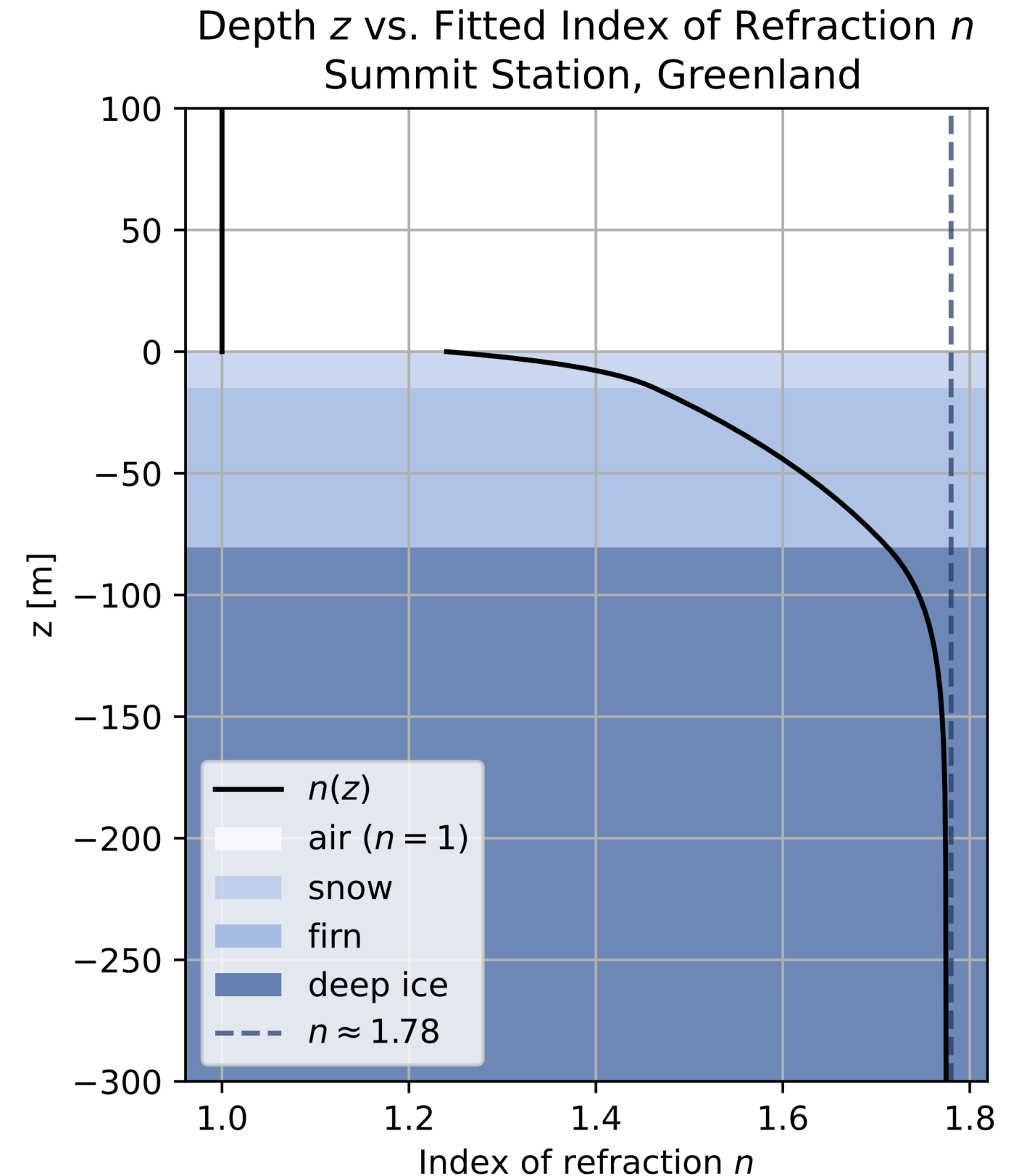


Computing signal propagation effects in polar ice for radio neutrino detectors

Marissa Boucher | *IceCube Particle Astrophysics Symposium 2026*

In-ice radio detectors allow us to target ultrahigh energy neutrinos ($>E_{\text{eV}}$) via the Askaryan effect.

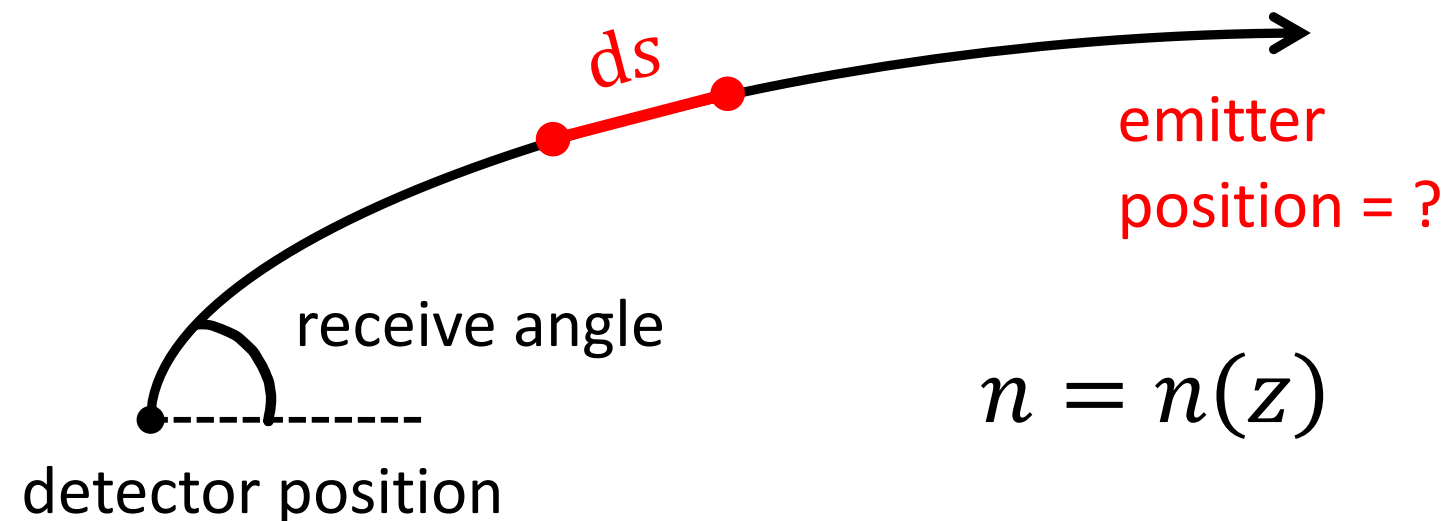
- Need to calculate propagation distance, time, path
- Radio signals can travel multiple kilometers to a detector $\Rightarrow$ ice refractive index significantly affects signal propagation



Geometric optics lets us compute these signal properties.

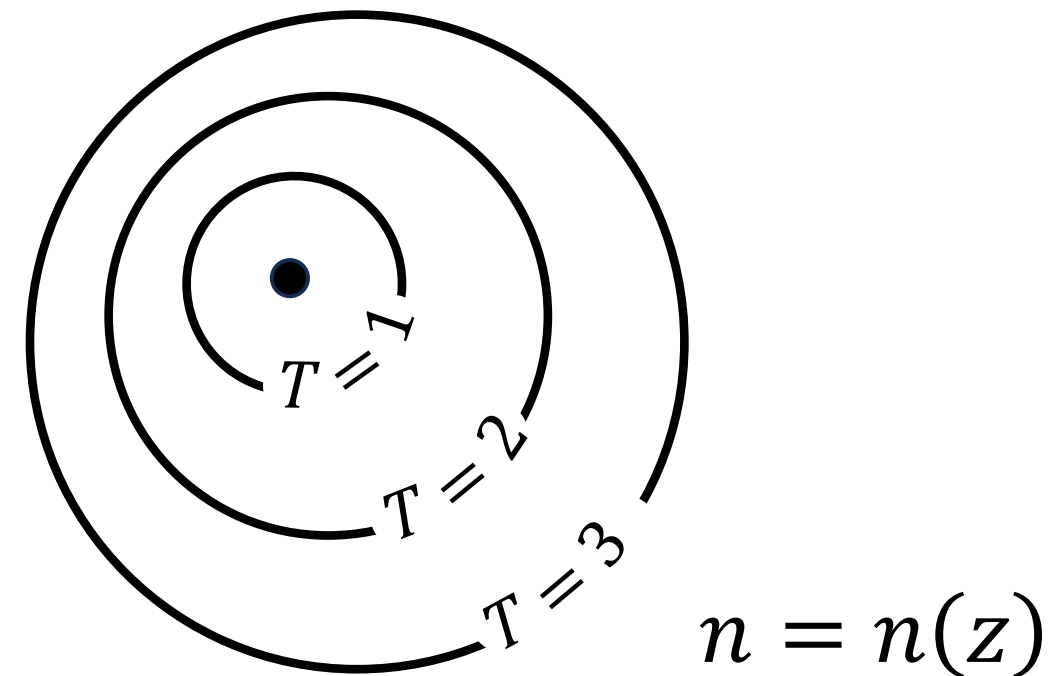
(infinite-frequency approximation)

