

Probabilistic modeling of Cherenkov emission from high-energy particle showers in ice and water

Ian Crawshaw, **Tianlu Yuan**, Emre Yildizci, Lu Lu, Anatoli Fedynitch

IceCube PA Symposium – Madison, WI

14 October 2026

Introduction

Neutrino telescopes target TeV+ energies using large volume detector

→ Expensive to track every particle

→ Challenging to simulate at scale

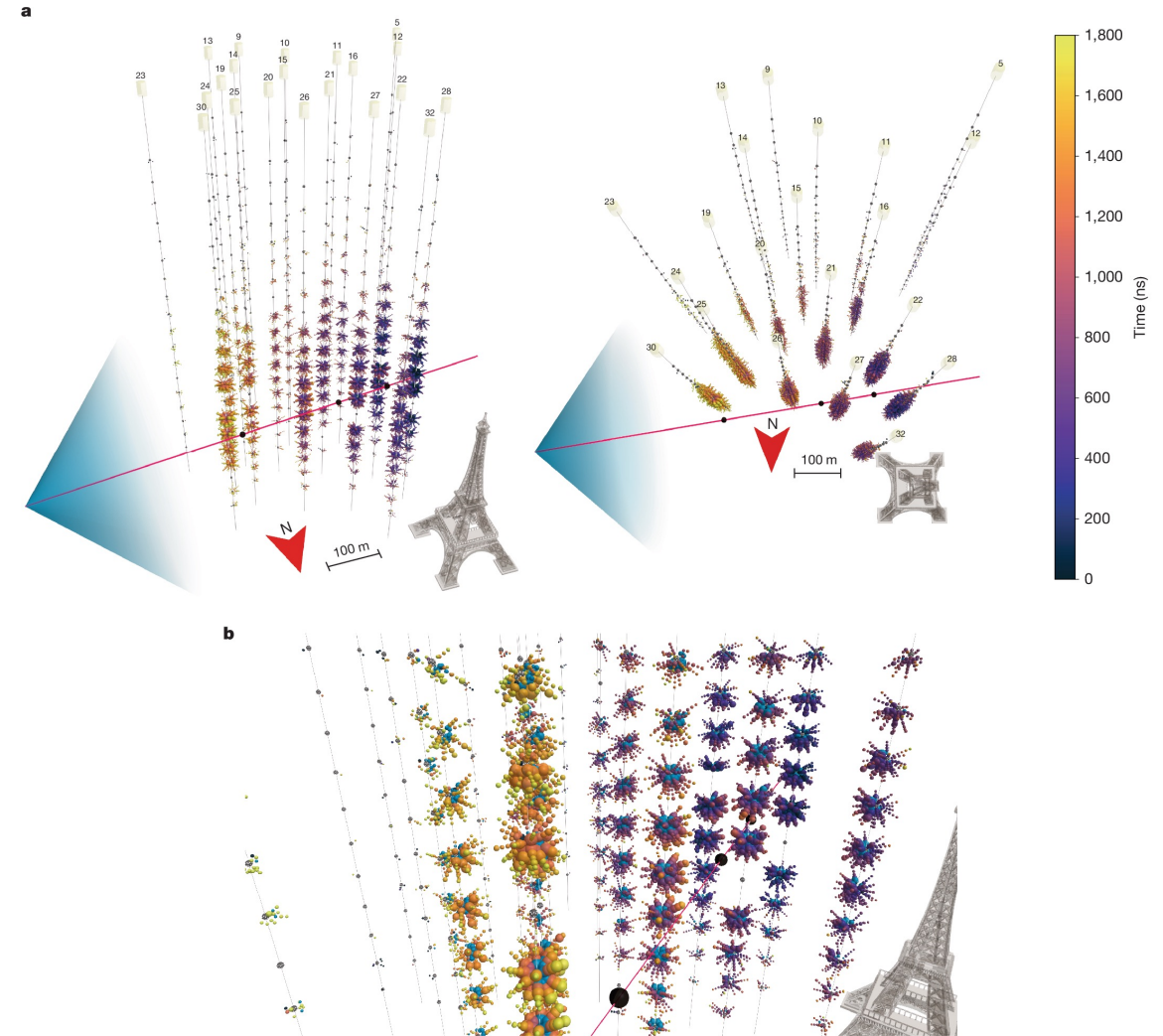
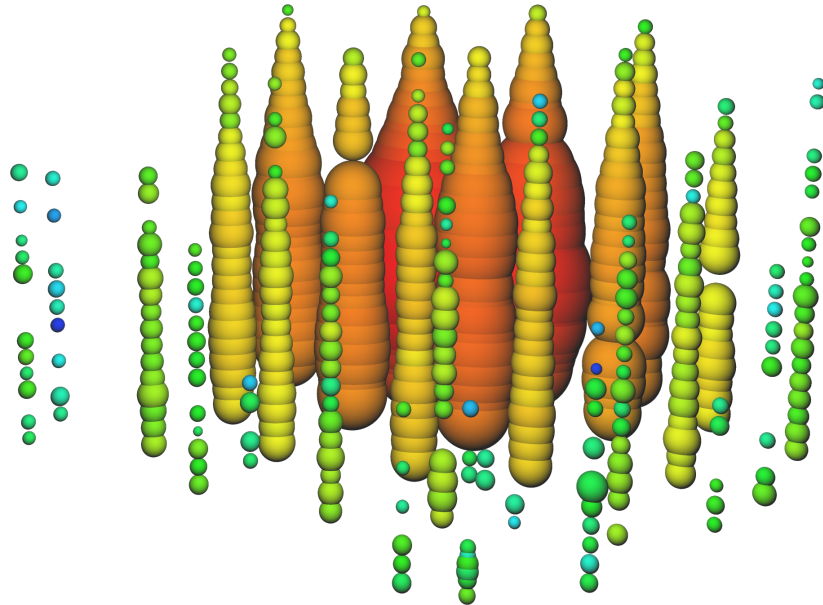
This talk:

→ Improved model for Cherenkov emission from particle showers

Neutrino observatories

Particle shower processes are similar in ice and water

- Multi-PMT sensors may be sensitive to shorter length scales, especially in water
- Important to model with care



[\(KM3NeT\) Nature 638, 376–382 \(2025\)](#)

High energy events

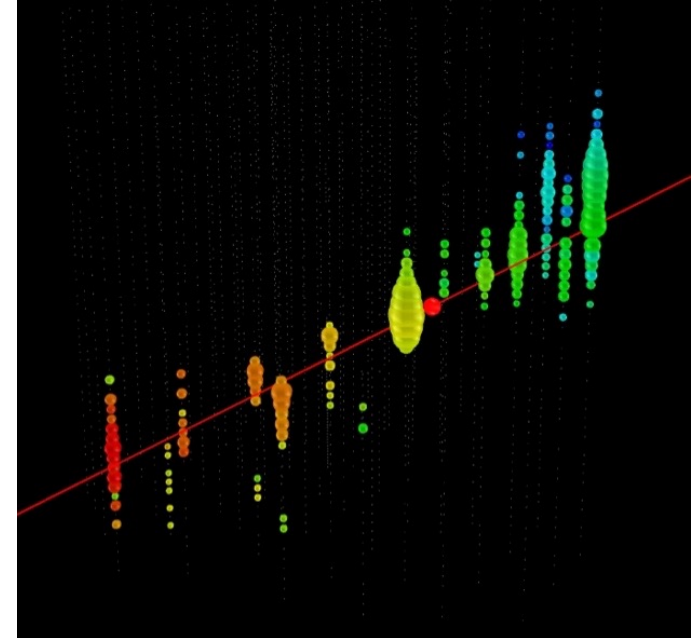
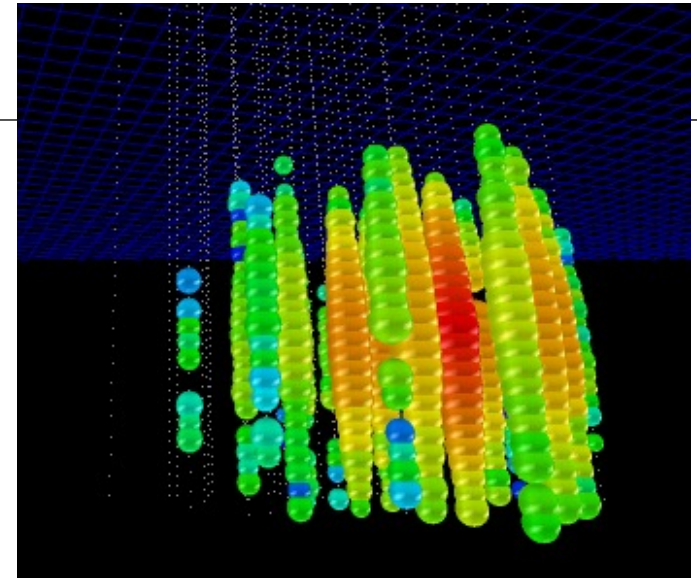
Electrons, photons initiate EM showers

Hadrons (π , K , p ...) initiate showers

- Contain EM components from $\pi^0 \rightarrow \gamma + \gamma$
- May contain low-energy (GeV) muons from decays

Muons $> \sim 1$ TeV lose energy via radiative processes

- pair production, Bremsstrahlung, photonuclear interactions (particle showers)



Cherenkov emission

Governed by Frank-Tamm formula:

$$\frac{d^2 N}{d\ell d\lambda} = \frac{2\pi\alpha z^2}{\lambda^2} \left(1 - \frac{c^2}{v^2 n^2} \right)$$

N : number of Cherenkov photons

λ : wavelength

ℓ : charged-particle track length

Approximation with gamma distribution

Idea (c.f. [Kowalski 2002](#), [Astropart. Phys. 44 \(2013\) 102](#)):

- parameterize **integrated** track length of entire shower as function of energy (normalization)
- parametrize **differential** track length along the shower axis with **gamma distribution** (shape)

$$\frac{d\hat{\ell}}{dx} = \hat{\ell}_{\text{tot}} \frac{b^a}{\Gamma(a)} x^{a-1} e^{-bx}$$

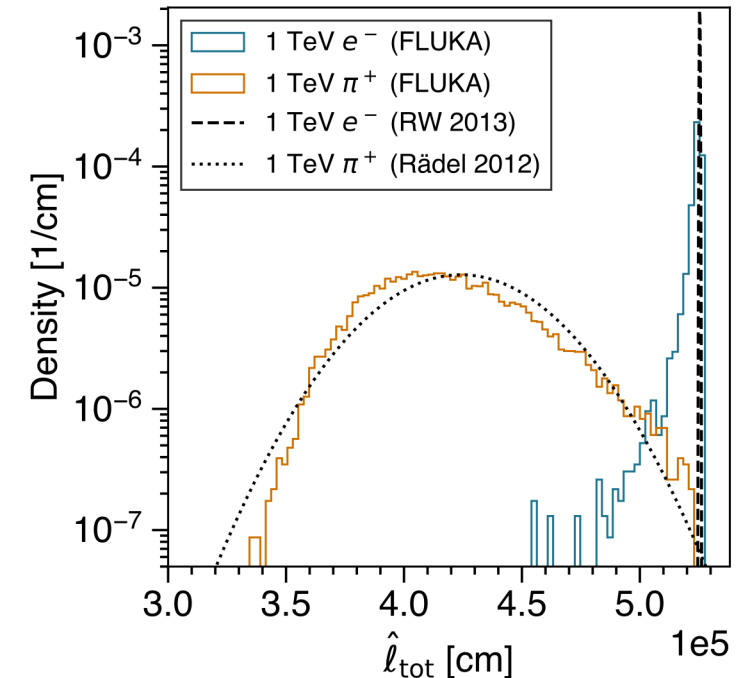
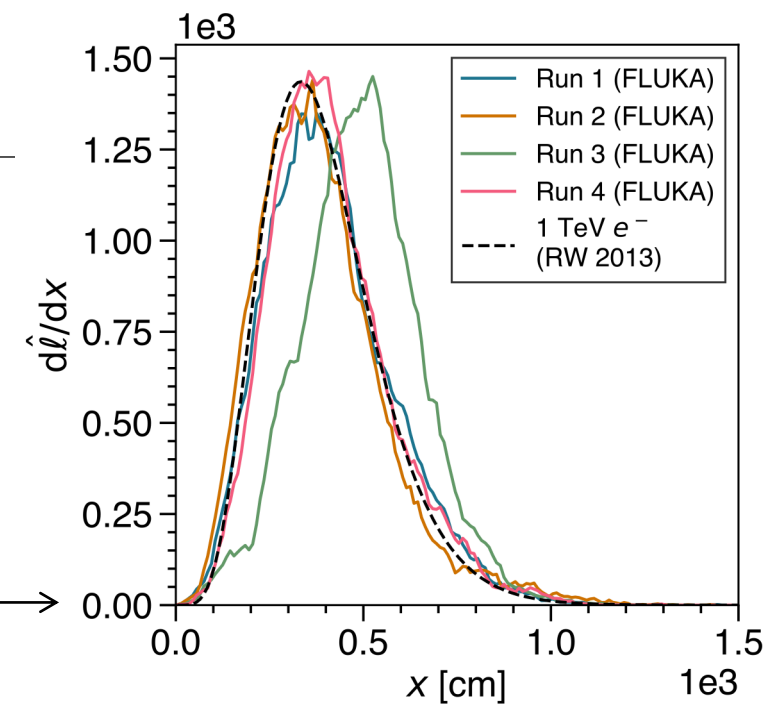
- $\hat{\ell}$ is the effective track length; as if all particles have $v = c$

Inaccuracies in prior work

Parametrizations from 2012/13 are fitted to an **average** shower shape at a given energy

Total track length for EM showers missing nuclear effects

particle direction →



Our approach using FLUKA (2025.1.0)

Initiate showers with 14 different particles
(e.g., e^- , γ , π^+ ...)

