Angular distribution of energetic particles scattered by strongly anisotropic MHD turbulence: understanding Milagro/IceCube results

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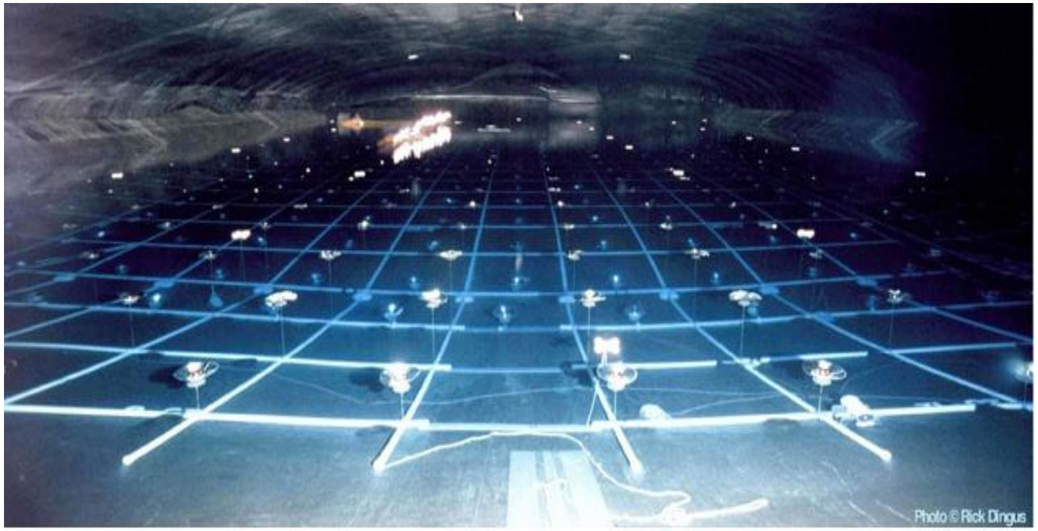
UCSD

New interest in an old problem:

Scattering of energetic particles by MHD waves

Milagro TeV-observatory newly sparked interest in energetic particle propagation after a remarkable discovery of a sharp CR arrival anisotropy

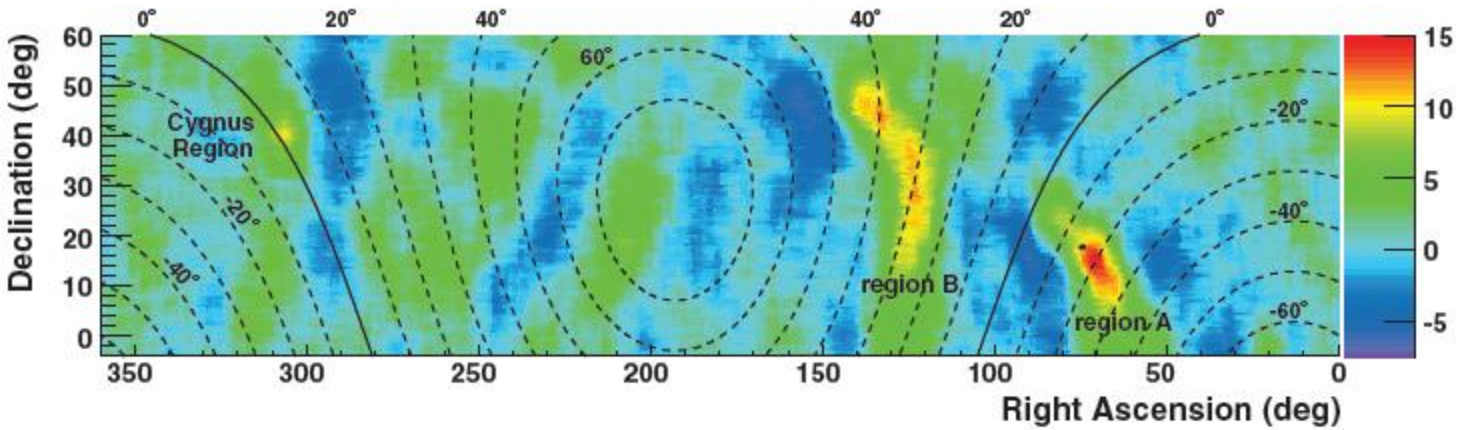


www.lanl.gov/milagro

Milagro TeV-observatory

- Water Cerenkov telescope with 723 photomultipliers
- Capability to distinguish between nuclei (protons) and gamma-rays
- < 1 deg angular resolution at 1 TeV
- 7 yr of data