$$\frac{d}{ds} \left(n \frac{d\mathbf{x}}{ds} \right) = \nabla n(\mathbf{x})$$



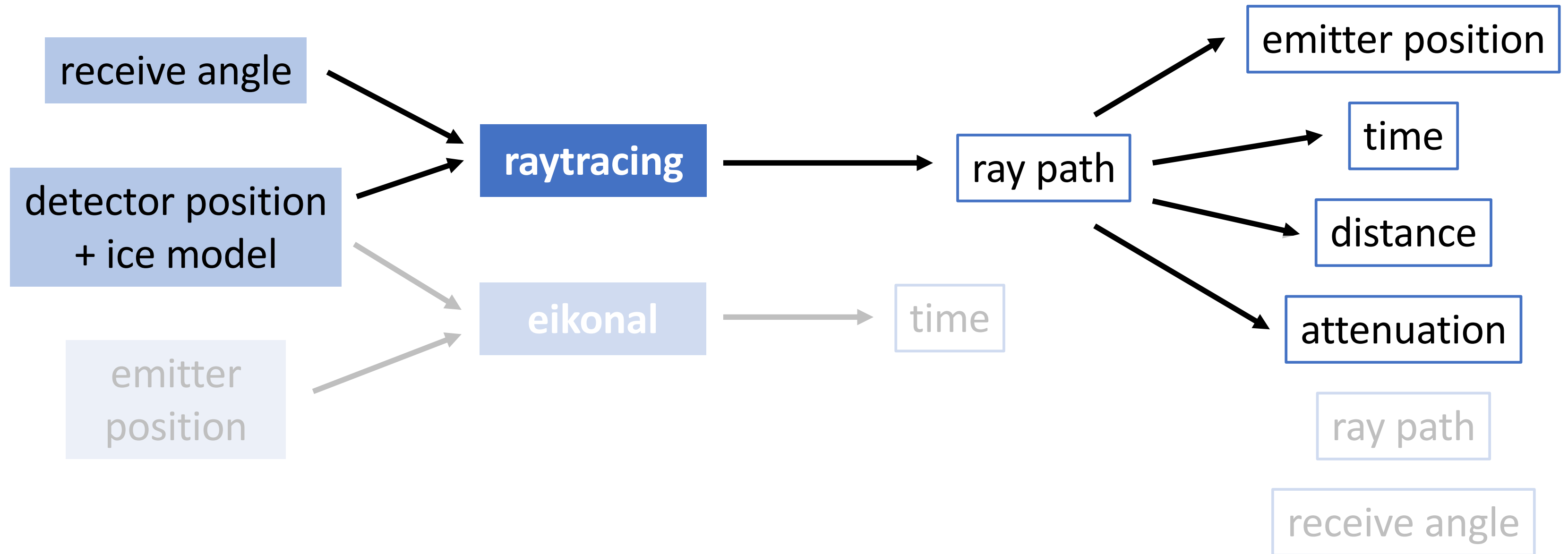
Approach 1: Raytracing

$$(\nabla T(\mathbf{x}))^2 = (n(\mathbf{x}))^2$$

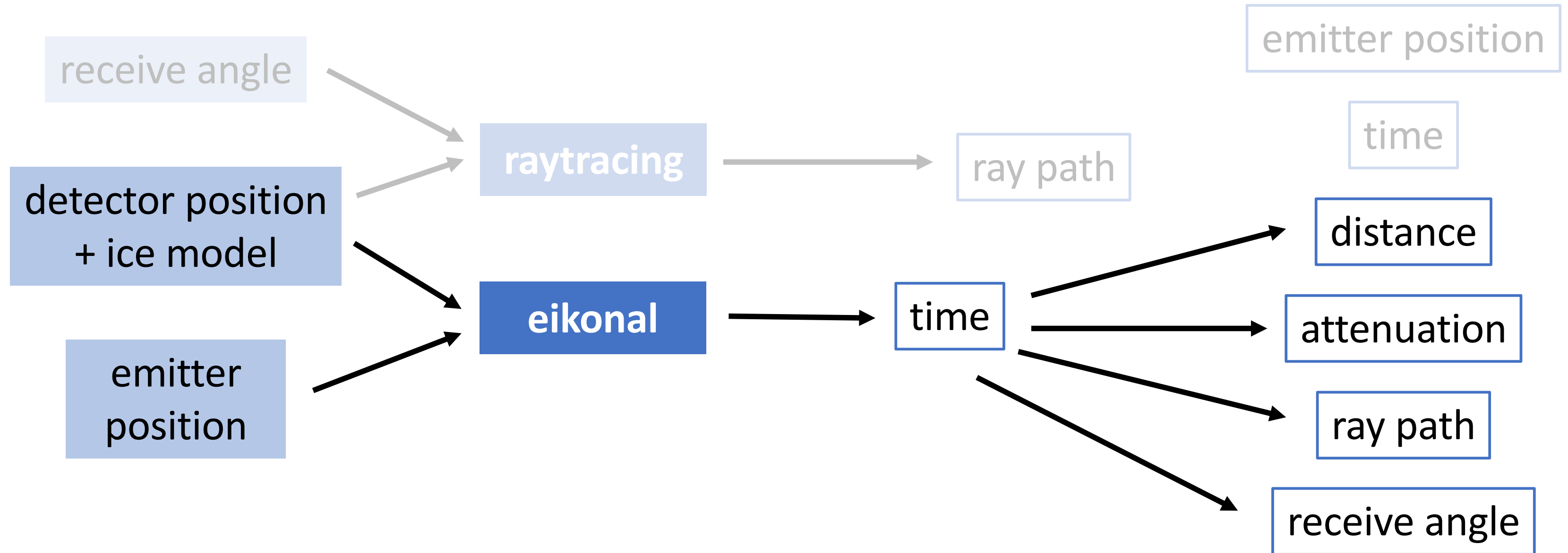


Approach 2: Eikonal

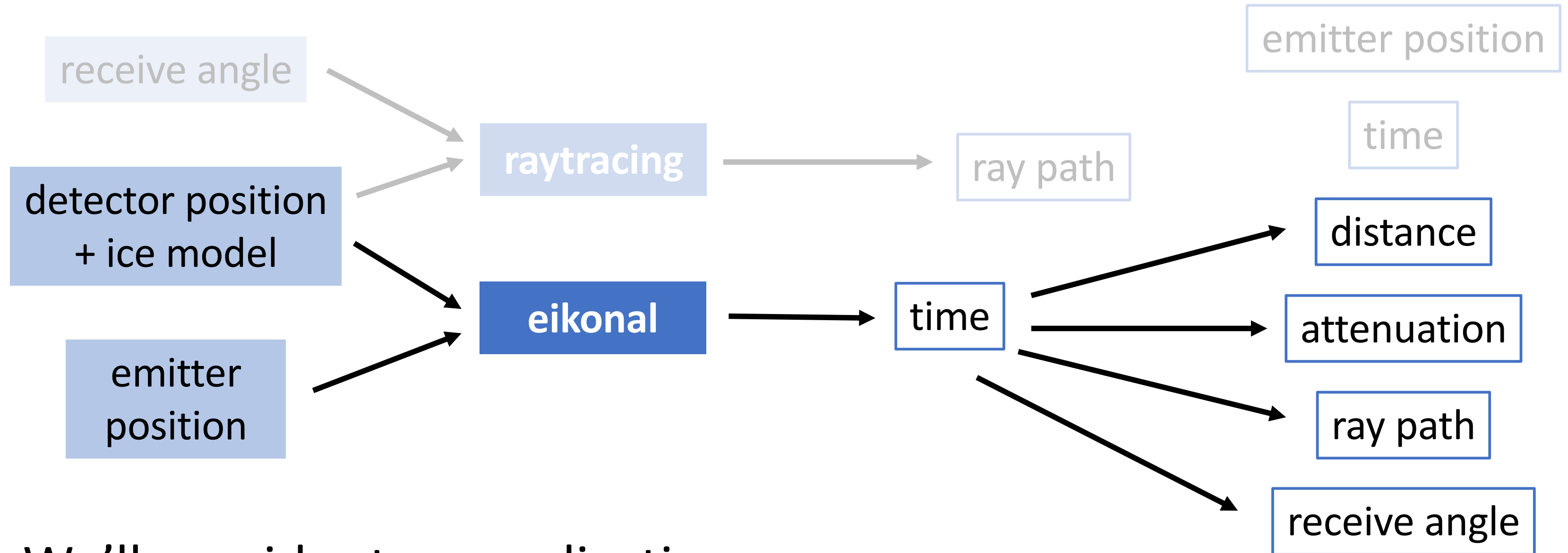
When should we use one or the other?



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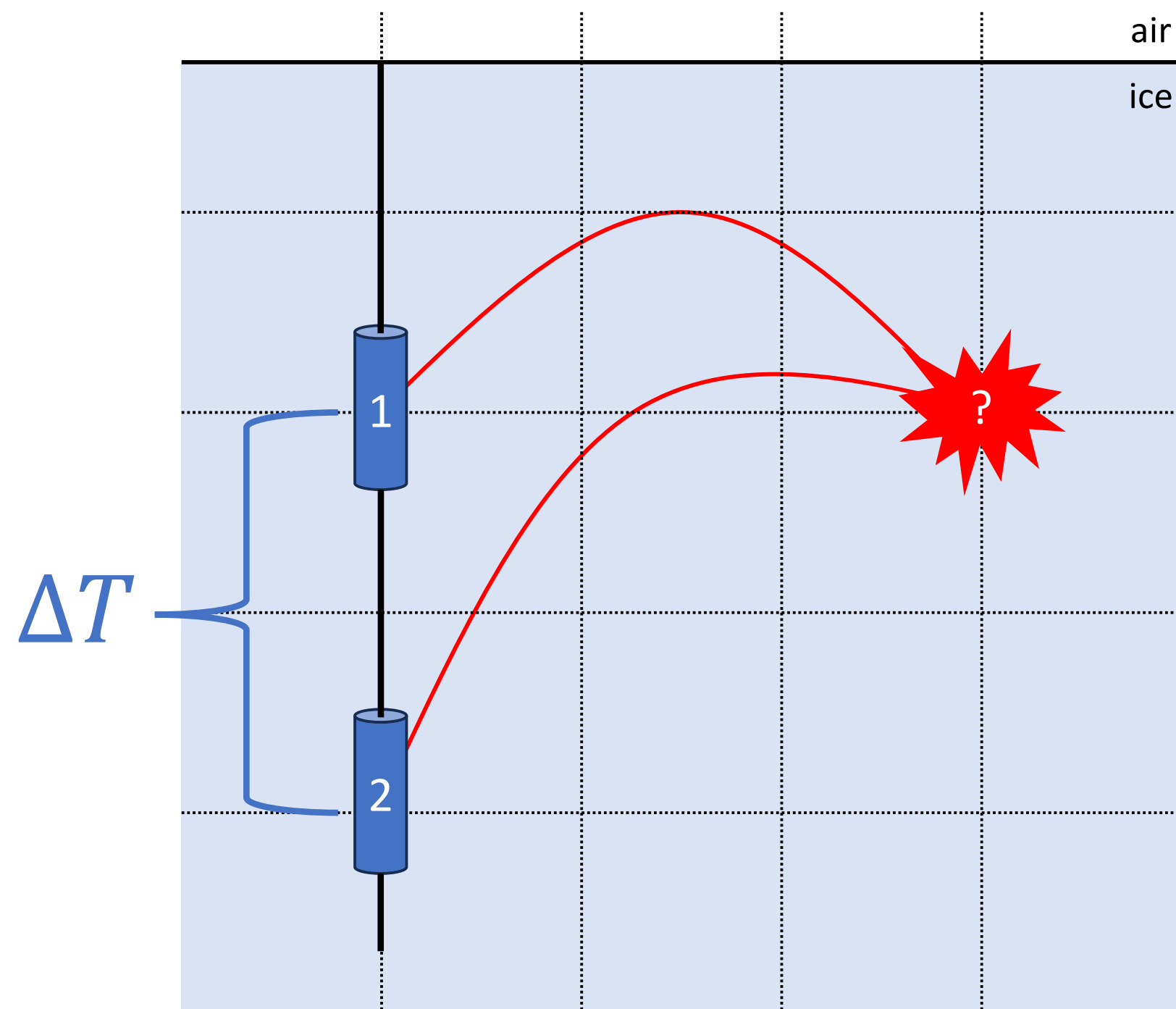


When should we use one or the other?



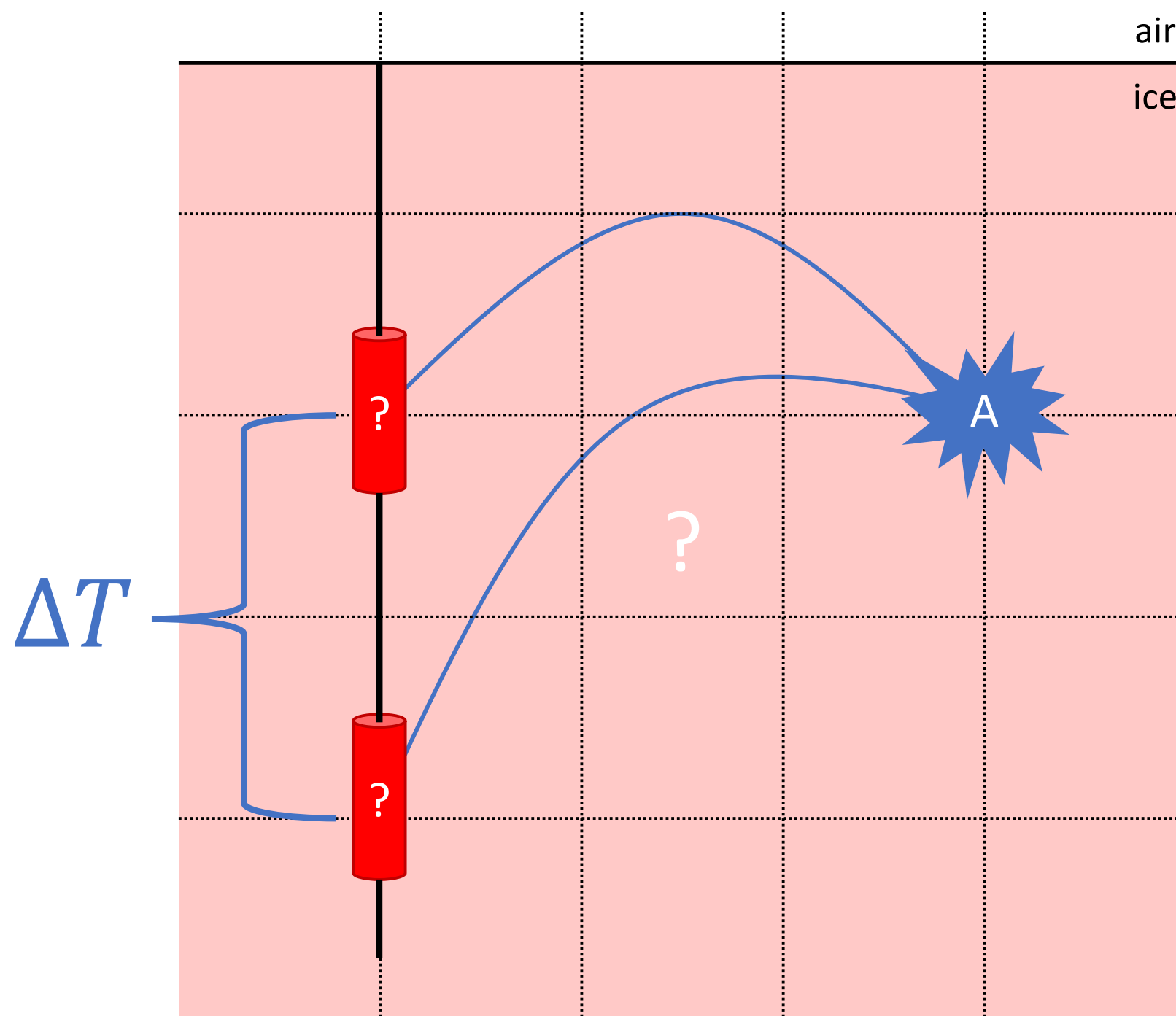
We'll consider two applications:
event reconstruction and **detector calibration**.

Event reconstruction



Known: arrival time difference,
ice model, detector position

Detector/ice calibration



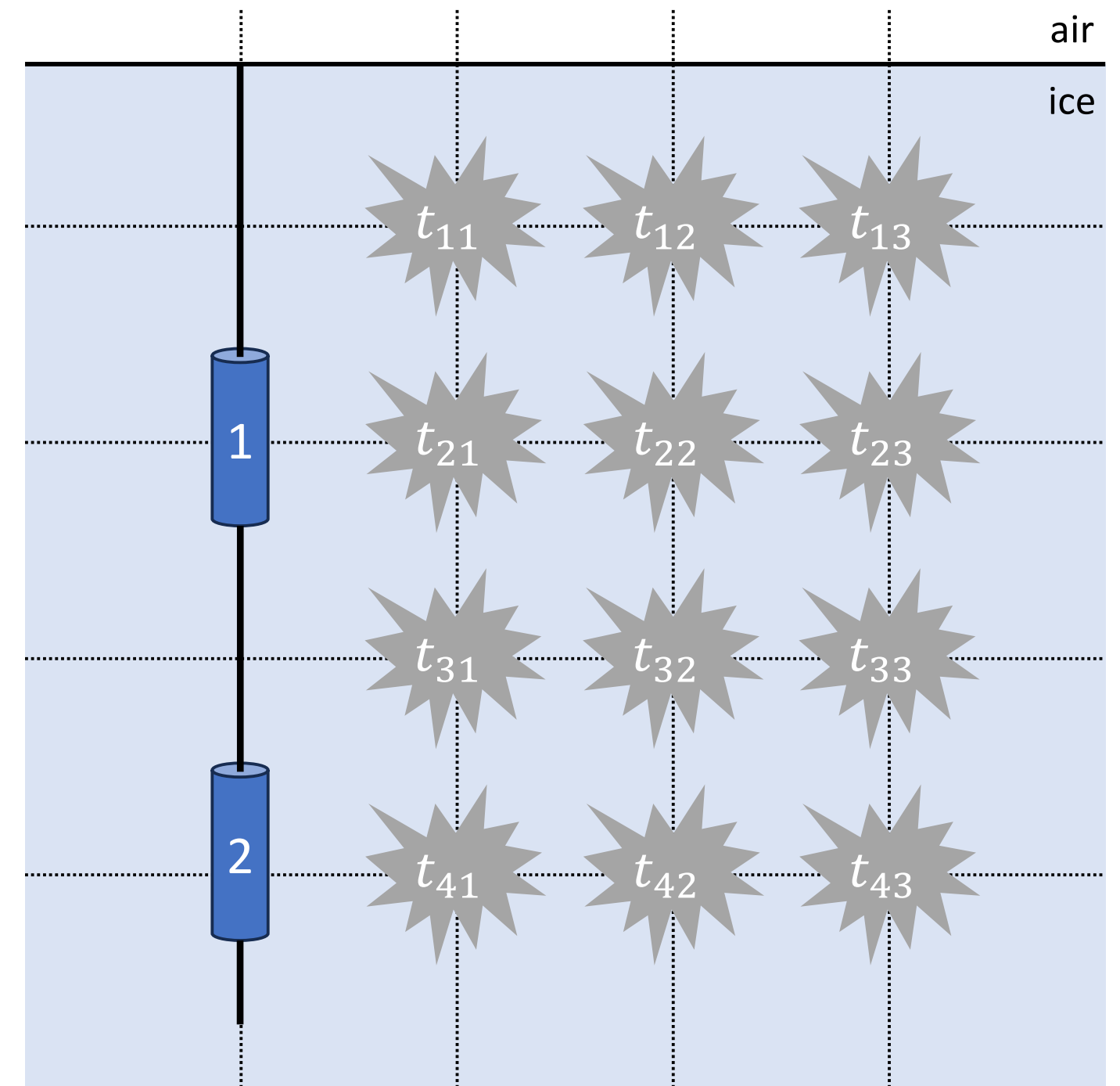
Known: arrival time difference,
emitter position

