CRA 2023

May 16-19, 2023
Loyola University - Chicago

sixth Cosmic Ray Anisotropy Workshop

WRAPPING UP AND COLLECTING THOUGHTS
On Cosmic Ray Astrophysics

Pasquale Blasi - Gran Sasso Science Institute

Date: May 19 2023

GRAN SASSO SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

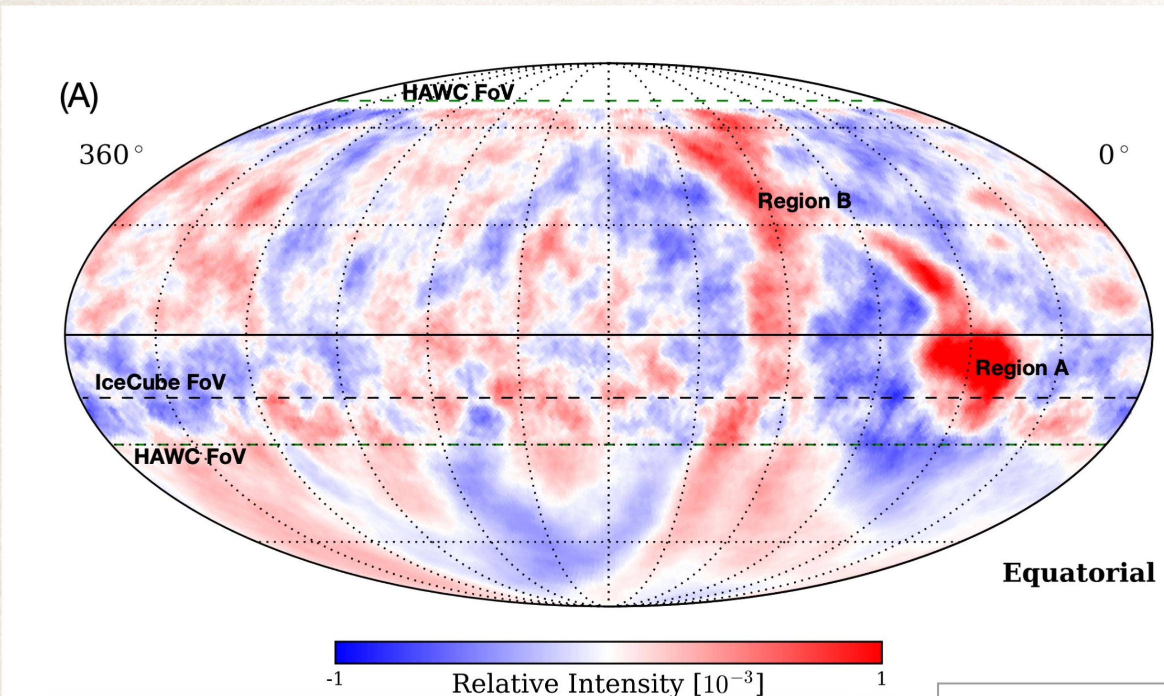
Once upon a time…

This workshop started in 2011 in the aftermath of the discovery of the small scale anisotropy in the high energy CR arrival directions

With time, the aim of the workshop broadened to include anisotropy at other energies and other phenomenological aspects of CR physics (transport, acceleration, …)

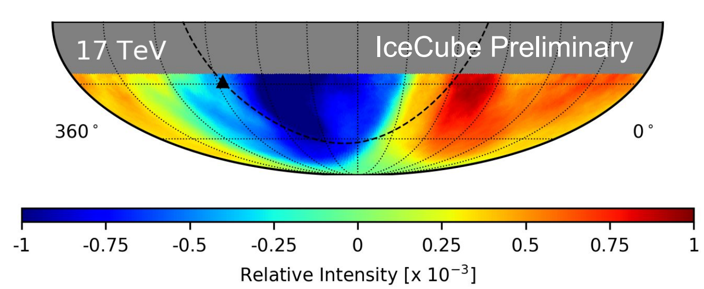
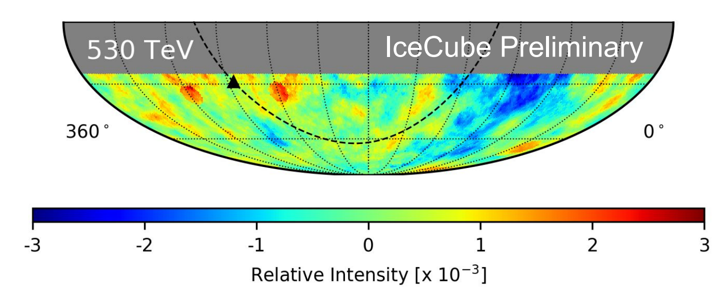
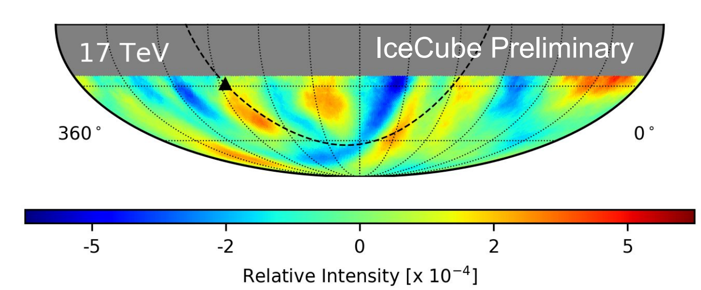
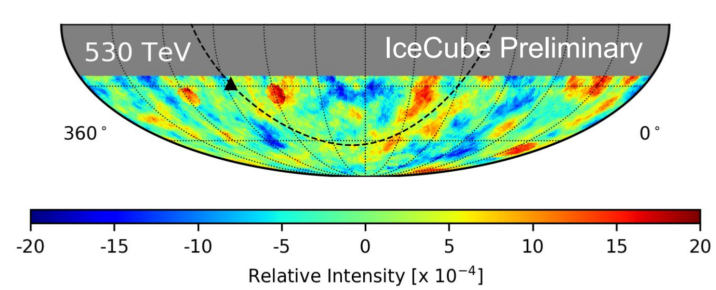
This happened mainly because the original phenomenon was kind of understood, at least in its statistical properties… though there may be several aspects that require further investigation, especially in terms of the connection with the helio-tail

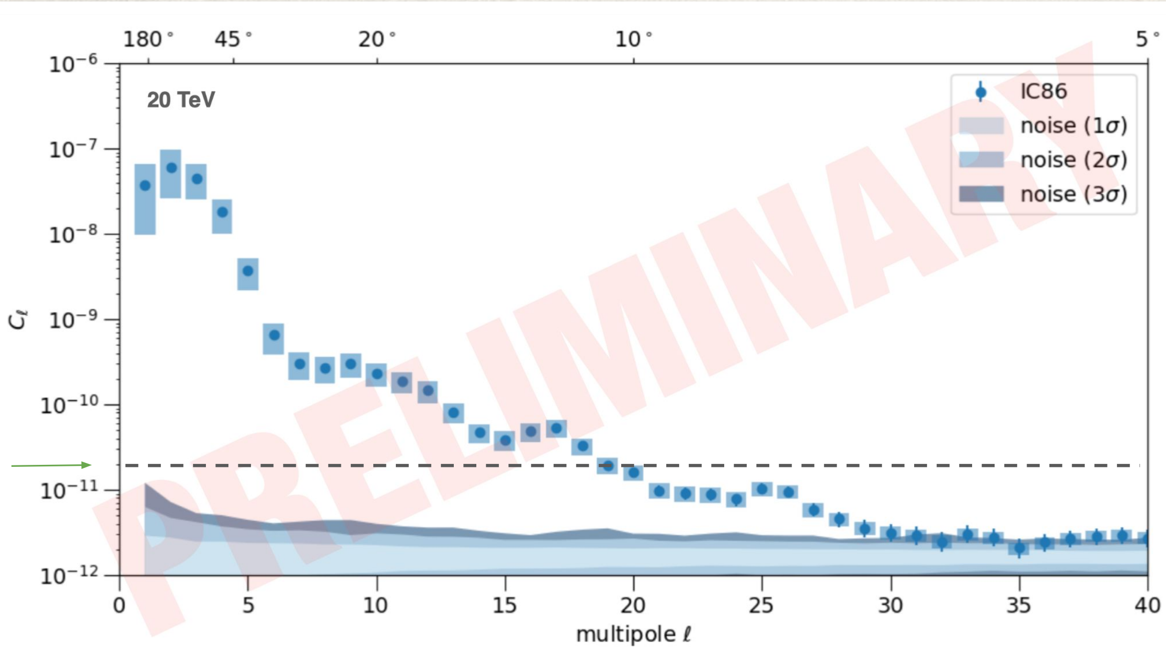
Dipole and Small Scale Anisotropy

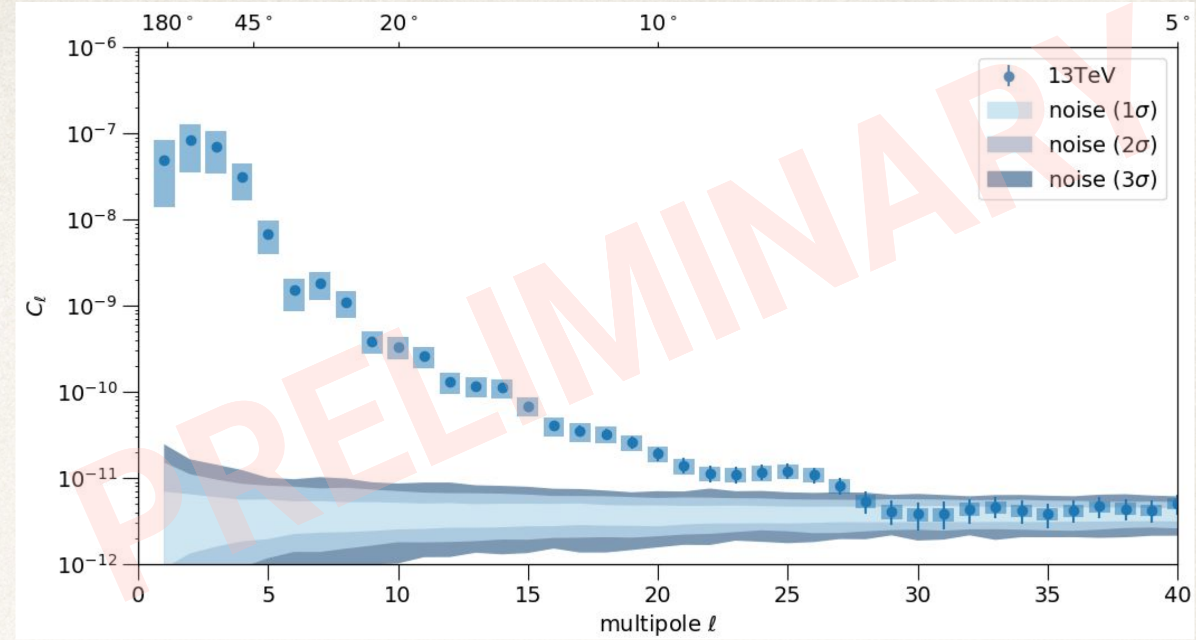


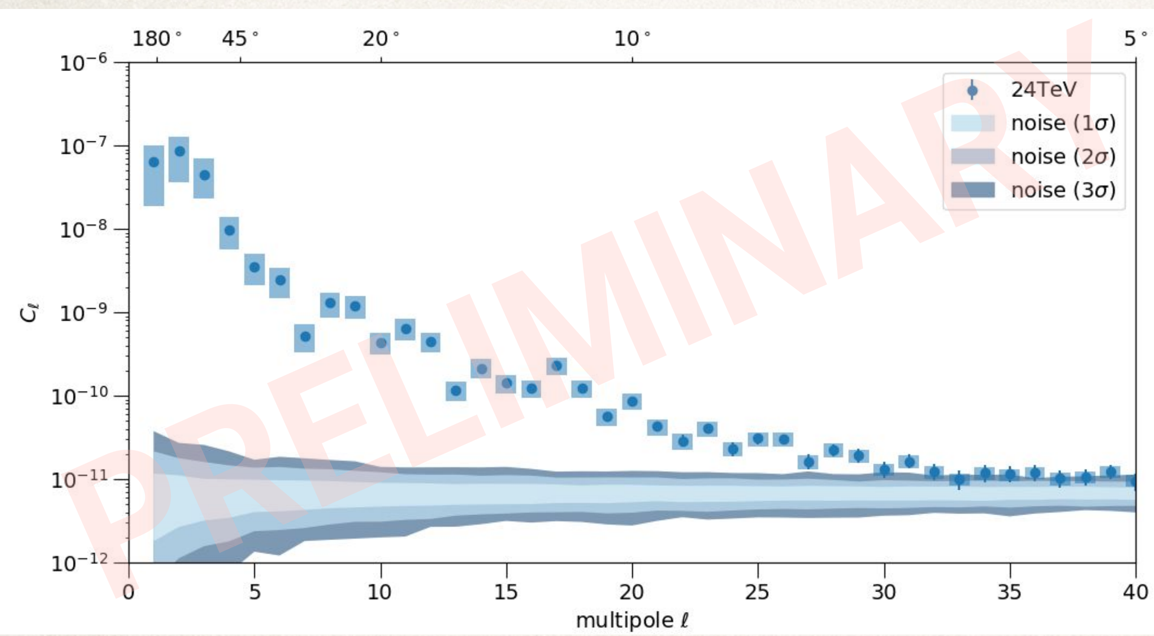
Many experiments joined in the search for the nature of anisotropies (IceCube, HAWC, IceTop, LHASSO …) to achieve full sky coverage and, equally important, extend the study to a wider range of energies

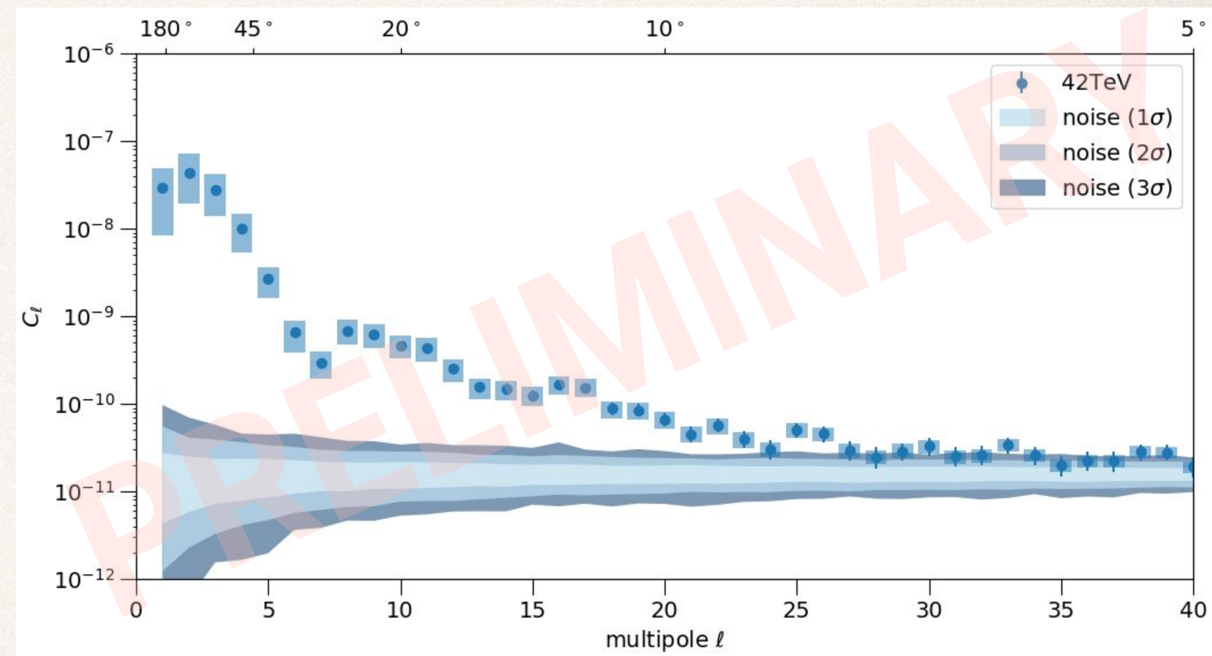
Talk by F. McNally

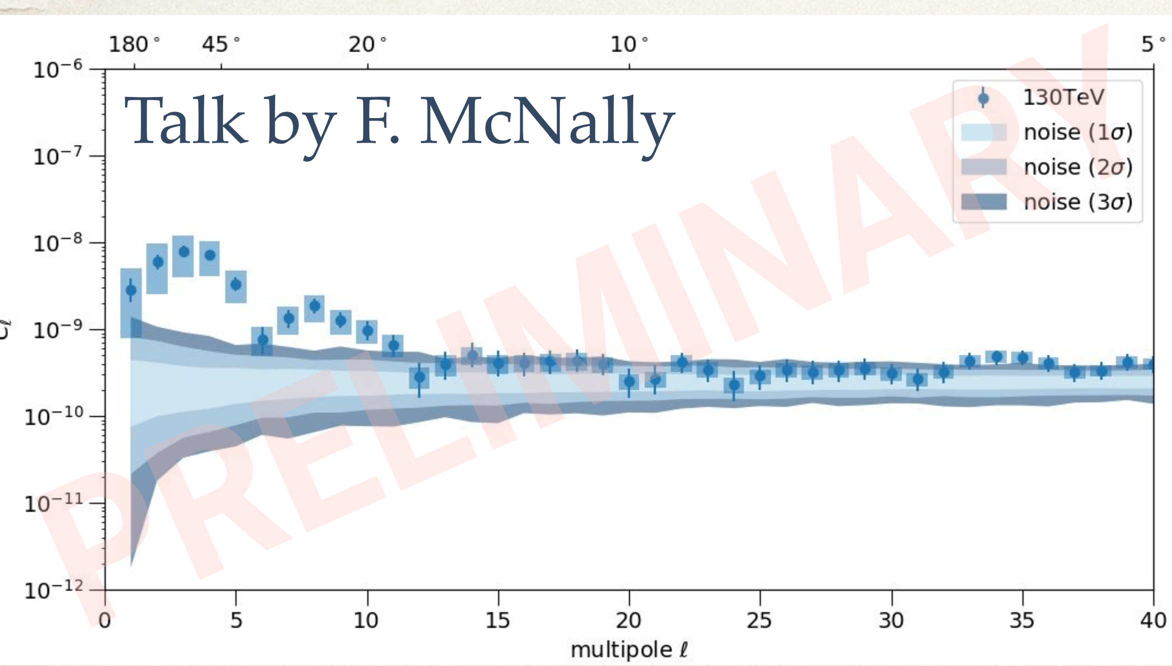
Relative Intensity	Low Energy	High Energy
Large-Scale Structure		
Small-Scale Structure		