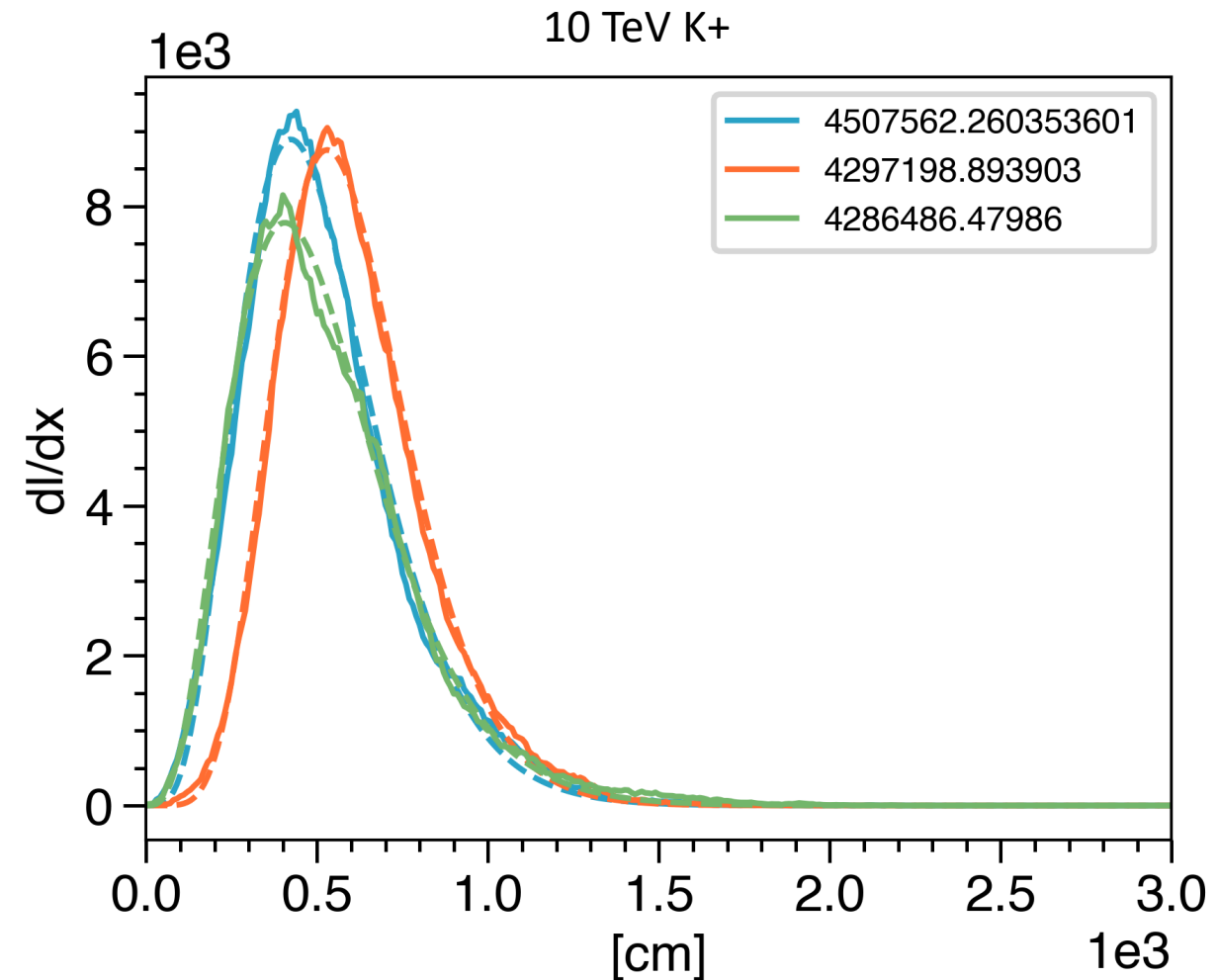
- Discrete steps in energy
- Interactions in ice

Histogram Cherenkov-weighted track length
along the beam axis

Fit $d\hat{\ell}/dx$ to gamma distribution, which can
be described by two parameters (a, b)

- Build distribution of (a, b) s

Fit **total** length distribution to skew-normal
(NIG) distribution for hadron (EM) showers



Solid line: FLUKA data

Dashed line: fitted distribution

Legend: total length [cm]

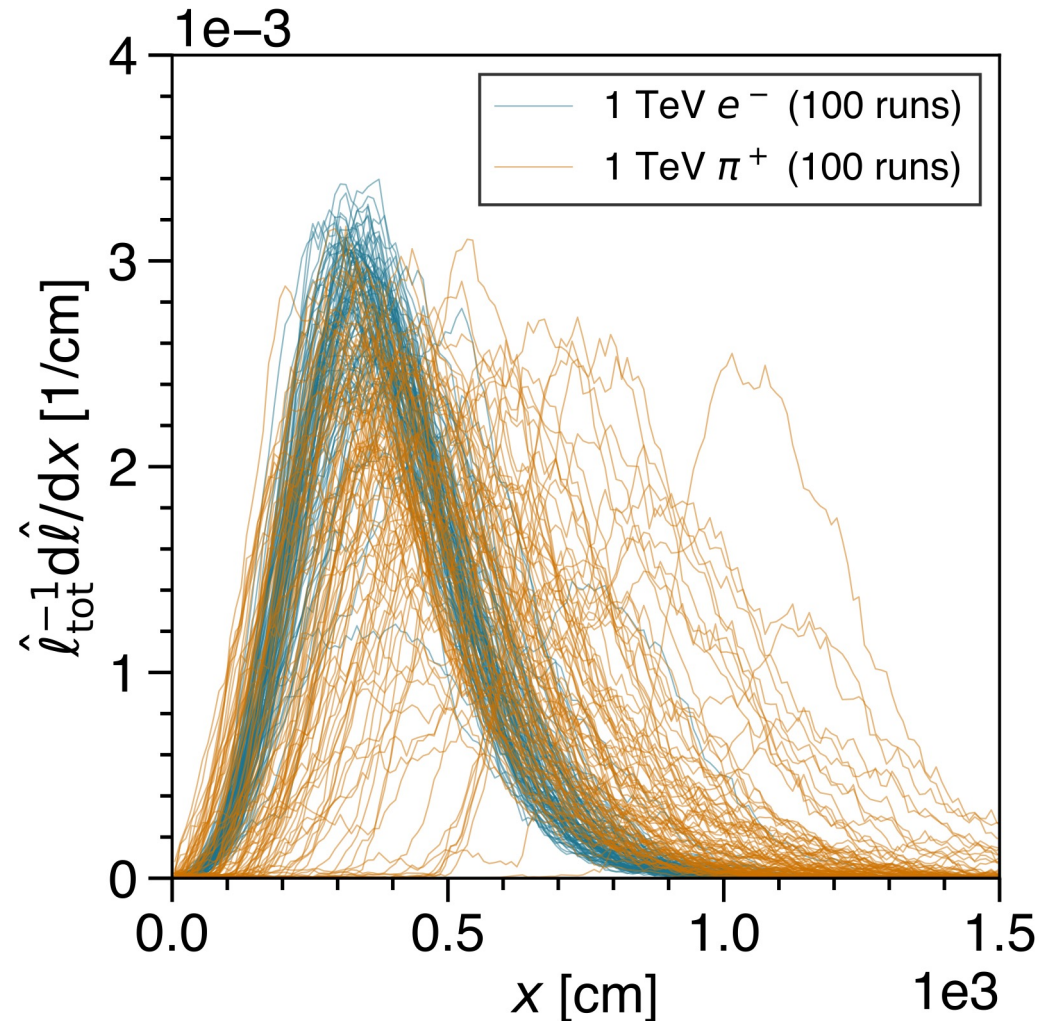
EM vs hadron-initiated shower

100 example FLUKA runs

→ Average profile cannot capture

→ hadron-initiated showers exhibit larger fluctuations in shape

In practice there are 10k runs at each discrete energy step

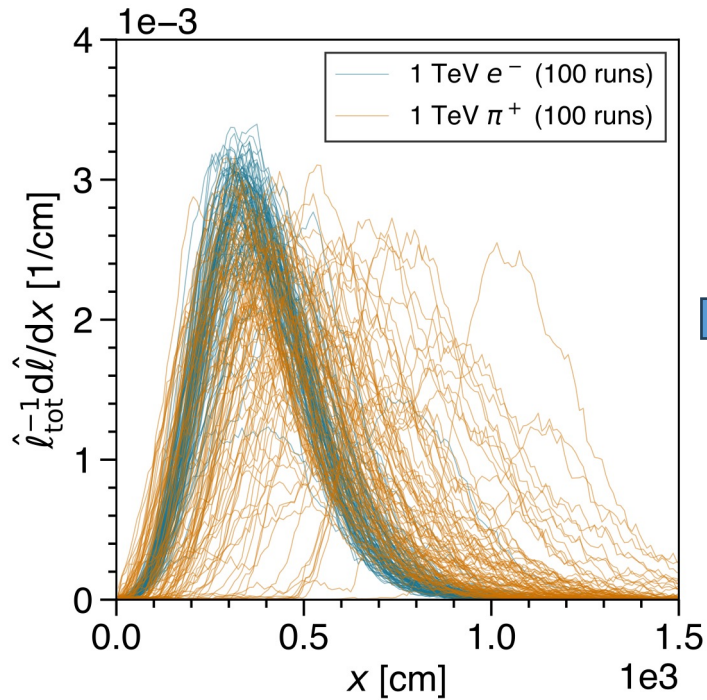


Constructing the model (shape)

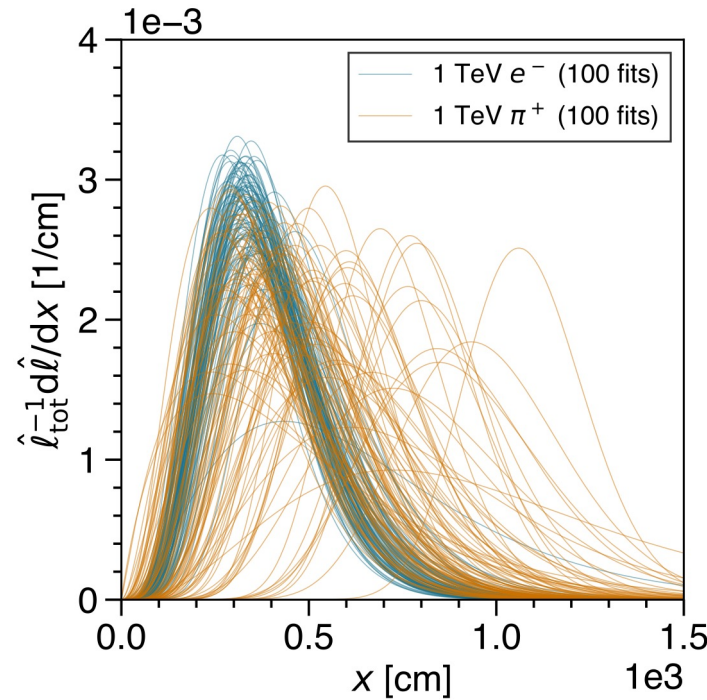
Blue: 100 1 TeV electron-initiated showers