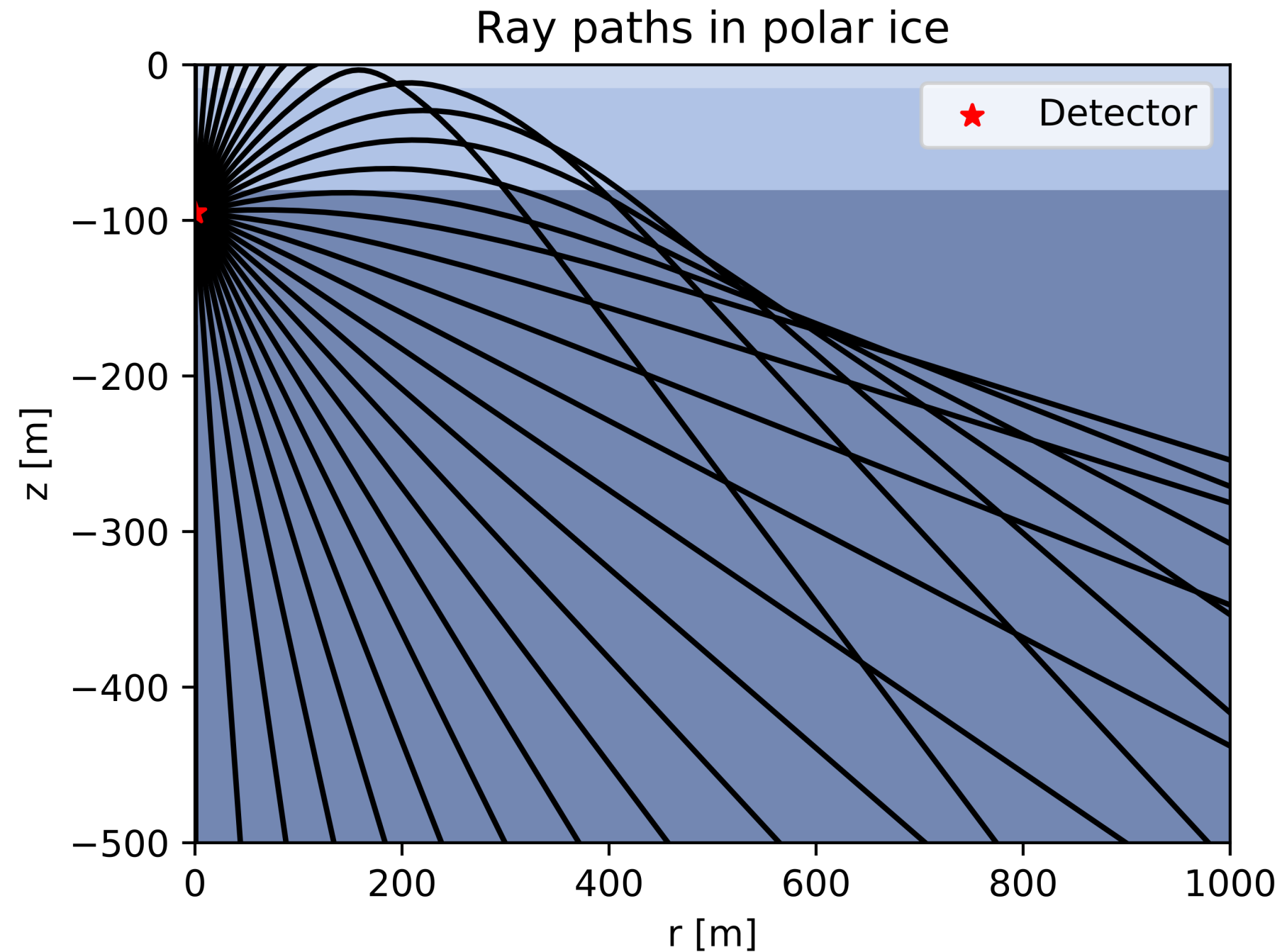
Both problems can be solved using **travel-time maps**: lookup tables with propagation times from possible emitter positions.

- Known: Ice model, detector position, N emitter positions
- Computing: travel-time map (N grid points)

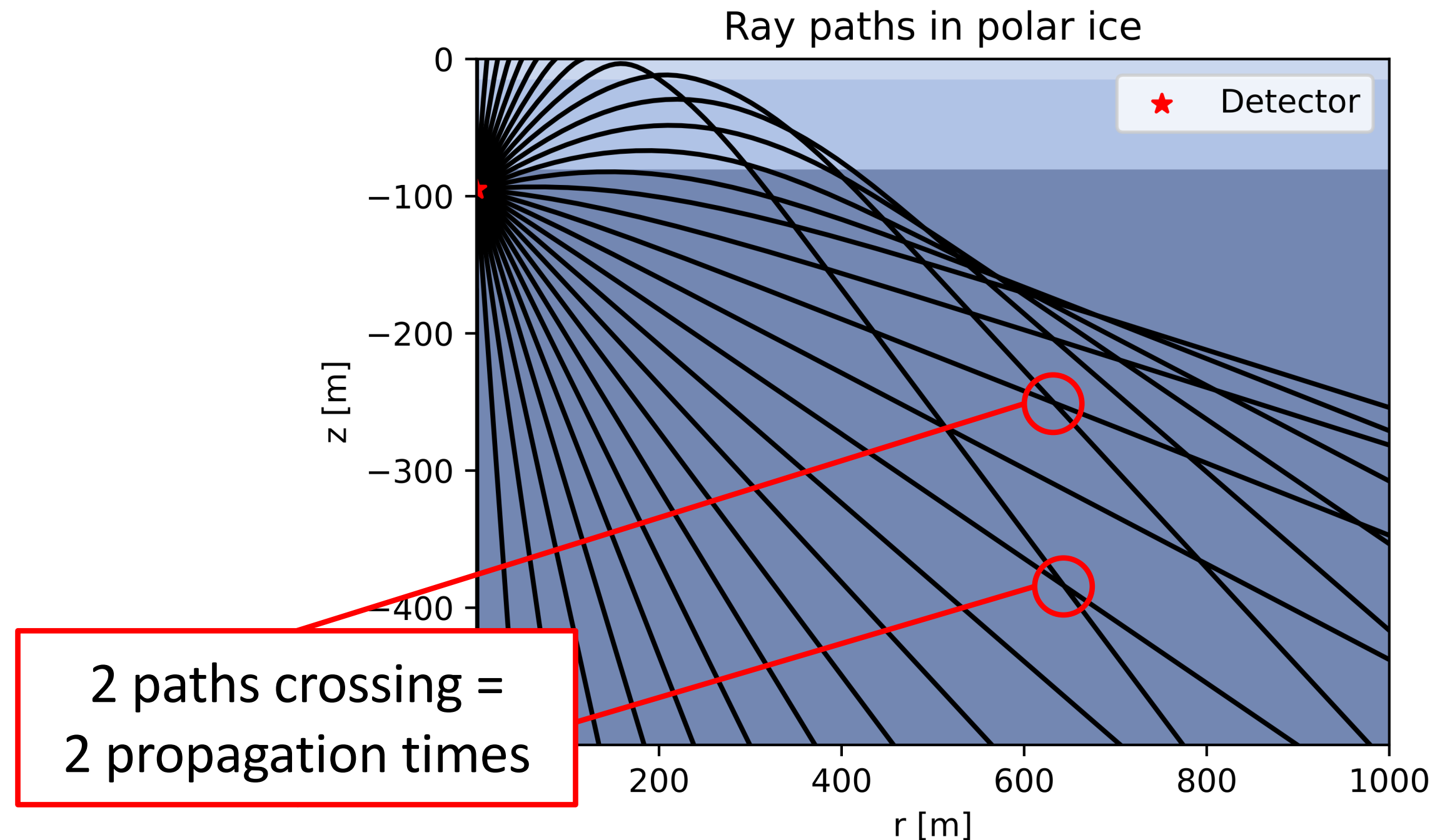
Raytracing $\rightarrow \mathcal{O}(N^2)$

Eikonal $\rightarrow \mathcal{O}(N \log N)$

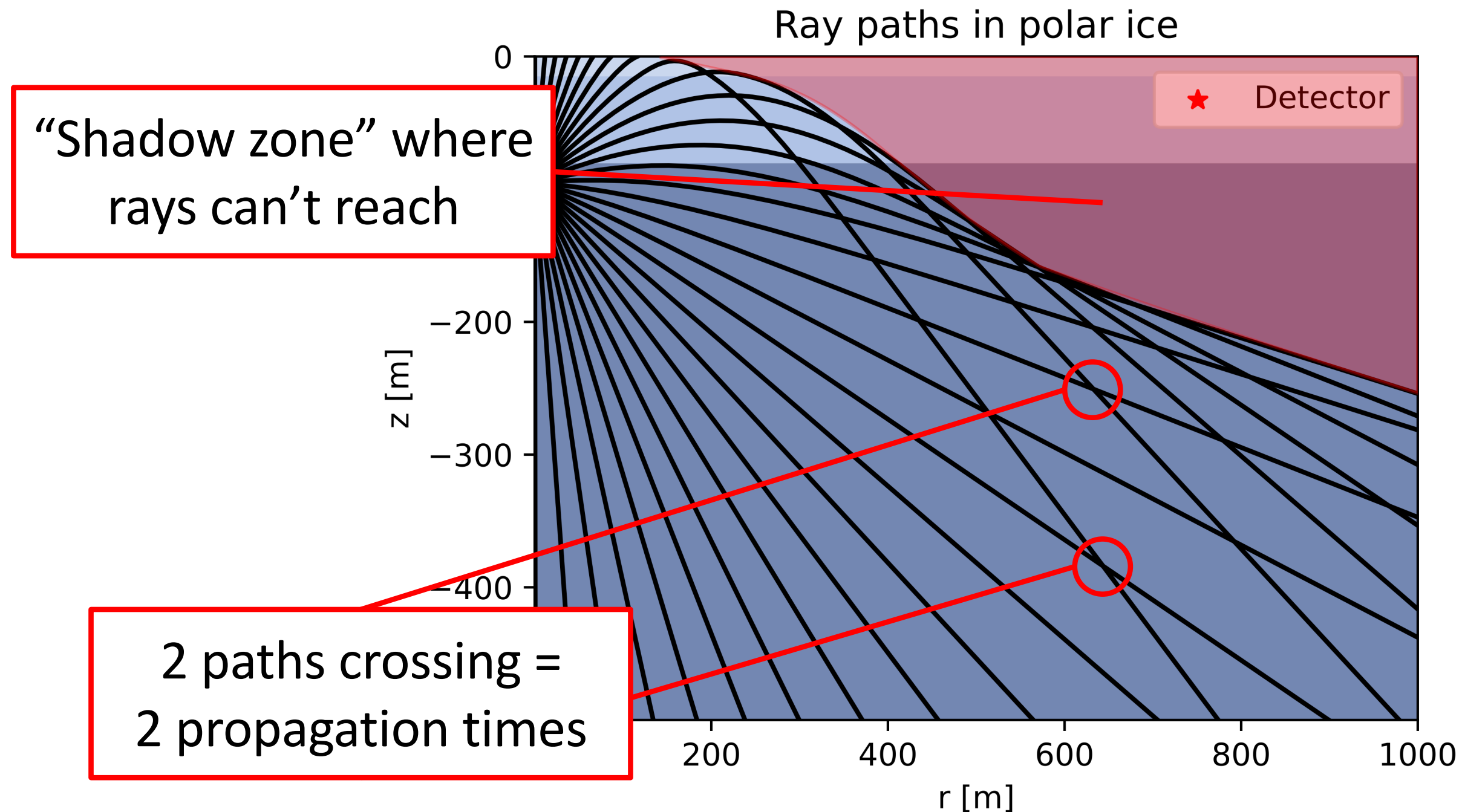




Raytracing can identify important features of the **ray family** connecting possible emitters to our detector.



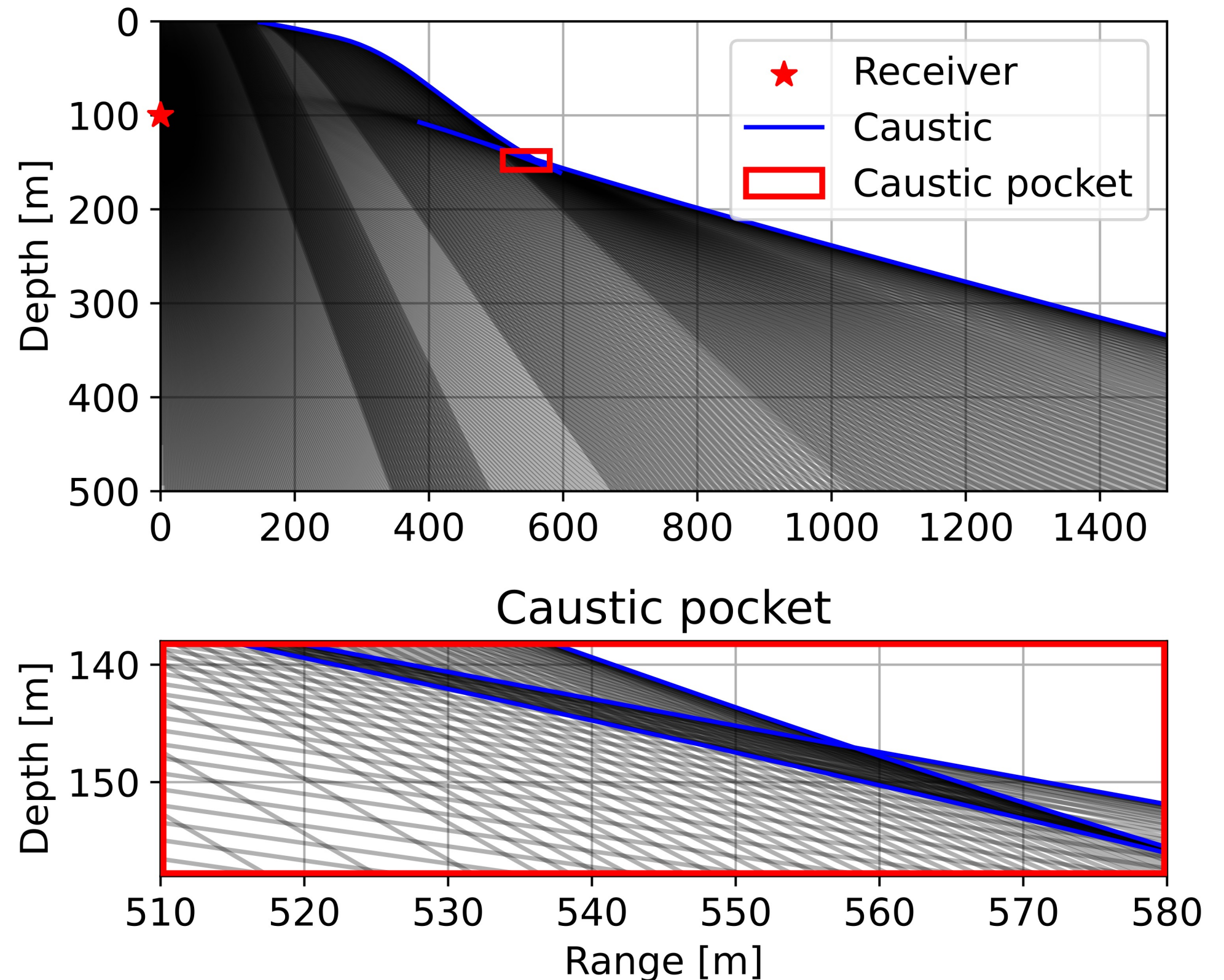
Raytracing can identify important features of the **ray family** connecting possible emitters to our detector.