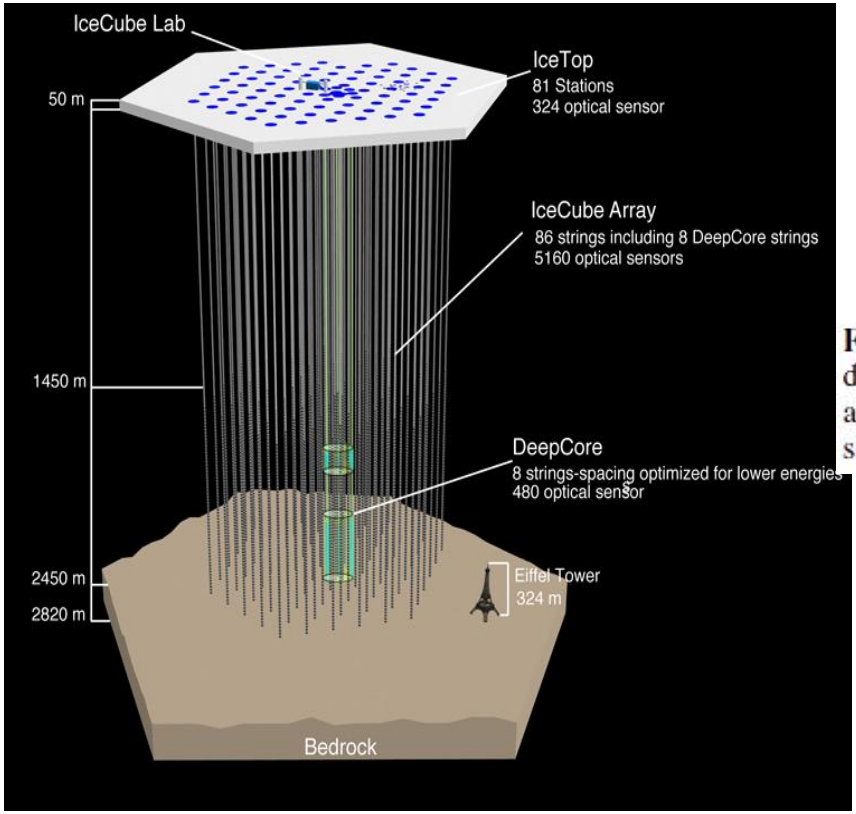
Abdo et al ‘08



Color =significance

IceCube

P. Desiati, and the IceCube Collaboration, *ArXiv e-prints:1007.2621* (2010).
S. Toscano, and The IceCube Collaboration, *Nuclea Physics B Proceedings Supplements* **212**, 201–206 (2011).