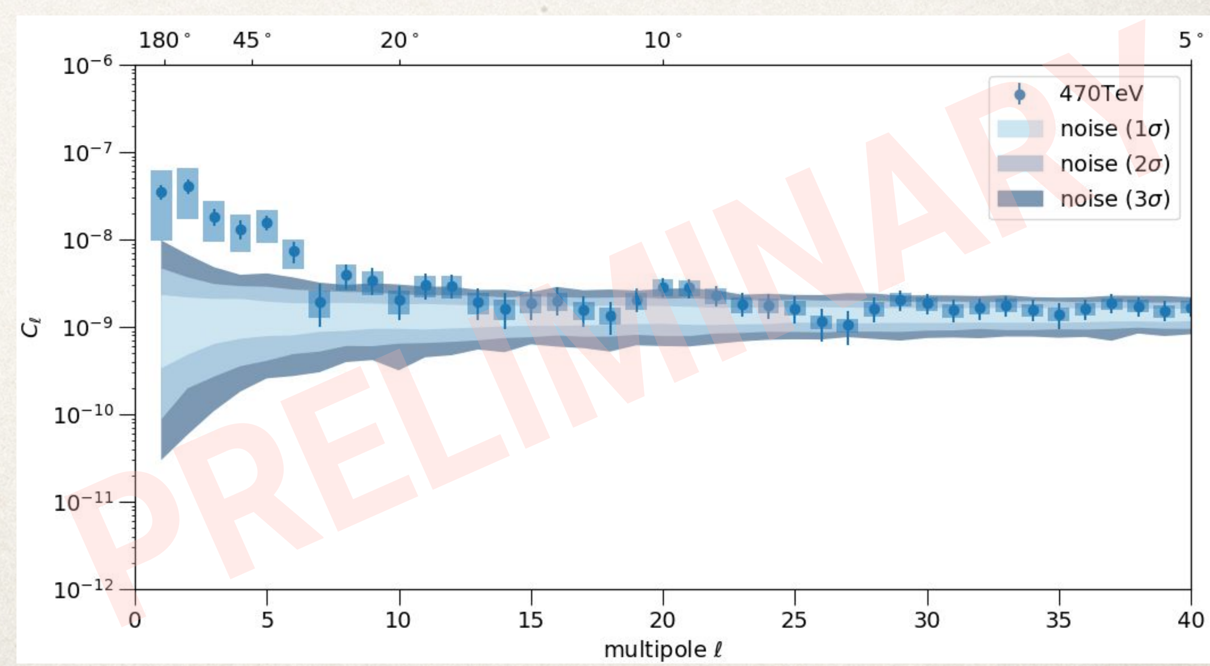

20 TeV
IC86
noise (1σ)
noise (2σ)
noise (3σ)
180° 45° 20° 10° 5°
C_ℓ
multipole ℓ
PRELIMINARY


13TeV
noise (1σ)
noise (2σ)
noise (3σ)
180° 45° 20° 10° 5°
C_ℓ
multipole ℓ
PRELIMINARY

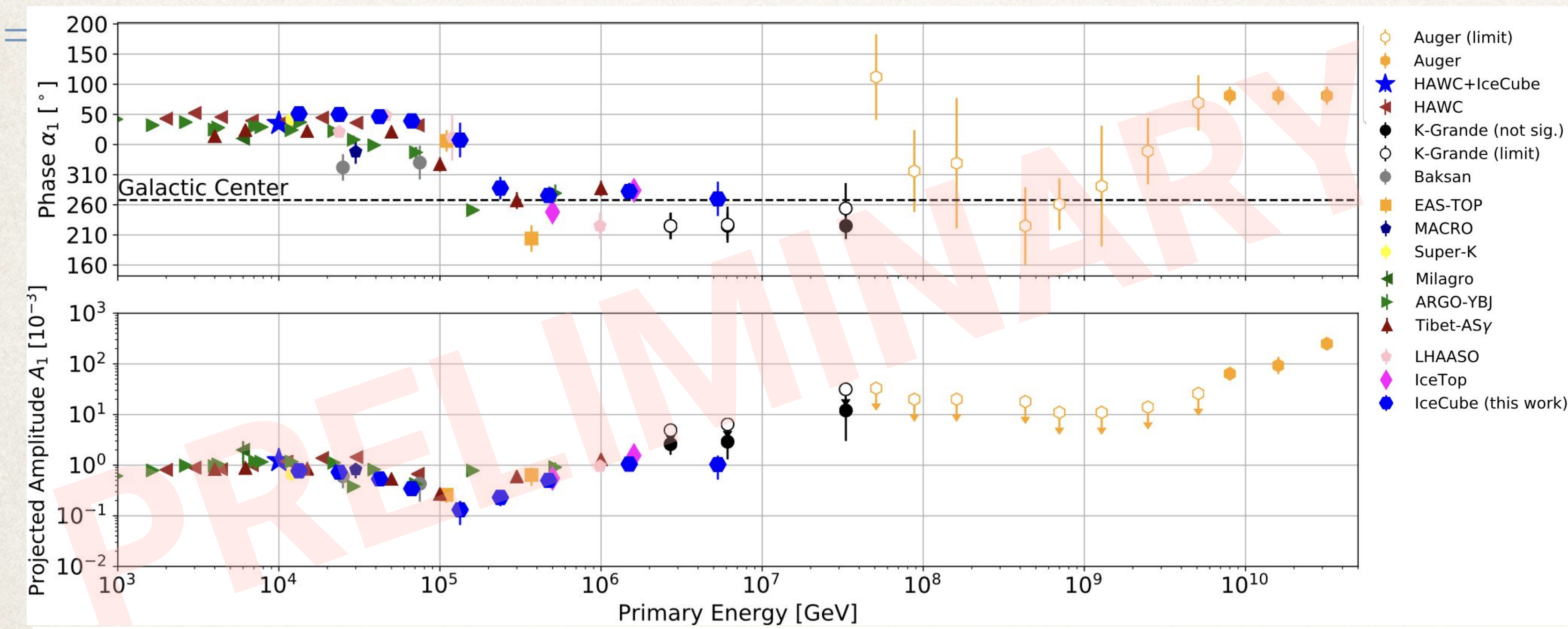

24TeV
noise (1σ)
noise (2σ)
noise (3σ)
180° 45° 20° 10° 5°
C_ℓ
multipole ℓ
PRELIMINARY


42TeV
noise (1σ)
noise (2σ)
noise (3σ)
180° 45° 20° 10° 5°
C_ℓ
multipole ℓ
PRELIMINARY


Talk by F. McNally
130TeV
noise (1σ)
noise (2σ)
noise (3σ)
180° 45° 20° 10° 5°
multipole ℓ
PRELIMINARY

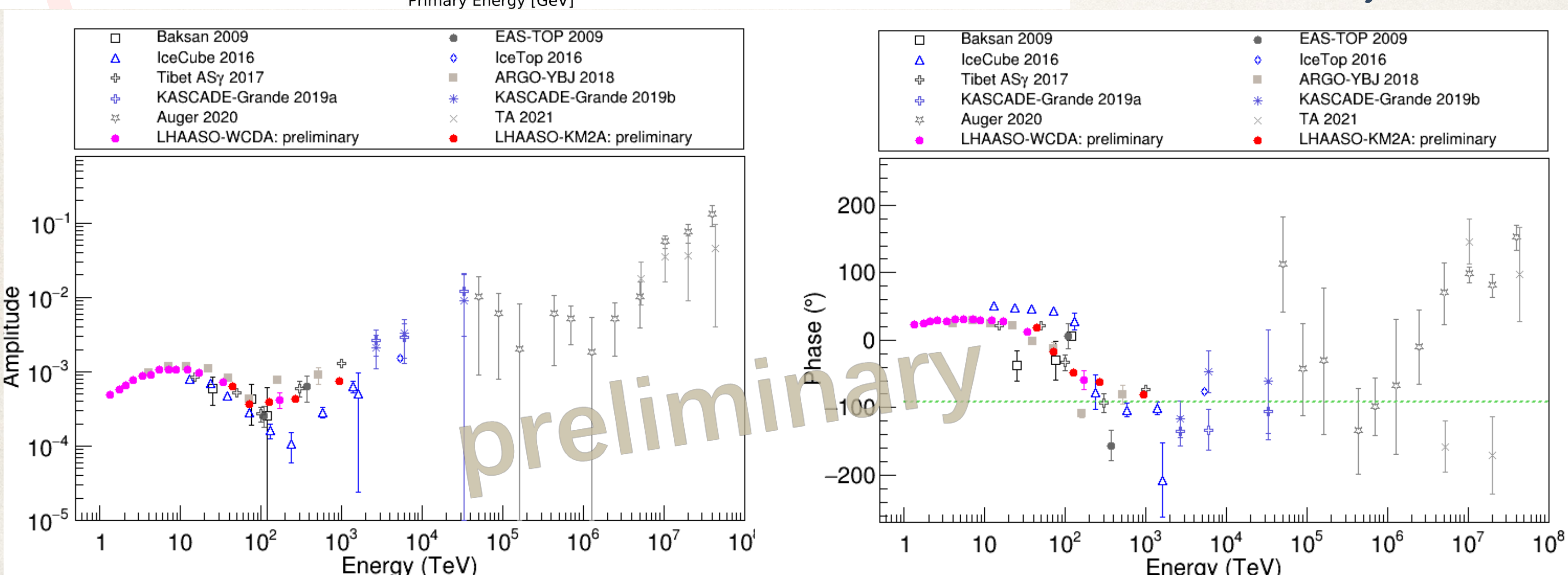

470TeV
noise (1σ)
noise (2σ)
noise (3σ)
180° 45° 20° 10° 5°
C_ℓ
multipole ℓ
PRELIMINARY

The Dipole



Talk by F. McNally

Talk by W. Gas



A few considerations about Galactic CR anisotropy

- The Dipole anisotropy is not very sensitive to the overall spatial distribution of the CR sources (PB&Amato 2012)
- The Dipole amplitude is dominated by the most recent and closest CR source (Lee 1979, Ptuskin+ 2006, PB&Amato 2012)
- The Dipole phase reflects the projection of the global dipole (due to the closest source) on the direction of the local magnetic field—in other words: do not look for the source in the direction of the phase (Alhers & Mertsch 2015)
- Small Scale Anisotropies are a byproduct of the propagation of CRs in the last mile (Giacinti & Sigl 2012, Alhers & Mertsch 2015)

Fundamental physics of CR transport

The advection-diffusion equation that we all love and use is actually an approximated version of a more fundamental equation, the Vlasov equation:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \dot{\mathbf{r}} \cdot \nabla_{\mathbf{r}} f + \frac{q\mathbf{v}}{c} \times (\langle \mathbf{B} \rangle + \delta \mathbf{B}) \cdot \nabla_{\mathbf{p}} f$$

From the Vlasov equation you get the diffusion equation for <f> if you make the ansatz that

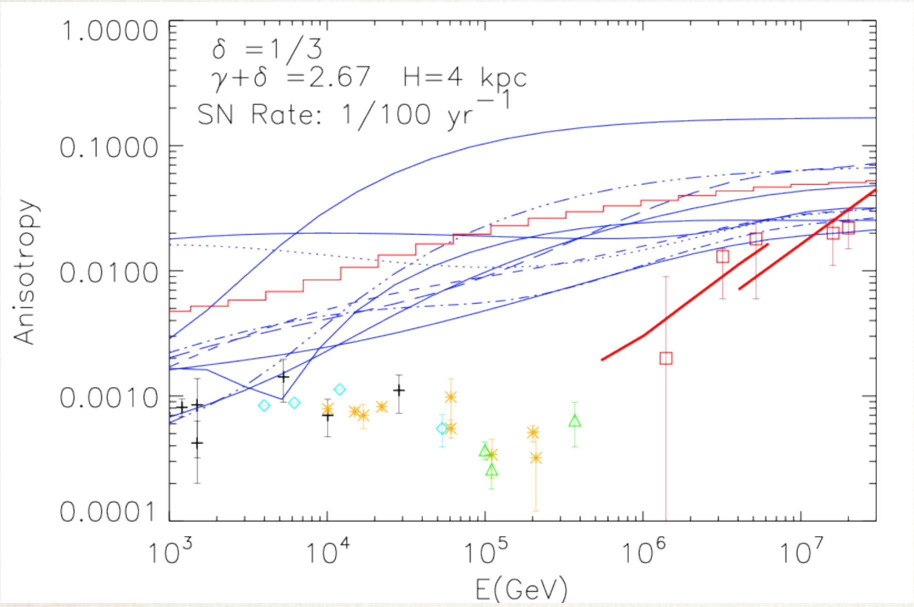
$$\langle f \rangle (p, \mu, t) = g(p, t) + h(p, \mu, t)$$

and you take only the first term of h expanded in Legendre polynomials (namely the dipole term)

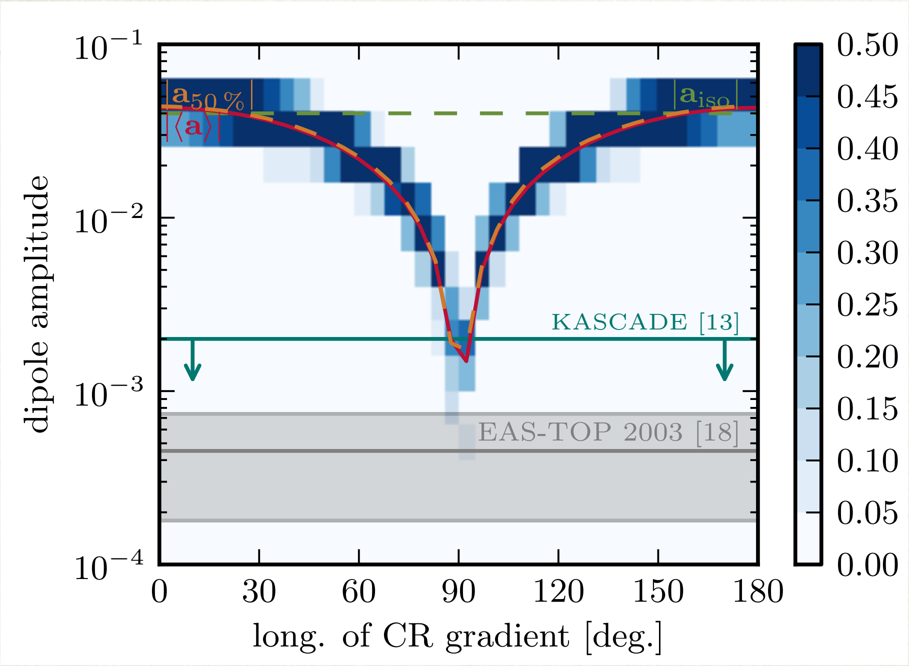
The dipole term on the other hand only depends on the gradient of <f>, so that once you know the <f> (from the diffusion equation), you know the dipole term…

You can continue this game and calculate the higher terms as well **(INTERMEDIATE AND SMALL SCALE ANISOTROPIES!)**