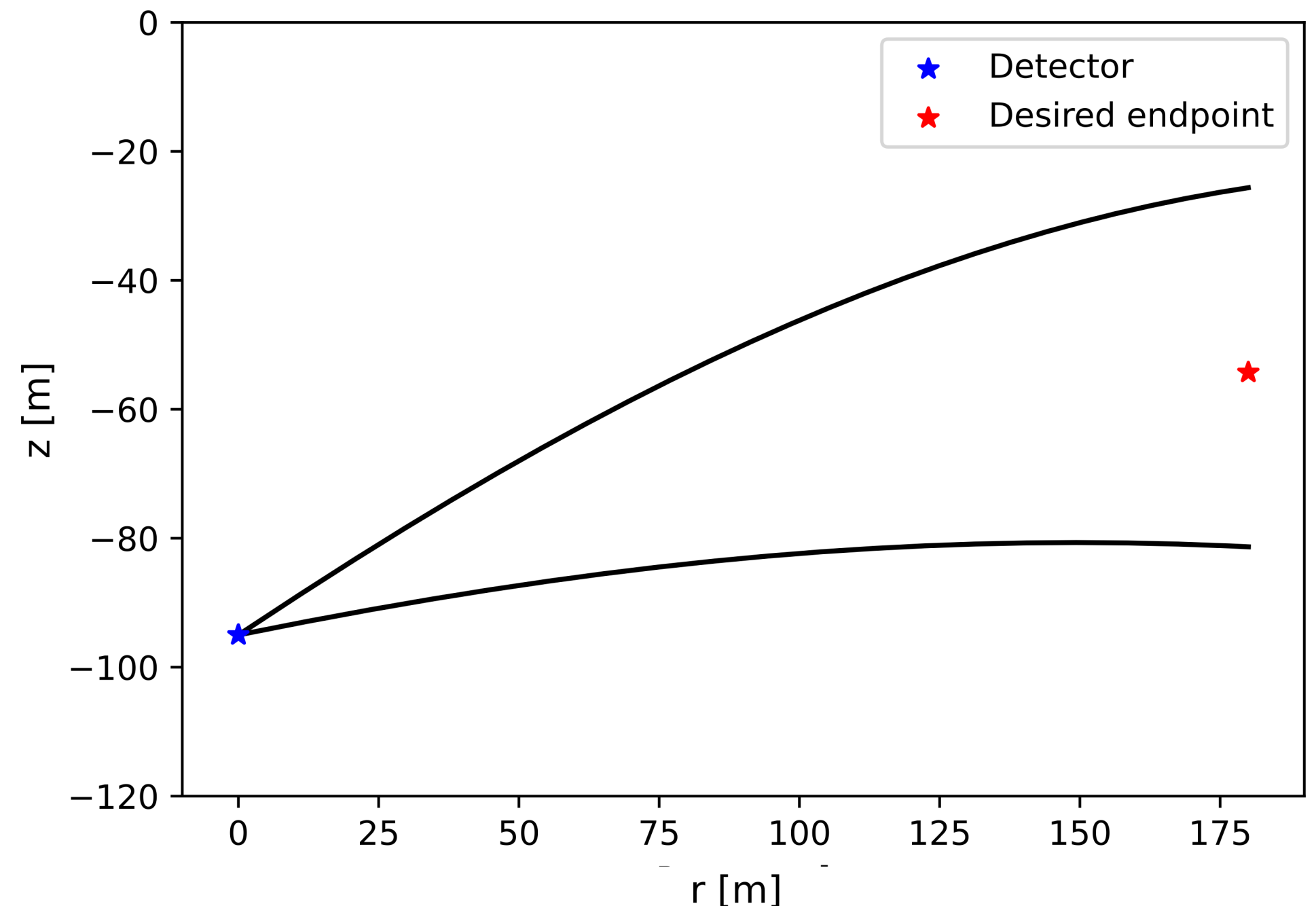
Raytracing can identify important features of the **ray family** connecting possible emitters to our detector.

Throwing more rays shows us the **caustic**.

- Curve where rays get infinitely close to each other
- Associated with infinite focusing in geometric optics
- Contains the shadow zone boundary
- May also form a four-solution “caustic pocket”

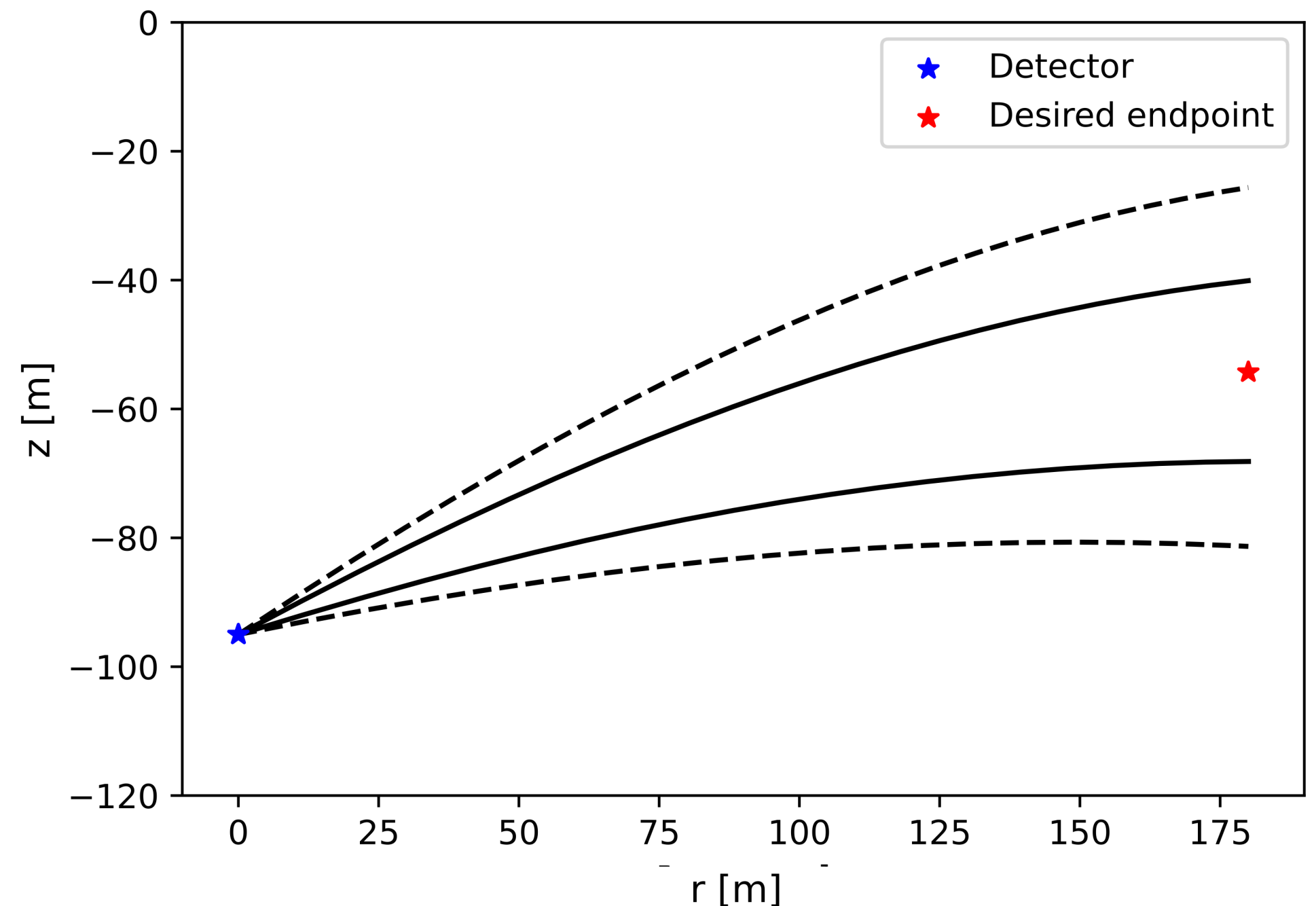


Raytracing lets us visualize solutions quickly, but is slow at calculating propagation times on a large grid.



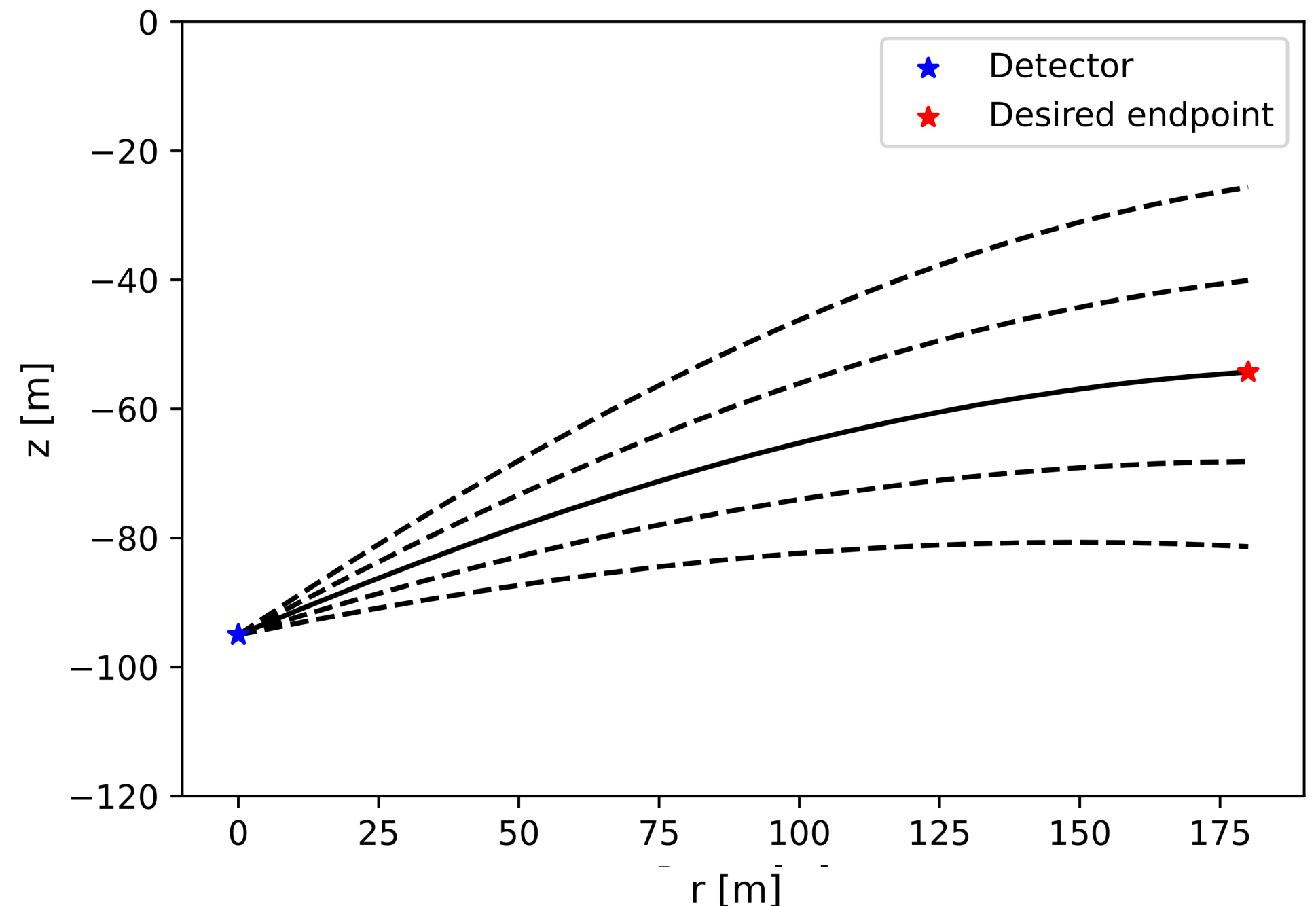
Expensive minimization required for each calculated propagation time ($O(N^2)$).

Raytracing lets us visualize solutions quickly, but is slow at calculating propagation times on a large grid.



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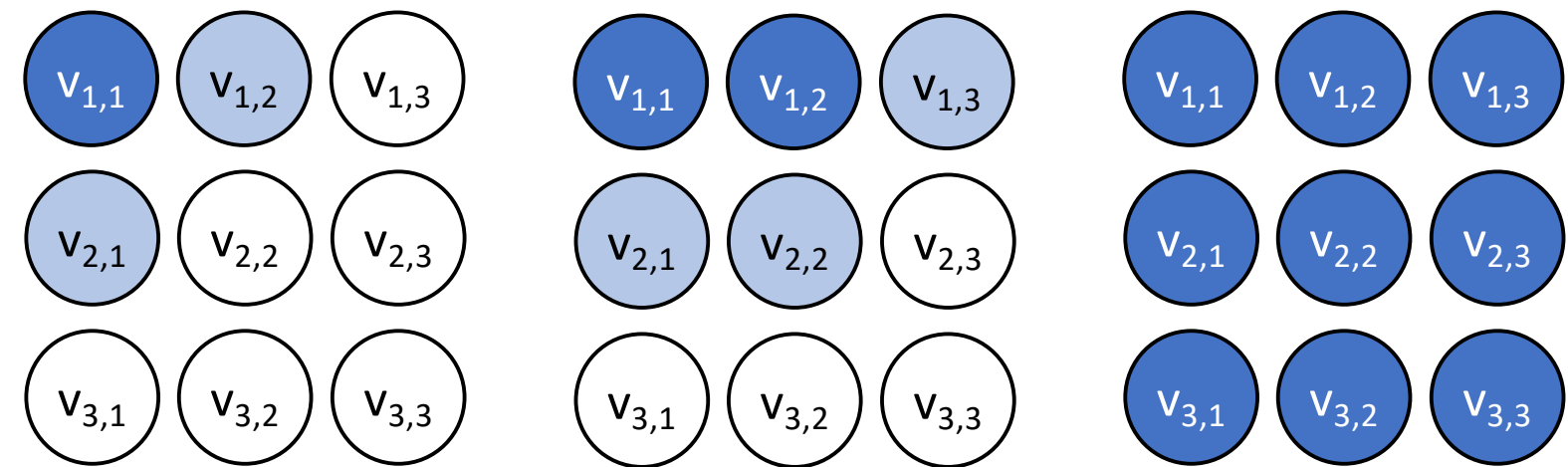
Raytracing lets us visualize solutions quickly, but is slow at calculating propagation times on a large grid.



