a great deal of effort has already gone

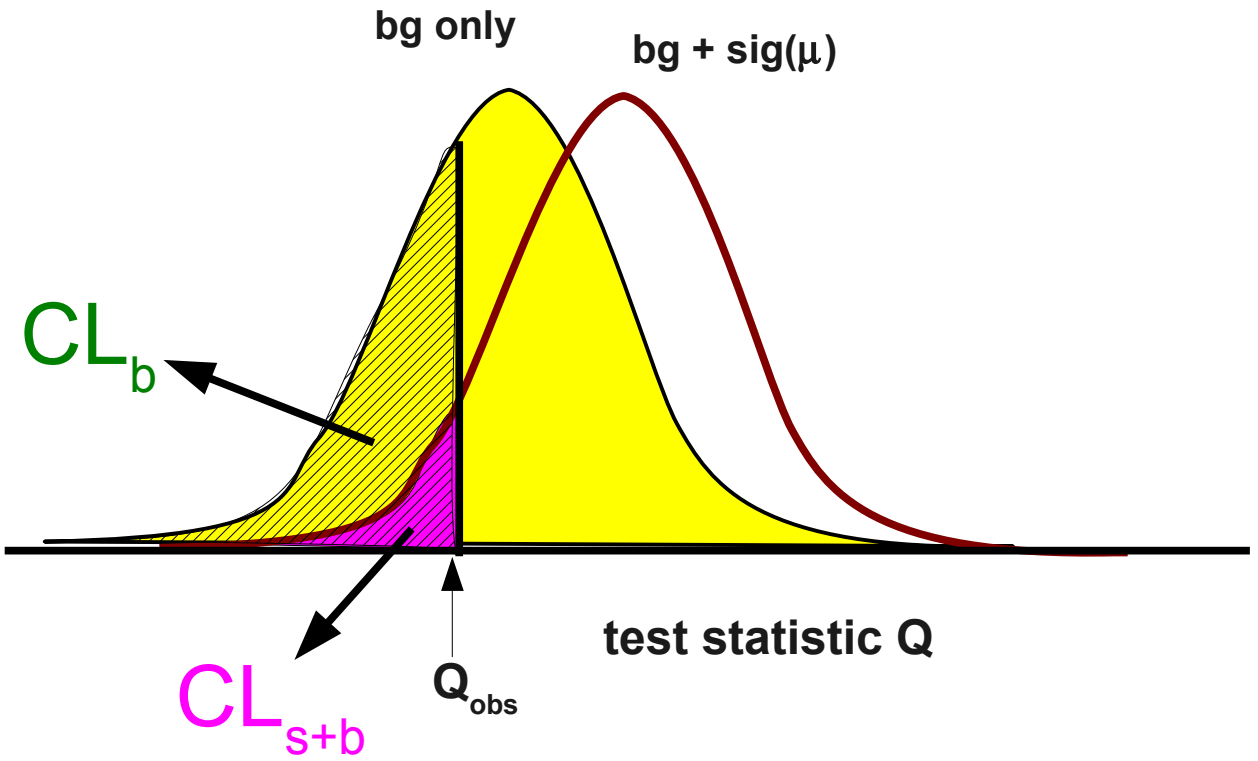
from PDG

exclusion of a parameter value that could result from a statistical fluctuation in situations where one has no sensitivity, *e.g.*, at very high Higgs masses.