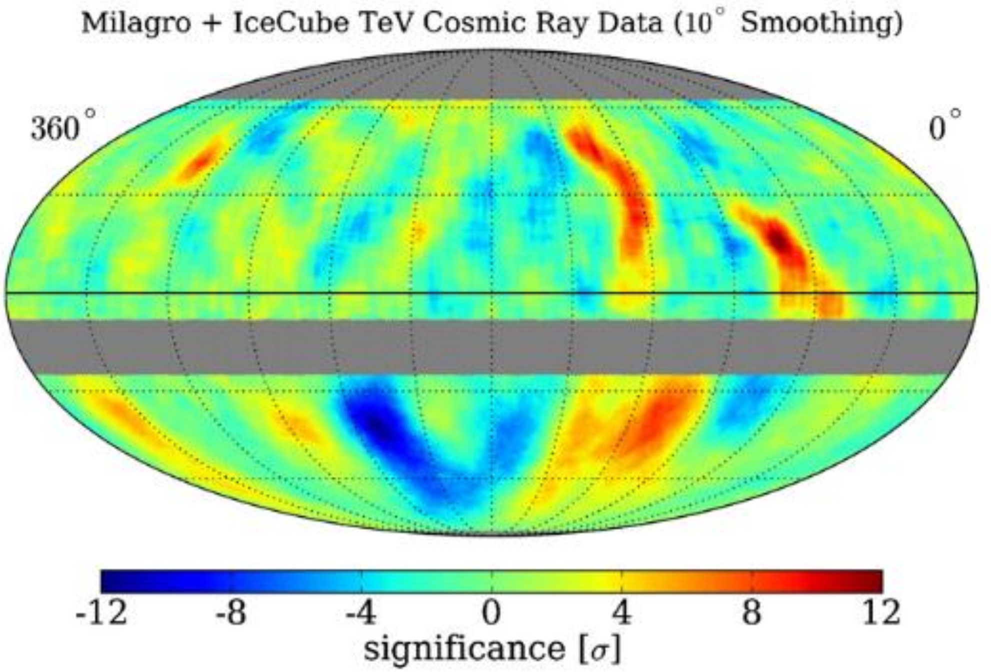


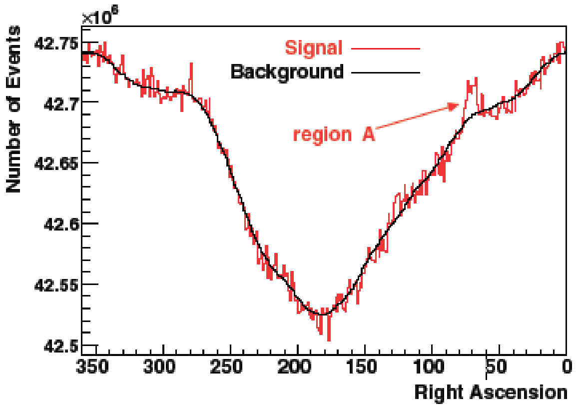
Figure 17. Combined map of significances in the cosmic ray arrival direction distribution observed by Milagro in the northern hemisphere (Abdo et al. 2008) and IceCube in the southern hemisphere (this analysis). Both maps have been smoothed with a 10° radius.

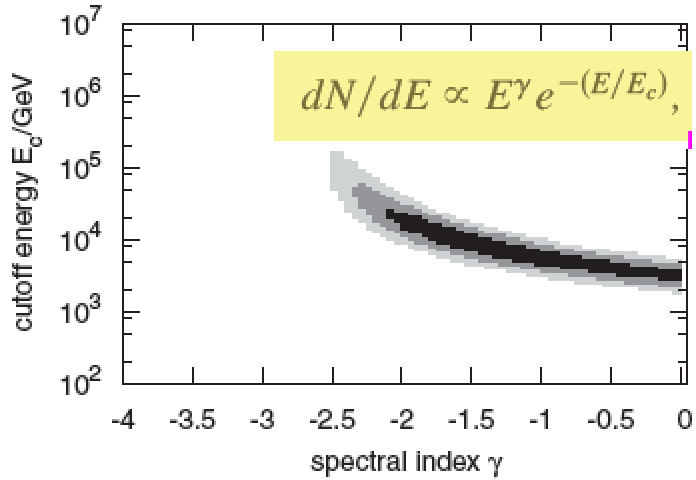
R. Abbasi, Y. Abdou, T. Abu-Zayyad, J. Adams, J. A. Aguilar, M. Ahlers, D. Altmann, K. Andeen, J. Auffenberg, and et al., *ArXiv e-prints 1105.2326* (2011).

(ApJ, Oct . 2011)