Orange: 100 1 TeV π^+ -initiated showers

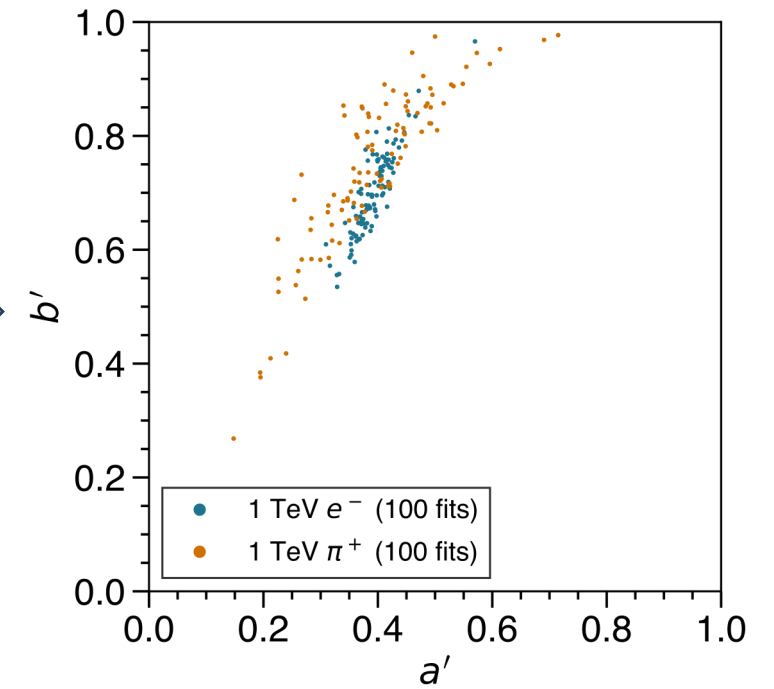
FLUKA



Gamma fits



Parameter space

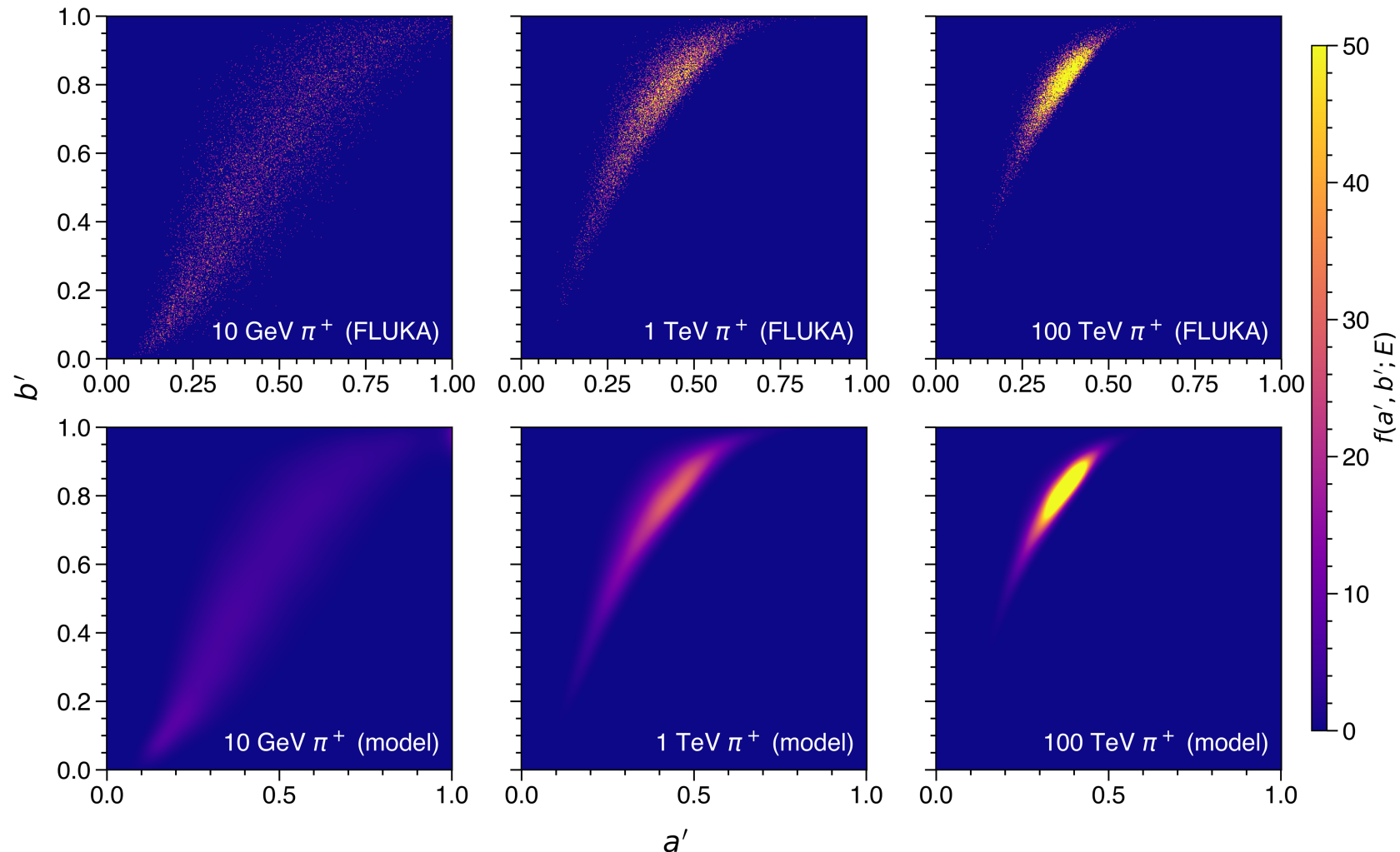


Transformation $a \rightarrow a', b \rightarrow b'$
projects onto unit grid for ease
of fitting and sampling

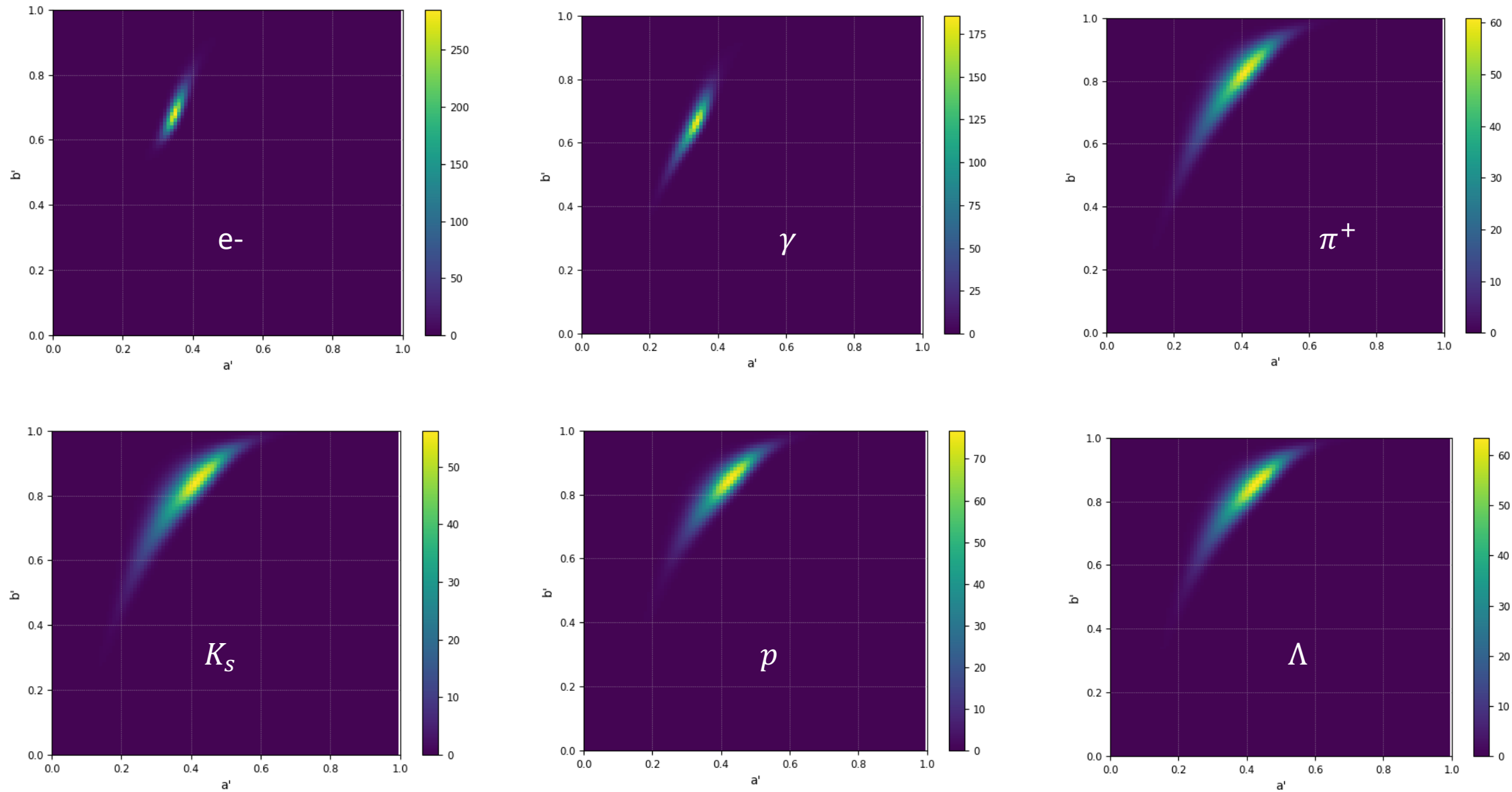
Penalized B-Spline model of gamma distributions

Histogram scatter points at each discrete energy step to get PDF

Fit with penalized B-splines over (a', b', E) dimensions



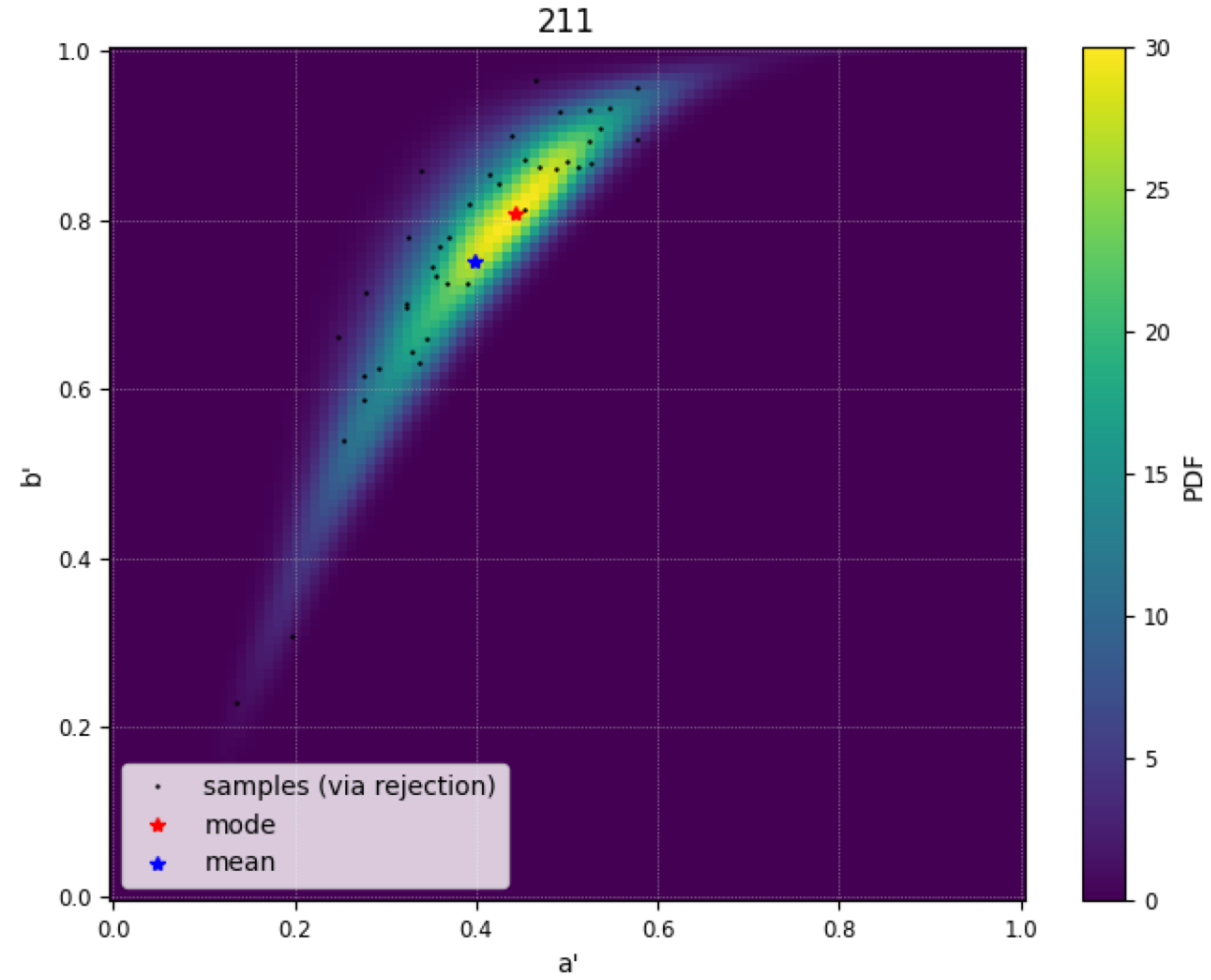
Comparison of fitted distributions @ 10 TeV