happens in 10% of the cases. i.e. ~sure to happen in a candidate sourc search

Modified Frequentist (a.k.a. CL_s) method

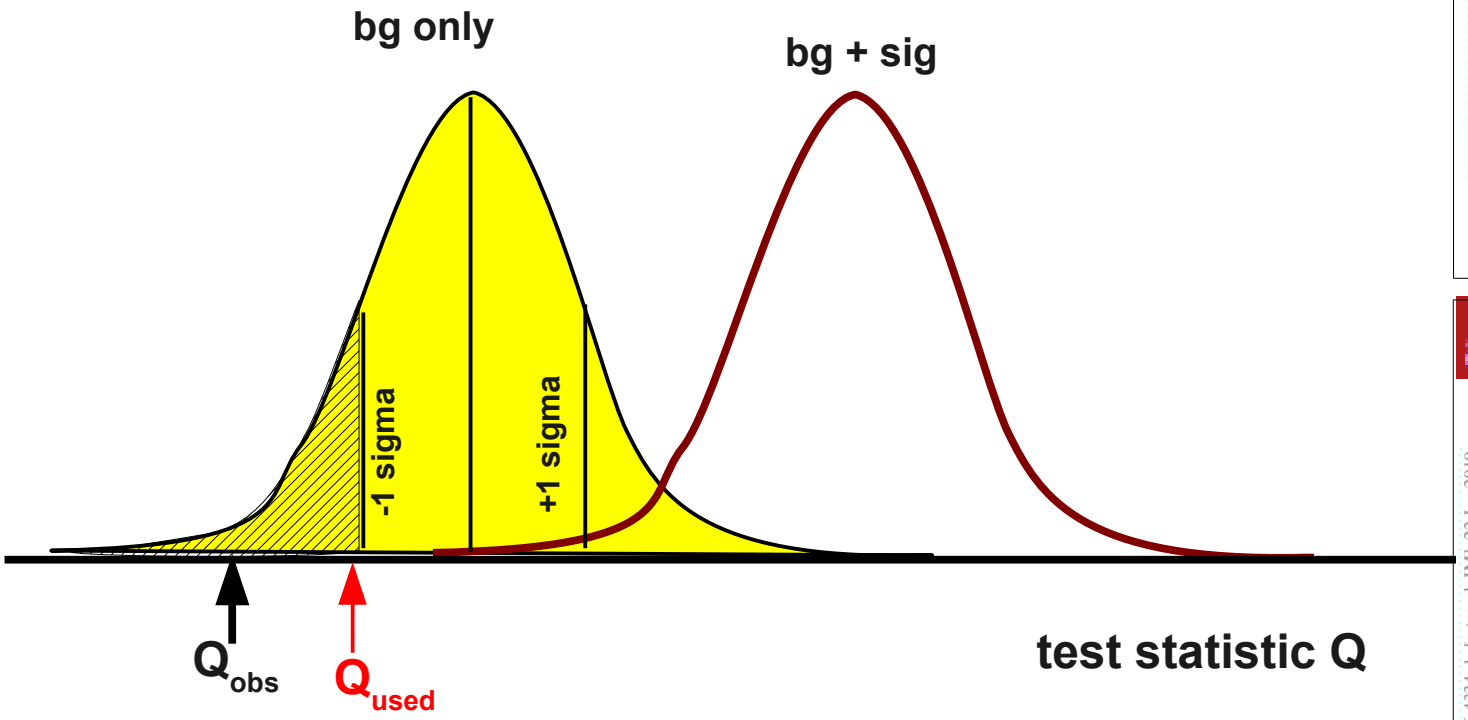
define:
$CL_s = CL_{s+b} / CL_b$
and require $CL_s(\mu) = 10\%$
for a 90% 'CL' limit



- If m = 0, CLs = 1 → never exlude this
- Only exclude values for which there is some ability to observe them
- Overcoverage : limits are 'worse'
  - nevertheless quite widely used: LEP, Tevatron, LHC...
- easy to implement
- unpopular with statisticians :
  - CLs is not a confidence level