Dipole Anisotropy



PB&Amato 2012

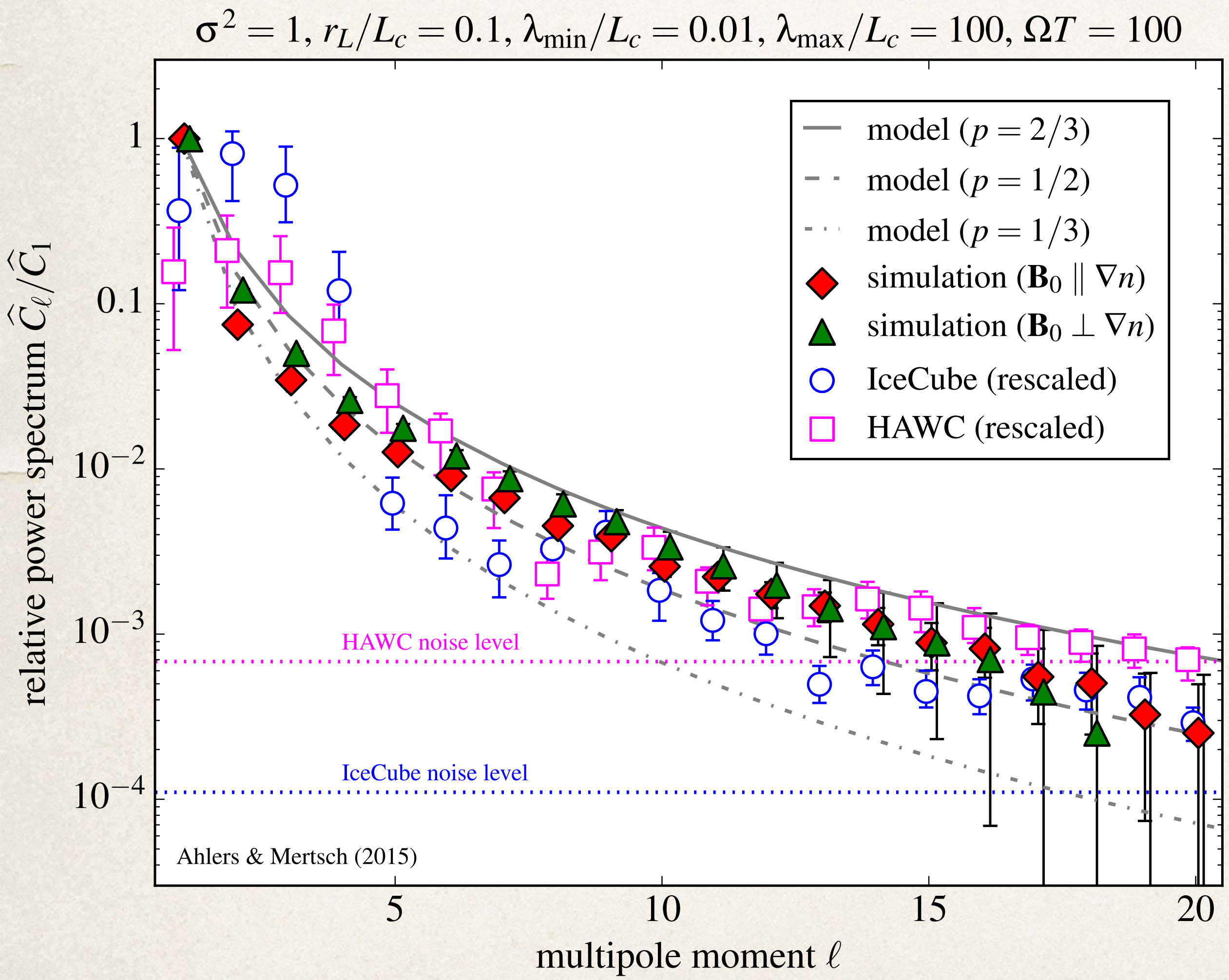


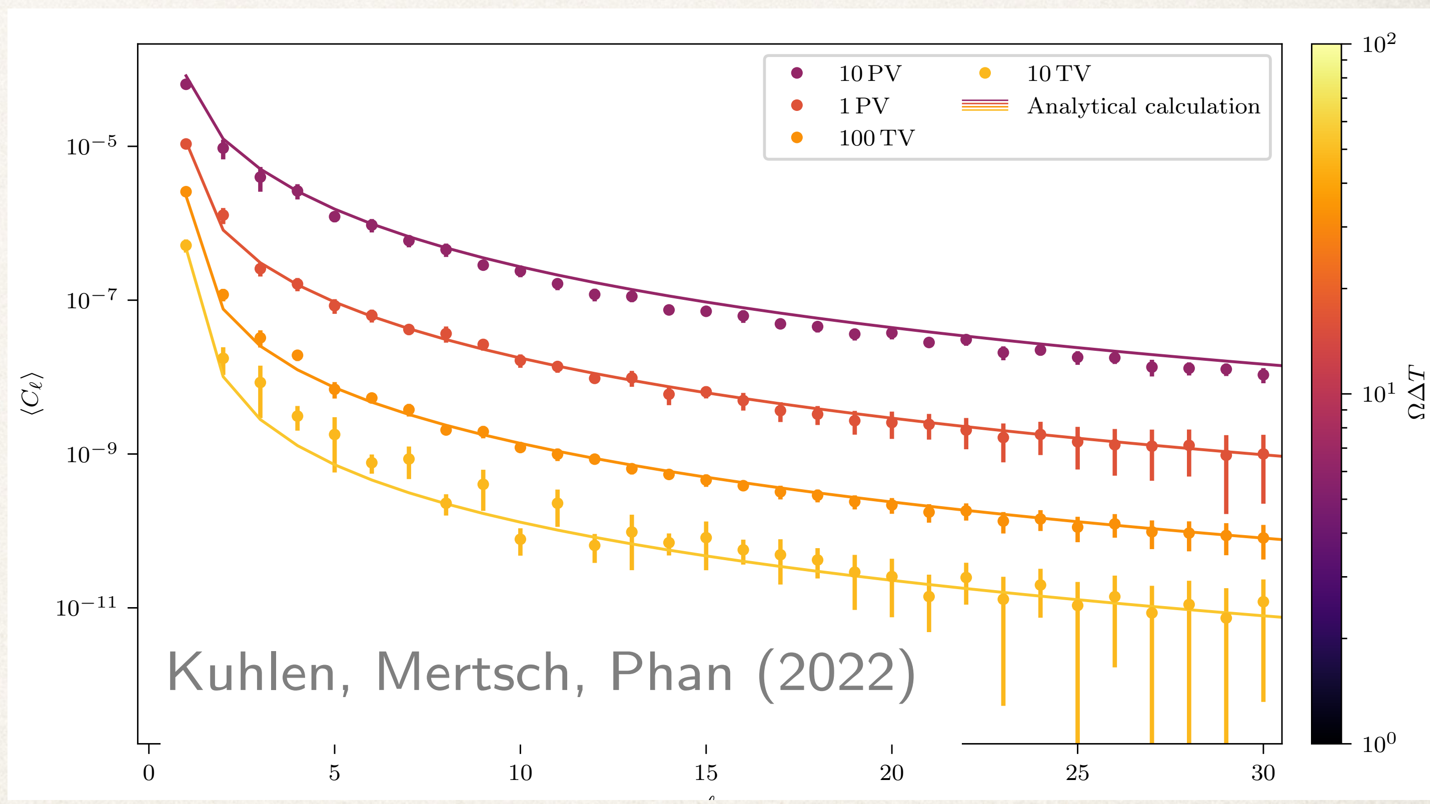
Mertsch & Funk 2014

- Although the mean anisotropy amplitude in the diffusion model is well defined, the fluctuations are divergent! In other words, the observed anisotropy depends on the specific realisation of sources and it is dominated by the closest and most recent source!
- Notice that this implies that it can depend erratically upon energy
- Even more interesting, you expect the phase to suffer sudden changes at energies where a source leaves room to another source…
- Finally, it was pointed out that the observed phase depends on the projection of the dipole on the direction of the local B field.
- At very high energies (path length larger than correlation length) it is less so…

Small Scale Anisotropies

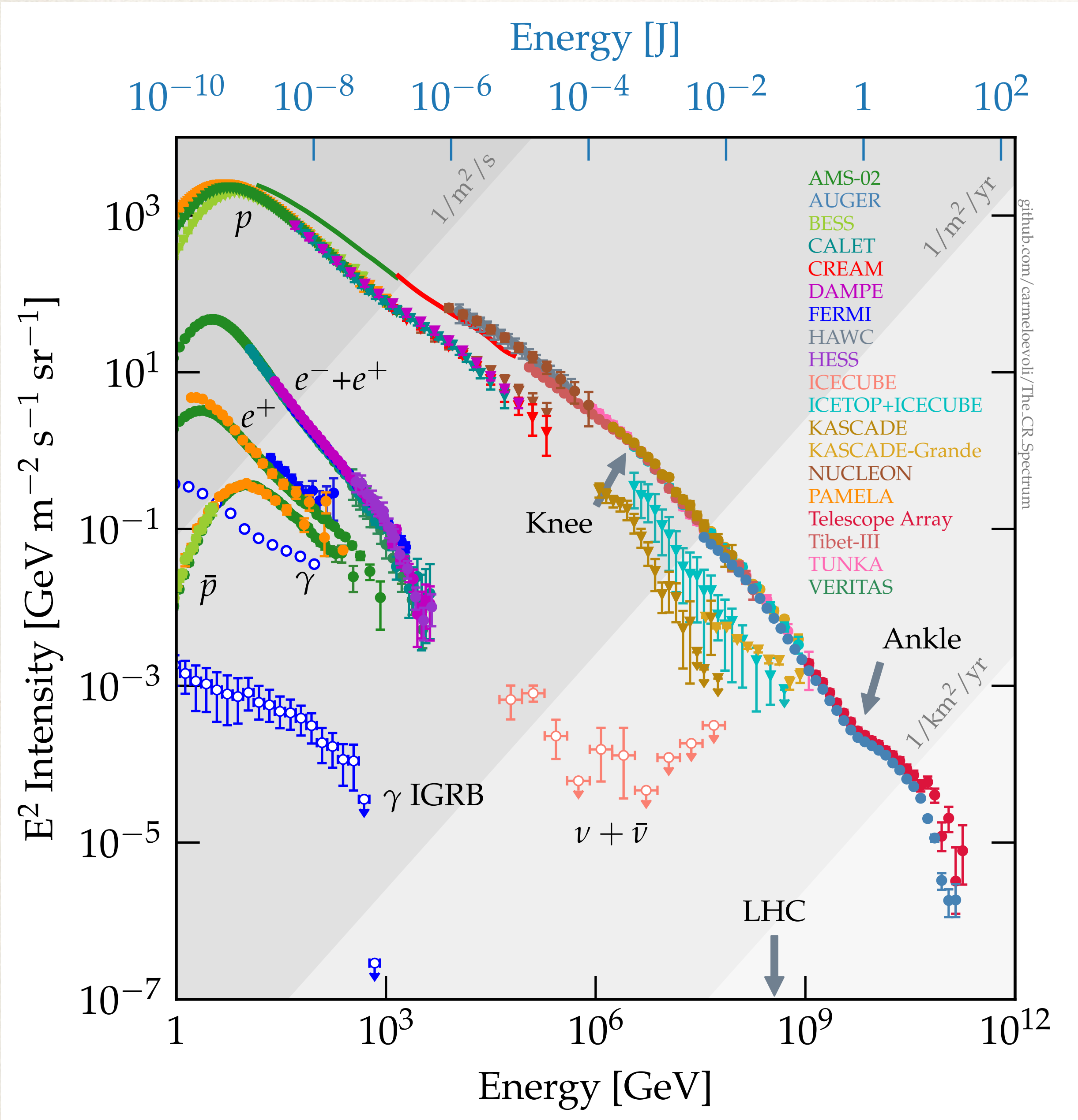
If the flux arriving at about one path length away from us has a dipole anisotropy, then the flux we get at Earth is also anisotropic on smaller scales





Alhers & Mertsch 2015

Spectra…



Talk by C. Evoli

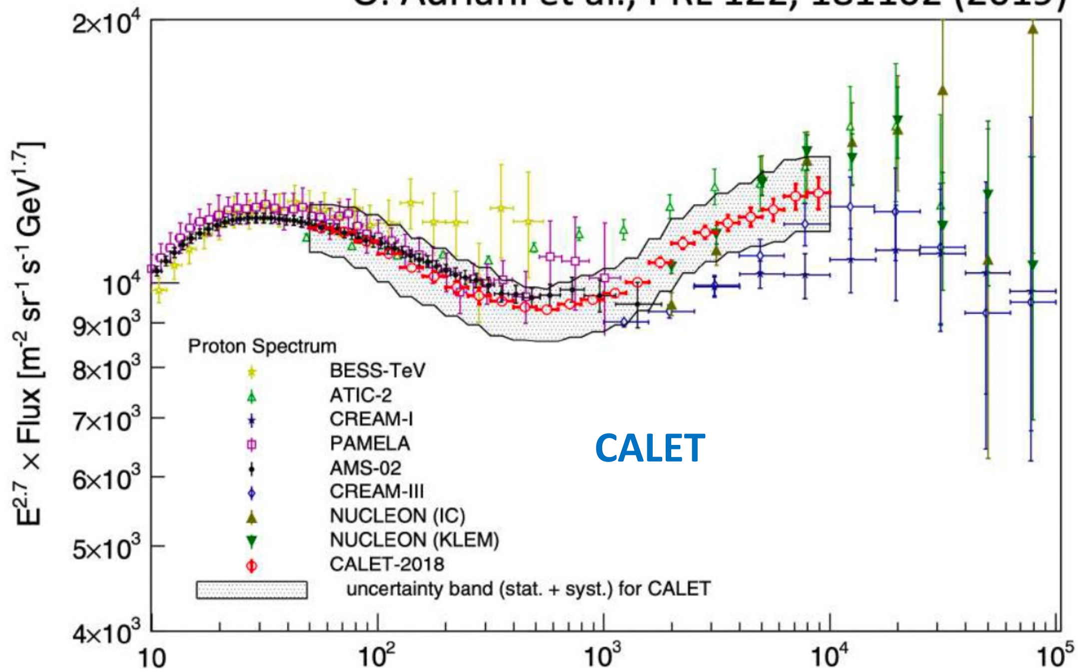
Several speakers have stressed how the situation that measurements are revealing is at odds with the standard model of CR origin

But the theoretical aspects of that model are very simple while this field develops in a very data driven way — it is obvious that while data get better, we understand more of the fine details of the standard model - that is why we are carrying out measurements

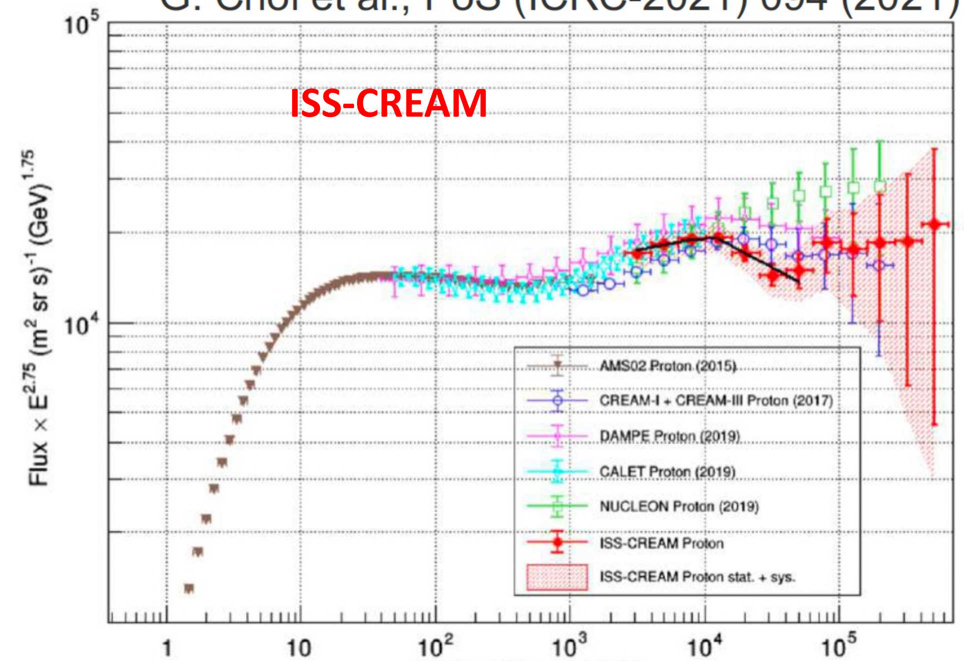
Especially important: power laws do not contain scales — it is only when we see deviations (breaks) that we identify scales (remember the knee?)

SPECTRAL BREAKS: CONSISTENT APPEARANCE OF FEATURES IN THE SPECTRA

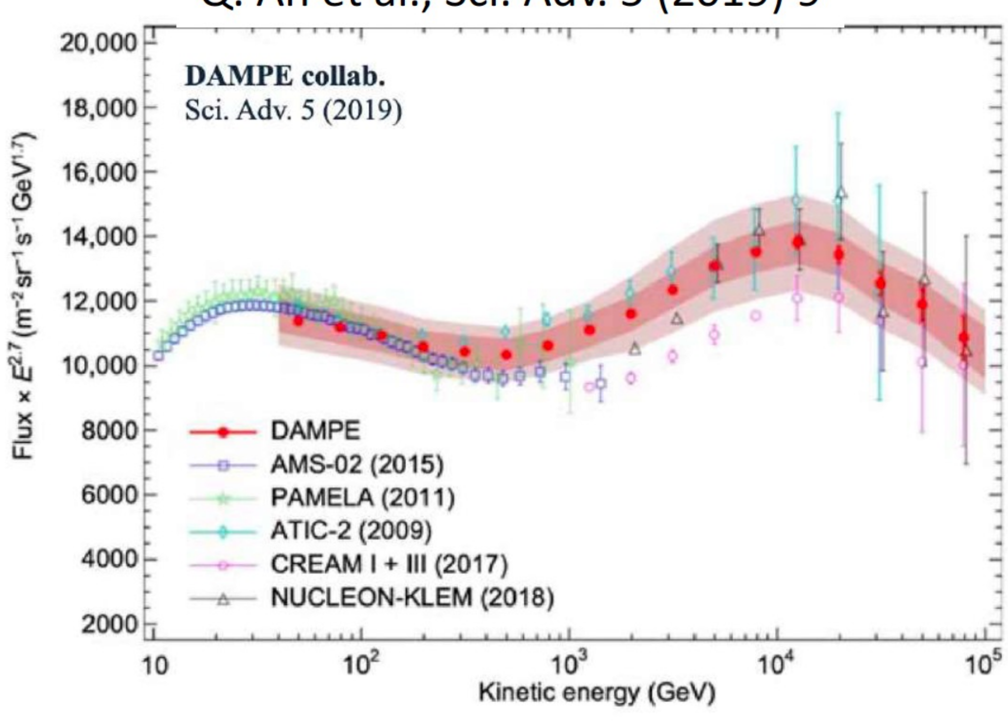
O. Adriani et al., PRL 122, 181102 (2019)



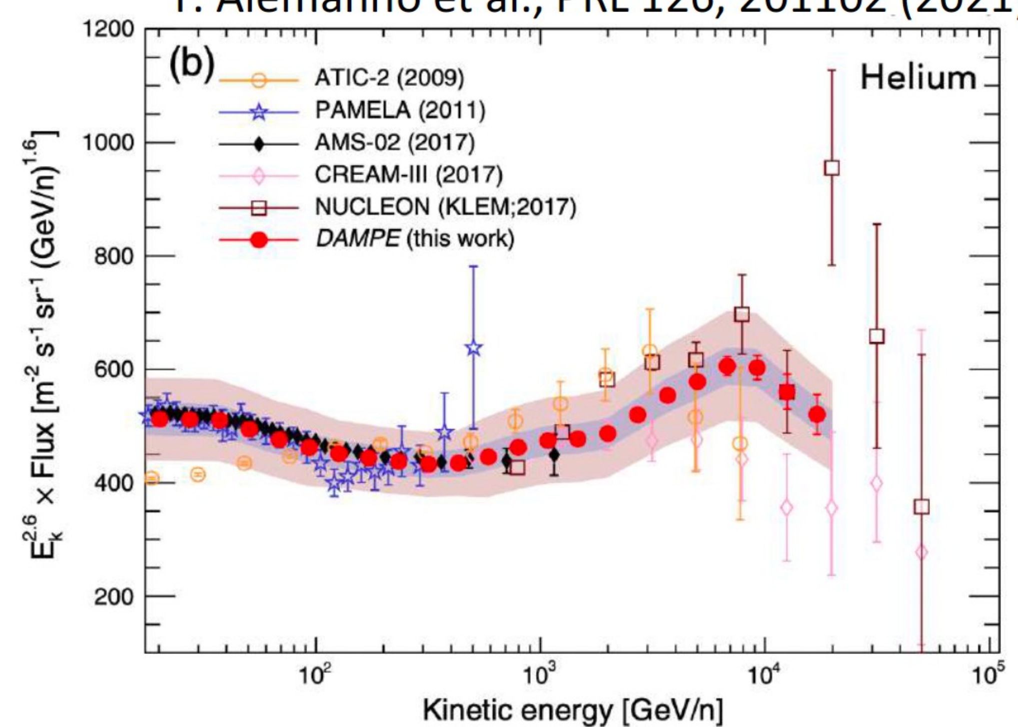
G. Choi et al., PoS (ICRC-2021) 094 (2021)



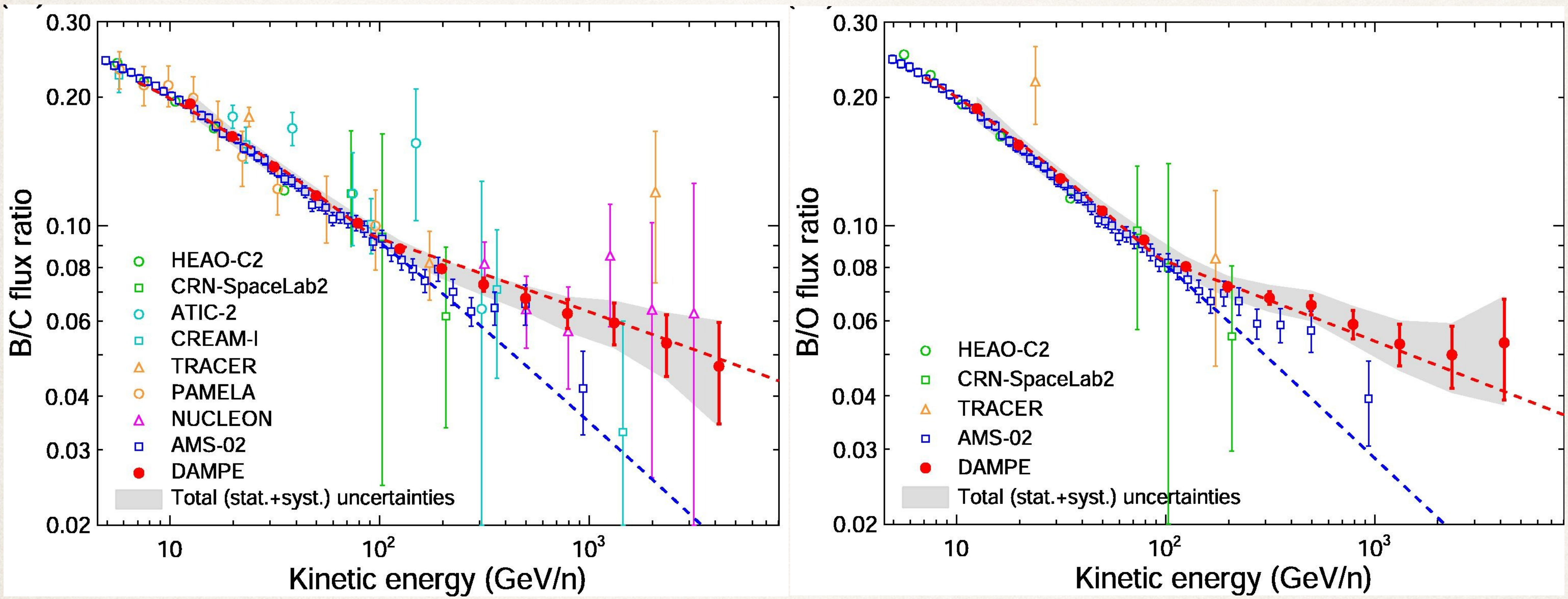
Q. An et al., Sci. Adv. 5 (2019) 9



F. Alemanno et al., PRL 126, 201102 (2021)



Secondary/Primary ratios



Talk by B. Schroer

The Spectral Break

At 300 GV all species we measure show a change of slope…

We know that this phenomenon is also present in the secondary/primary ratios, hence this feature is intrinsic in the way particle diffuse in the Galaxy