Sampling

BSpline model gives full description of shape as a function of energy

Rejection sampling yields individual shower realizations

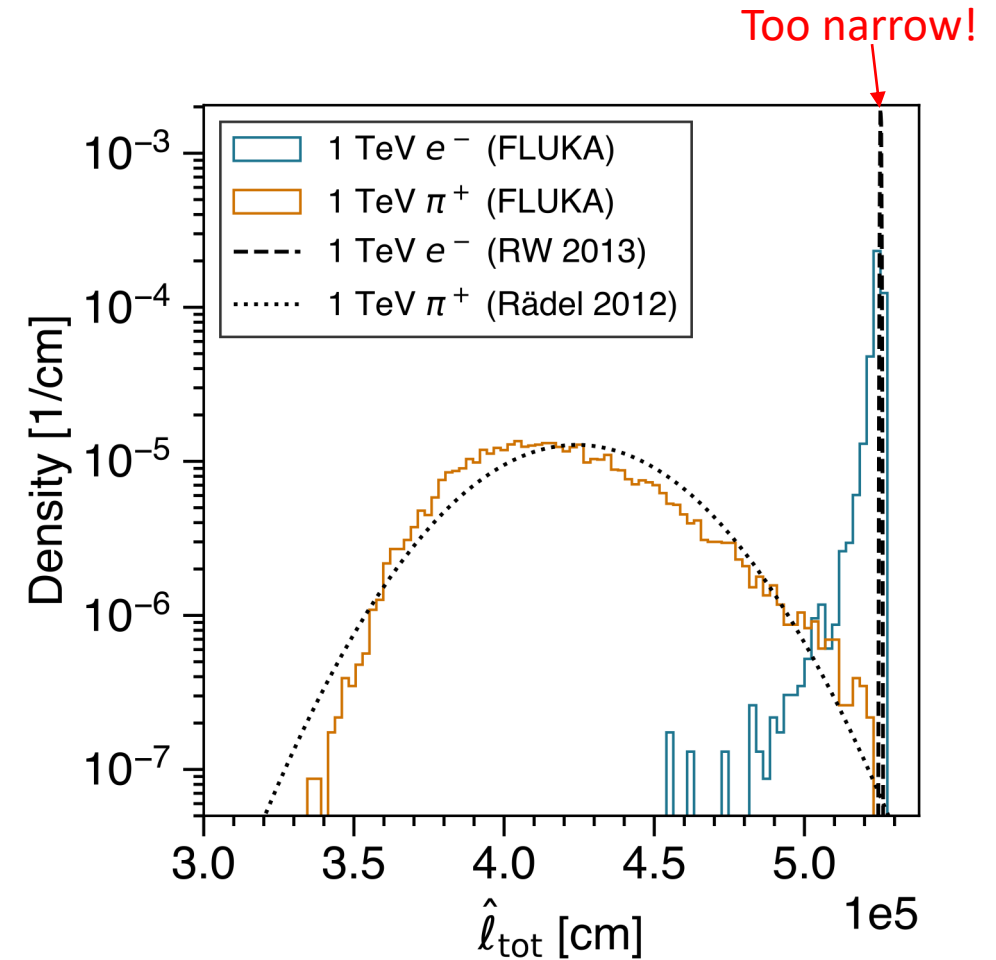


$\hat{\ell}_{\text{tot}}$ differences vs prior work

Noticed **differences** with the existing $\hat{\ell}_{\text{tot}}$ distribution (RW 2013)

For EM showers, FLUKA results in **10x larger** variance vs previous gaussian fits (dashed black)

- Differences understood
- Photo/electro-nuclear effects decreases of Cherenkov budget



$\hat{\ell}_{\text{tot}}$ parametrization

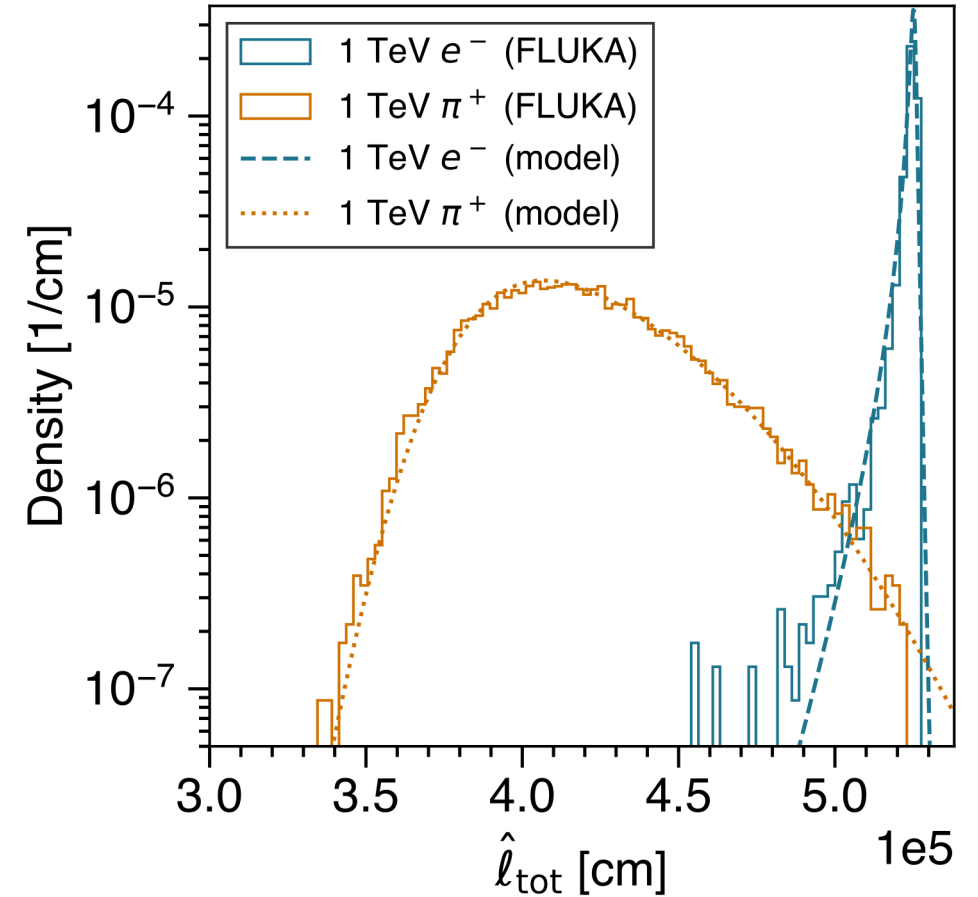
EM vs hadron showers result in two different distributions

→ Use skew-normal (SN) for hadrons

$$\text{SN}(x; \alpha, \xi, \omega) = \frac{2\phi(z)\Phi(\alpha z)}{\omega}$$

→ Use normal-inverse gaussian (NIG) for EM

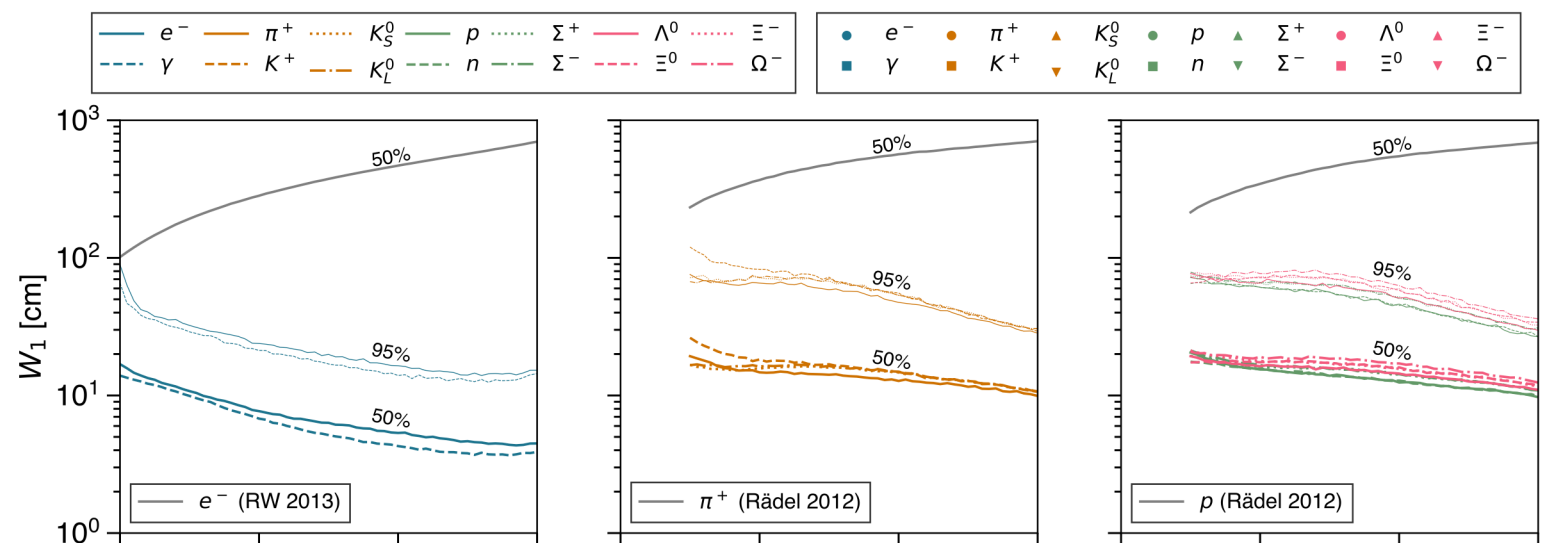
$$\text{NIG}(x; \alpha, \beta, \xi, \omega) = \frac{\alpha K_1(\alpha\sqrt{1+z^2})}{\omega\pi\sqrt{1+z^2}} \exp(\sqrt{\alpha^2 - \beta^2} + \beta z)$$



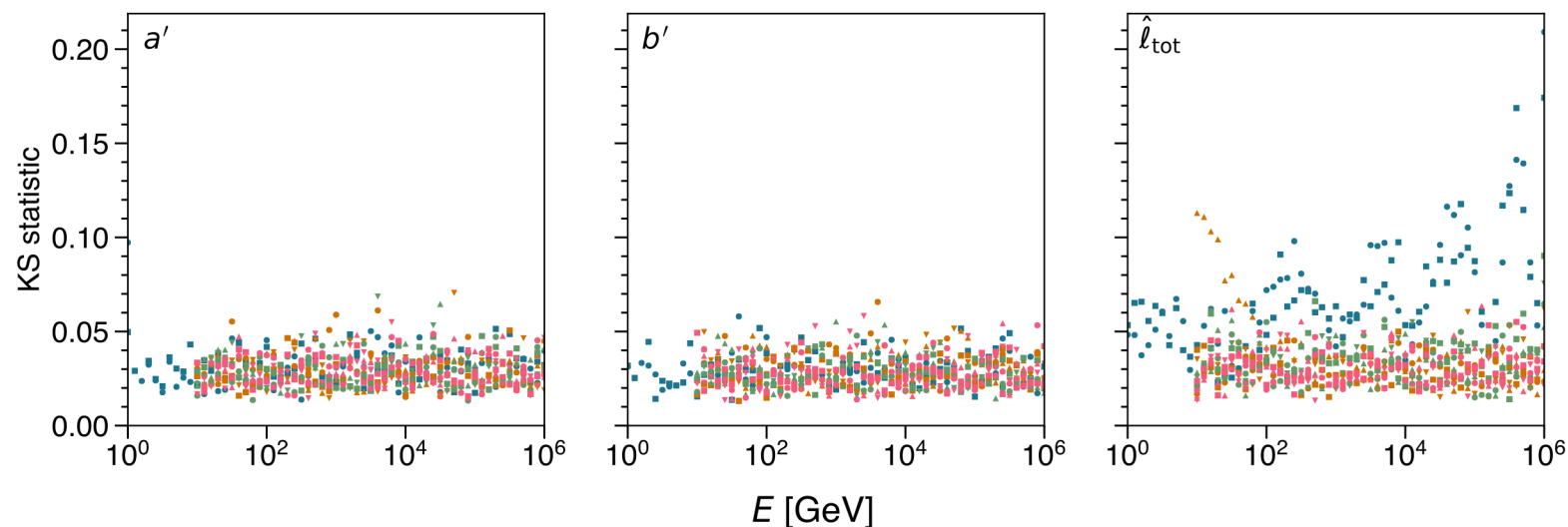
Accuracy

Overall, shape is much improved when compared to existing model

W_1 : Wasserstein distance to FLUKA;
shape comparison (lower is better)



KS statistic between sampling from
our model and values from FLUKA



Neutrino-DIS simulation in ice (w. PYTHIA8)

Simulate neutrino interaction off proton with PYTHIA8

- handles the fragmentation and hadronization
- outputs a list of final state (FS) particles with user-configurable decay modes
- π^0 s are common and decayed to photons; note that this would be part of what we think of as “hadron shower”, but it’s EM