Expensive minimization required for each calculated propagation time ($O(N^2)$).

We solve the eikonal equation using the Fast Marching Method, fixing boundary values rather than initial conditions.

$$\begin{array}{c}
 \text{---} \bigcirc_{v_1} \xrightarrow{\Delta x} \bigcirc_{v_2} \text{---} \\
 T(x + \Delta x) \approx T(x) + v_1 \Delta x
 \end{array}$$

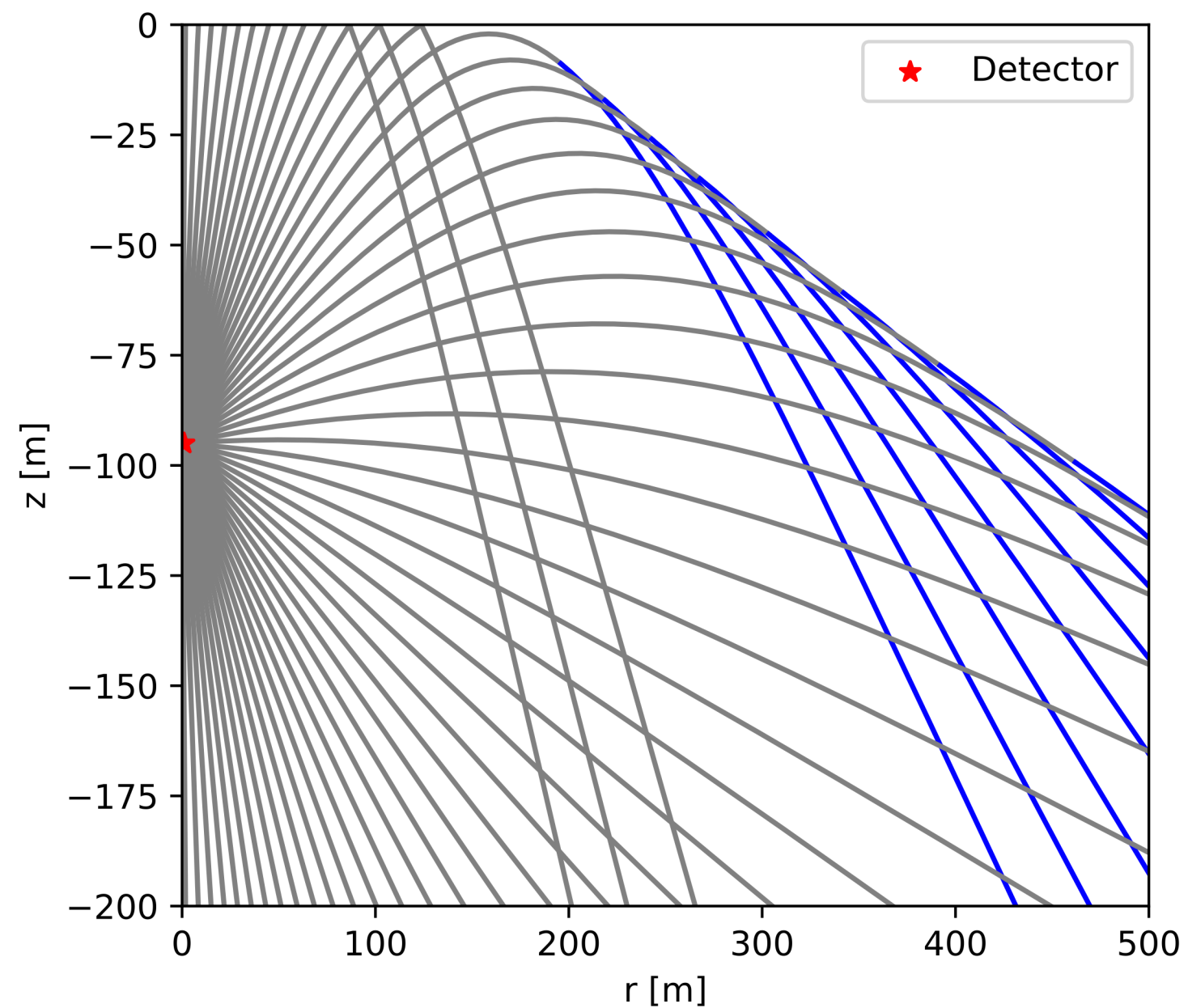


Works in $O(n \log n)$ for n nodes,
but only recovers earliest travel-times when rays intersect.

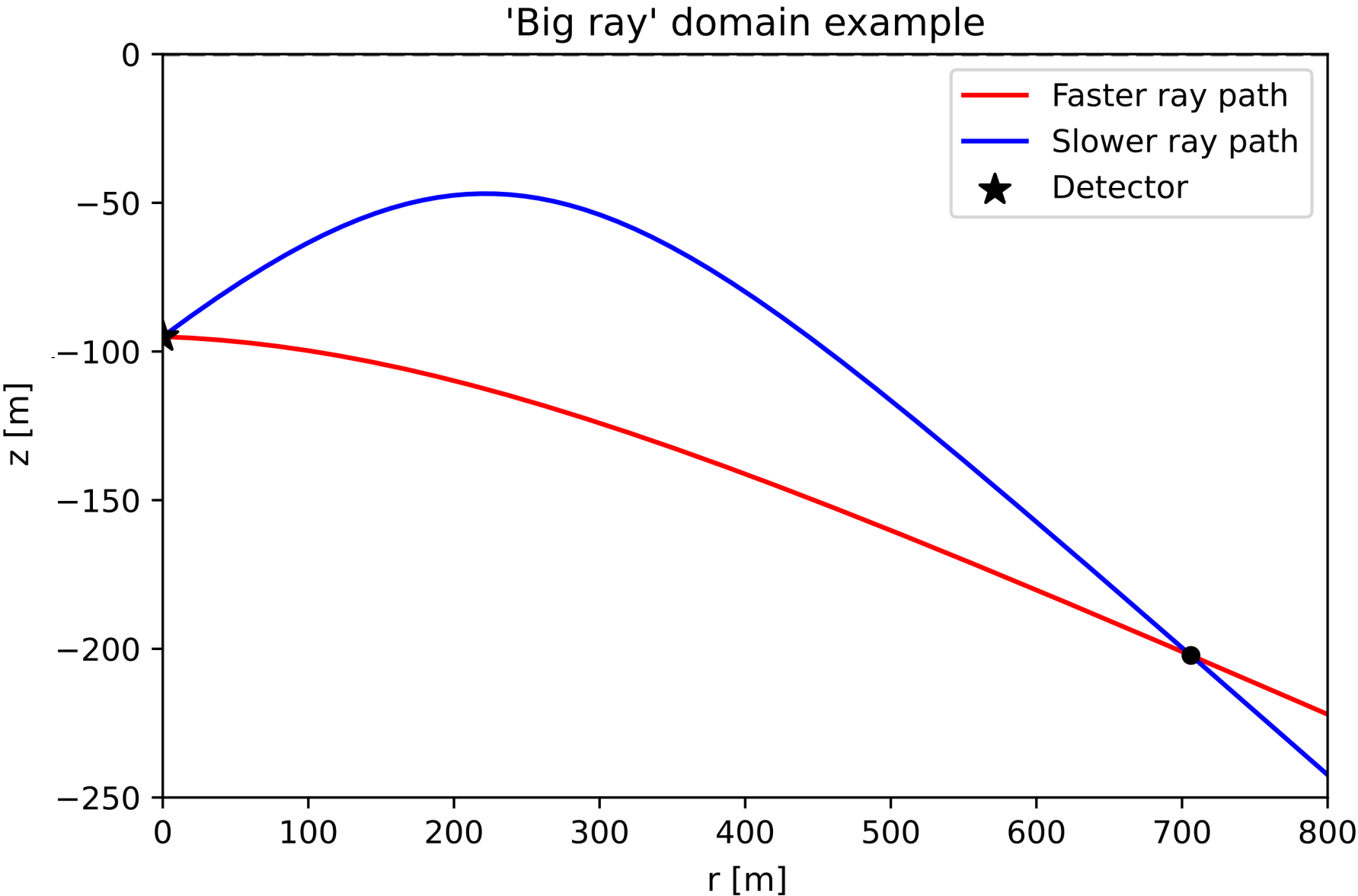
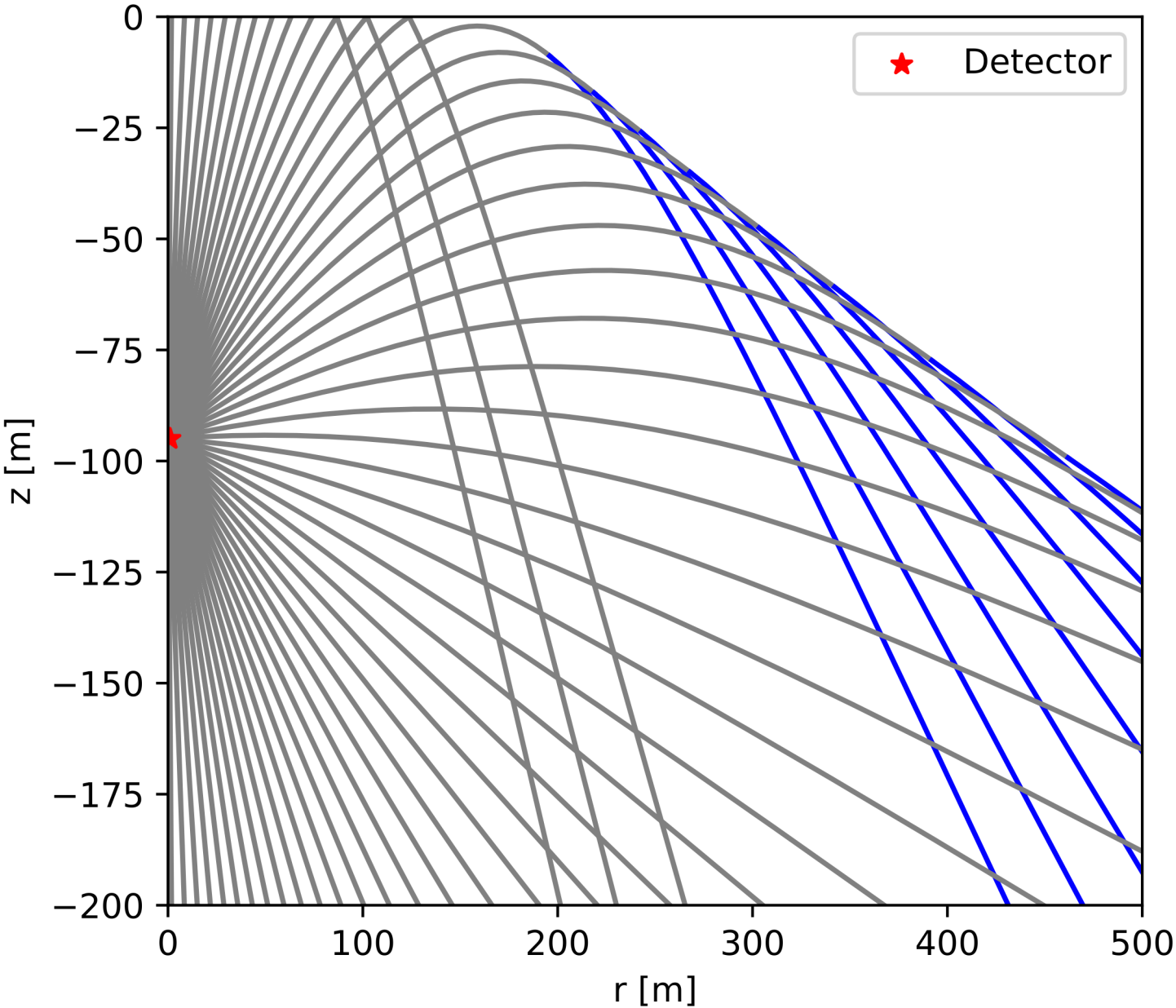
The FMM: James A. Sethian. *Level set methods and fast marching methods*. 1999.

Pykonal: Malcolm C. A. White et al. In: *Seismol. Res. Lett.* 91.4 (2020), pp. 2378–2389.

Example: Refracted rays turn over within the ice due to the decreasing refractive index.