DUE TO THE TRANSITION FROM A SELF-GENERATED TURBULENCE TO A PRE-EXISTING TURBULENCE (PB+2012, …)

NON TRIVIAL SPATIAL DEPENDENCE OF D(E,Z) ON THE HEIGHT UPON THE DISC (Tomassetti 2012, …)

This boils down to understanding why CRs scatter in the Galaxy

Turbulence

At low energies (<300 GV) we can count on self-generation for particle scattering: it is generated at all scales hence it does not suffer of the pathologies of MHD turbulence

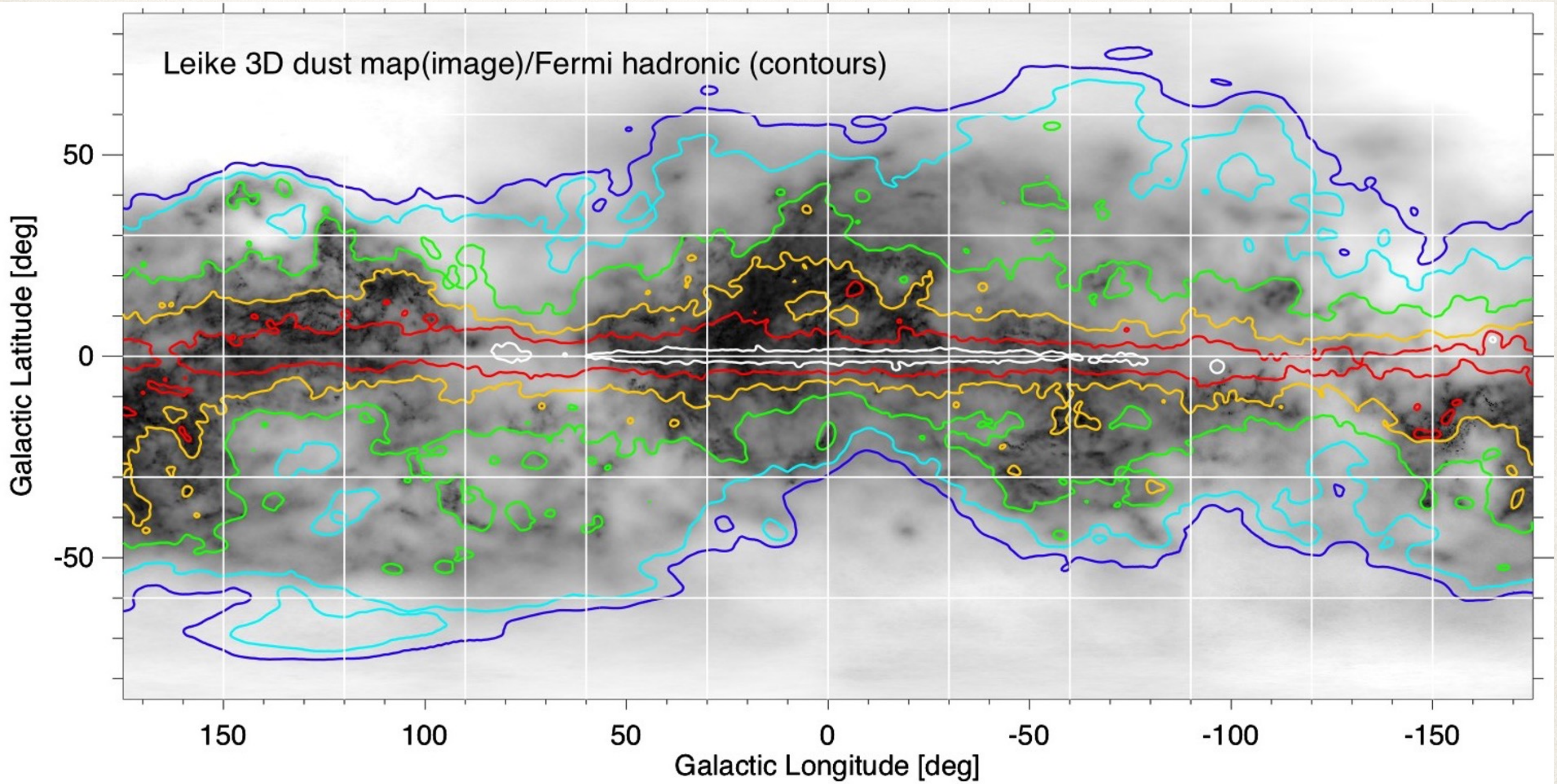
At higher energies we are sailing are in stormy waters: Alfvenic (and slow MS) turbulence cascades anisotropically (Talk by A. Lazarian) - no effective scattering

Fast modes are isotropic, but when dampings are accounted for the resulting diffusion coefficient looks nothing like what we infer from B/C

The addition of other effects (such as mirroring) may result in more likeable results but very model dependent and very sensitive to environmental conditions

Conclusion: despite much sophisticated theories of turbulence, **we do not know yet how high energy particles diffuse in the Galaxy**

Turbulence-CR scattering and targets



Talk by R. Benjamin

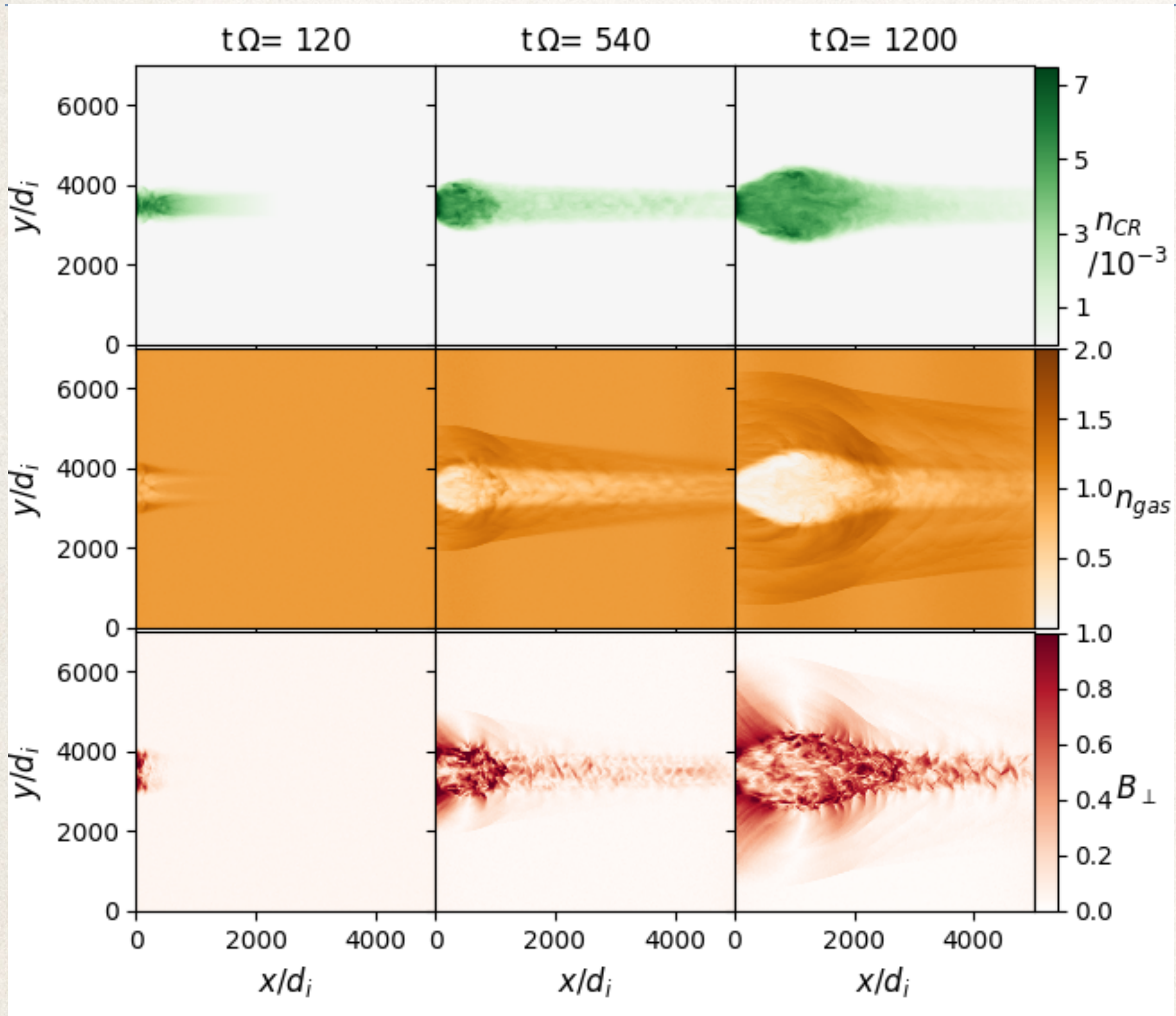
Self-Generated scattering

Original idea dates back to the pioneering work by Kulsrud&Pearce (1969), Skilling (1975) and Holmes (1975), but it has been studied recently in terms of the spectral break (PB+2012, Aloisio+2013, Aloisio+ 2015, Evoli&PB 2018)

The effect is based on the excitation of a streaming instability. In its basic form its growth rate is proportional to the CR density gradient — effective both on Galactic scales and near sources

In special conditions (energy density carried by the CR current larger than the local magnetic field energy density) a fast growing branch of this instability (Bell 2004) is activated.

High Energy Particles leaving a source severely change the medium around



Schroer+, 2021, *Dynamical effects of cosmic rays leaving their sources*

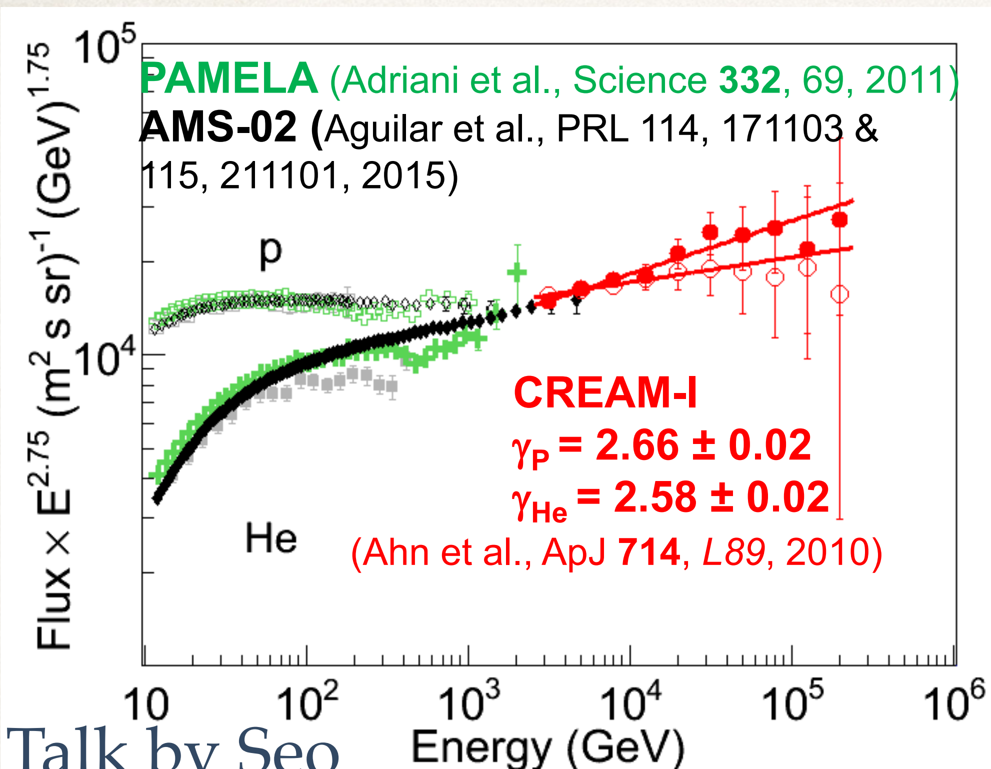
- PARTICLES ESCAPING A SOURCE REPRESENT AN ELECTRIC CURRENT, UNDER SOME CONDITIONS IT EXCITES A STREAMING INSTABILITY THAT LEADS TO STRONG PARTICLE SCATTERING

- THE PRESSURE GRADIENT THAT DEVELOPS CREATES A FORCE THAT LEADS TO THE INFLATION OF A BUBBLE AROUND THE SOURCE

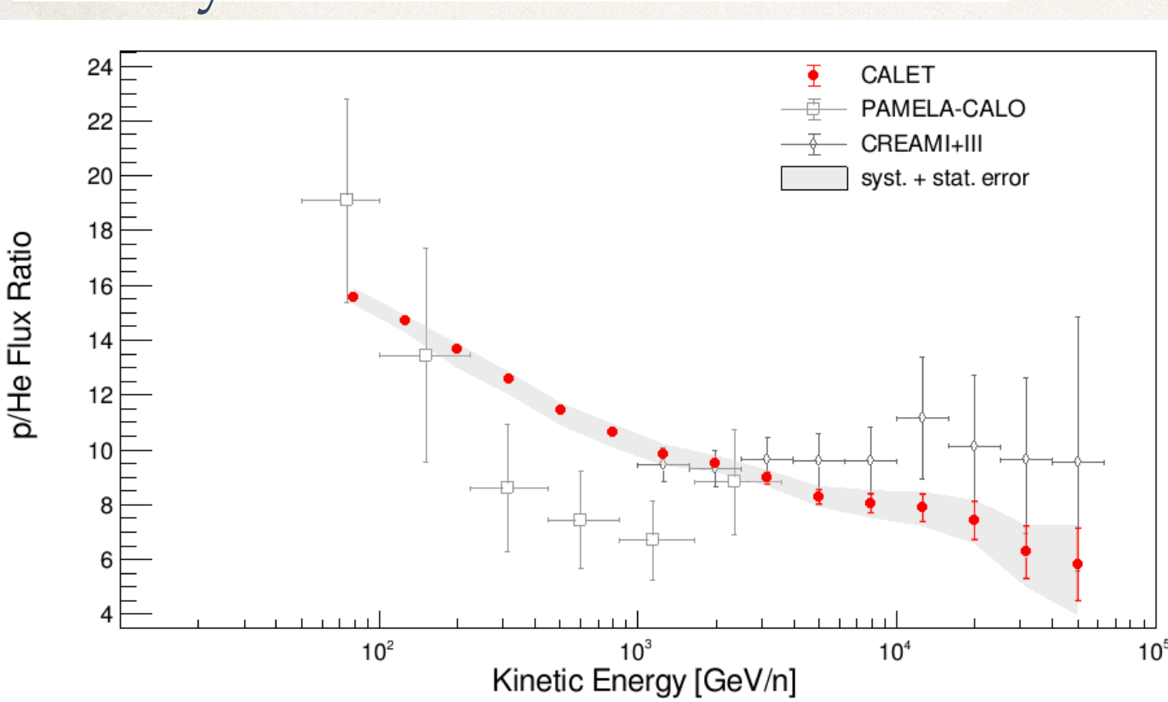
- THE SAME FORCE EVACUATES THE BUBBLE OF MOST PLASMA

- THERE IS NO FIELD IN THE PERP DIRECTION TO START WITH, BUT CR CREATE IT AT LATER TIMES (**SUPPRESSED DIFFUSION, about 10 times Bohm**)

Discrepant hardening

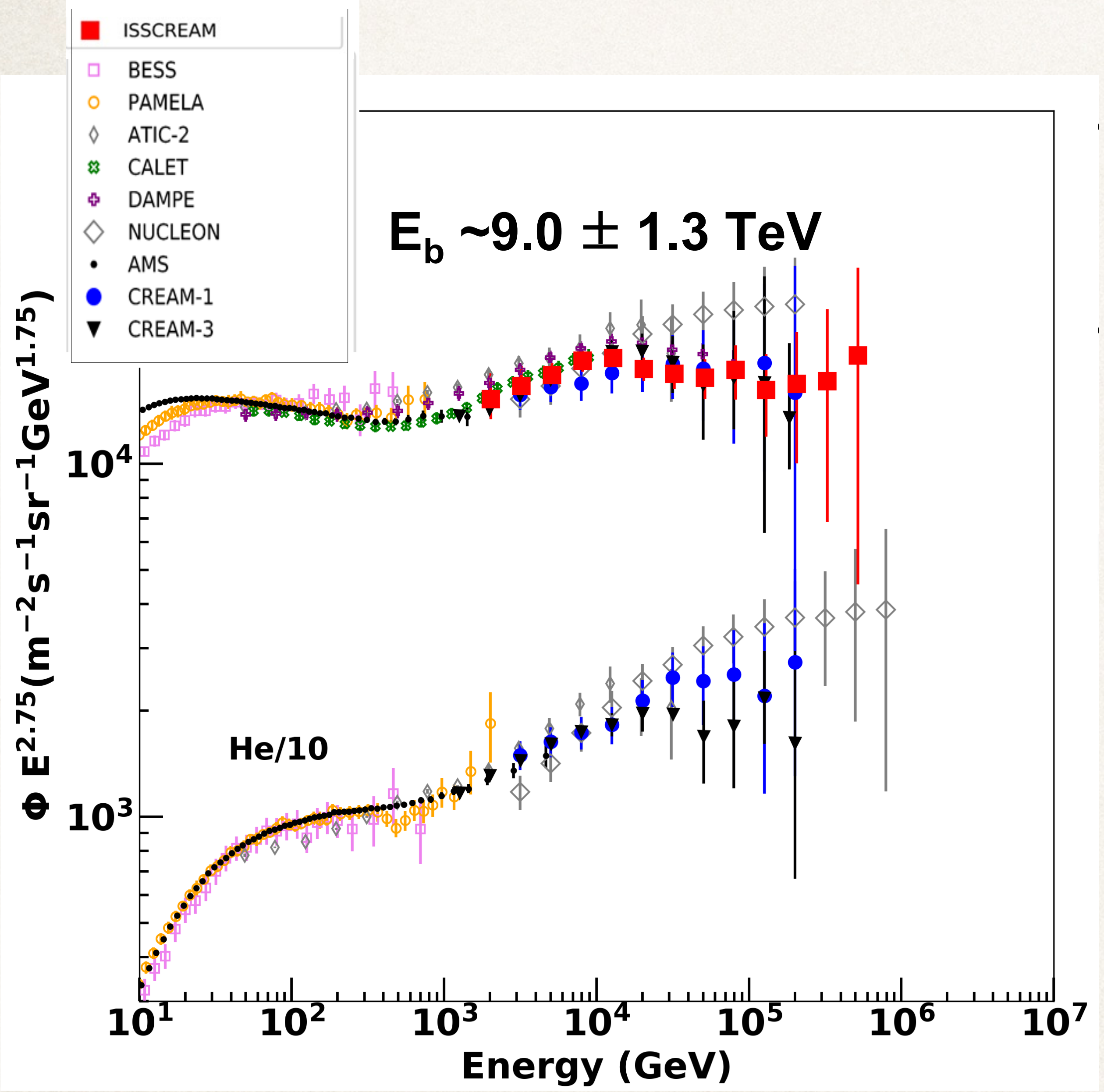


Talk by Seo



- H and He not only have different observed spectra, they require different source spectra
- The spallation of 4He to 3He does not help in explaining the difference since the experiments measures the sum
- This finding is at odds with the purely rigidity dependent nature of DSA
- The only possibility that jumps to mind is severe spallation inside sources, but…
- The discrepant hardening would suggest a He dominated knee (or intermediate mass dominated)