http://icecube.wisc.edu/

Some Details of Milagro observations





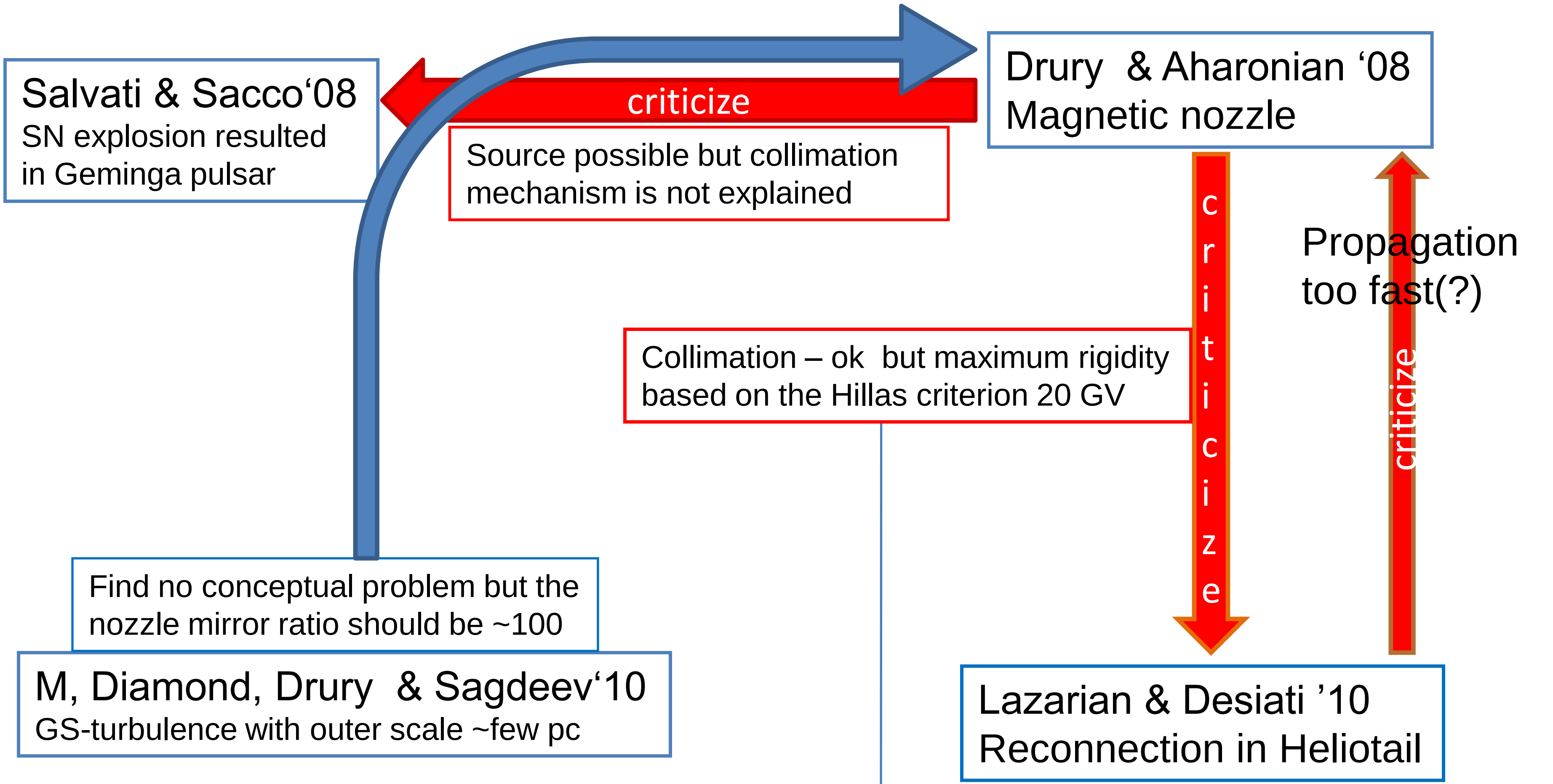
- Sharp beam~10º (beam A)
- Large scale angular anisotropy ~10^{-3}
- Sharp anisotropy ~10^{-4} (fractional excess)
- Energy range of the beam 1-10TeV
- The spectrum is somewhat flatter than the background

Characteristics of the beam to explain

- The width ~10^{o} (beam A)
- Fractional excess (~0.1 of the large scale angular anisotropy)
- Cut-off momentum 10 TeV
- Spectral index of the beam (harder than that of the background CR)