For now, assume low charm energies where decay dominates

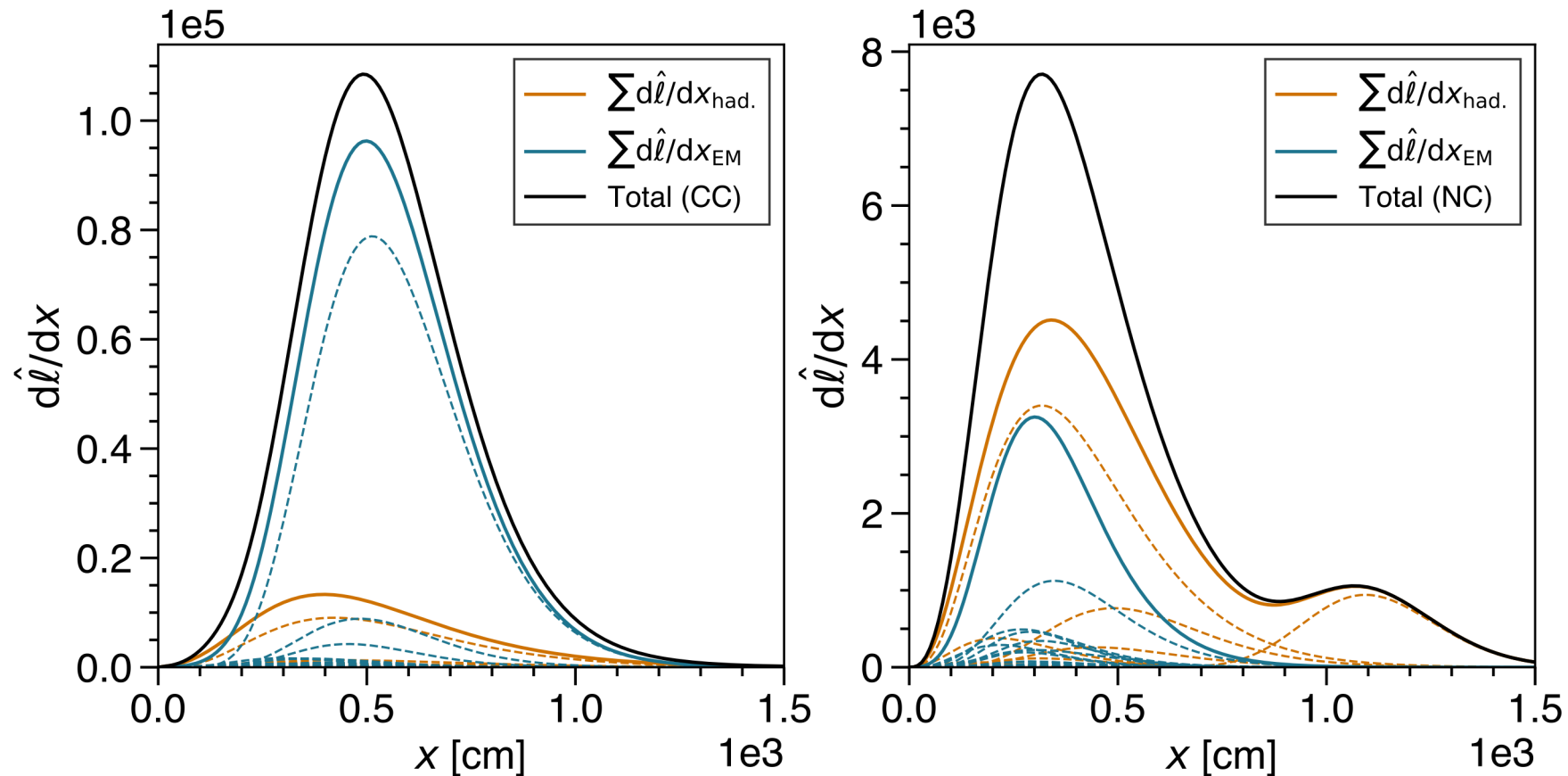
- decay products are passed through to parameterization

Application

100 TeV electron neutrino DIS off proton

Left: Charged current

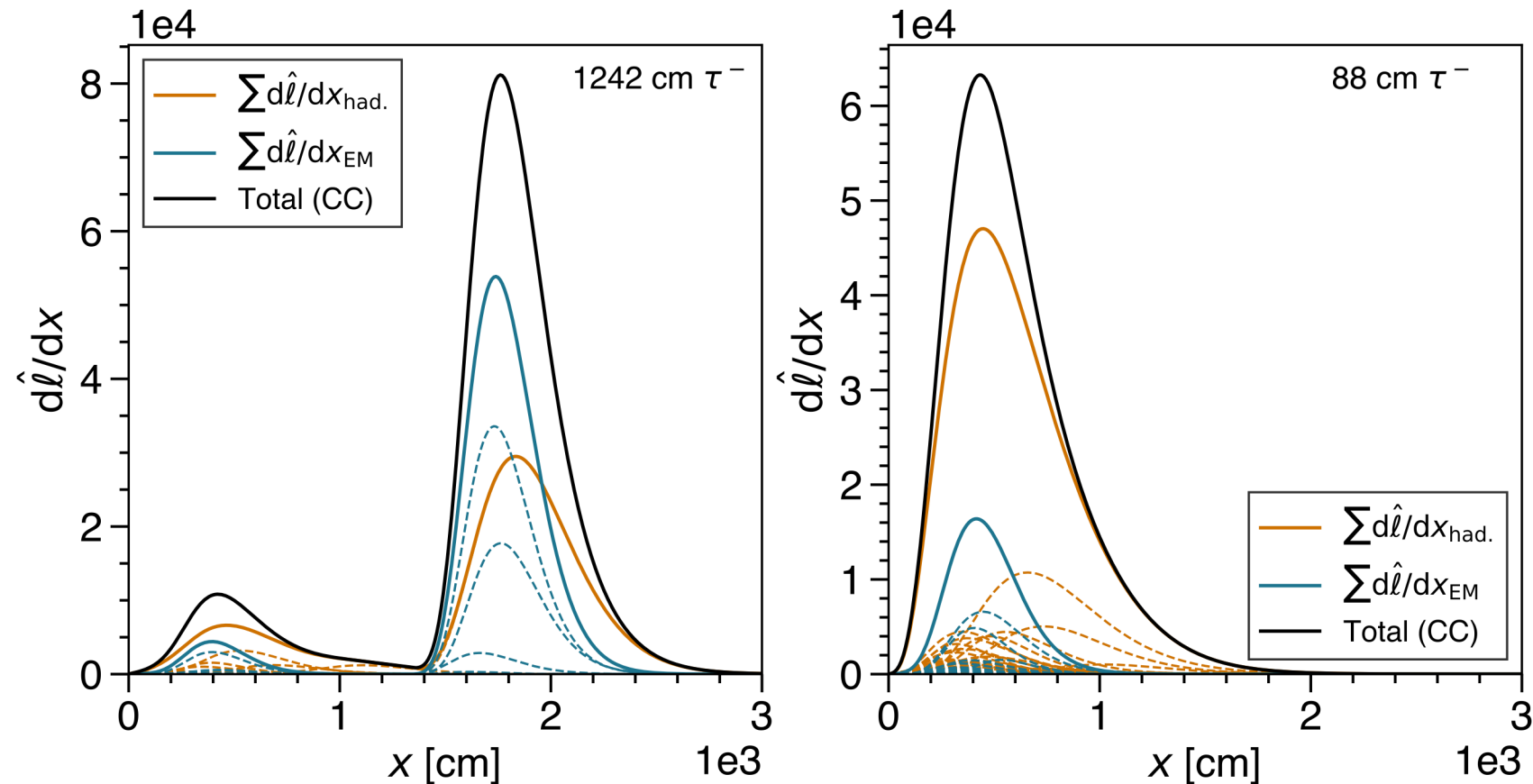
Right: Neutral current



Application

100 TeV tau neutrino DIS off proton

Both: Charged current

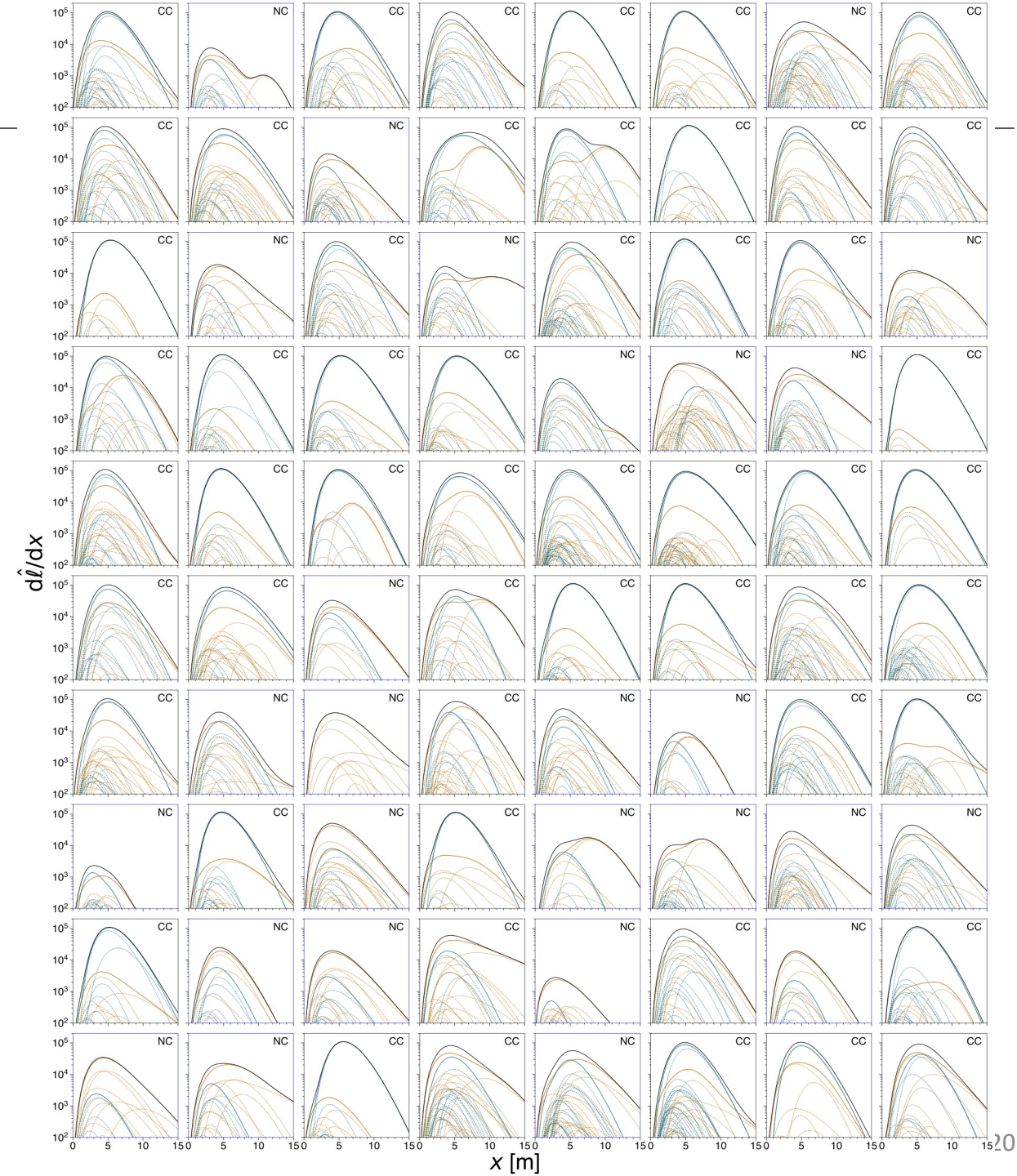


More examples

Results from 80 PYTHIA **nuE** runs with profiles sampled from our model

NC interactions highlighted in blue

Variations in shape should be included in simulations (WIP)