The so-called DAMPE feature



The DAMPE feature is now found in H and He spectra by CALET and ISS-CREAM

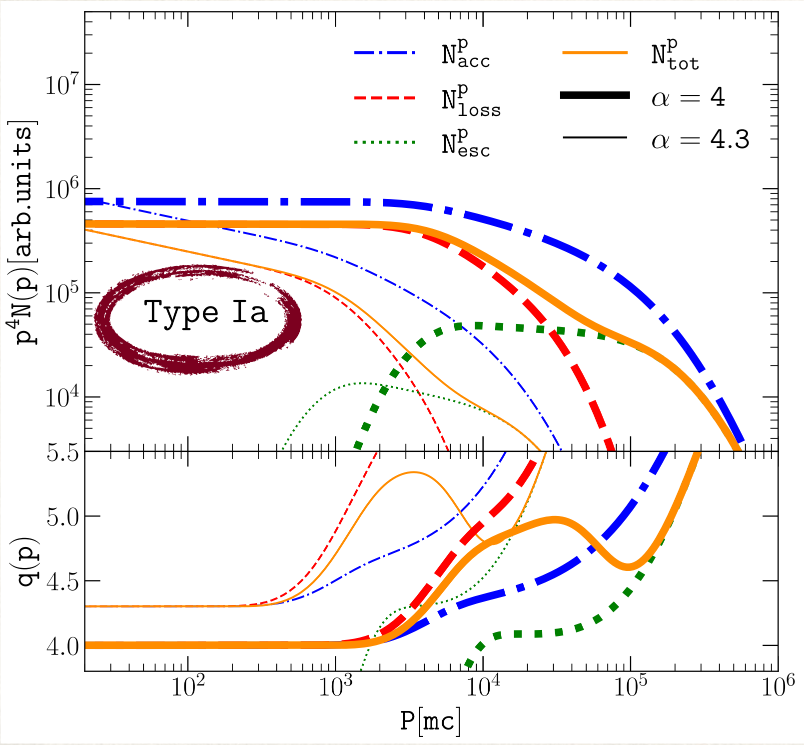
Its origin is clearly still unclear, but it might be the signature of accelerators running out steam and being replaced by less common, more luminous sources

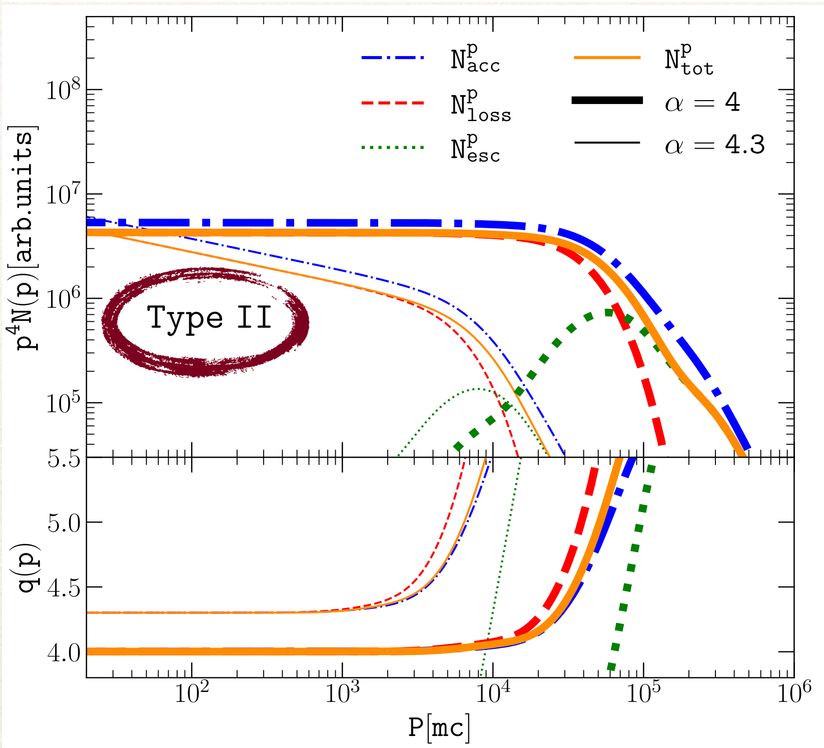
Talk by Seo

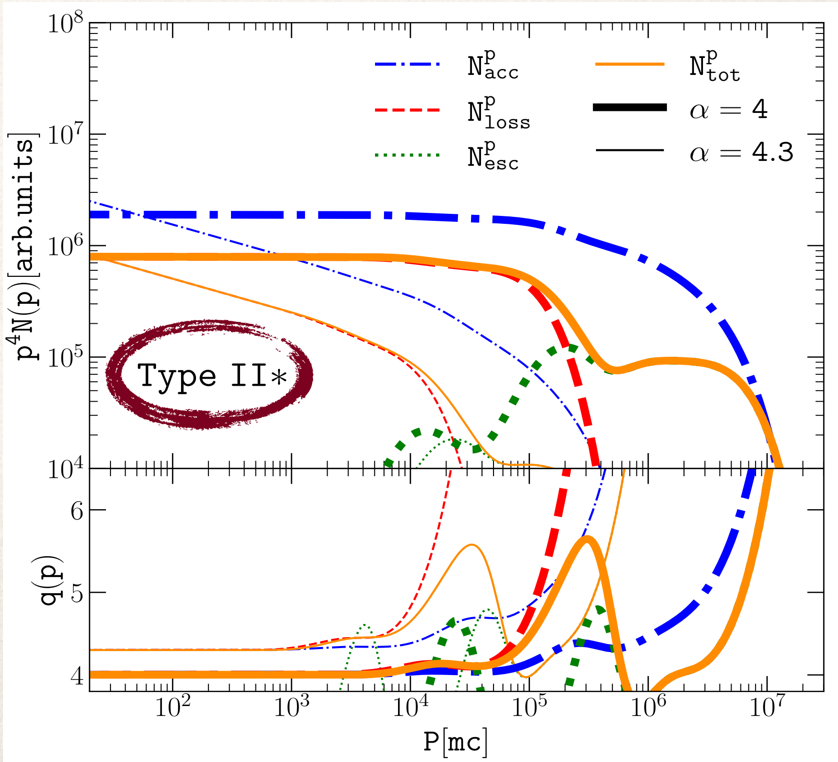
Acceleration/sources

- SNR are effective accelerators, as also shown by the large B field in the X-ray rims. The highest effective E_{max} is reached at the beginning of Sedov phase
- For SN-Ia E_{max} is typically around 100 TeV
- For SN-II exploding in the wind of the pre-SN star E_{max} can be a bit higher but still <<knee
- Only in rare, very energetic core collapse SNe one can get up to the knee region
- But the spectrum is all but trivial

Cristofari, PB & Caprioli 2021, Cristofari, PB & Amato 2020



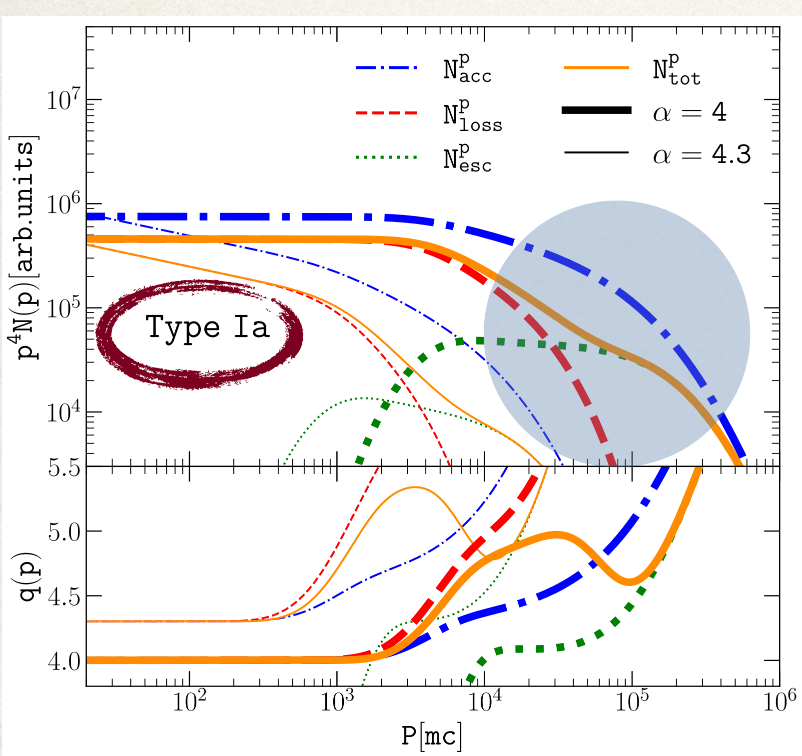


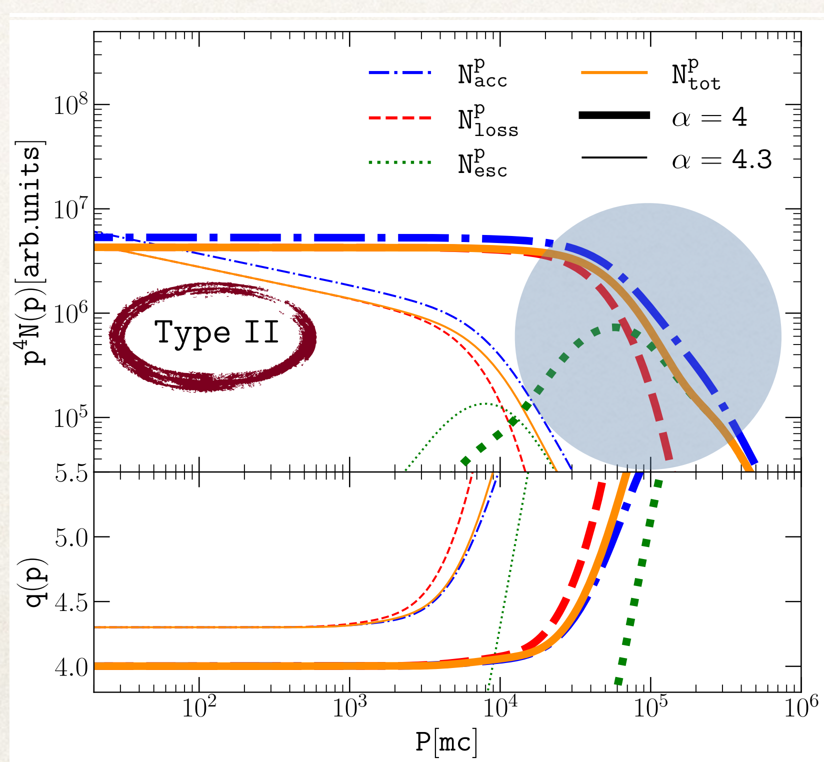


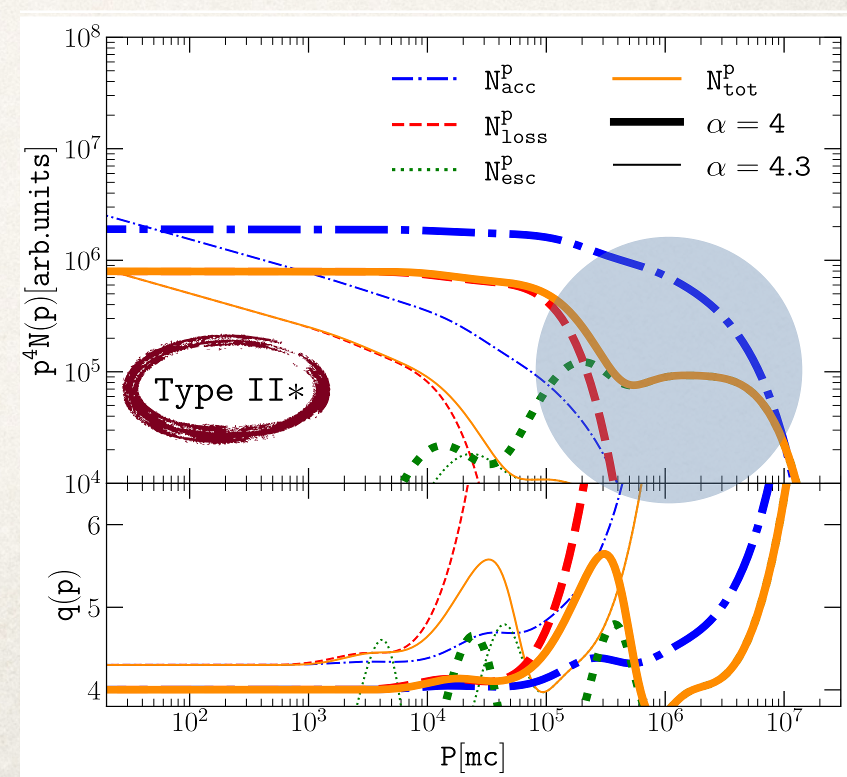
Acceleration/sources

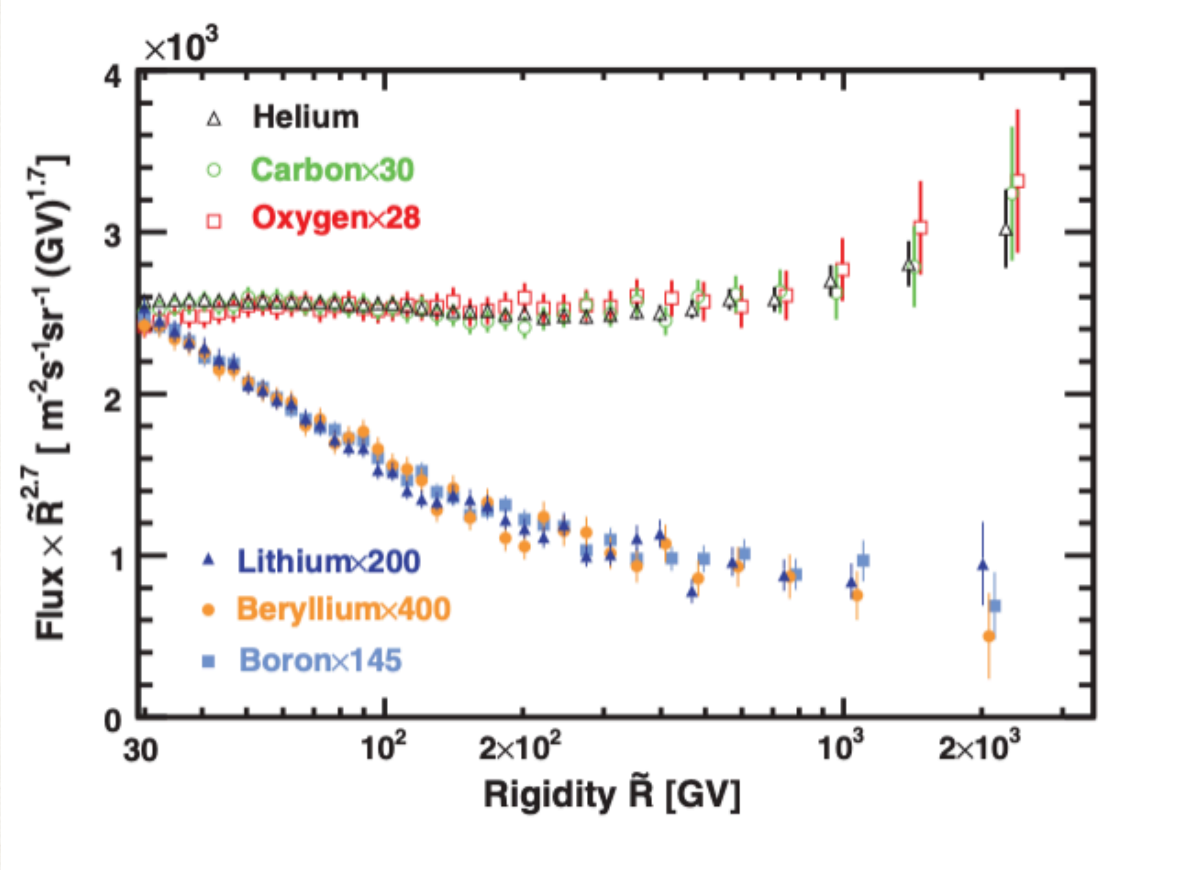
- SNR are effective accelerators, as also shown by the large B field in the X-ray rims. The highest effective E_{max} is reached at the beginning of Sedov phase
- For SN-Ia E_{max} is typically around 100 TeV
- For SN-II exploding in the wind of the pre-SN star E_{max} can be a bit higher but still <<knee
- Only in rare, very energetic core collapse SNe one can get up to the knee region
- But the spectrum is all but trivial