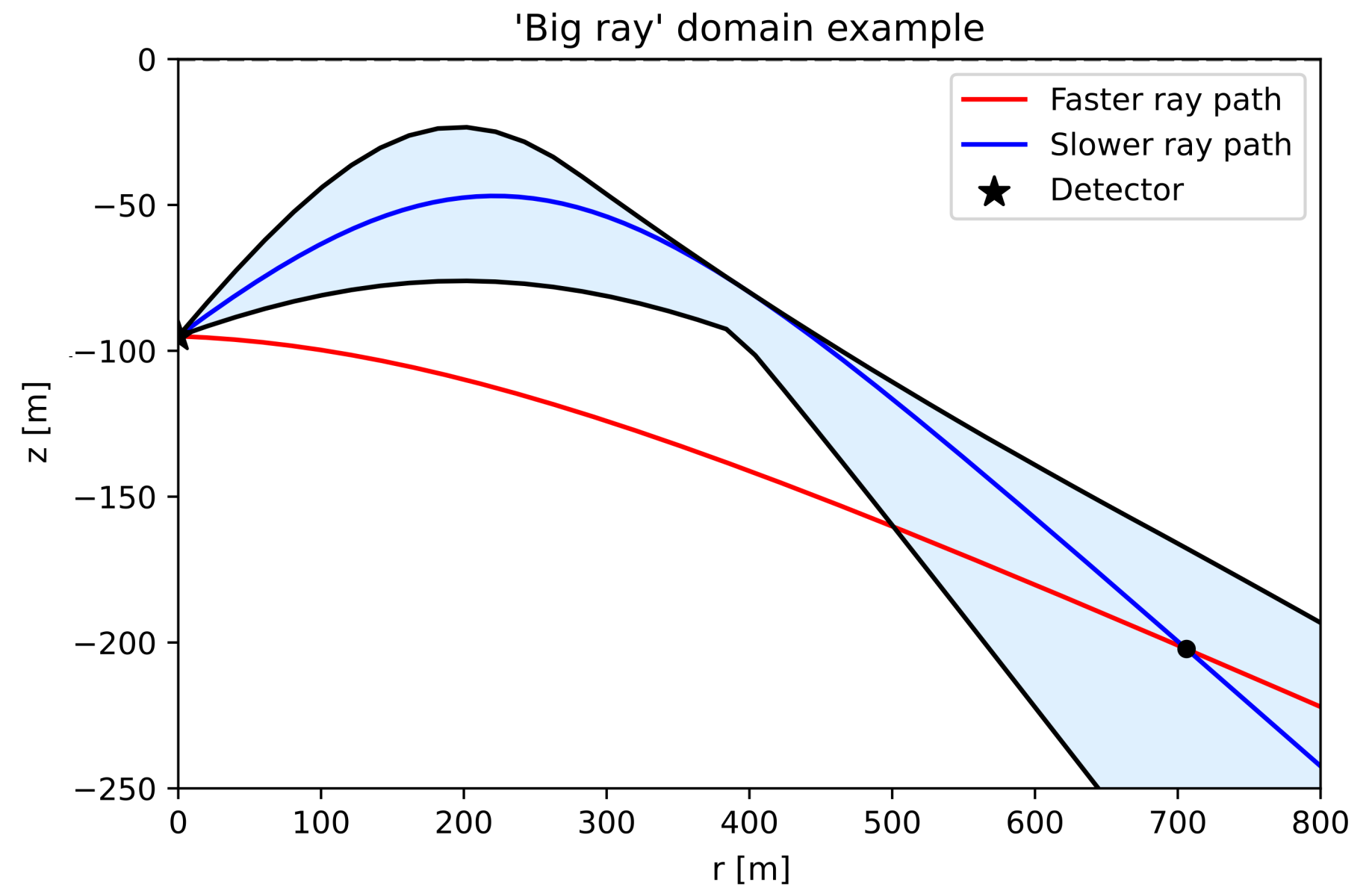
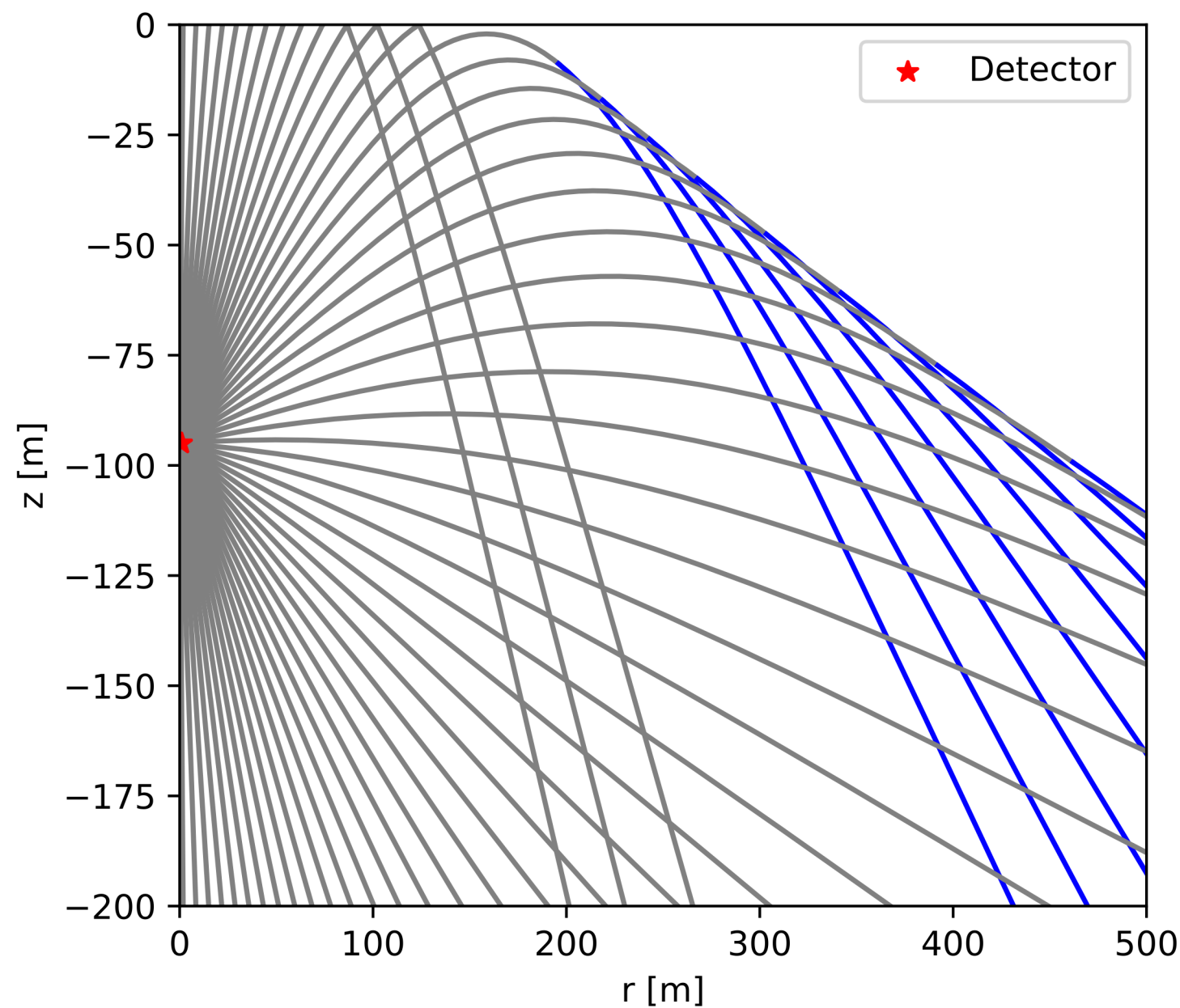


Example: Refracted rays turn over within the ice due to the decreasing refractive index.



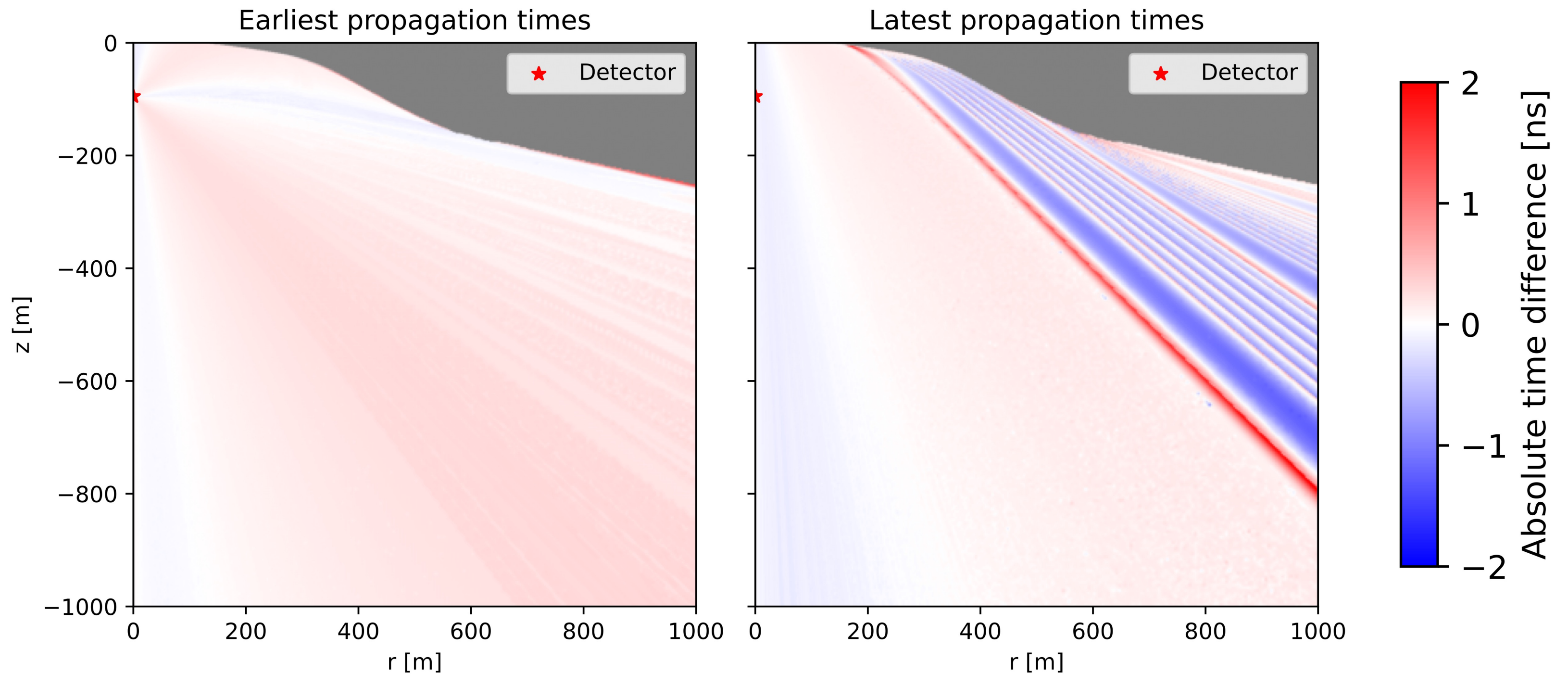
Red ray = global minimum, calculated by FMM
Blue ray = local minimum, missed by FMM

Example: Refracted rays turn over within the ice due to the decreasing refractive index.



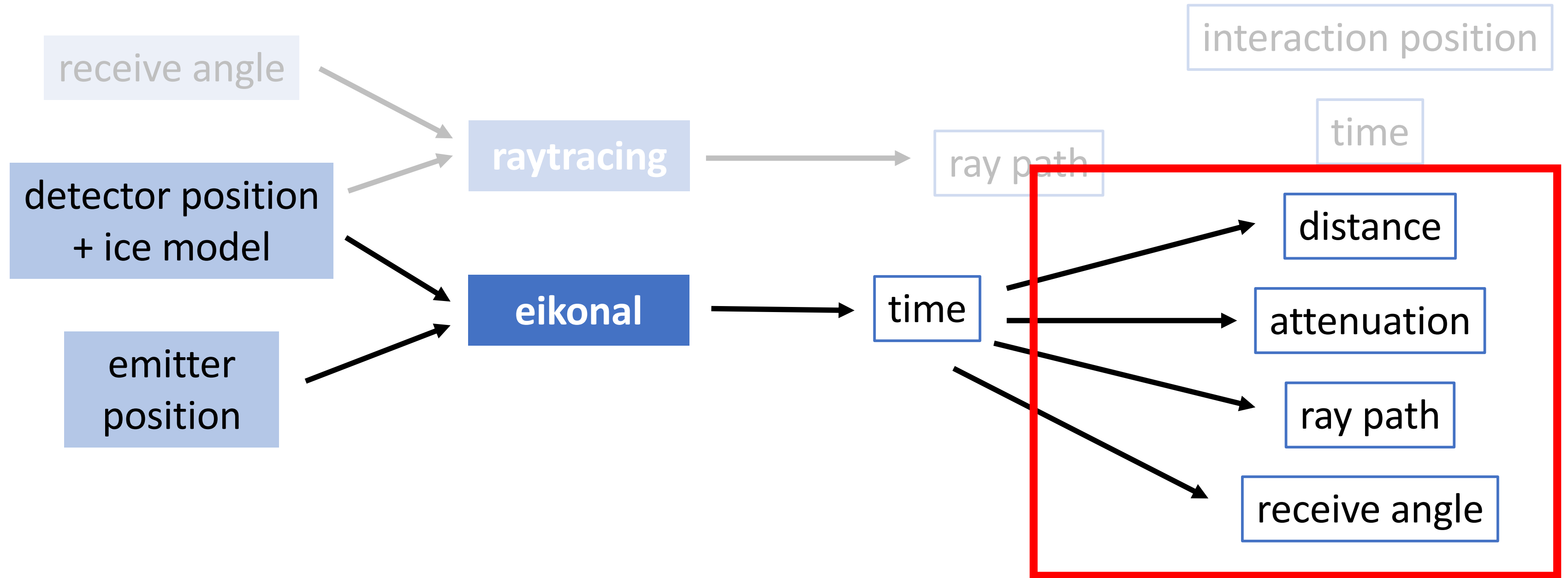
Global minimum outside domain – not calculated
Local minimum inside domain – calculated!

Big rays: Jean-David Benamou. In: *J. Comput. Phys* (1996).