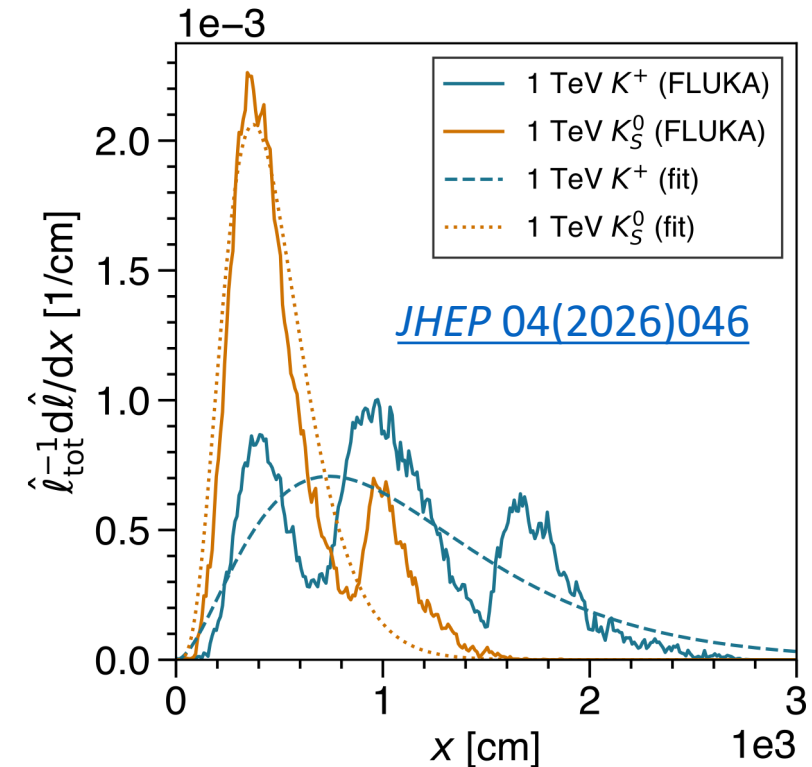
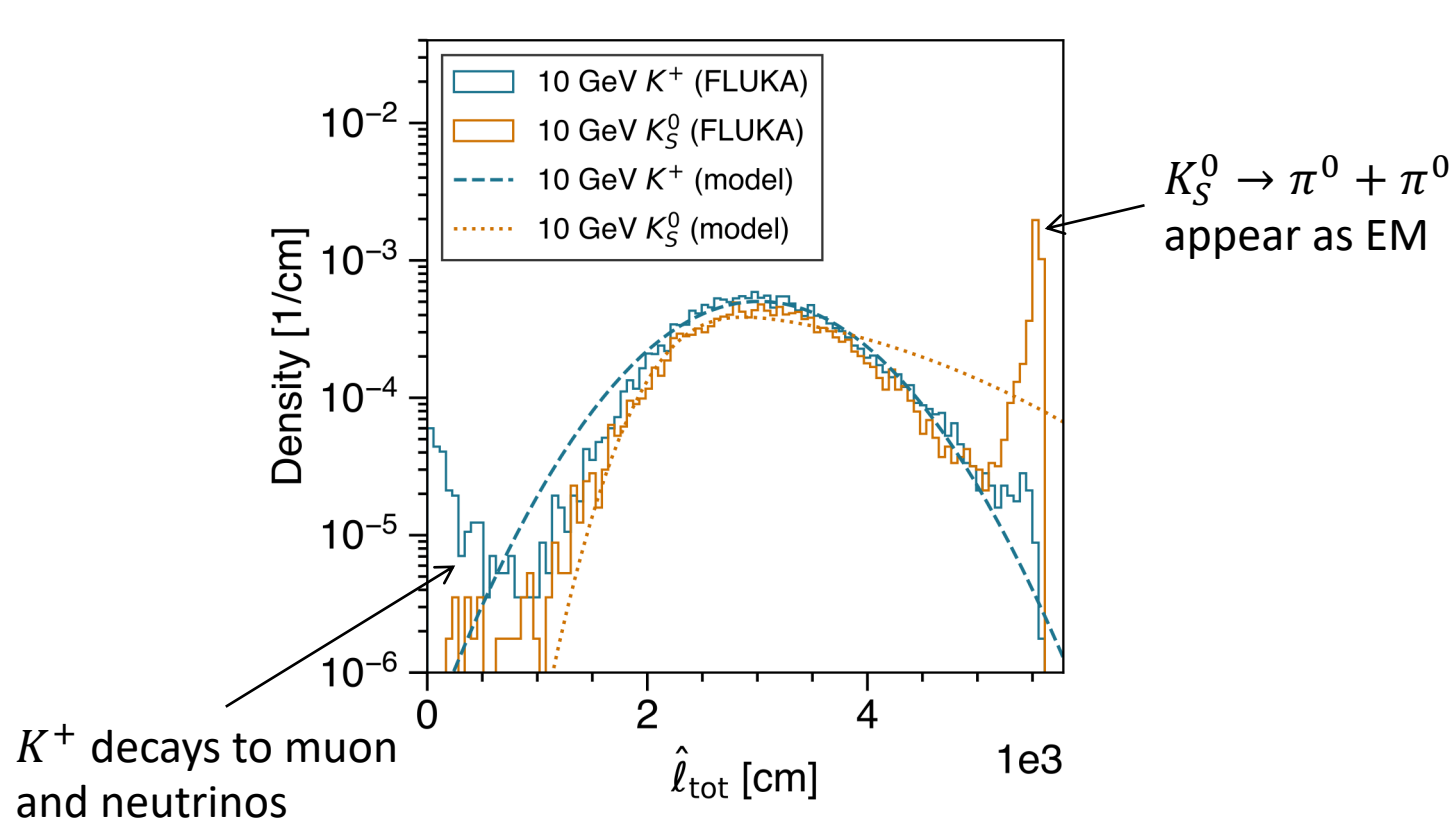
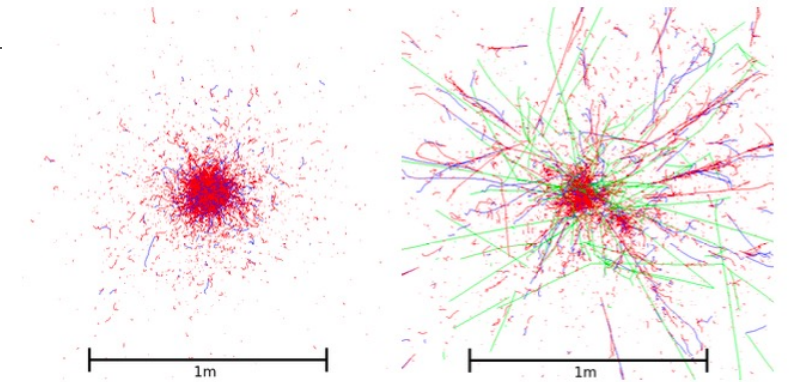
Limitations

MSc's thesis L. Rädcl

Transverse spread not captured by **1D** profile

Outliers in shape and normalization

Charm requires separate treatment



Takeaways

Parameterizations to capture shower-profile fluctuations

- Shape fitted to gamma distributions; parameter space modeled with B-splines

Total Cherenkov-weighted track length from FLUKA fitted to SN (hadron) or NIG (EM) distribution

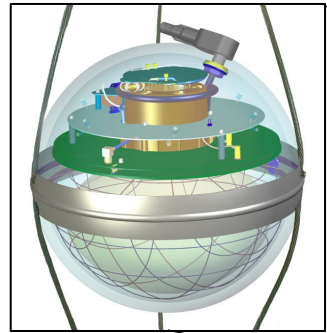
- EM differences with existing model tracked down to nuclear effects which were not simulated/included in previous G4 study

Python package on pypi: <https://pypi.org/project/acem/>

“Approximate Cherenkov Emission Model (ACEM)”

Backups

IceCube



Digital Optical
Module (DOM)

IceCube Lab

125 m string spacing

IceTop
81 Stations
324 optical sensors

IceCube Array
86 strings including 8 DeepCore strings
5160 optical sensors

17 m DOM spacing

DeepCore
8 strings-spacing optimized for lower energies
480 optical sensors



Eiffel Tower
324 m

5000+ DOMs

Fully operational by 2011

~2 km deep

50 m

1450 m

2450 m

2820 m

Bedrock

Neutrino detection

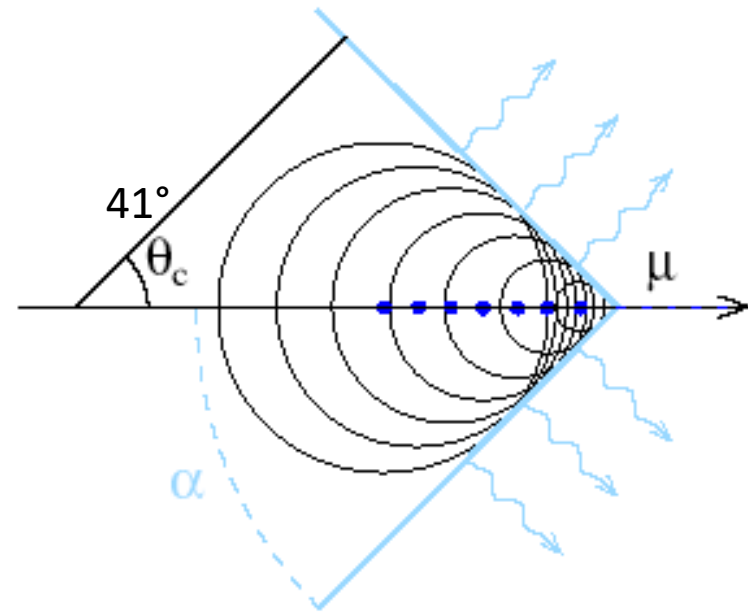
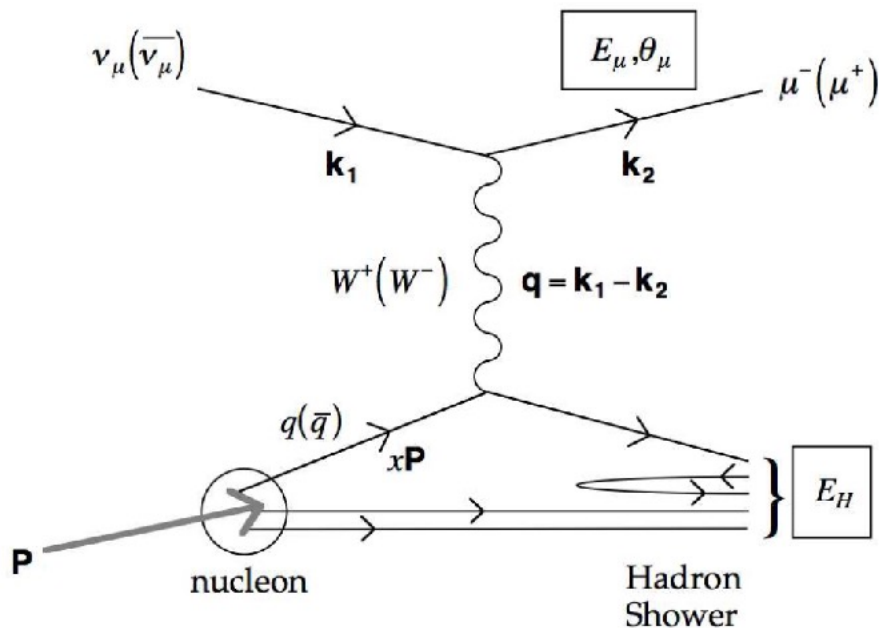
Neutrino interacts with targets in media such as ice and water

- At energies relevant for this talk, **primarily DIS off nucleons**
- W-boson resonance with atomic electrons at $E_\nu \sim 6.3$ PeV

Charged outgoing particles emit Cherenkov radiation detectable by PMTs

- Large boost factor $\rightarrow$ align with neutrino direction

Rev. Mod. Phys. 84, 1307



Shower MC challenges

At lower energies, possible to run full Geant4 tracking

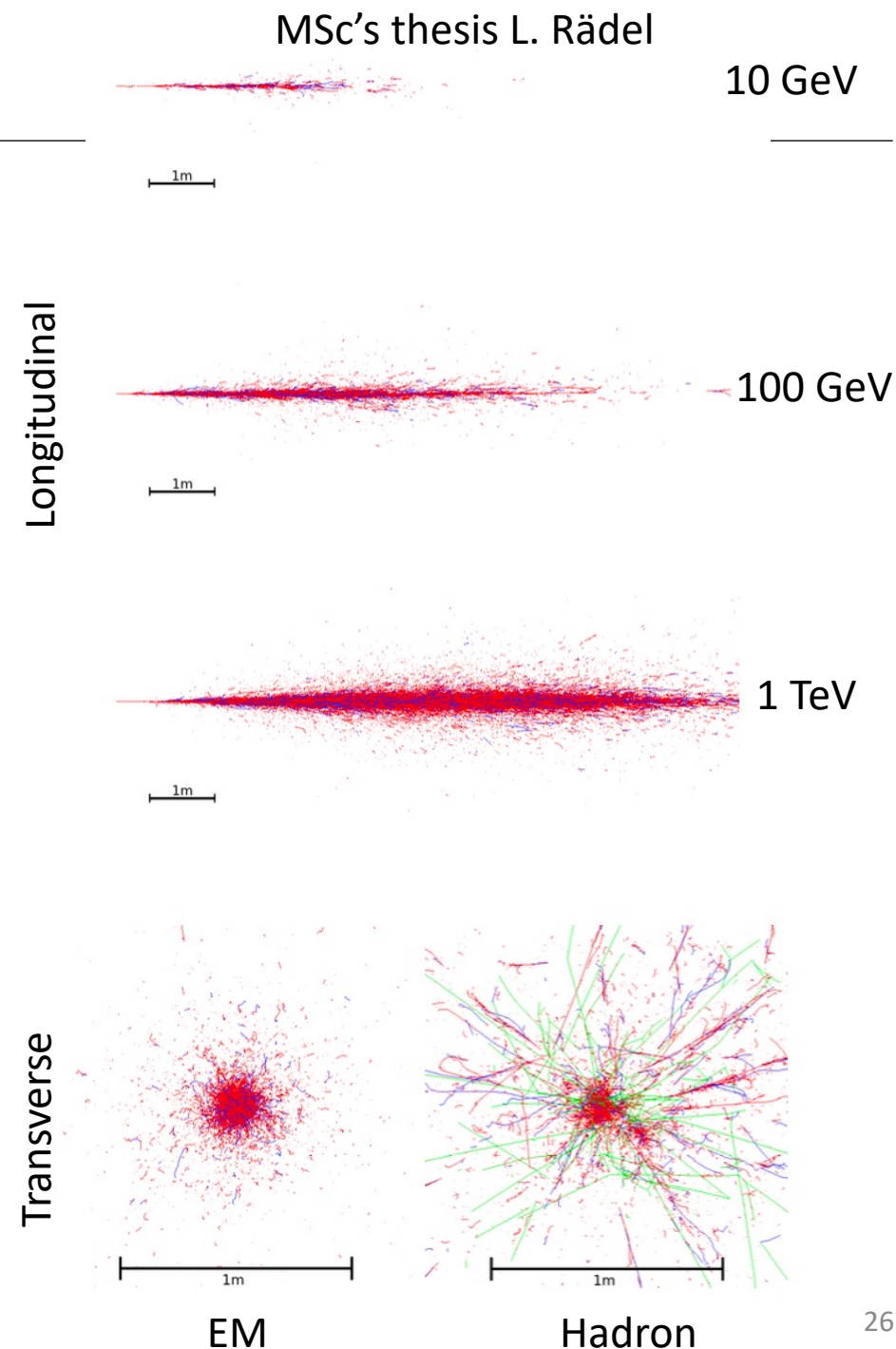
Doesn't scale well to TeV+ energies

Instead resort to parametrizations of G4 MC

→ Parameterize longitudinal profile, angular emission profile of Cherenkov yield*