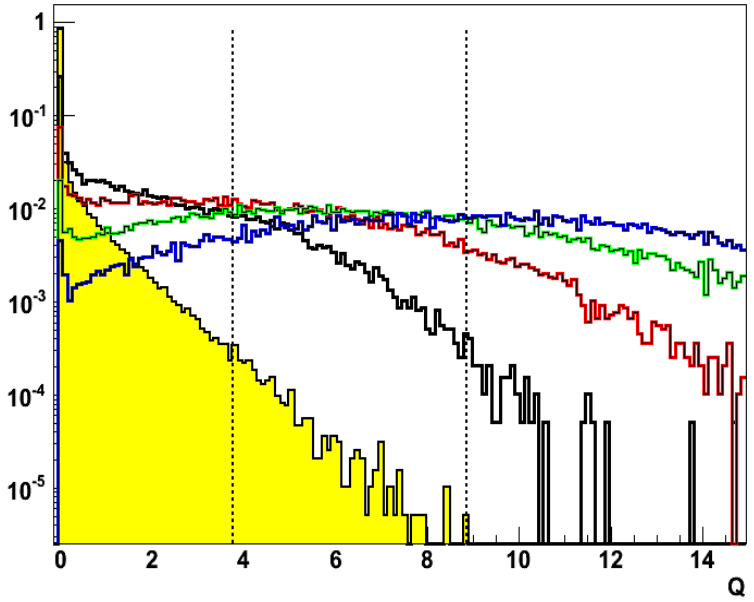
Power constrained limits

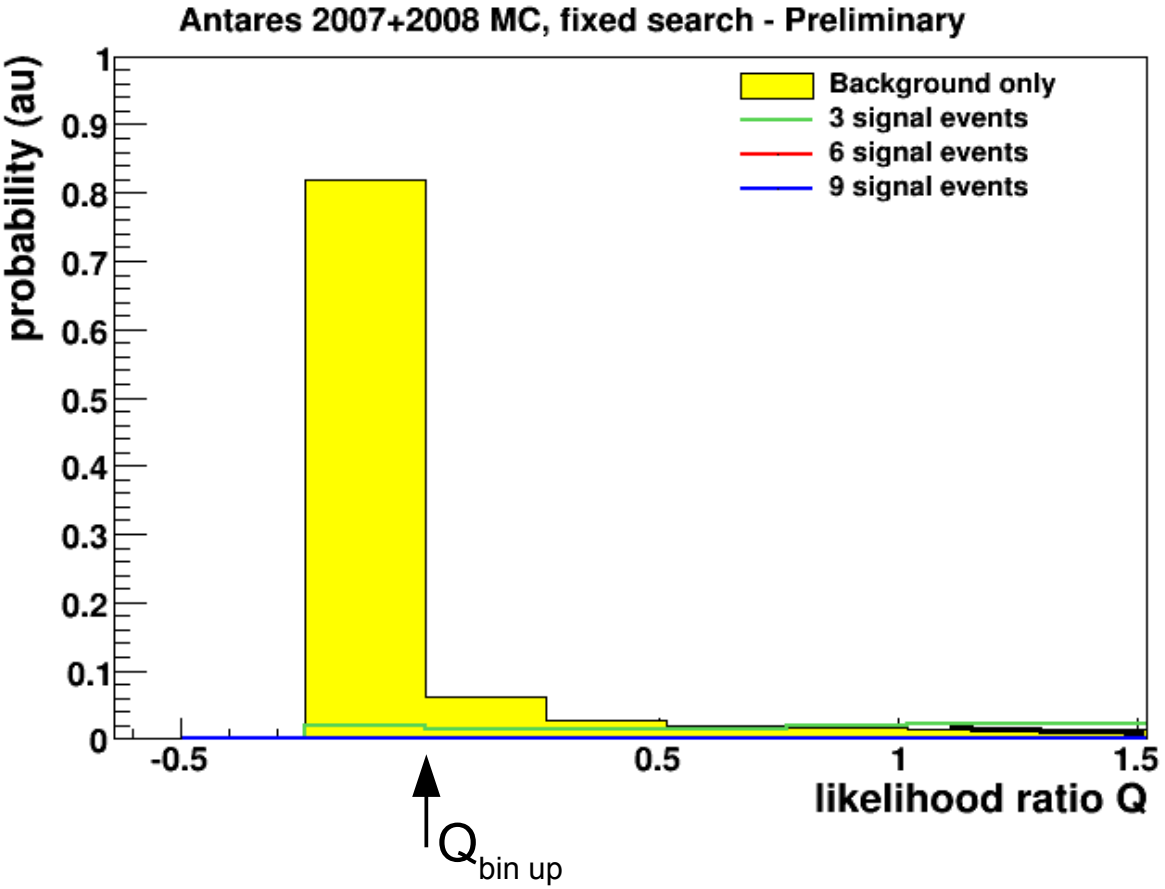
- If the observed limit is lower than some threshold, the actual limit is reported for the threshold value.
- The threshold is determined from the bg-only distribution



nb: one can easy do something like this by accident...
… by binning

'Power constrained' by accident?





doing this:

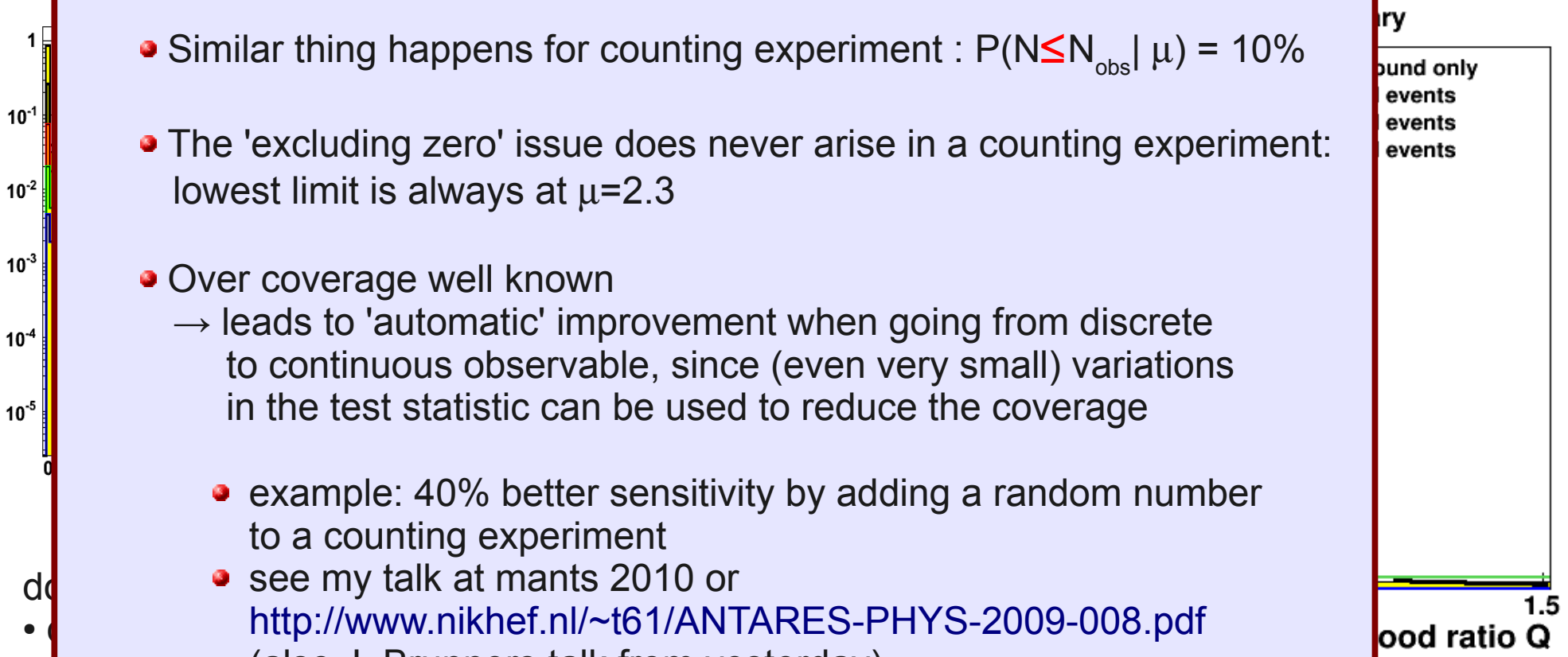
- over-covers (badly)
  → higher limits that needed for coverage
- can solve/hide the problem of excluding zero
- result depends on binning chosen (probably not desirable)

what happens depends on details of the code, but for events in 1st bin likely to amount to:

$$P(bin \leq bin_{obs}) = P(Q < Q_{bin\ up} \mid \mu) = 10\ \%$$

'Power constrained' by accident?

- Similar thing happens for counting experiment : $P(N \leq N_{obs} | \mu) = 10\%$
- The 'excluding zero' issue does never arise in a counting experiment: lowest limit is always at μ=2.3
- Over coverage well known
  → leads to 'automatic' improvement when going from discrete to continuous observable, since (even very small) variations in the test statistic can be used to reduce the coverage
  - example: 40% better sensitivity by adding a random number to a counting experiment
  - see my talk at mants 2010 or http://www.nikhef.nl/~t61/ANTARES-PHYS-2009-008.pdf (also J. Brunners talk from yesterday)



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what happens depends on details of the code, but for events in 1st bin likely to amount to:

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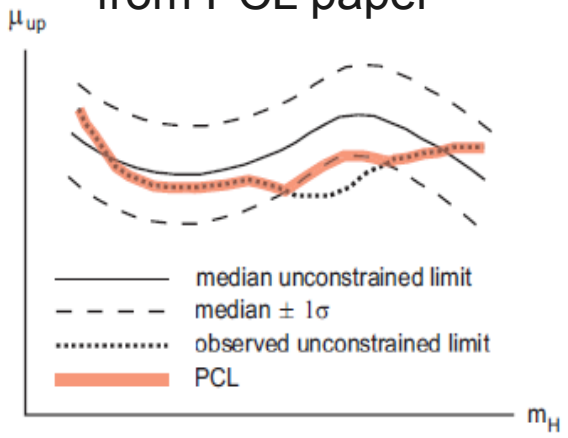
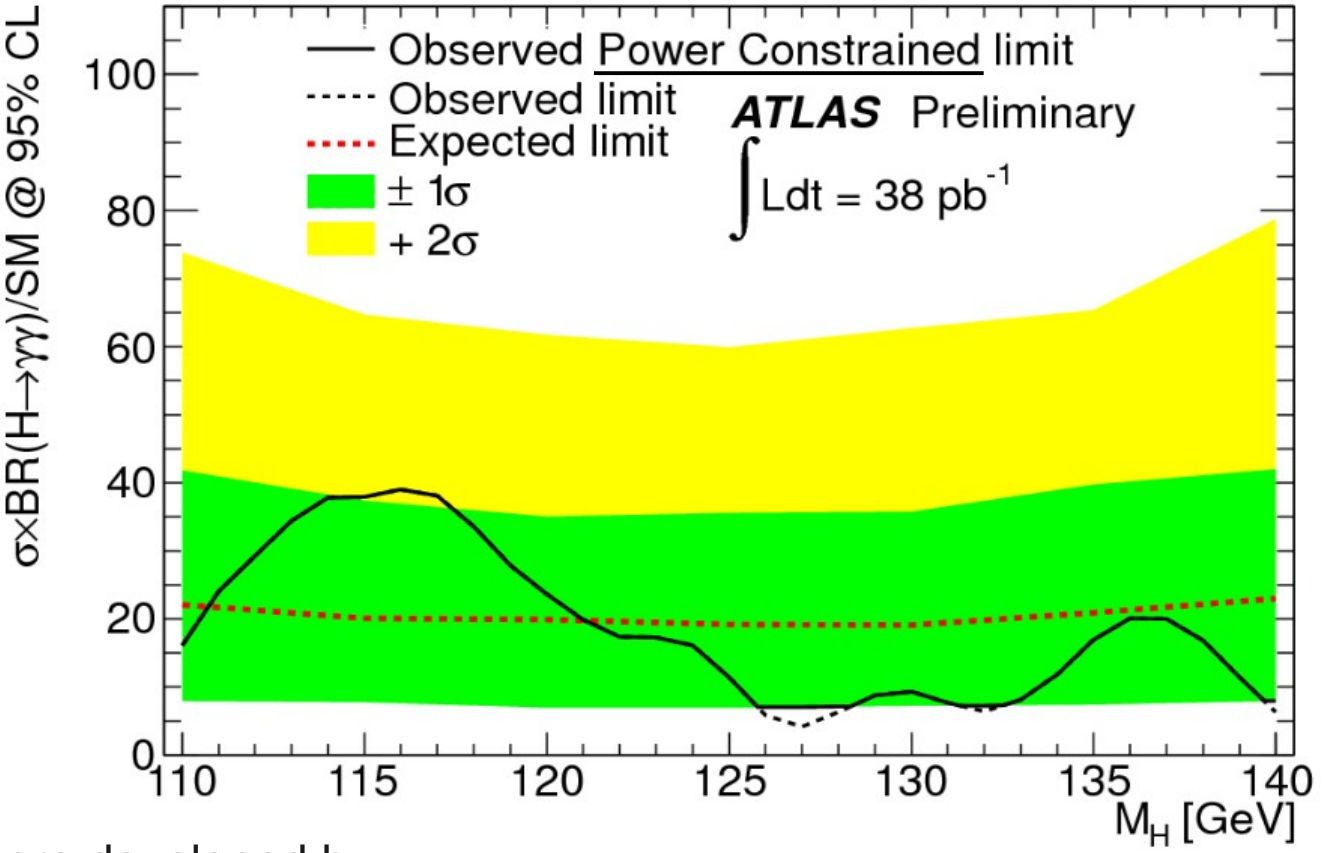
Meanwhile at the LHC...