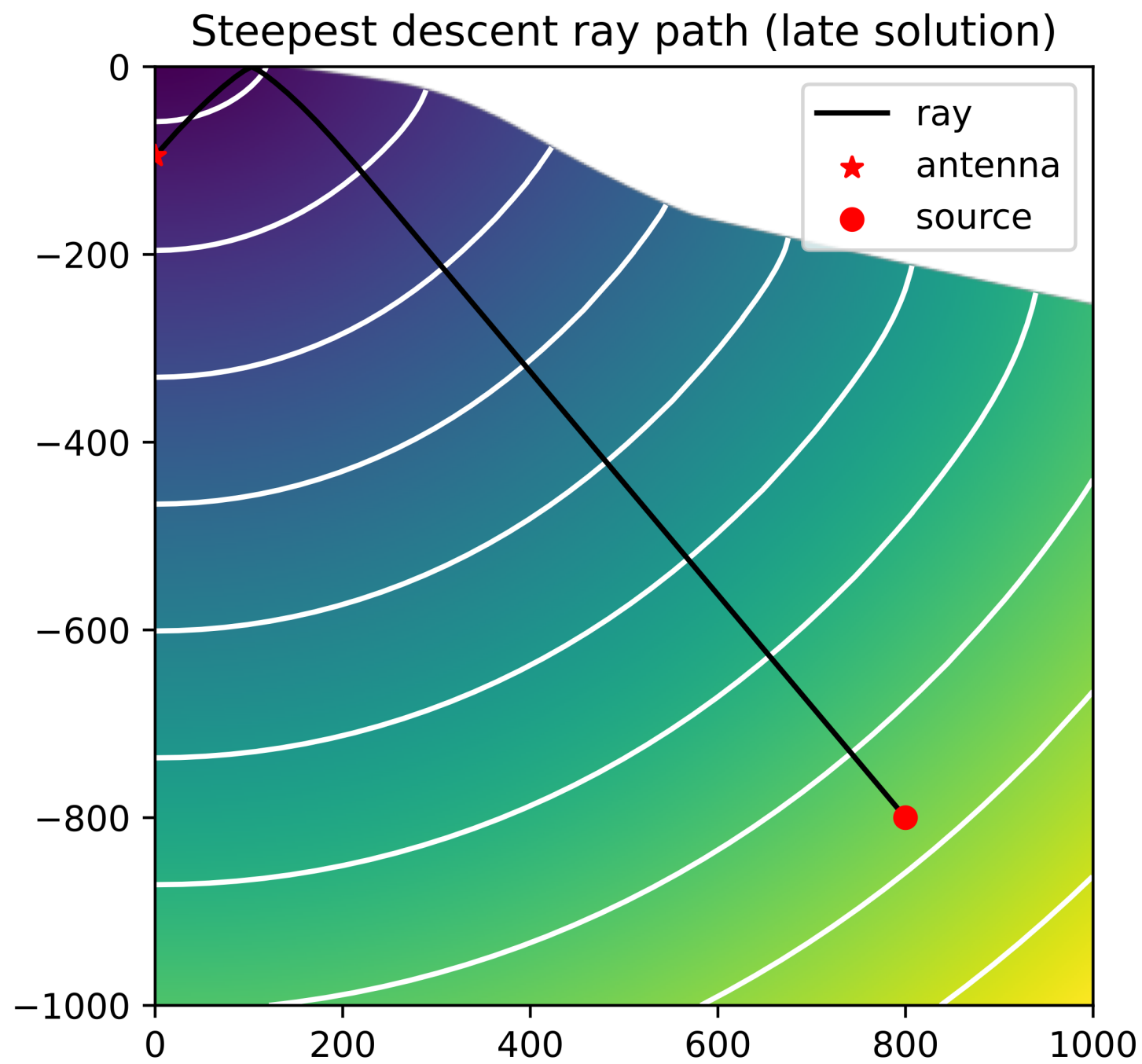


Check against raytracing method: <2 ns agreement,
significant speedup

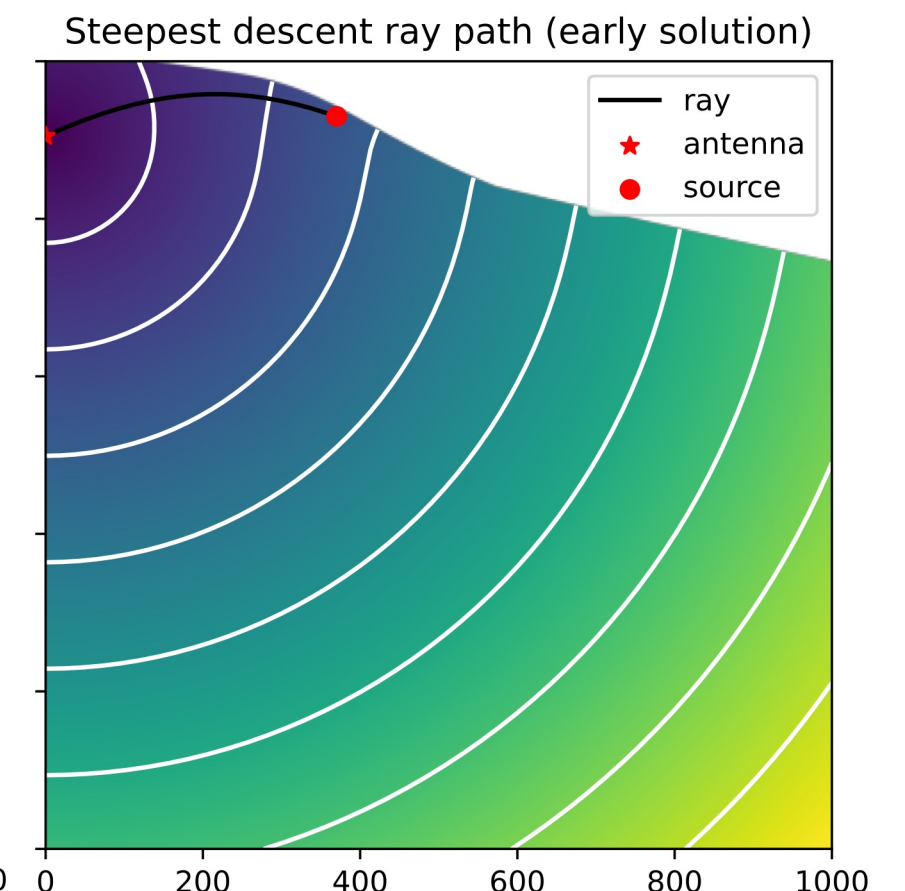
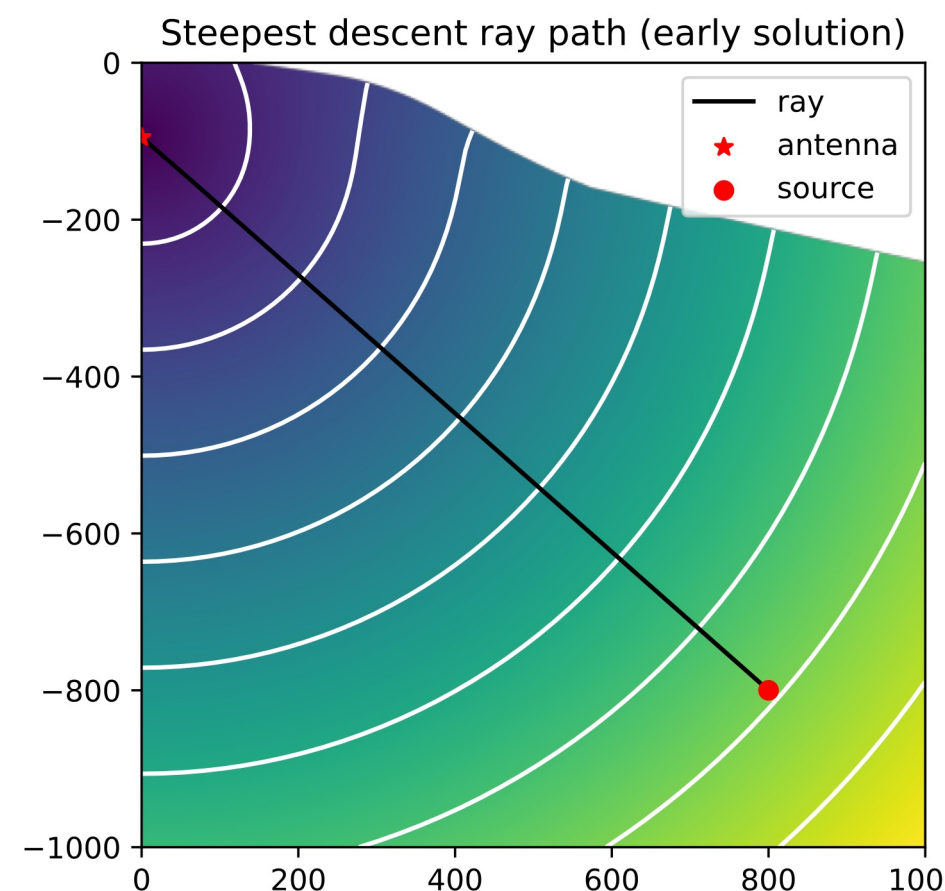
Closing the loop: calculating secondary quantities from travel-time maps



Closing the loop: calculating secondary quantities from travel-time maps



Gradient of travel-time map $\nabla T(\mathbf{x})$
= direction of a ray intersecting $\mathbf{x}$



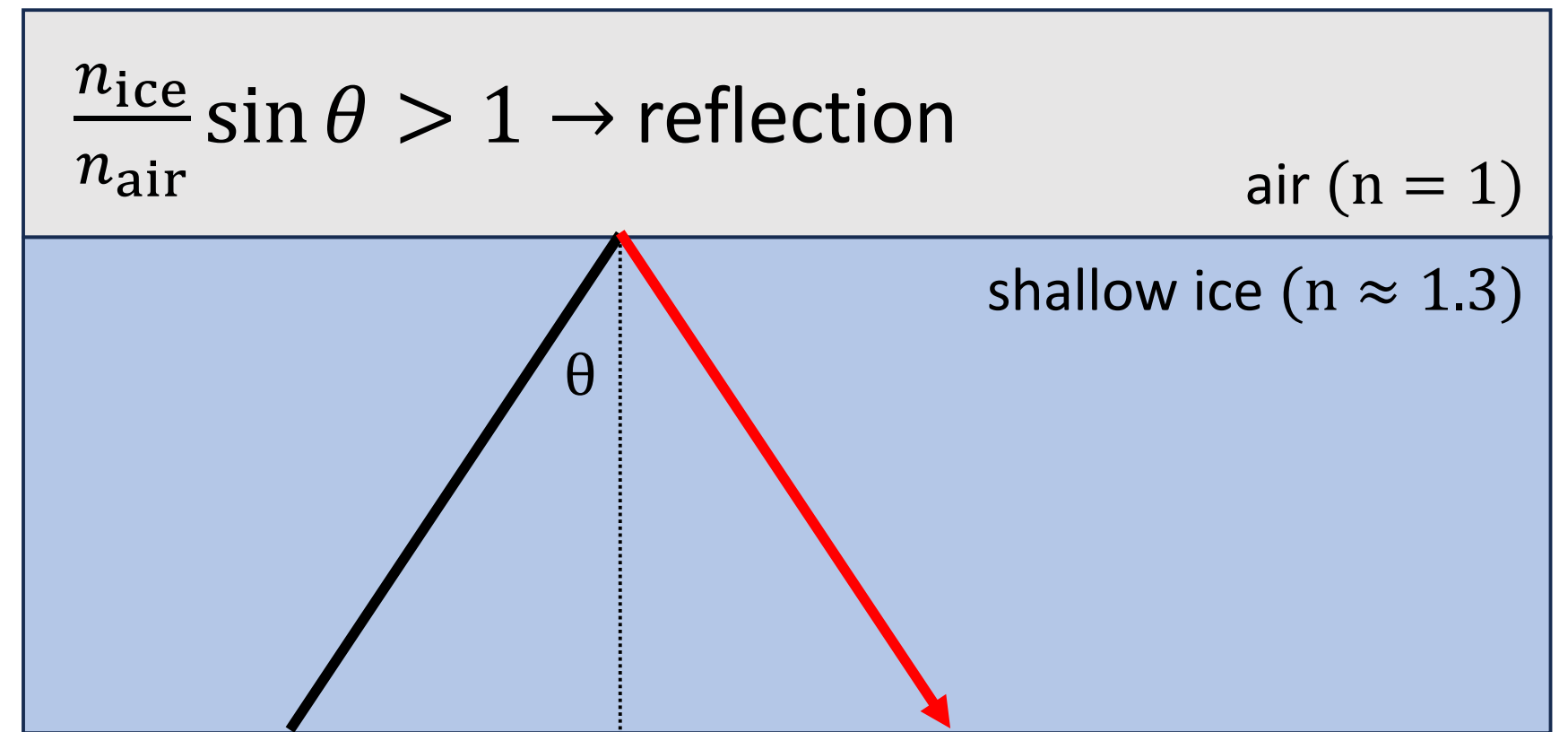
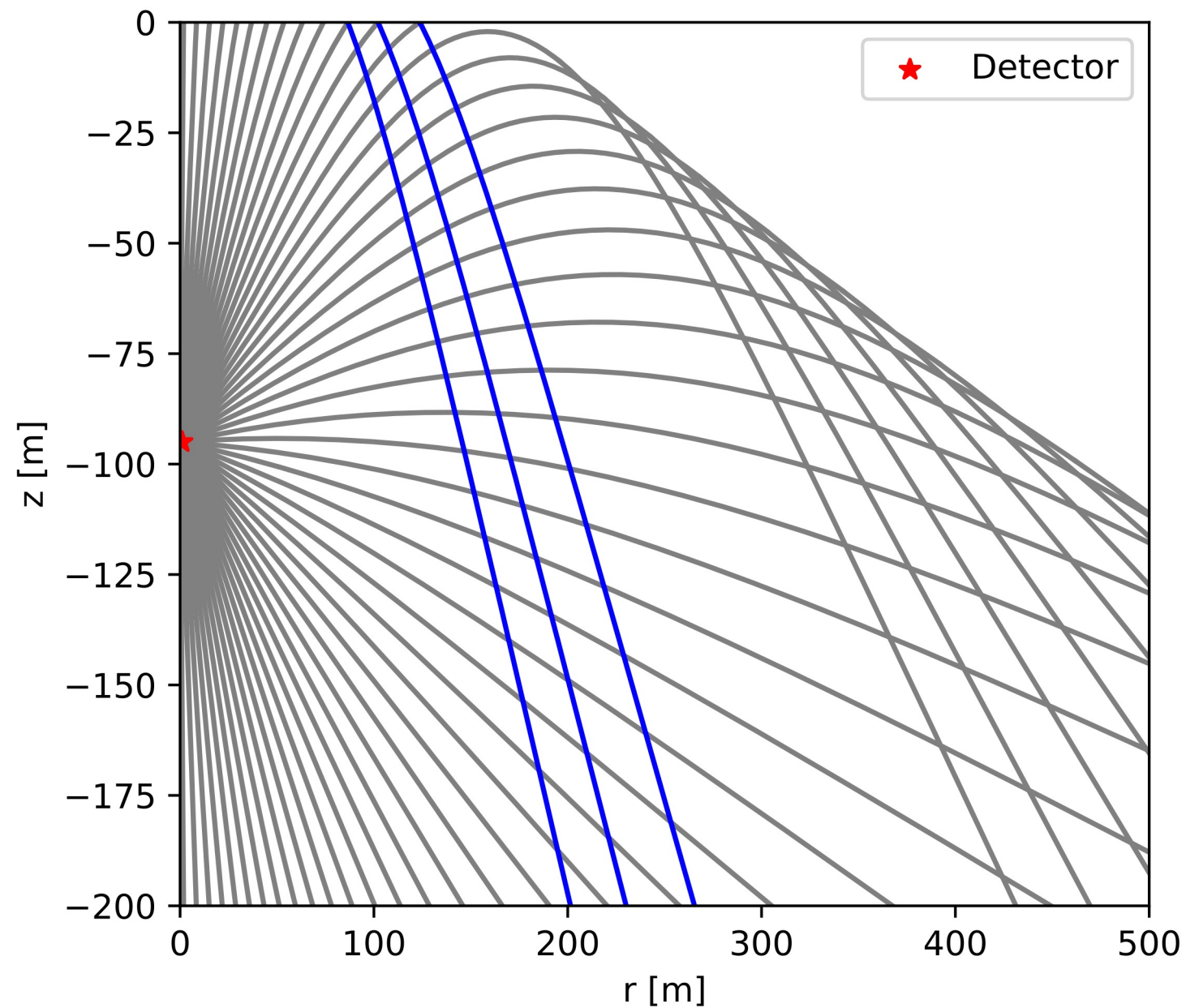
Summary

- As radio-based neutrino detection experiments grow in scale, computationally efficient methods for simulating signal propagation become more important.
- Raytracing is quick and useful as a first pass to visualize the solution set, but inefficient on large scales.
- Implemented the more efficient eikonal method with adaptations to radio detection