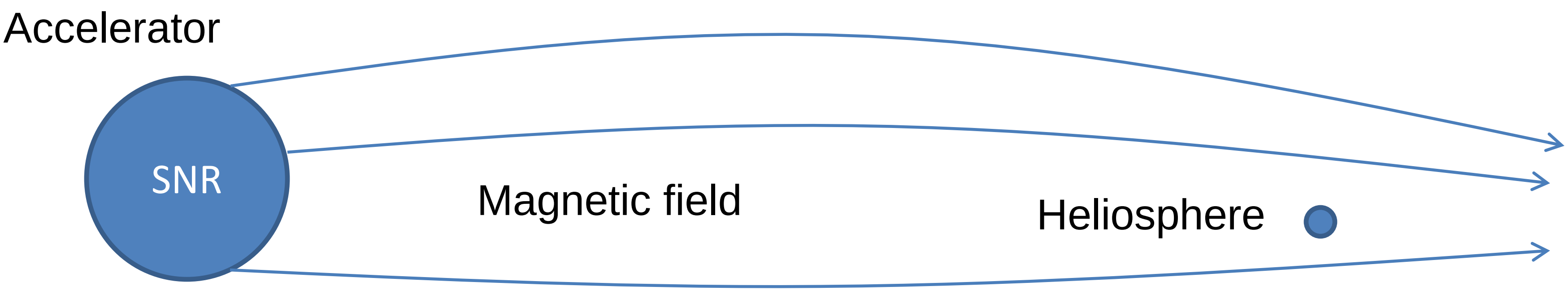
➢ Beam phase space density, integrated over its angular distribution

Suggested explanations



Hillas criterion (E<BVR): not written in stone: M, Sagdeev, Diamond 2011
arXiv:1101.4958 cycling through acceleration zone of size R ('betatron' acceleration)

Basic ideas



Assumptions

- Earth is magnetically connected with unspecified CR accelerator, such as a SNR shock or any 'accelerator'; even a large scale anisotropy may suffice
- Flux tube is filled with CRs and MHD waves associated with the accelerator (waves may also be from the ISM background turbulence); only the outer scale of the cascade matters
- MHD cascade is strongly anisotropic (GS)

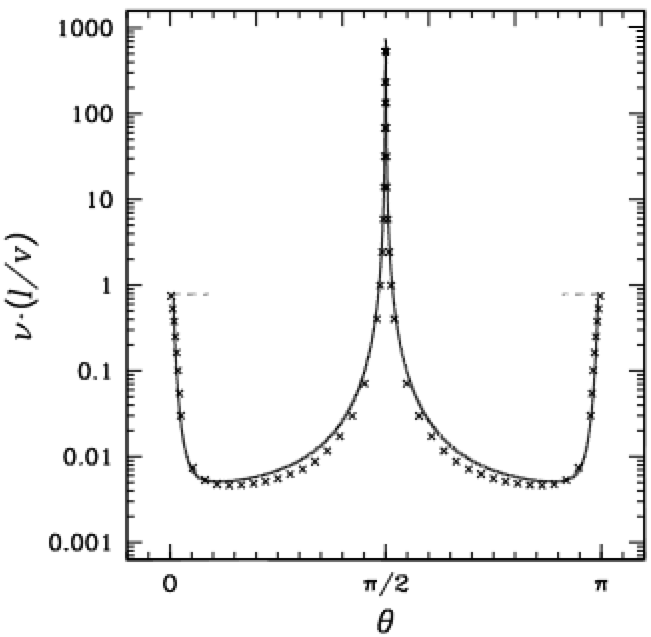
(Goldreich & Sridhar '95;Montgomery & Turner 81; Shebalin, Matthaeus & Montgomery 83; Matthaeus, Bieber, & Zank'95)

Particle transport in MHD turbulent plasma

- Studied starting from early 60's: Sagdeev & Shafranov '61; Vedenov, Velikhov, Sagdeev '62; Rowlands, Shapiro & Shevchenko '66
  - ➢ Jokipii '66 – relevant astrophysical context
- Anisotropic turbulence (GS) modifies transport: Chandran '00

$$I = \frac{1}{6\pi} k_\perp^{-10/3} l^{-1/3} g\left(\frac{k_\parallel l^{1/3}}{k_\perp^{2/3}}\right) e^{-\tau/\tau_k}$$