from PCL paper



from Moriond



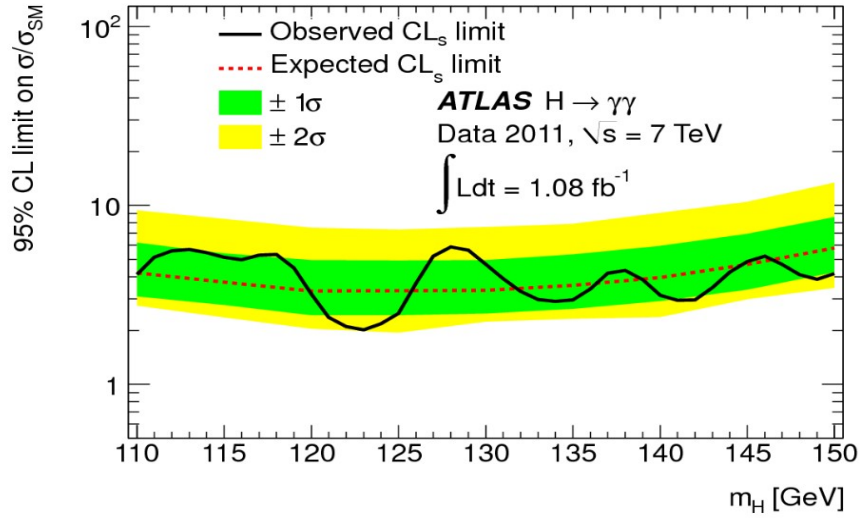
- Power-constrained limits were developed by Atlas member and addopted as 'official'
- Used for several Atlas analyses (Moriond 2011)

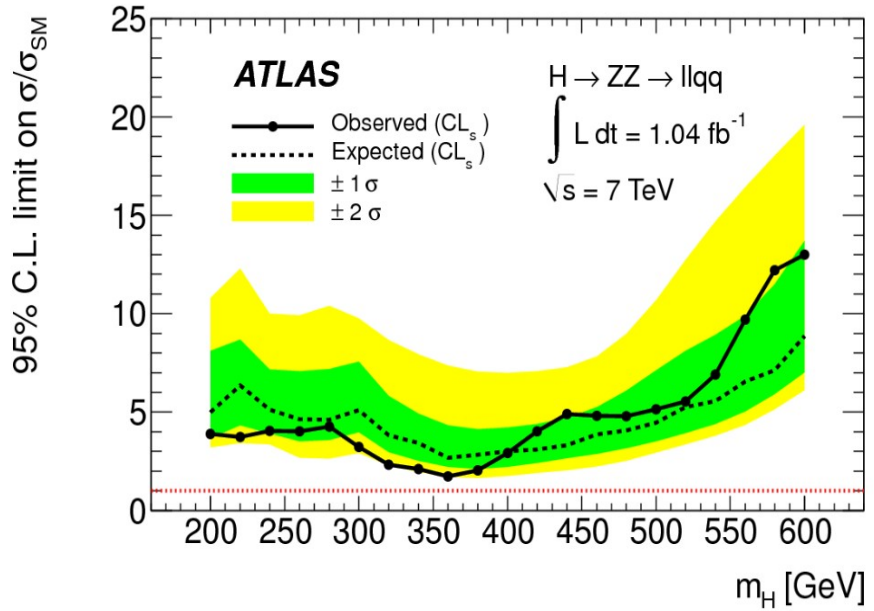
- note: they use threshold = median -1σ
  could also use: threshold = median (Juergen would like that...)

however..