→ Ref: Masters' thesis of L. Rädcl (2012) and [*Astropart. Phys.* **44** \(2013\) 102](#)

*muons treated separately



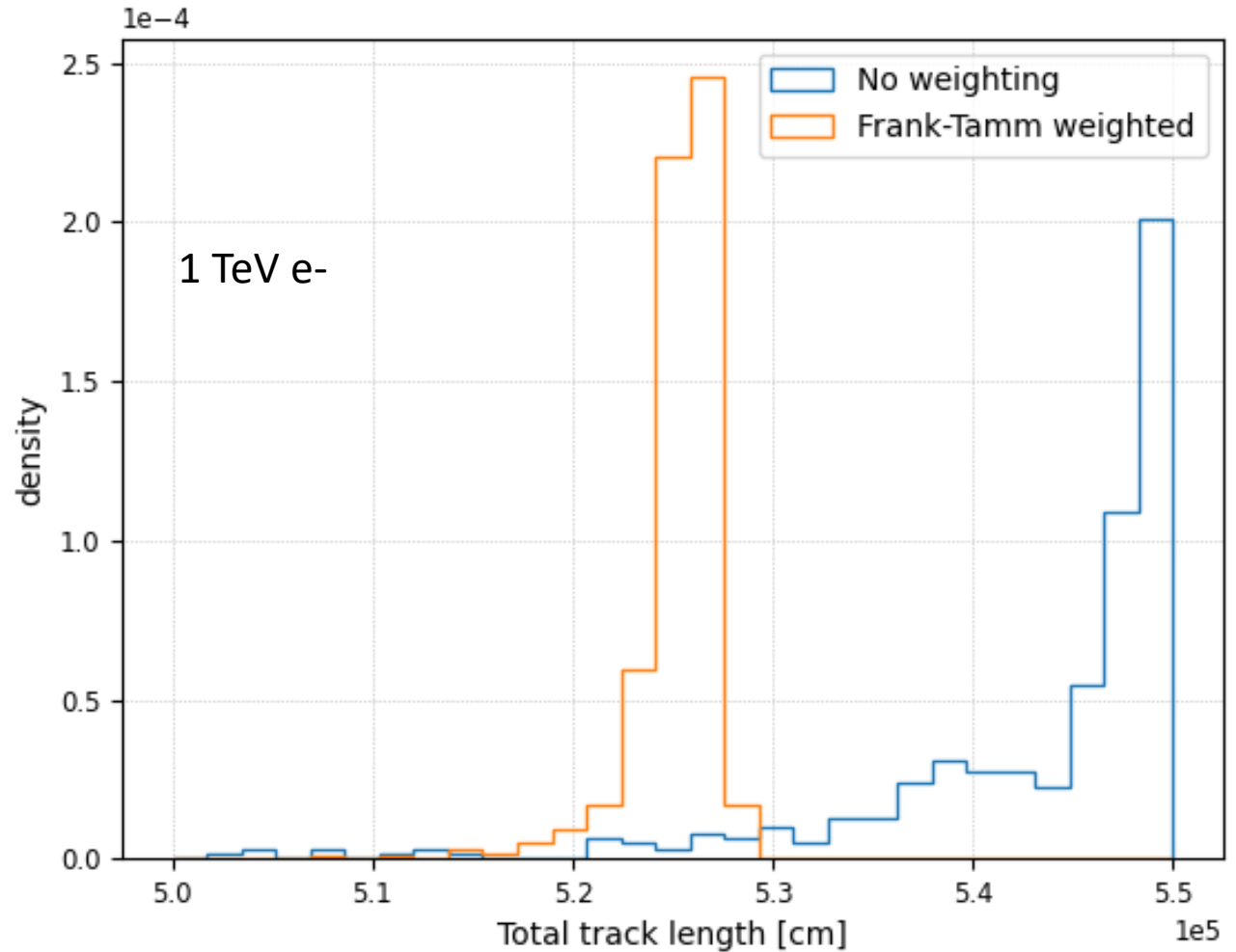
Effective track length

$$\hat{\ell}_i \equiv \ell_i \frac{n^2 - c^2/v_i^2}{n^2 - 1}$$

Simplifies Frank-Tamm to be independent of v

Allows for rescaling to different densities and refractive indexes

→ Applicable to water and ice



Electro-nuclear in FLUKA

[1] <https://fluka-forum.web.cern.ch/t/electro-nuclear-interactions-in-fluka/6294>

[2] <https://doi.org/10.1051/epjconf/201714612005>

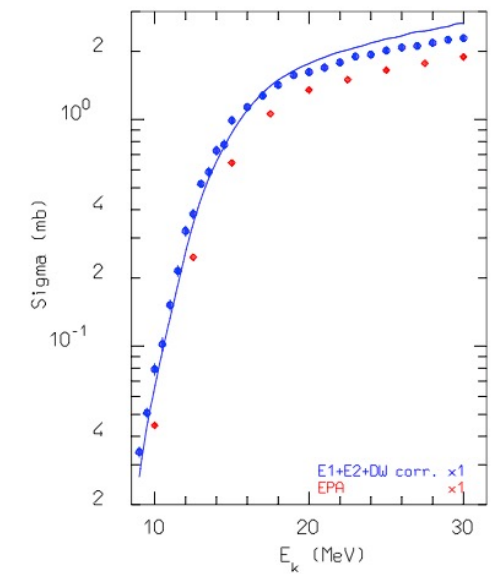
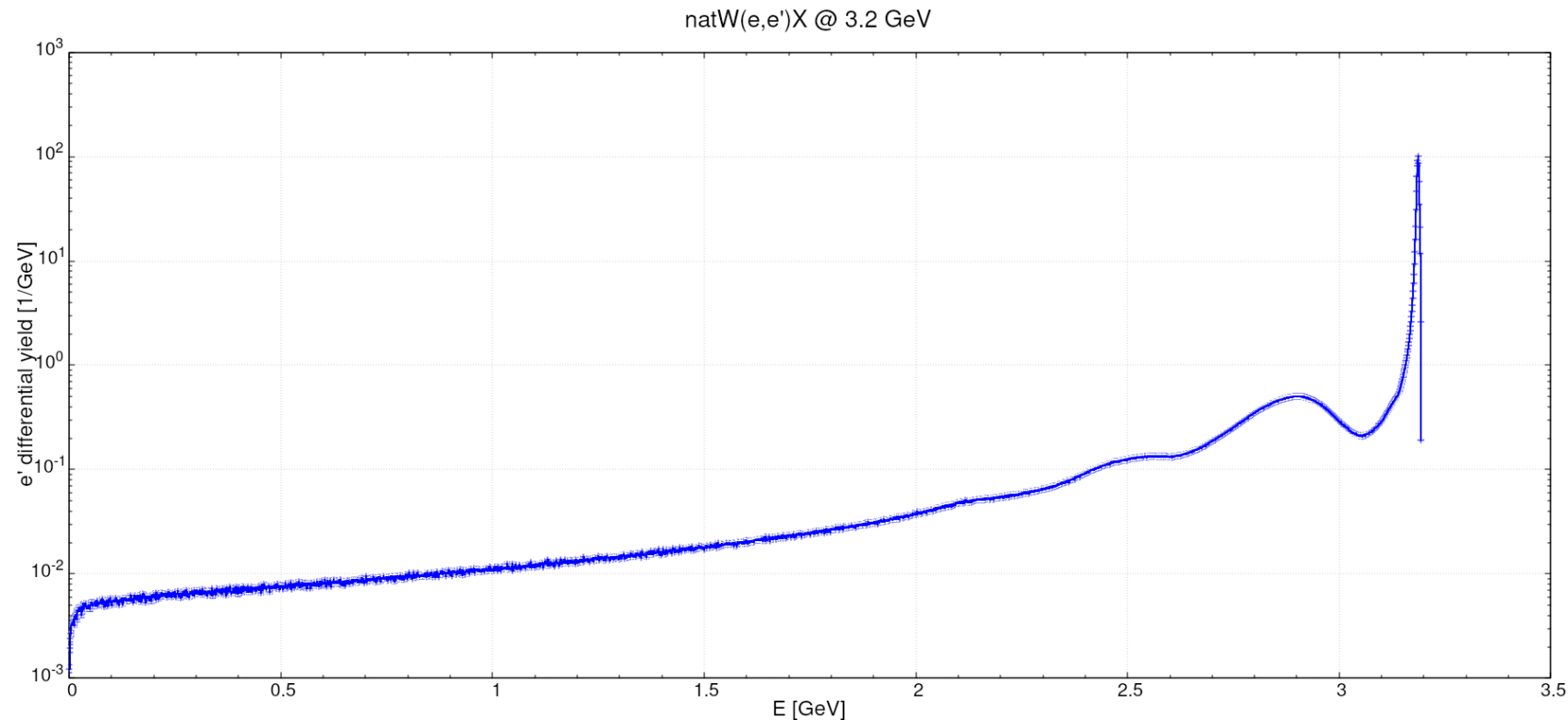


Figure 3. Electronuclear cross section for $^{181}\text{Ta}(e^-, n)^{180}\text{Ta}$ as a function of e^- energy in the few-tens of MeV range. Blue dots are experimental data from [16], red dots are FLUKA simulations with EPA, the blue line are FLUKA results with the full approach described in the present paper.

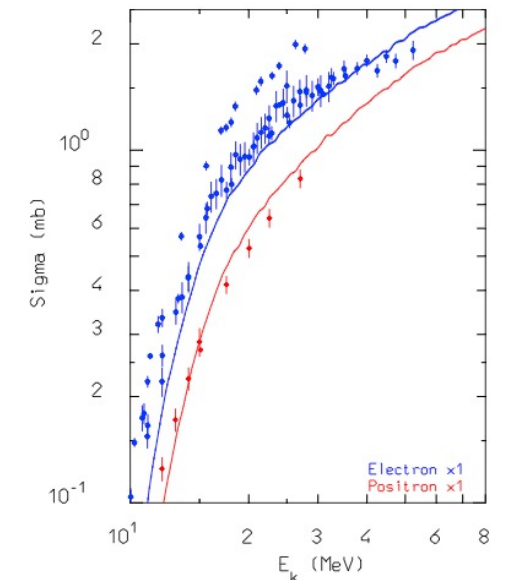


Figure 4. Fission cross section for electrons (blue) and positrons (red) on ^{238}U as a function of electron energy. Symbols: experimental data from [18–20], lines: FLUKA EMD.

G4 physics list used previously

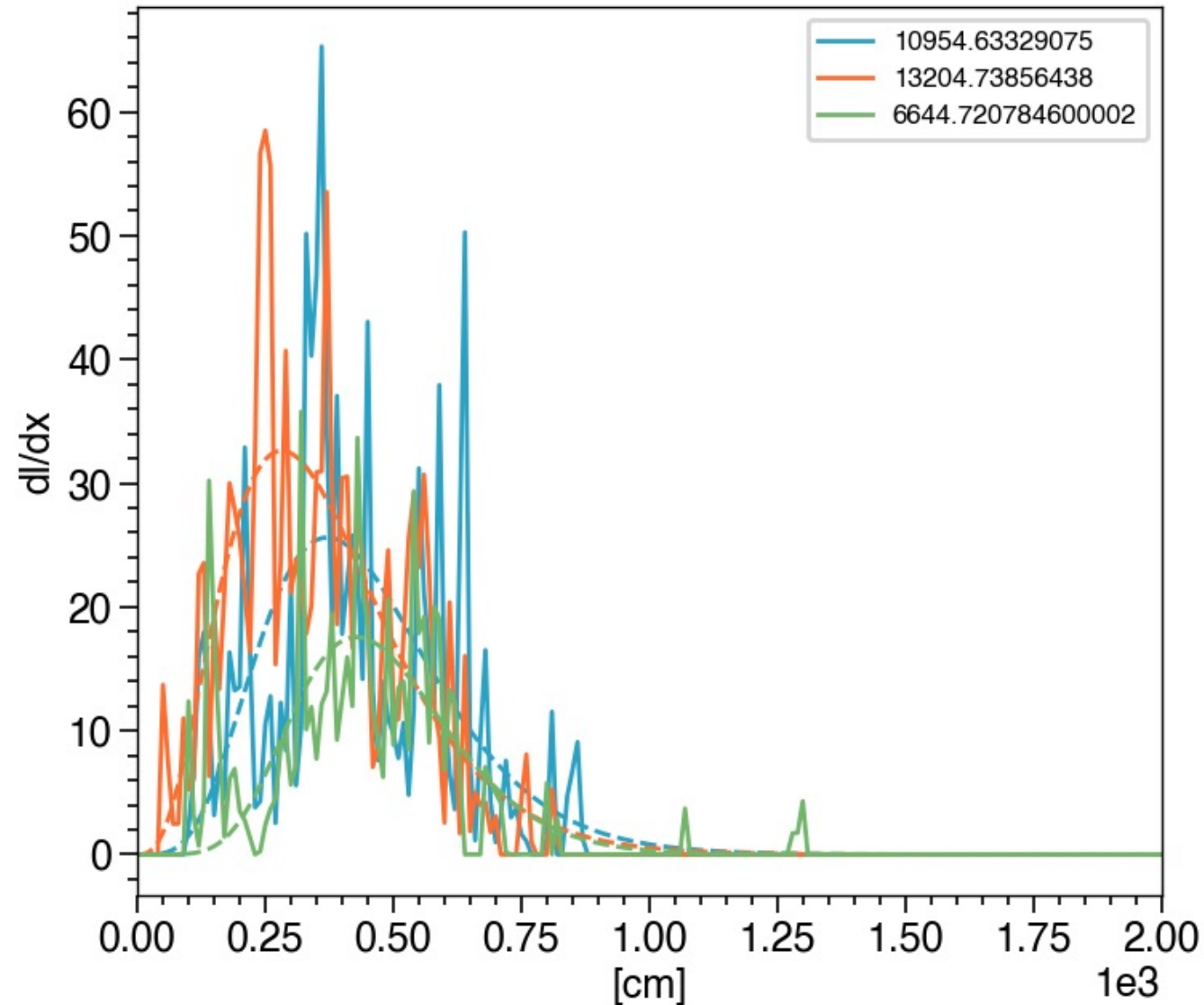
Does not include photo/electro-nuclear reactions

Table A.1: Physics processes of most important particles used in the simulation. If no model is specified the default model is used. For hadrons and ions that are not listed Coulomb scattering and ionization are defined.