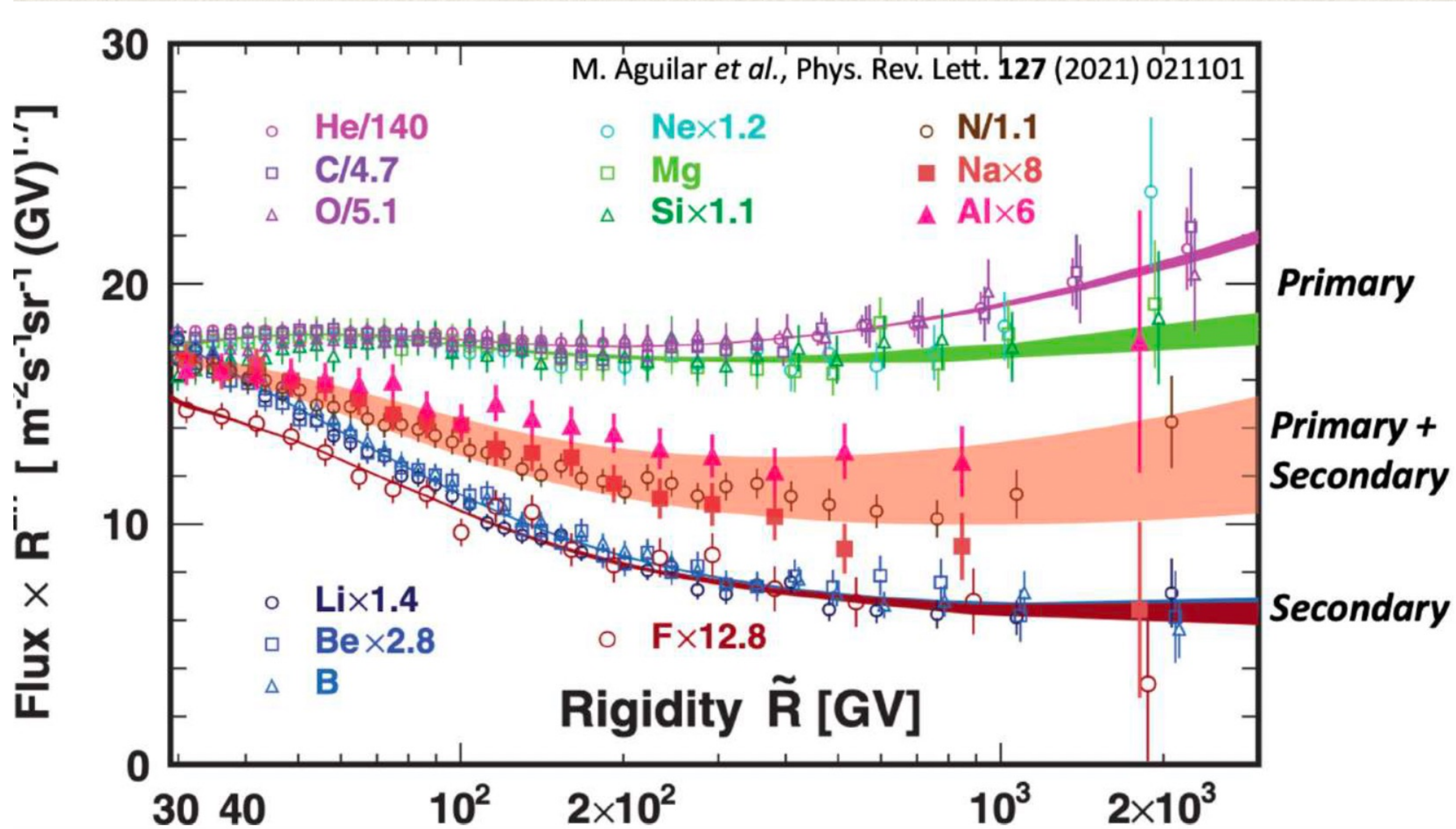
Cristofari, PB & Caprioli 2021, Cristofari, PB & Amato 2020











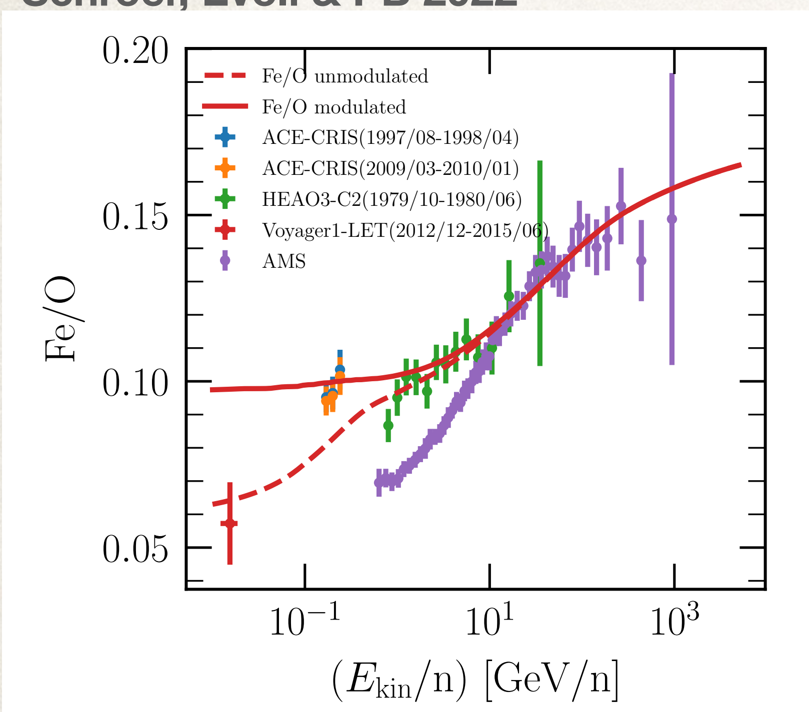
1) Virtually all elements have a spectral break at few hundred GV rigidity, though less evidence in heavier nuclei, due to a more prominent role of spallation at low energy

2) Care must be used in the classification of elements in primary and secondary: virtually all elements are not pure, especially the intermediate mass ones

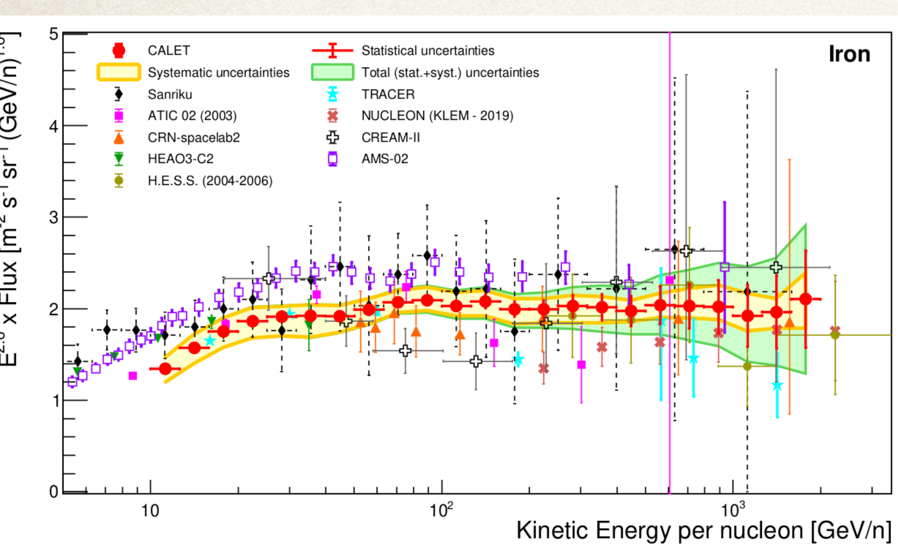
3) unacceptably large dependence of the conclusions on partial cross sections that are uncertain (some of them) at the level of 30-50%, while data are much more accurate (Talks by Schroer, Evoli, …)

THE CASE OF IRON: THE FE/O RATIO

Schroer, Evoli & PB 2022



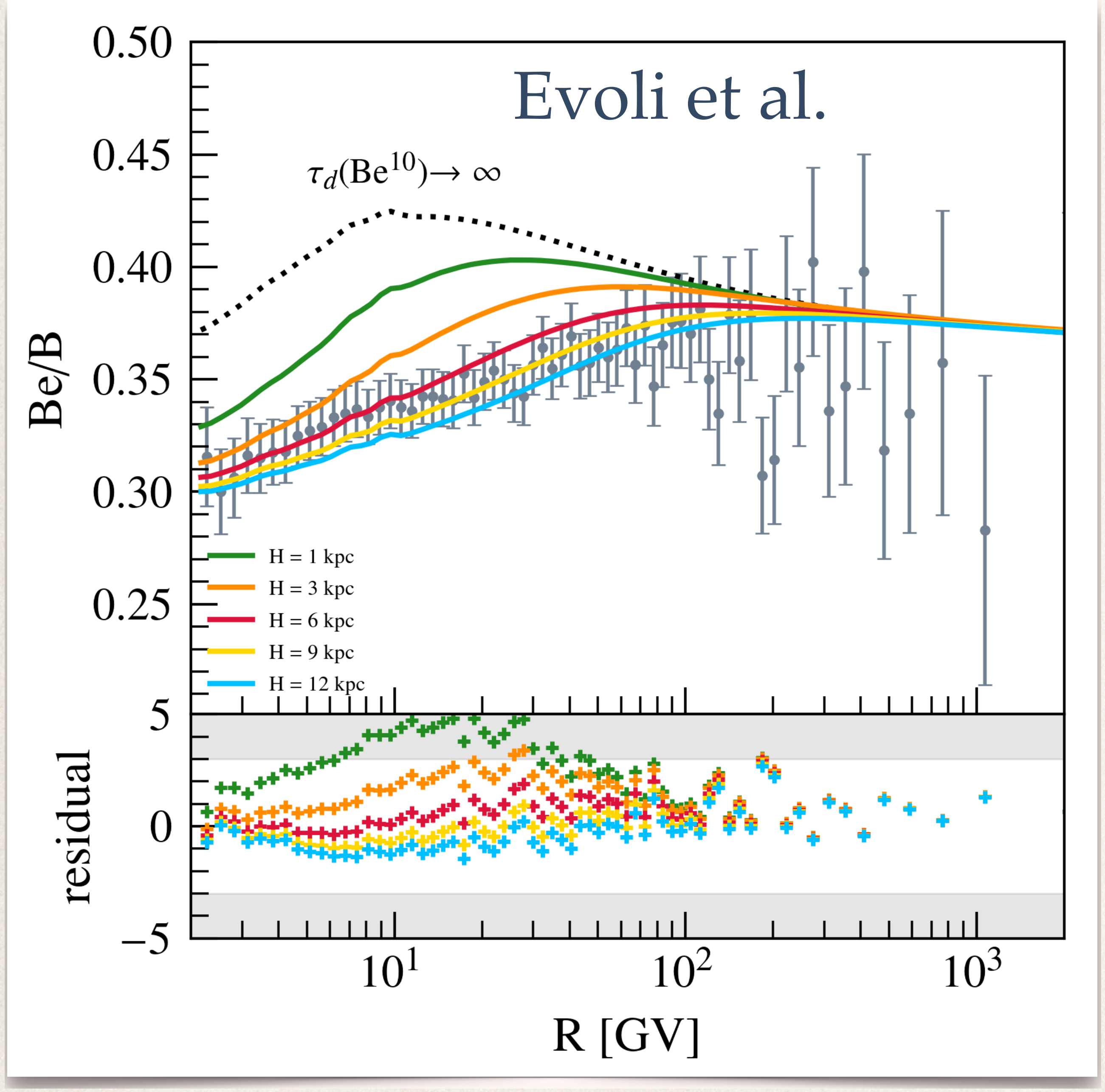
1. *THE CALCULATED RATIO OF MODULATED FLUXES IS IN BAD AGREEMENT WITH AMS-02 RESULTS BELOW A FEW TENS GV*
2. *HOWEVER IT IS IN EXCELLENT AGREEMENT WITH PREVIOUS MEASUREMENTS, FOR INSTANCE BY ACE-CRIS AND HEAO03*
3. *THE RATIO OF UNMODULATED FLUXES CAN ALSO BE COMPARED WITH VOYAGER DATA, AND AGAIN IT SEEMS IN GOOD AGREEMENT*



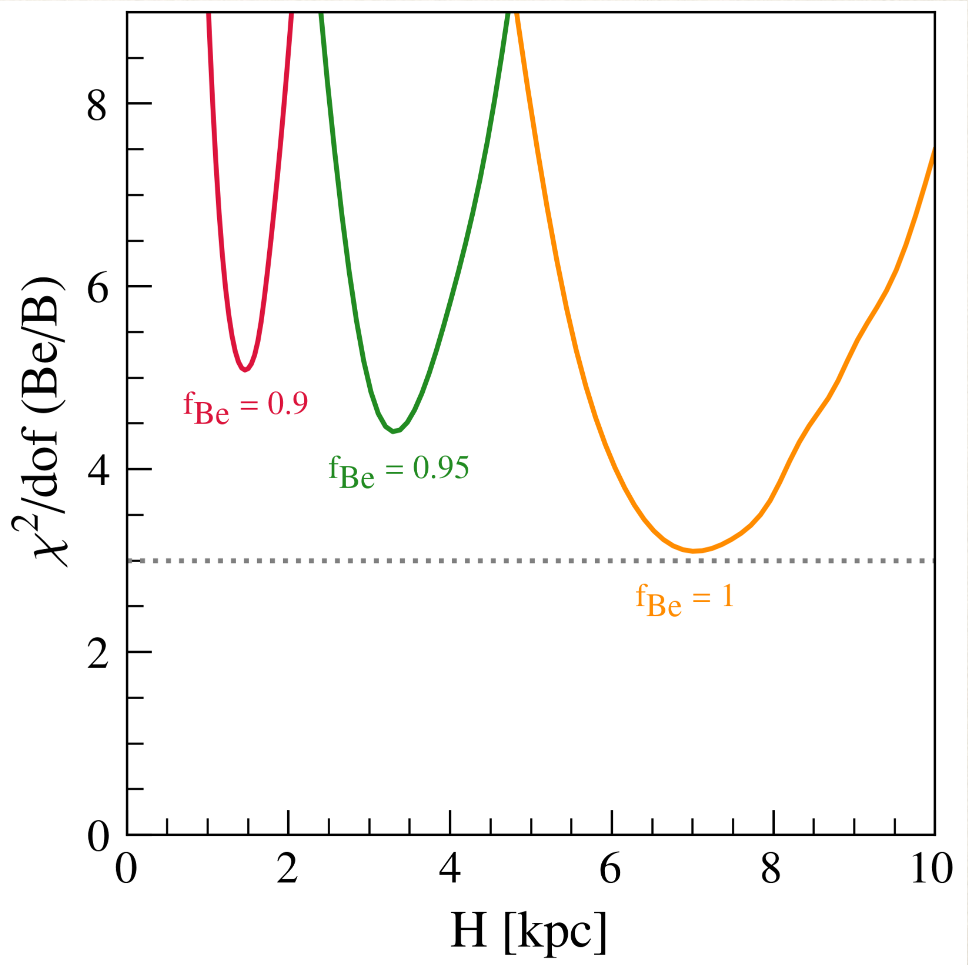
NONE OF THE THEORETICAL UNCERTAINTIES TURNS OUT TO BE ABLE TO ACCOUNT FOR DATA

IT IS WORTH STRESSING THAT FOR IRON THE EFFECTS OF INTERACTIONS IN THE APPARATUS ARE VERY SERIOUS…

ON THE IMPORTANCE OF MEASURING CROSS SECTIONS

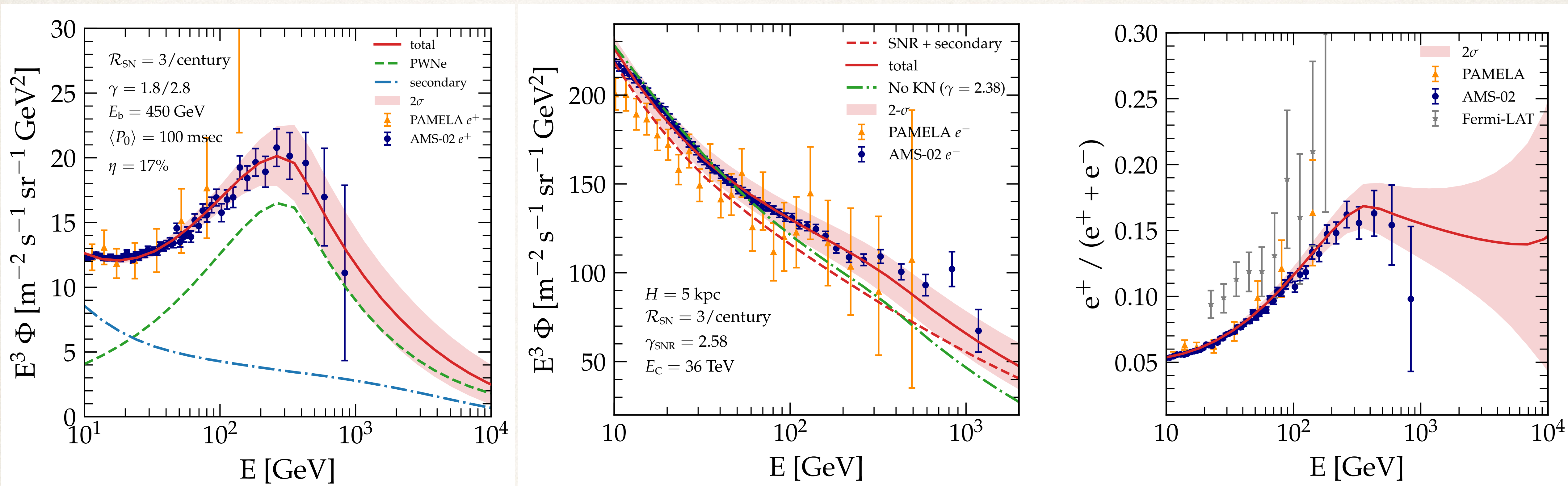


THERE ARE MANY INSTANCES IN WHICH THE UNCERTAINTIES IN THE CROSS SECTIONS LIMIT OUR ABILITY TO INFER PHYSICAL INFORMATION. ONE SUCH INSTANCE IS THE PRODUCTION OF BE AND B FROM HEAVIER ELEMENTS —> LIMITS ON HOW WELL WE CAN DERIVE THE SIZE OF THE MAGNETIZED HALO OF THE GALAXY



Positrons

Talk by C. Evoli

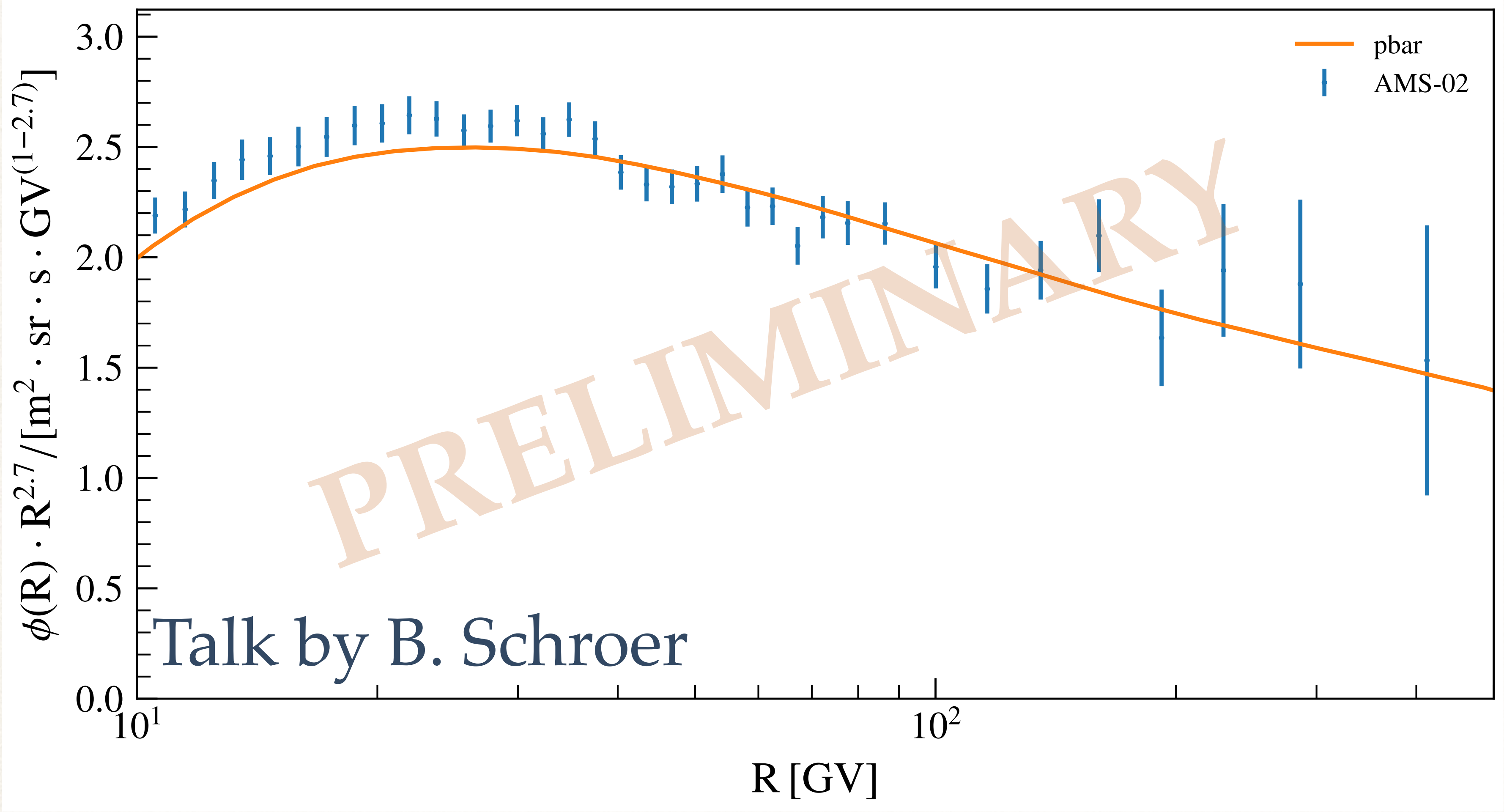


There is little doubt that the observed positron flux and positron ratio requires a source other then secondary production