$$g(x) = H(1-|x|),$$



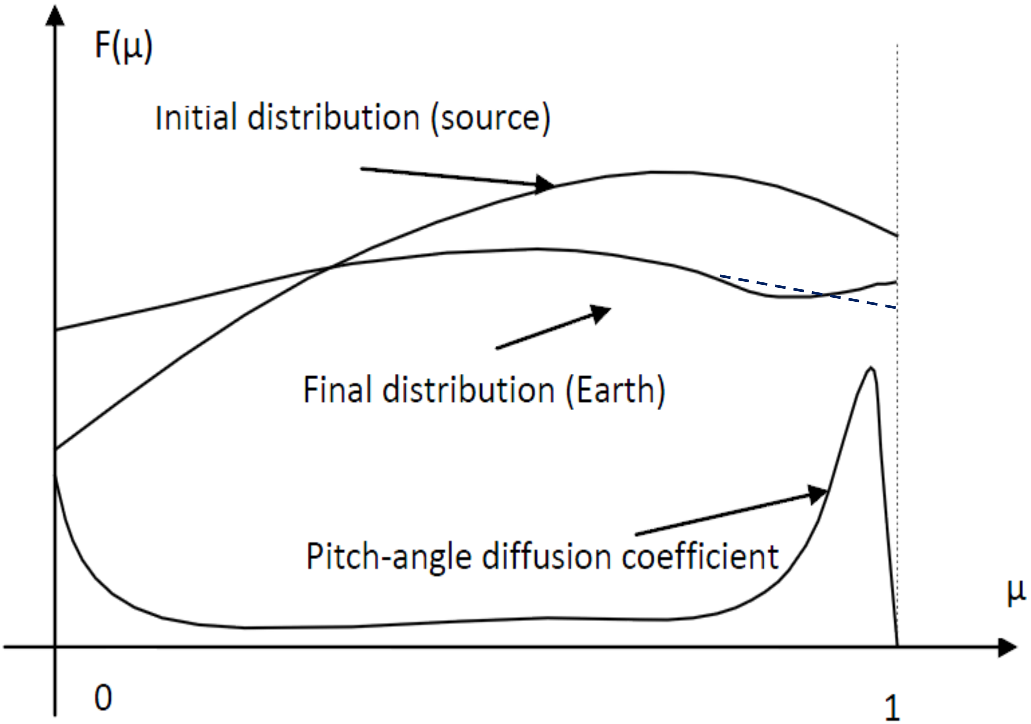
$$D_{\mu\mu} = \Omega^2 (1-\mu^2) \sum_{\mathbf{k},n} \frac{n^2 J_n^2(\xi)}{\xi^2} \times \int_0^\infty I(k_\parallel, k_\perp, \tau)\, e^{i(k_\parallel v_\parallel + n\Omega)\tau} d\tau$$

← Pitch-angle scattering frequency

$$\xi = k_\perp v_\perp / \Omega$$

Can peaked pitch-angle scattering collimate a beam?

- seems unlikely: for long propagation, even weak or peaked scattering should smear out all sharp anisotropies: transient effect



❑ spectral problem of particle propagation has singular points at $|\mu|=1$

❑ $\ln(1-\mu)$ term should appear just outside $\mu\approx1$ region (outer solution, while inner solution remains regular)

❑ $\mu\approx1$ has deficit of particles in the regular part of eigenfunction (large scale anisotropy)

❑ ln –term fills this dip locally which appears as a bump on the large scale anisotropic distribution

Key steps of analysis

- sum up Bessel series

$$D_{\mu\mu} = -\left(1-\mu^2\right)\sum_{\mathbf{k}} \frac{1}{\xi^2}\int_0^{\infty} I\left(k_{\|}, k_{\perp}, \tau\right) e^{ik_{\|}v_{\|}\tau} d\tau \frac{\partial^2}{\partial \tau^2} J_0\left(2\xi \sin\frac{\Omega\tau}{2}\right)$$