Code base: github.com/philippwindischhofer/Reconal/tree/big_rays

BACKUP

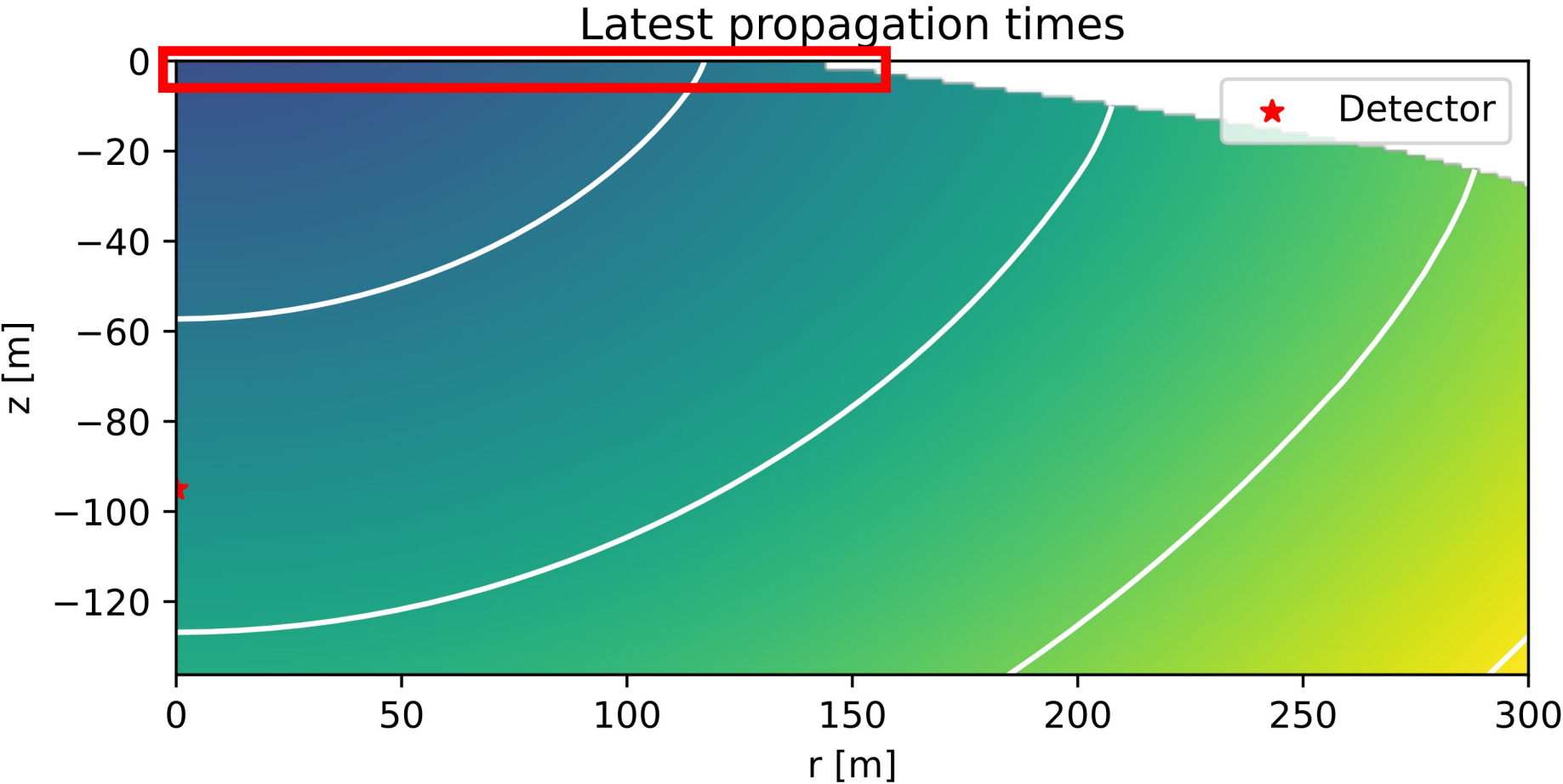
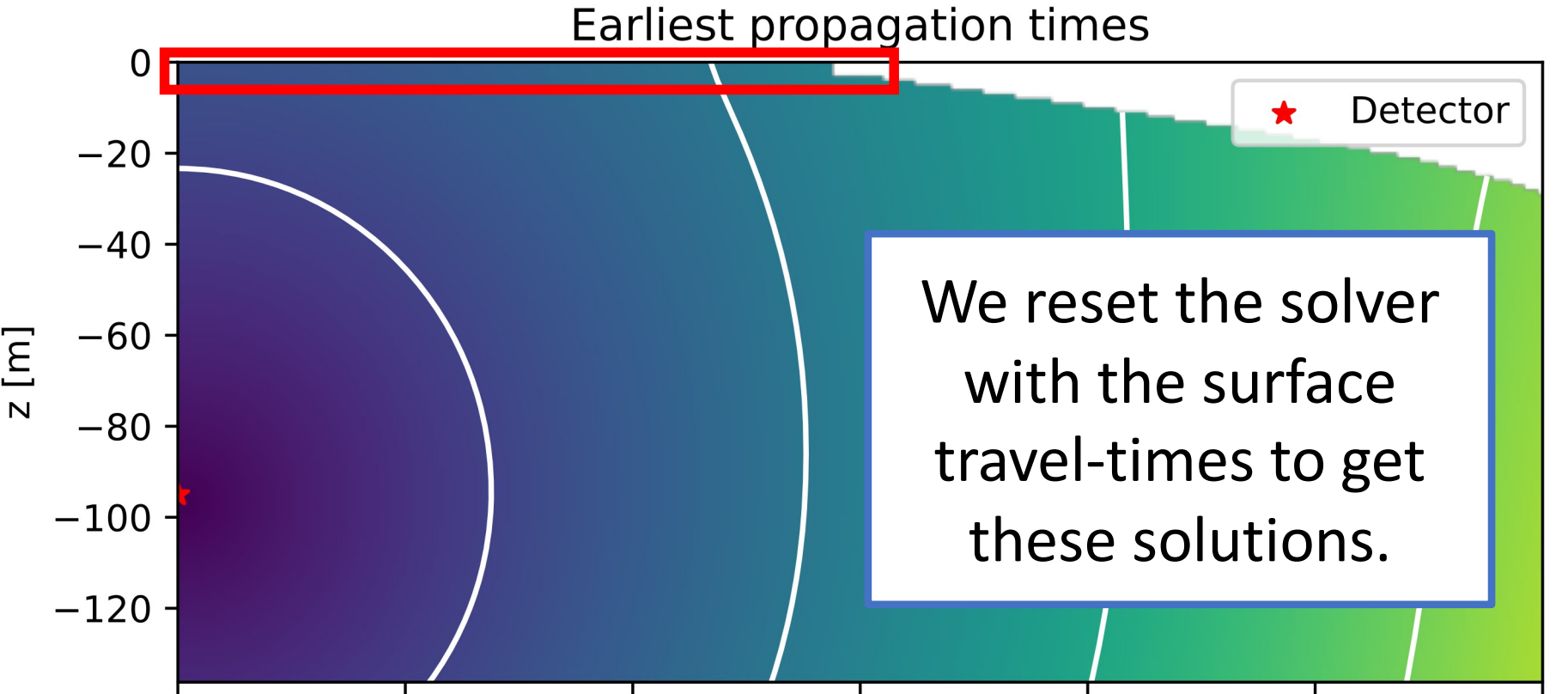
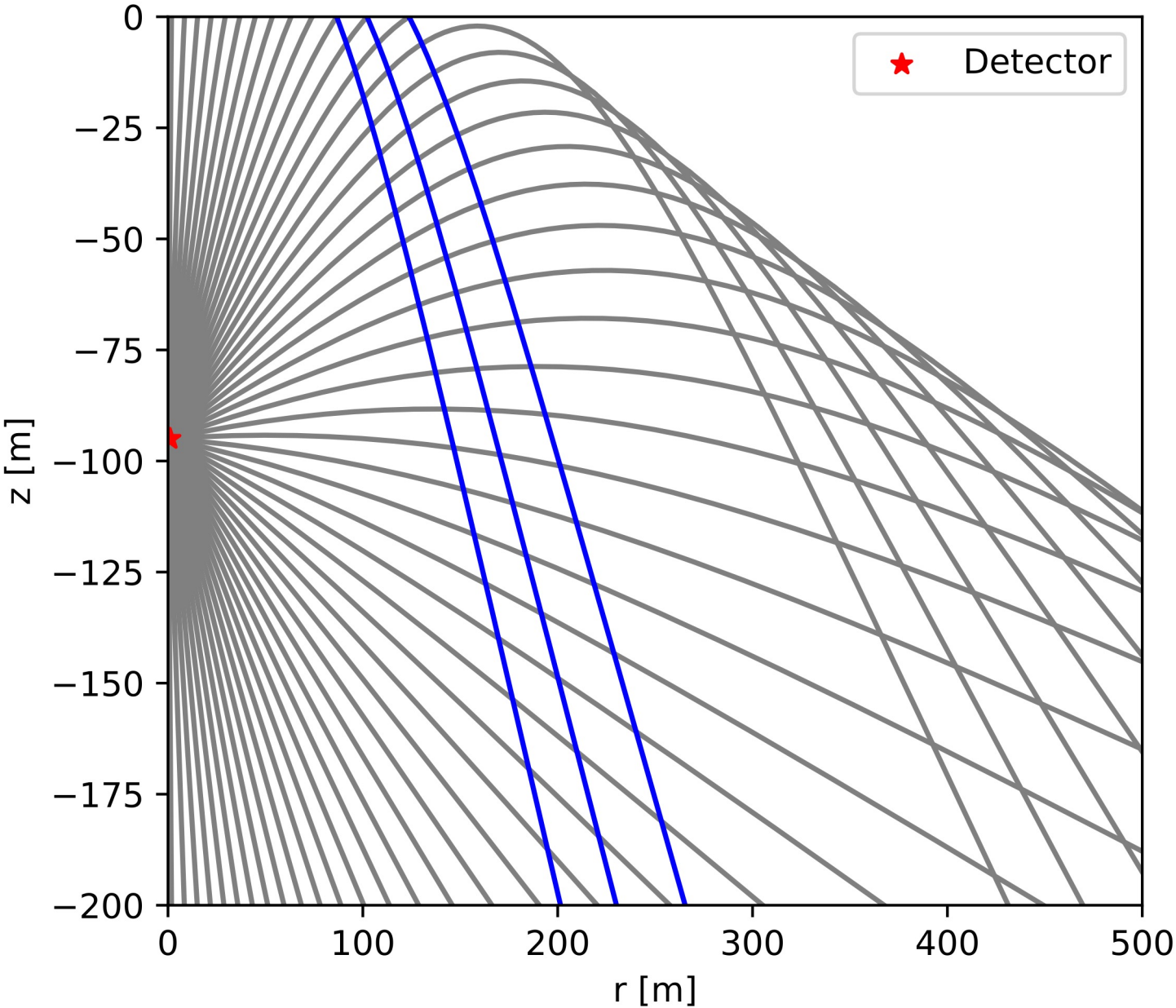
Some ray paths reflect off the air-ice discontinuity and propagate downward.



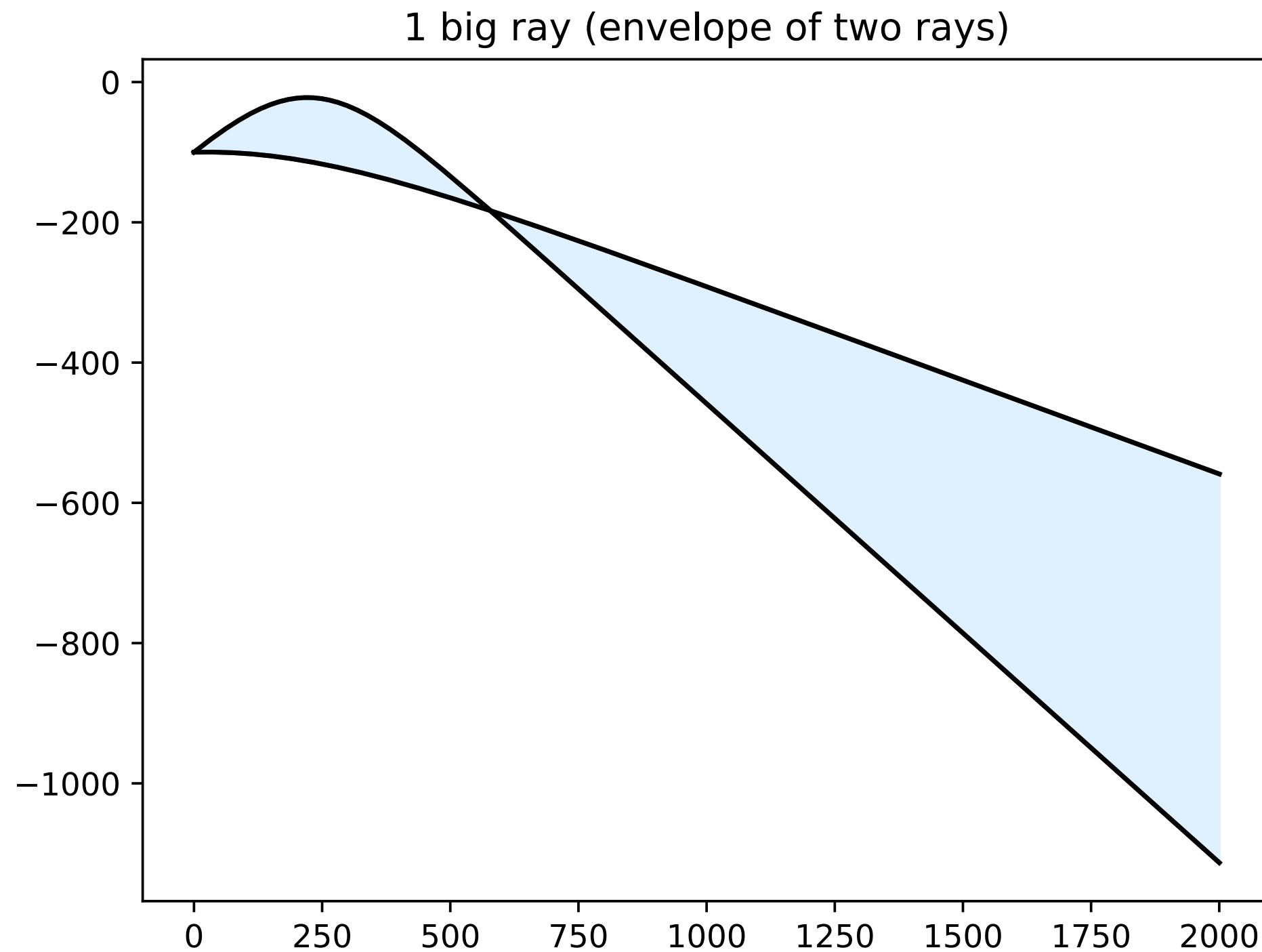
Pre-reflection: early travel-time solutions

Post-reflection: late travel-time solutions
— not calculated by FMM

Some ray paths reflect off the air-ice discontinuity and propagate downward.



Choosing the right big rays



Choosing the right big rays