Particle	Process	Model
γ	G4PhotoElectricEffect G4ComptonScattering G4GammaConversion	G4PEEffectFluoModel G4KleinNishinaModel
e^-	G4CoulombScattering G4eIonisation G4eBremsstrahlung	
e^+	G4CoulombScattering G4eIonisation G4eBremsstrahlung G4eplusAnnihilation	
μ^+, μ^-	G4CoulombScattering G4MuIonisation G4MuBremsstrahlung G4MuPairProduction G4MuNuclearInteraction	
π^+, π^- , K^+, K^-, p^+	G4CoulombScattering G4hIonisation G4hBremsstrahlung G4hPairProduction	
α, He^{3+}	G4ionIonisation G4CoulombScattering G4NuclearStopping	G4IonCoulombScatteringModel
all unstable particles	G4Decay	

p+n track lengths from 1 TeV e-

photonuc enabled → larger variance (no Cerenkov weighting in this plot)



Disabling photonuc reproduces G4 variance

Again, 1 TeV e-; total track length shown; units are cm

```
In [1]: b=np.loadtxt('DataOutputs_ELECTRON/job100_ELECTRON_1.00000E3.csv', delimiter=',')
```

```
In [2]: b[:,501].mean()
```

```
Out[2]: 522350.0815459986
```

PHOTONUC and ELECTNUC on

```
In [3]: b[:,501].std()
```

```
Out[3]: 3686.1084049979067
```

```
In [1]: b=np.loadtxt('DataOutputs_ELECTRON/job90_ELECTRON_1.00000E3.csv', delimiter=',')
```

```
In [2]: b[:,501].mean()
```

```
Out[2]: 528236.1264581786
```

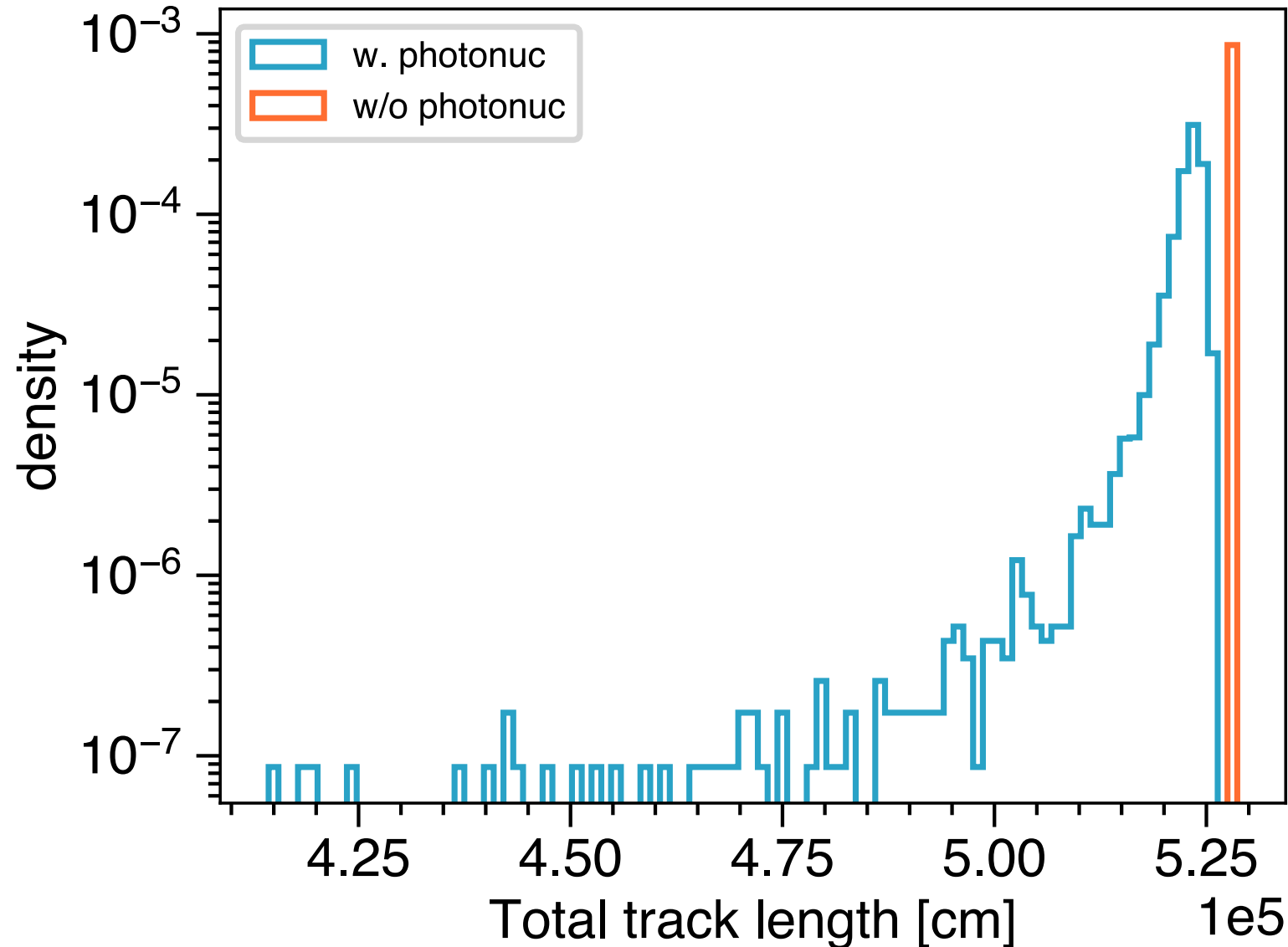
PHOTONUC and ELECTNUC off

```
In [3]: b[:,501].std()
```

```
Out[3]: 151.51195490132537
```

Impact of photonuclear effects on EM shower

1 TeV e- ... w/o photonuc very close to RW2013 result



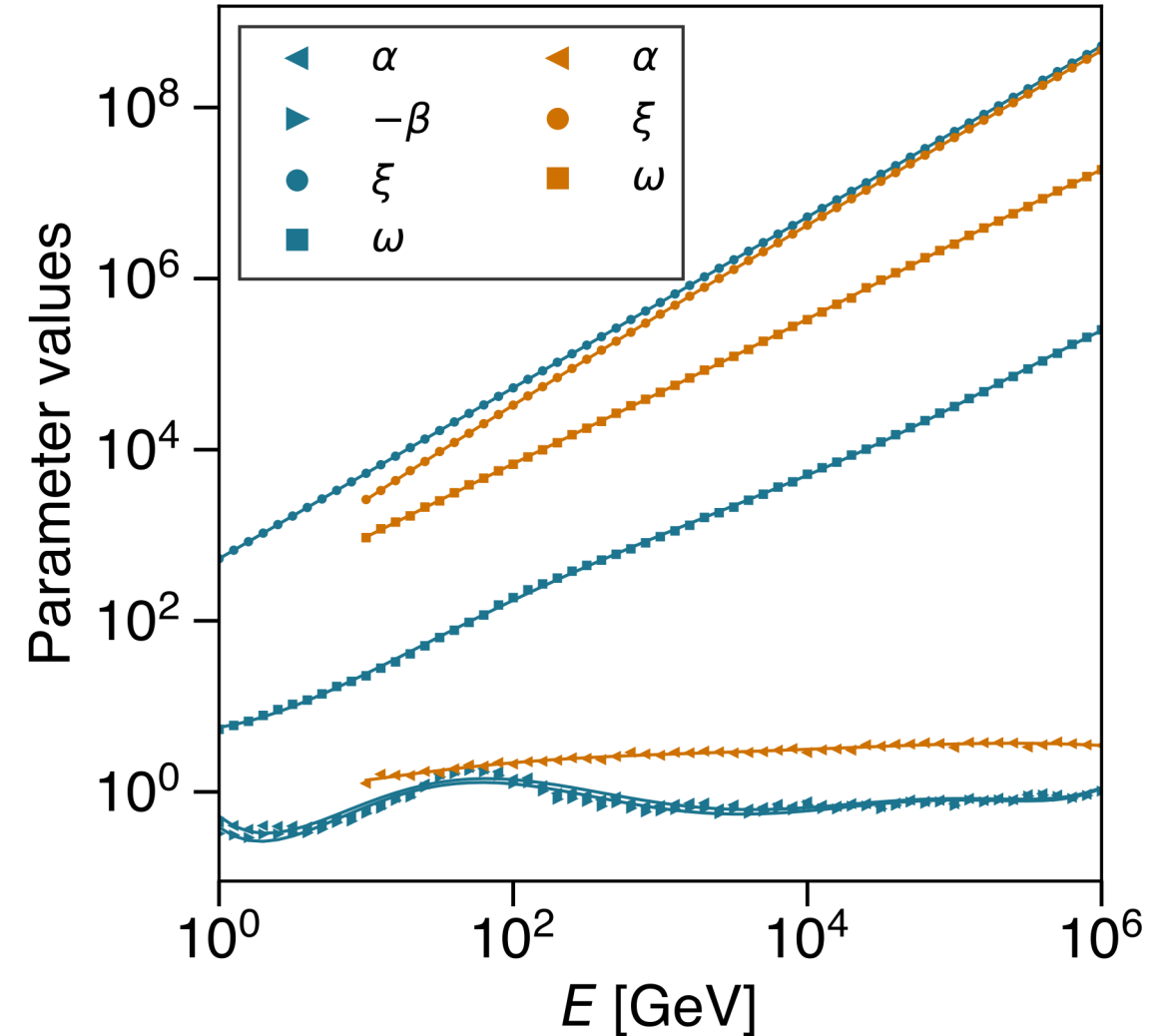
Parametrization vs energy

Need to model parameters governing SN and NIG as a **function of energy**

Fit parameter values at discrete-energy intervals with sixth-order polynomial

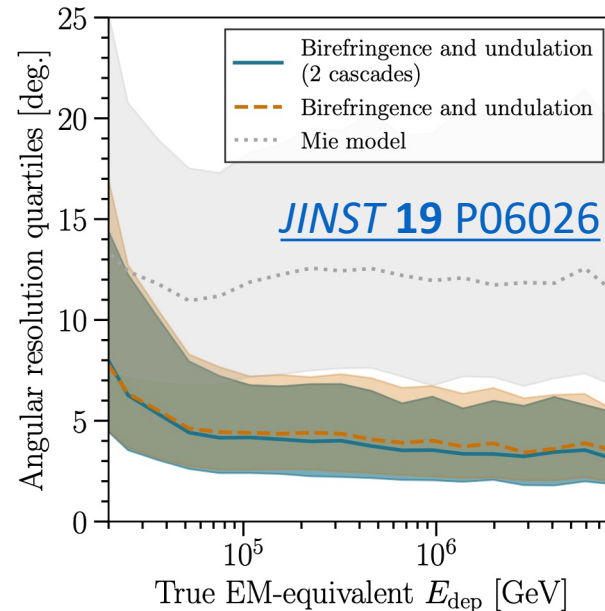
$$g_p(E; \mathbf{t}_p, s_p) \equiv s_p \exp \left\{ \sum_{i=0}^6 t_{p,i} (\log_{10} E)^i \right\}$$

See paper for details



Implications for reconstruction and classification

Using model that approximates extension yields reco. improvements



Realistic longitudinal extensions can further improve directional reconstruction

Possibilities to distinguish more hadronic showers (NC) from CC

Charm

Heavy mesons can have comparable interaction and decay lengths

MC+parametrization approaches are likely the way to go, similar to PROPOSAL for muons

- Simulate charm meson secondary interactions and decay products, then apply parameterized shower profile on those
- Ongoing work on this in SIREN [Comput.Phys.Commun. 316 \(2025\)](#)