Meanwhile at the LHC...

lepton-photon





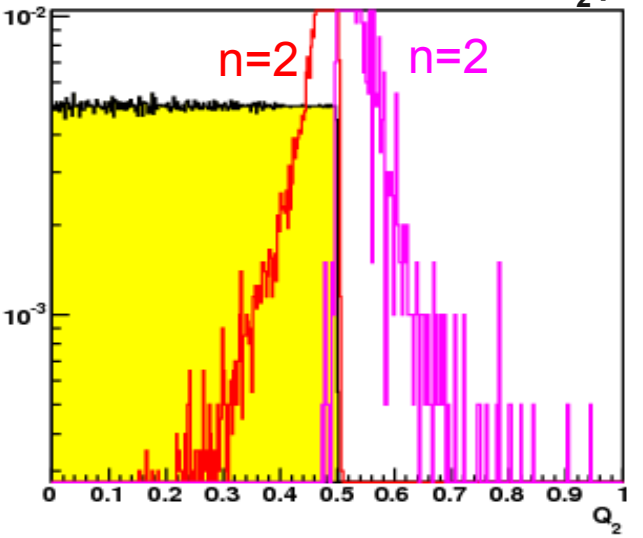
Atlas has now decided that it will produce CLs -type limits for its results.. (as a temporary solution).

- **after discussion with CMS → allows to compare directly**
- **No power constrained limits shown for recent (lepton-photon) results.**
- **Bayesian methods also still allowed (I have not talked about them .. ..they're especially popular in CMS)**
- **seems CLs is not going away easily (but plan is still to use PCL in the future)**
- **Feldman & Cousins seems not to be on their radar**

Feldman & Cousins

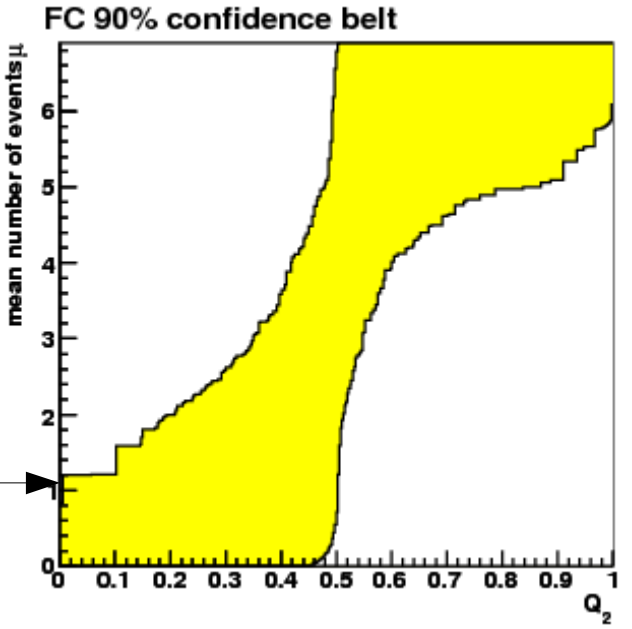
- Prevents excluding zero (by spending coverage on lower limit)
- produces double sided interval (we don't really care)
- Can be difficult to implement:
  - likelihood ordering requires many pseudo-experiments to work well..
  - a transformation of the test statistic can help, but still

transformed test statistic $Q_2(Q)$



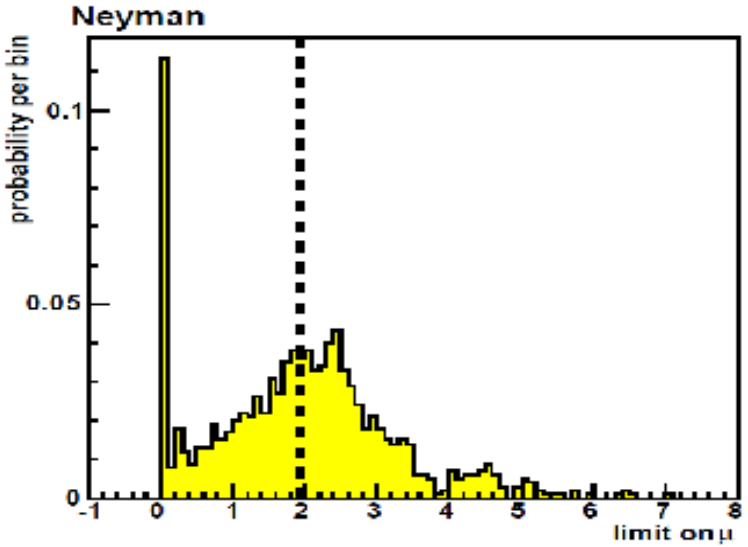
we chose it because:

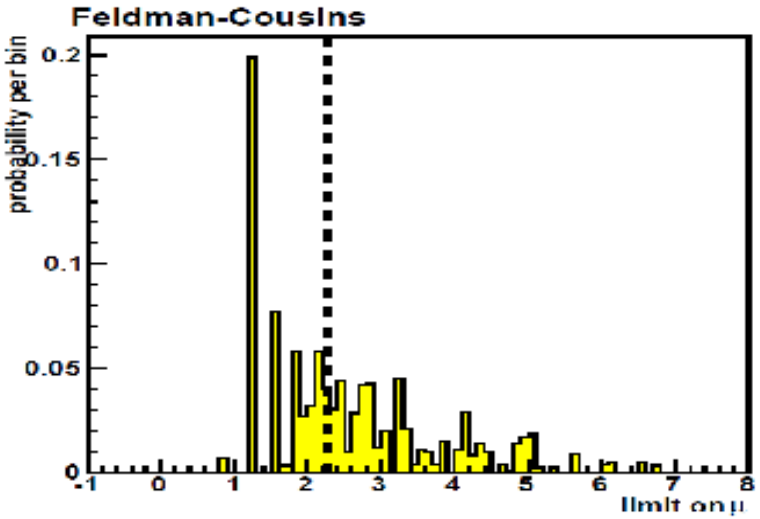
- IceCube uses it
- allows use of full range of continuous variable without the need for additional measures (like power-constraining or something that depends on the binning)
- better coverage (lower limits) than CL**s**

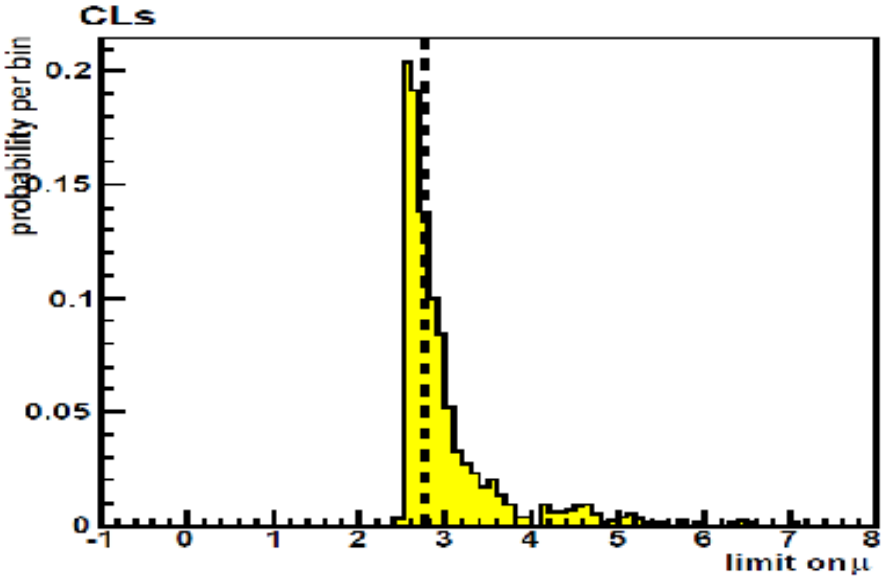


lowest possible limit around 1 event (not unreasonable)

Comparison of methods







- Neyman:
  - Easy to implement, exact coverage = lowest possible limits
  - non-physical limits (undesirable)
- Feldman-Cousins
  - tedious to implement (for continuous variable)
  - modest overcoverage
  - no unphysical limits
- CLs / Modified Frequentist (CERN-OPEN-2000-205)
  - easy to implement
  - limit does not depend on bg-only fluctuations that do not look like signal
  - severe overcoverage -> high values for limit
- **Power constrained limits**
  - easy modification of 'Neyman'
  - not yet widely accepted (but maybe soon)
  - threshold is somewhat arbitrary

Questions and thoughts

- Do we desire to use a single limit setting method
  - across experiments (Antares/IceCube/others?)
  - different measurement (e.g. do we care if the point sources use another type of limit than the diffuse flux analysis.. this is currently the case)
- Do we treat the very bg-like events in the same way?
  - limit distribution suggests that we do not
    (ic40 result looks like there are very few points below the sensitivity)
- For point sources: do we want to change from F&C to..
  - Power constraint limits (fine, but perhaps a bit too new for some readers)
  - CLs (used very widely still in HEP despite that statisticians don't like it)
  - something else?