The best physically justified sources are pulsars for which there is independent evidence of appropriate spectra and presence of positrons

Dark matter interpretations unreasonably epicyclical (x-sections enhanced by Sommerfeld effect, boosting effect, leptophilic, …)

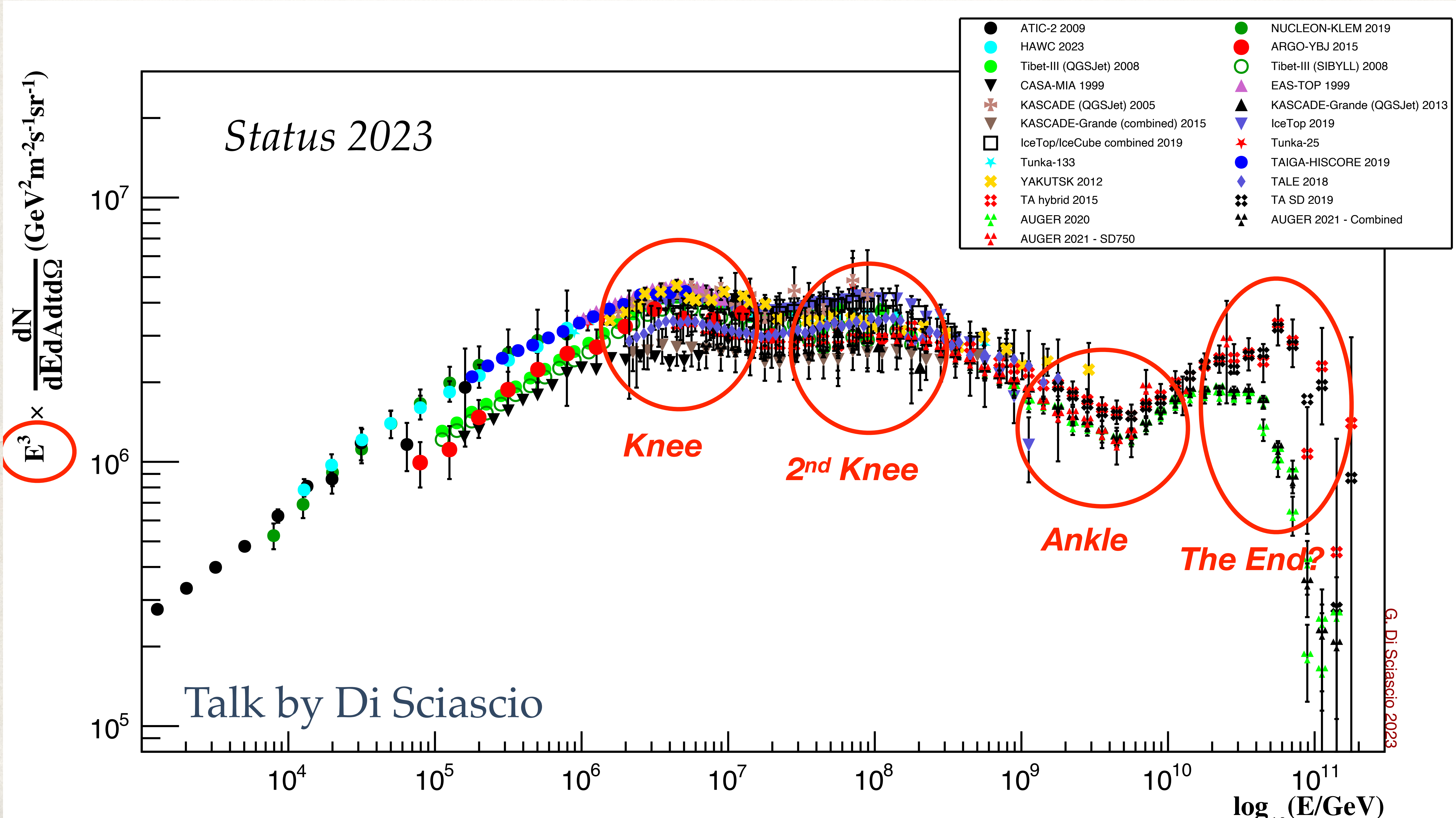
The case of antiprotons



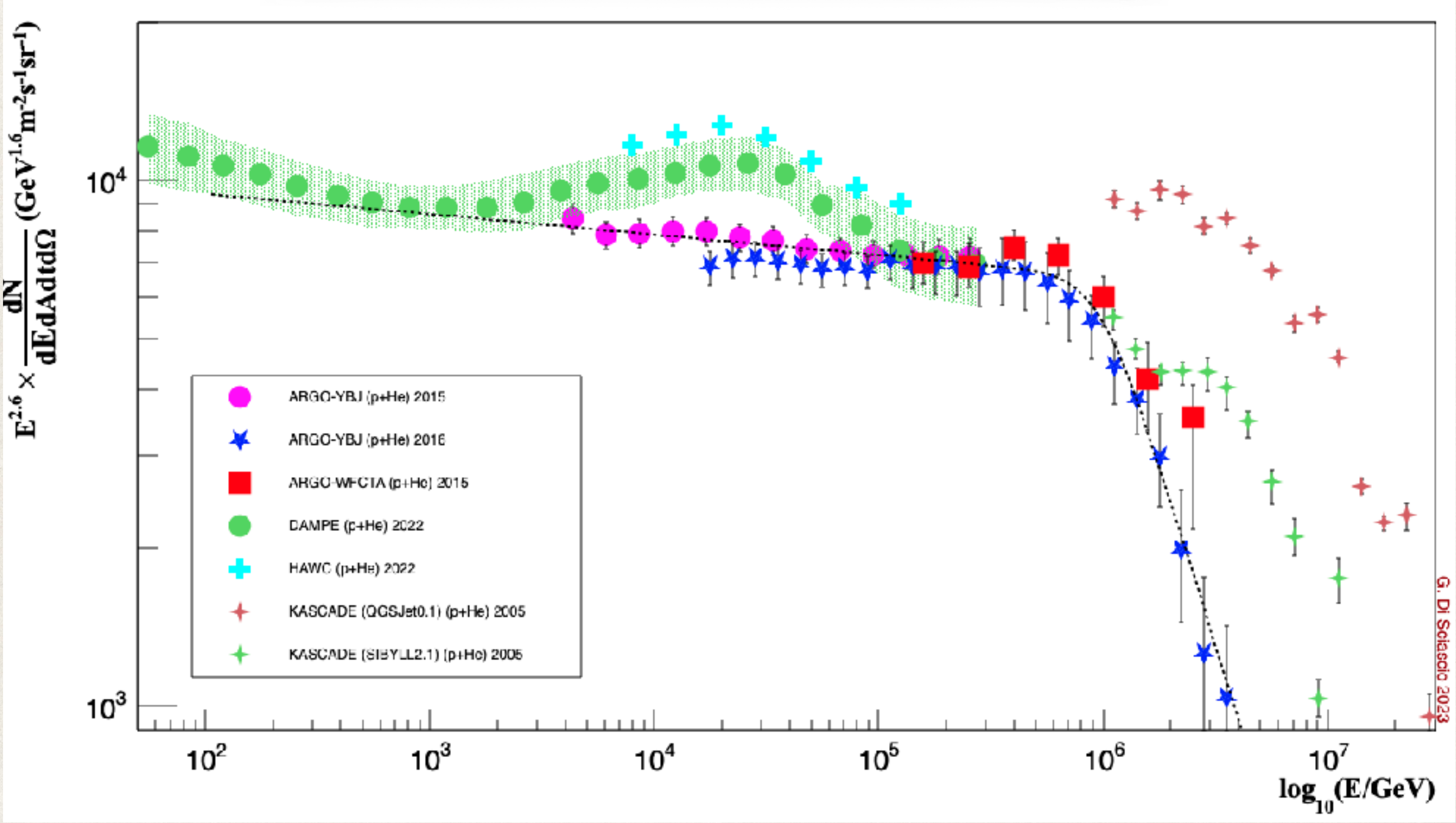
The production of antiprotons in CR is historically one of the most important indicators of transport

With recent determinations of the pbar production cross section, there seems to be no need for new physics

Moving to higher energies…



Moving to higher energies…



Moving to higher energies…

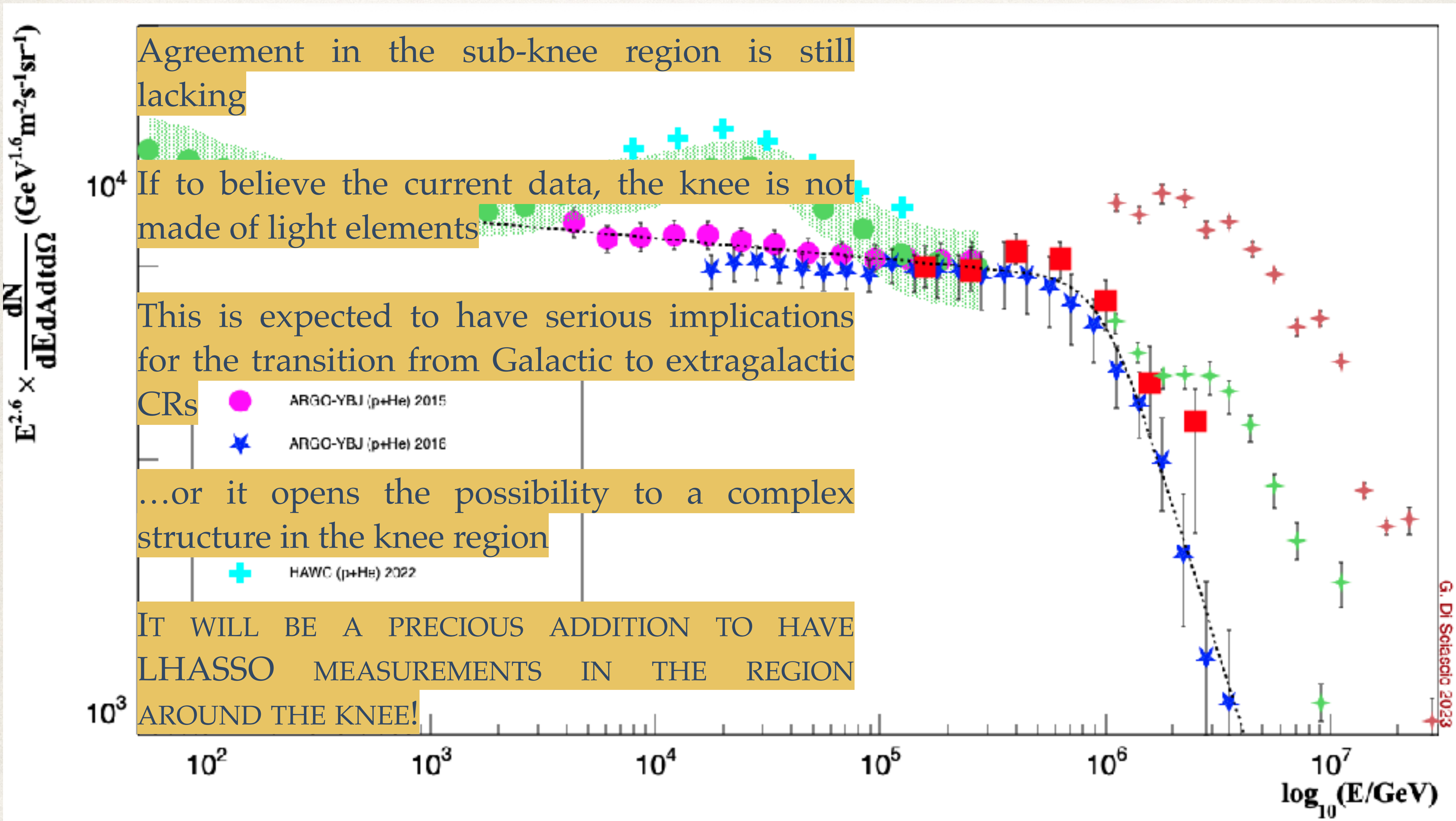
Agreement in the sub-knee region is still lacking

If to believe the current data, the knee is not made of light elements

This is expected to have serious implications for the transition from Galactic to extragalactic CRs

…or it opens the possibility to a complex structure in the knee region

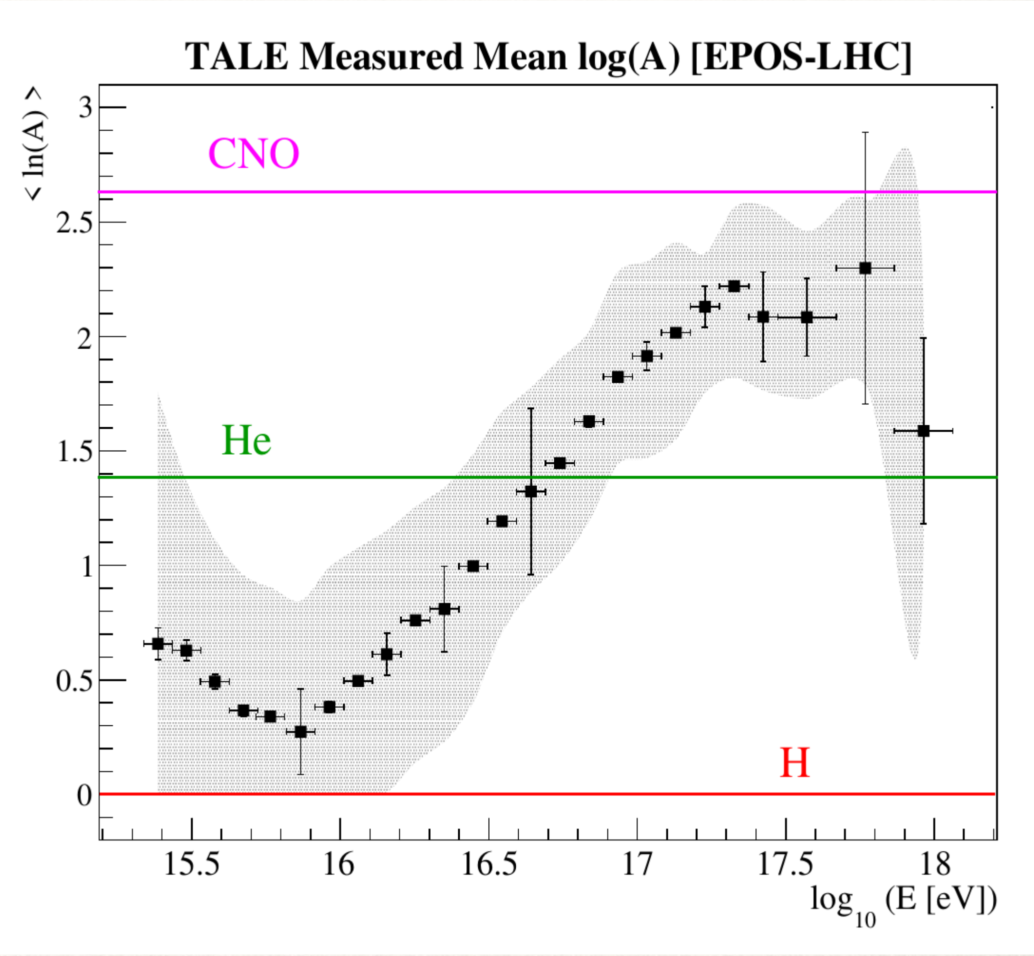
It will be a precious addition to have LHASSO measurements in the region around the knee!