- get uniformly valid representation of D including μ≈1

$$D_{\mu\mu} = \frac{\pi}{2}\frac{v}{l}\left(1-\mu^2\right)\left[\frac{J_1^2(y)}{y^2} + r y^{4/3}\right] \qquad y = \sqrt{\left(1-\mu^2\right)/\varepsilon},\ \varepsilon = v/l\Omega \qquad r \sim 10^{-2}$$

l -outer scale

- consider transport problem

$$(u+\mu)\frac{\partial f}{\partial z} = \frac{\partial}{\partial \mu}\left(1-\mu^2\right) D(\mu) \frac{\partial f}{\partial \mu}$$

- convert to eigenproblem

$$\Psi(z,\mu,p) = f(z,\mu,p) - f_\infty(p)$$

by applying BC

$$\Psi = \begin{cases} \phi(\mu) = f_0(\mu) - f_\infty, & z = 0 \\ 0, & z = \infty \end{cases}$$

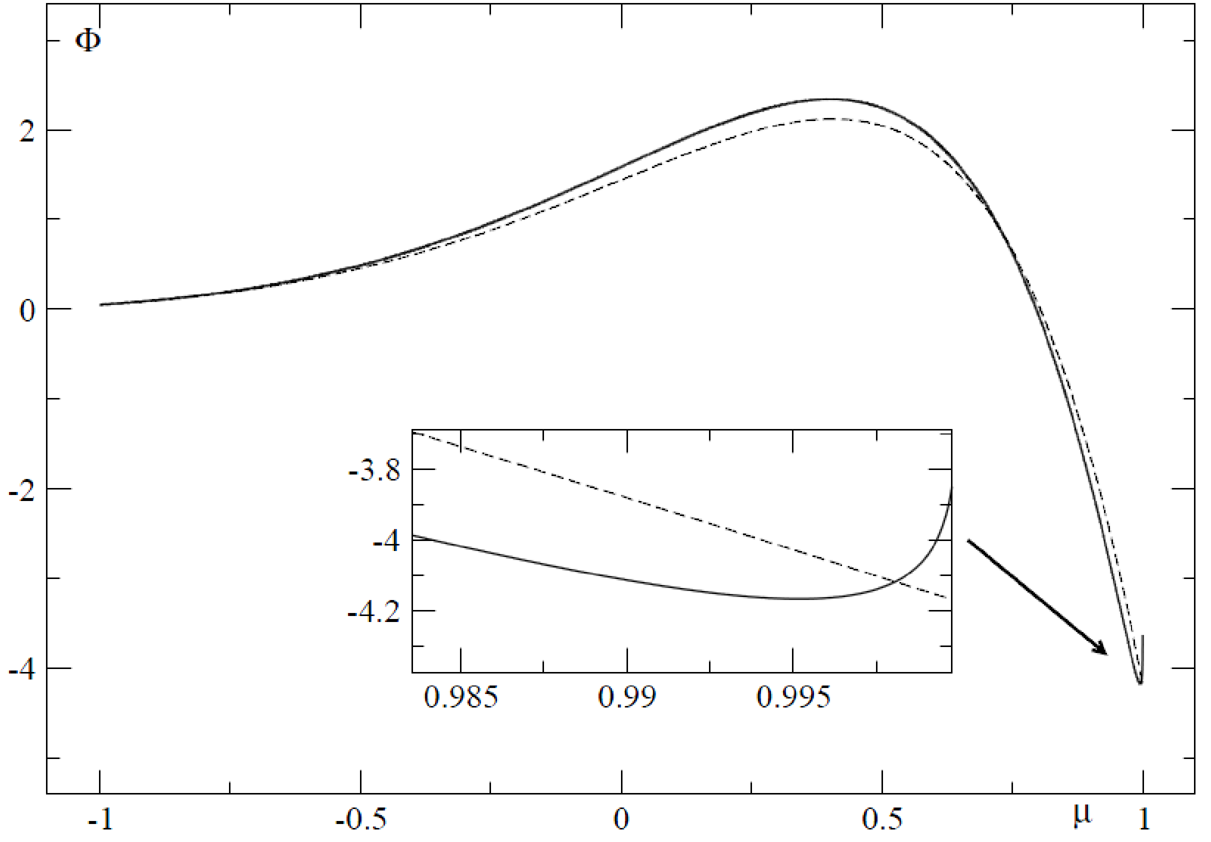
$$\Psi = \sum_\lambda C_\lambda \Psi_\lambda(\mu) e^{-\lambda z}$$

$$\boxed{\frac{d}{d\mu}(1-\mu^2) D(\mu) \frac{d\Psi_\lambda}{d\mu} + \lambda(u+\mu)\Psi_\lambda = 0}$$

- for D=1 well studied equation (Richardson 1918), also occurs in particle acceleration at relativistic shock, e.g. Kirk & Schneider ‘87

- complete set of orthogonal eigenfunction: $\{\Psi_\lambda\}_{\lambda_i=-\infty}^{\lambda_i=\infty}$

• once the set of eigenfunctions is complete, use it in solving perturbed problem



Distance to the source

Large scale anisotropy (first eigenfunction of the CR propagation problem) decays along the flux tube due to p-a scattering:

$$F_S(z,p) \sim F_S(0,p)\exp\left[-\mathscr{L}^{-1}\frac{z}{l}\right]$$

where $\mathscr{L}^{-1}(\varepsilon)$ is the inverse dimensionless particle scattering length

$$\mathscr{L}^{-1}(\varepsilon) = \frac{\lambda_1}{6}\left(\delta\ln\frac{1}{\varepsilon} + \varepsilon^{3/2}\right) \qquad \delta \equiv V_A/c = 10^{-4}$$

small parameter of the theory:

$$\varepsilon = \frac{r_g(p)}{l} \qquad \lambda_1 \approx 14.54 \qquad \lambda_2 \simeq 2\lambda_1$$

- max distance

$$L_{Smax} \approx \frac{6l}{\lambda_1 \varepsilon^{3/2}(p_{Bmax})} Ln \qquad L_{Smax} \simeq 130 \cdot Ln \cdot \left(\frac{10TeV}{E_{Bmax}}\right)^{3/2} pc.$$

Beam energy window

Why window? $\mathscr{L}^{-1}(\varepsilon)$ has a minimum $(\approx 1.7 \cdot 10^{-3})$

$$\text{at } \varepsilon = (2\delta/3)^{2/3} \simeq 1.6 \cdot 10^{-3} \text{ for } \delta \equiv V_A/c = 10^{-4}$$

- cyclotron instability may spread the beam
- but: CR isotropic background stabilizes

➢ beam energy window and spectral slope constrained

$$F_0(p) \leq A \frac{V_A}{c} \frac{l^2}{r_g^2(p)} F_C(p)$$

$$A = \frac{8}{\lambda_1 j_1^4 (q_c - 2)} \approx 10^{-3}$$

• beam curvature drift across the flux tube is weak for the distance limit inferred

Outer scale

Two approaches:

I. We obtained

- ✓ angular width
- ✓ fractional excess
- ✓ maximum momentum

as functions of the outer scale *l*

To agree with Milagro results all tree consistently indicate

$$l \sim 1\ \mathrm{pc}$$

II. Assume the turbulence is driven by escaping CR from a SNR at

$$l \sim r_g(p_{max})$$

To recover the same outer scale, it is necessary to assume

$$E_{max} \sim 3\ \mathrm{PeV}\ (\mathrm{for}\ B_{\mu G} \sim 3)$$

'knee' energy!

Results and conclusions

Assuming:

✓large scale anisotropic distribution of CRs (at a putative source, e.g. SNR)

✓anisotropic cascade of Alfvenic turbulence originating at scale *l*

Calculated:

- propagation of the CRs down their gradient along interstellar magnetic field

Surprising findings:

CR distribution develops angular shape consisting of

- large scale anisotropic part
- beam, tightly focused along local field direction
- large scale part has the spectral index of background CRs
- beam angular width

$$\Delta\vartheta \simeq 4\sqrt{\varepsilon}, \text{ where } \varepsilon = r_g(p)/l \ll 1$$

- beam fractional excess relative to background CRs

$$\simeq 50\varepsilon.$$

- If $l \sim 1$ pc all these parameters are consistent with Milagro findings and the beam maximum energy also matches Milagro's 10TeV cut-off

- required 1pc outer scale naturally occurs as a cyclotron instability scale of CRs at the 'knee' energy of 3 PeV

How to make three beams out of one?