Knee pain

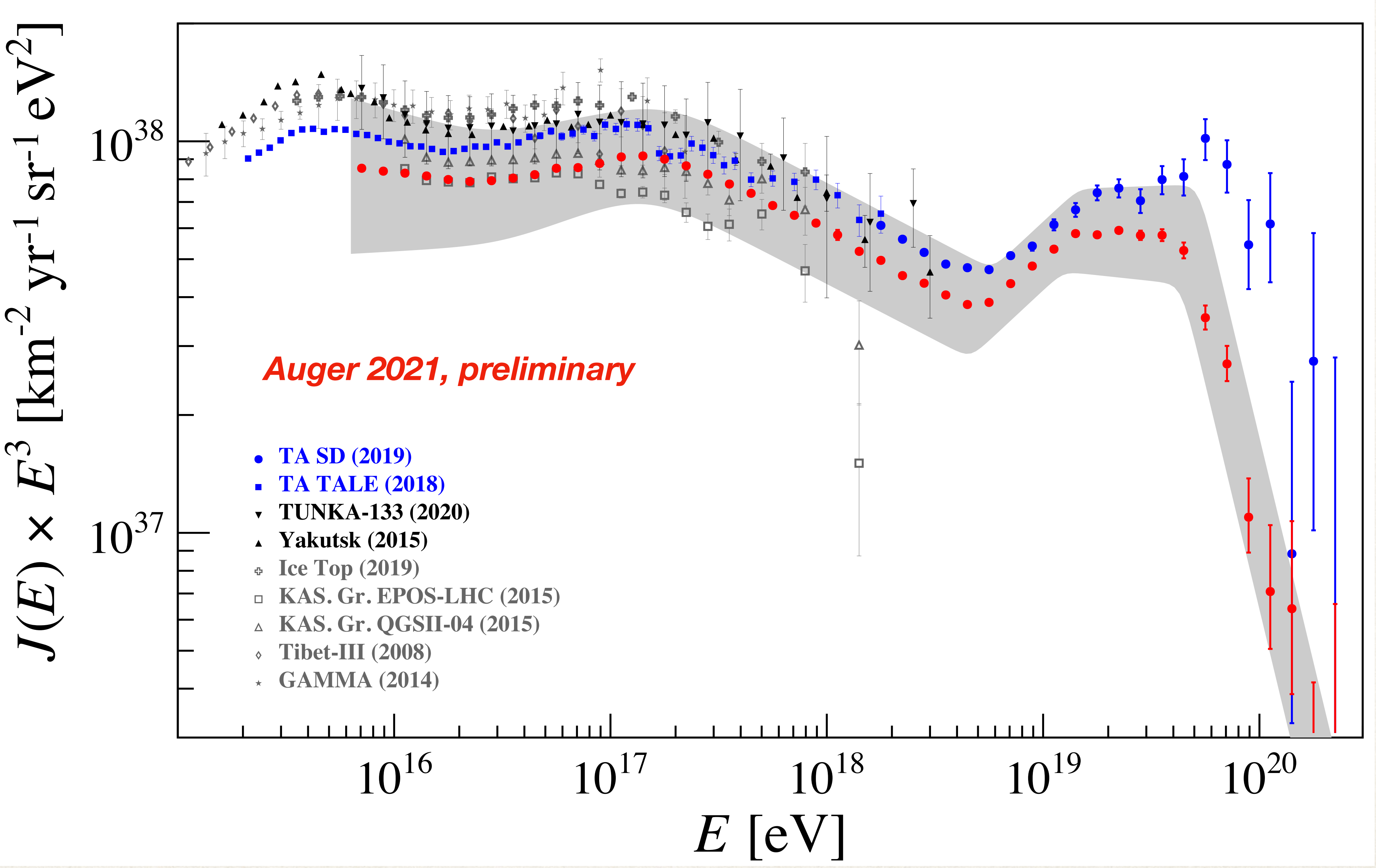
Indeed it is painful to admit that we are still unable to answer

1) whether the knee is light or intermediate mass
2) whether the knee is due to a superposition of cutoffs in the spectra of elements of different mass (aka Peter cycle) or to the transition to the small pitch angle scattering regime (D(E) prop E^2)
3) In the latter case, be aware that theorists will have even more sleepless nights in figuring out how to accelerate to >>PeV energies
4) These problems are of purely observational nature. It is everybody's responsibility to have reliable data with meaningful systematic uncertainties

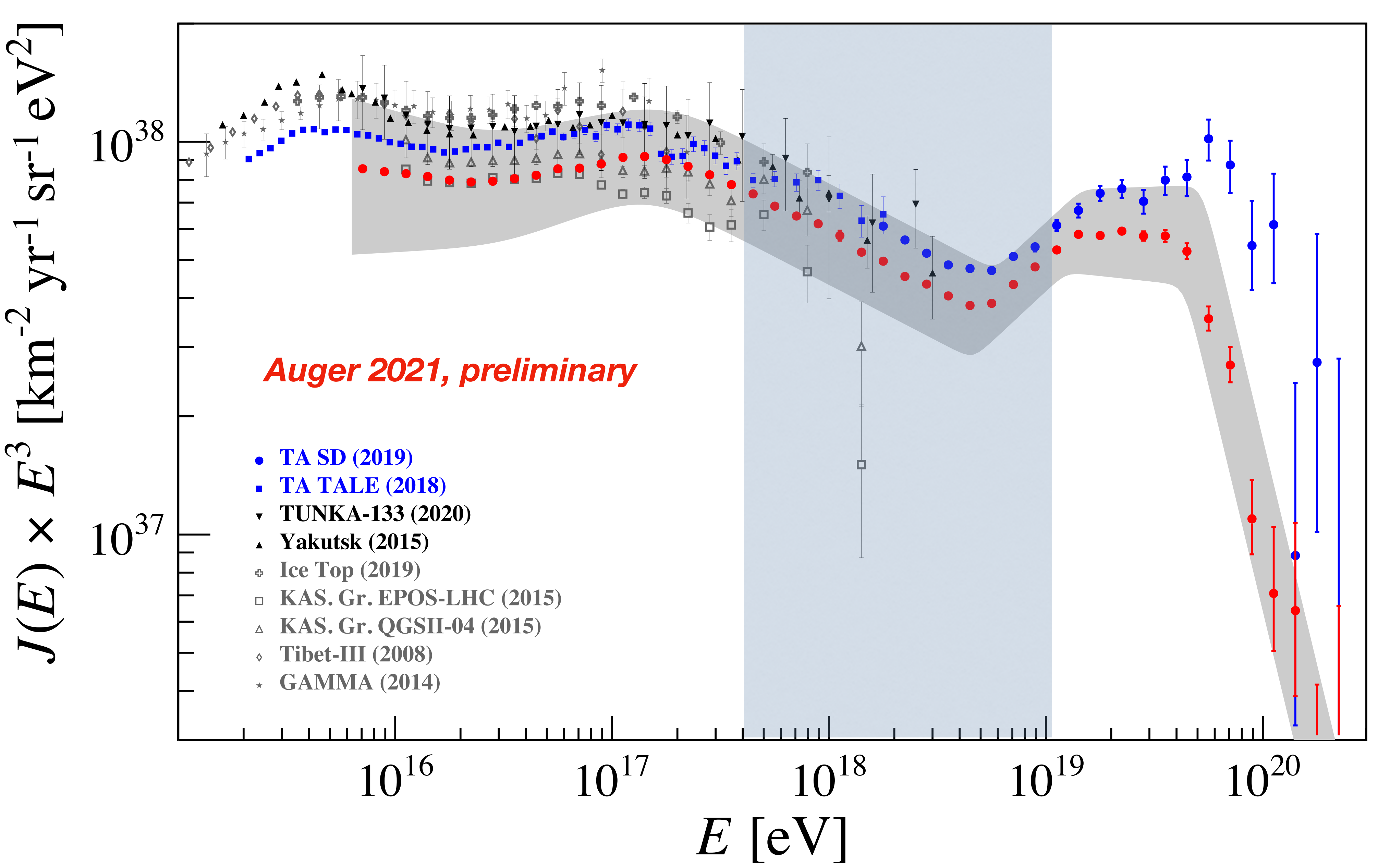


Talk by A. Zayyad

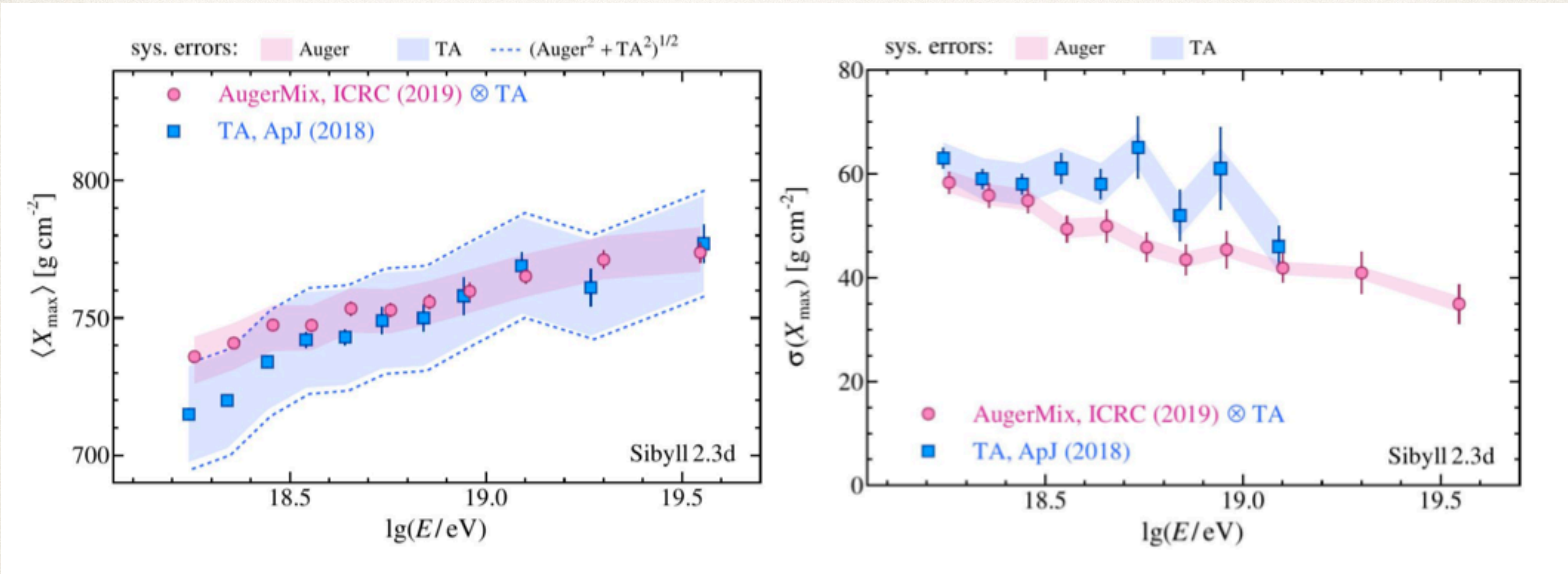
Moving our way out of the Galaxy



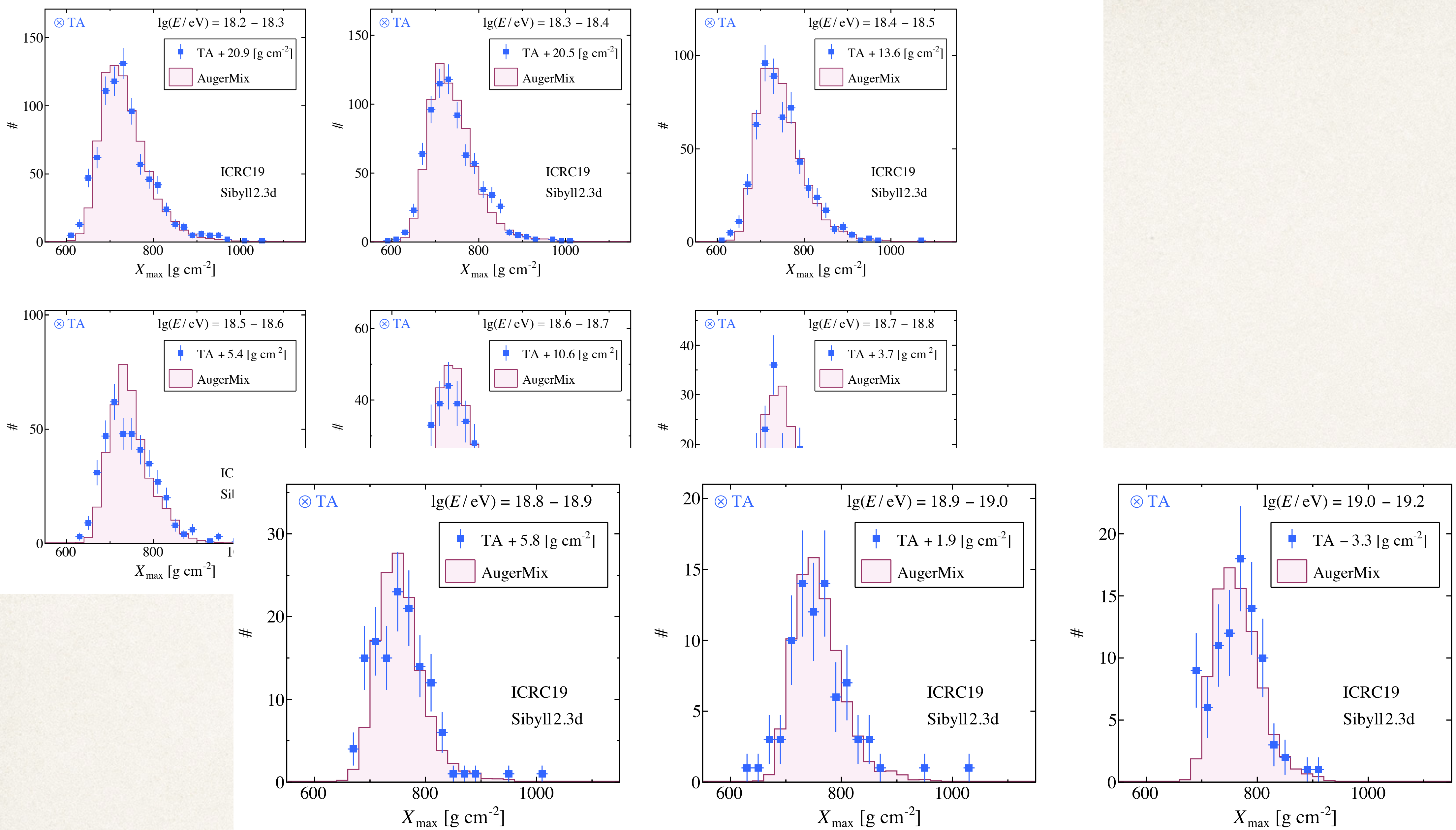
Moving our way out of the Galaxy

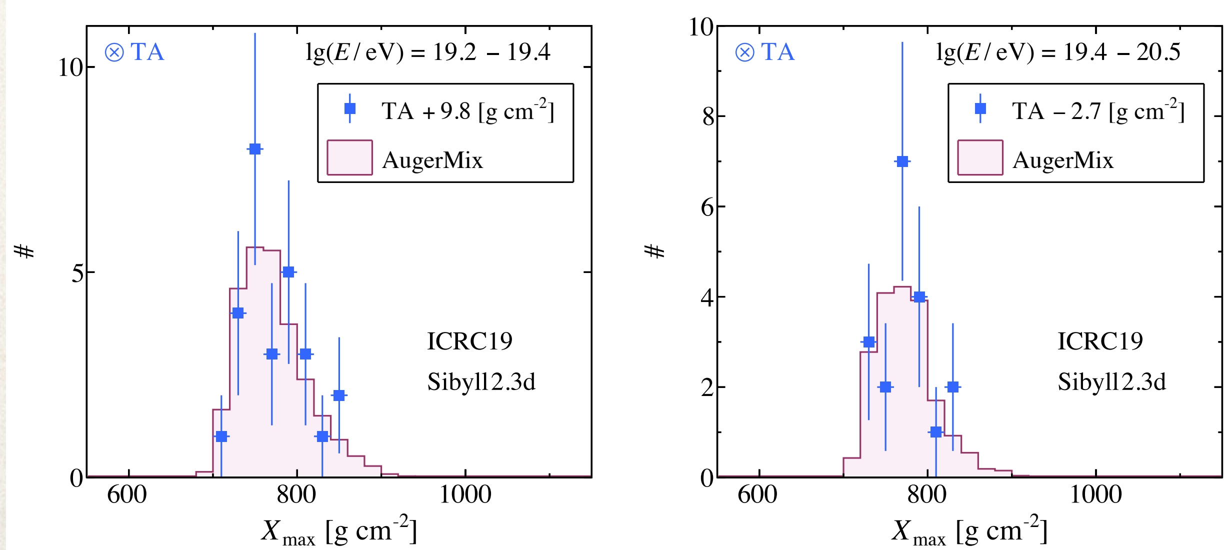


At last…"Agreed Upon" picture of mass composition?



Auger-TA comparison of Xmax distributions (i)



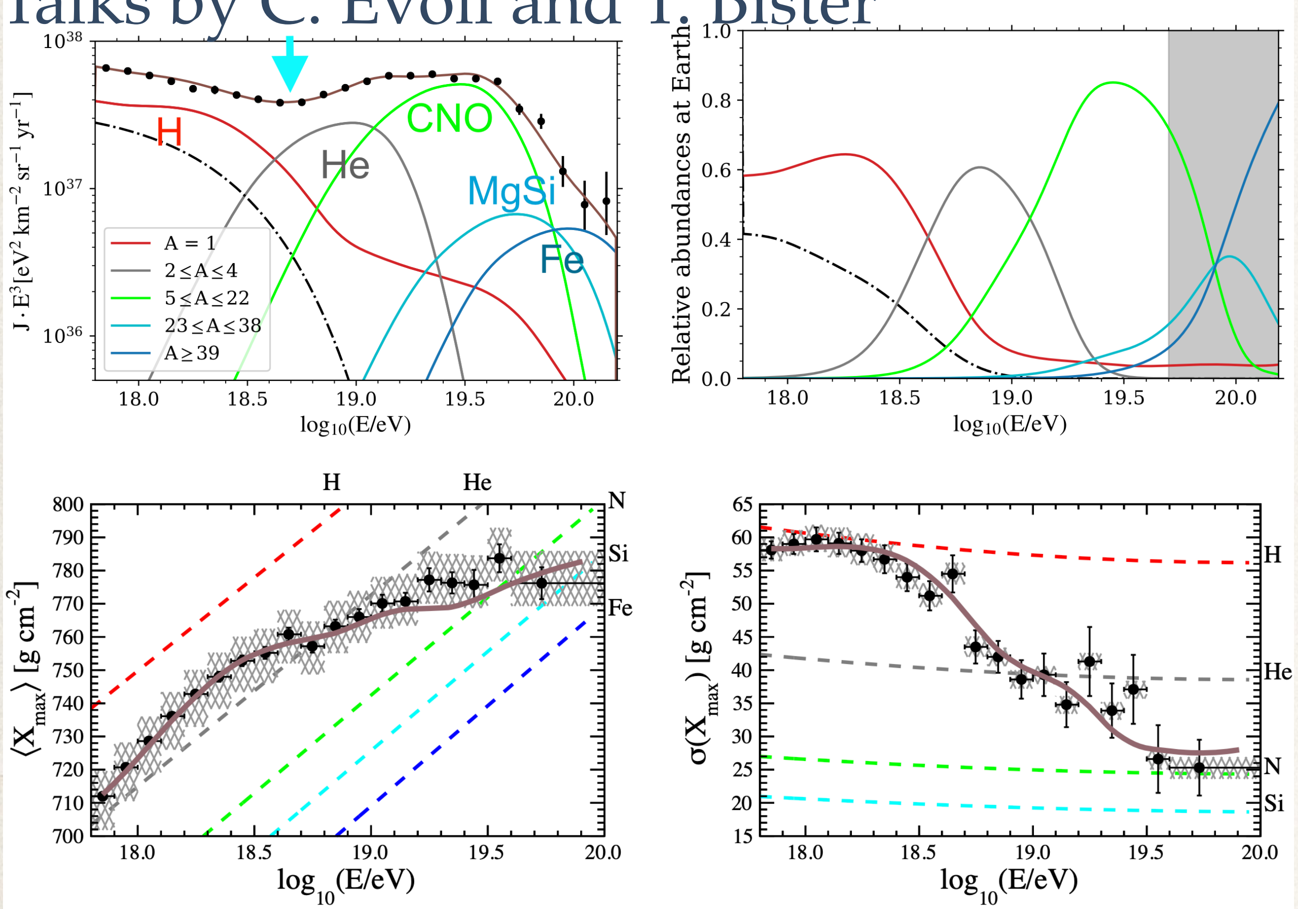


Mean Xmax shifted to agree between data sets for each energy bin

(UHECR 2022, D. Bergmann for the Auger-TA joint working group)

Spectra and Masses…

Talks by C. Evoli and T. Bister



Whoever believes that Nature reflects a sense for beauty, should try to fit Auger data

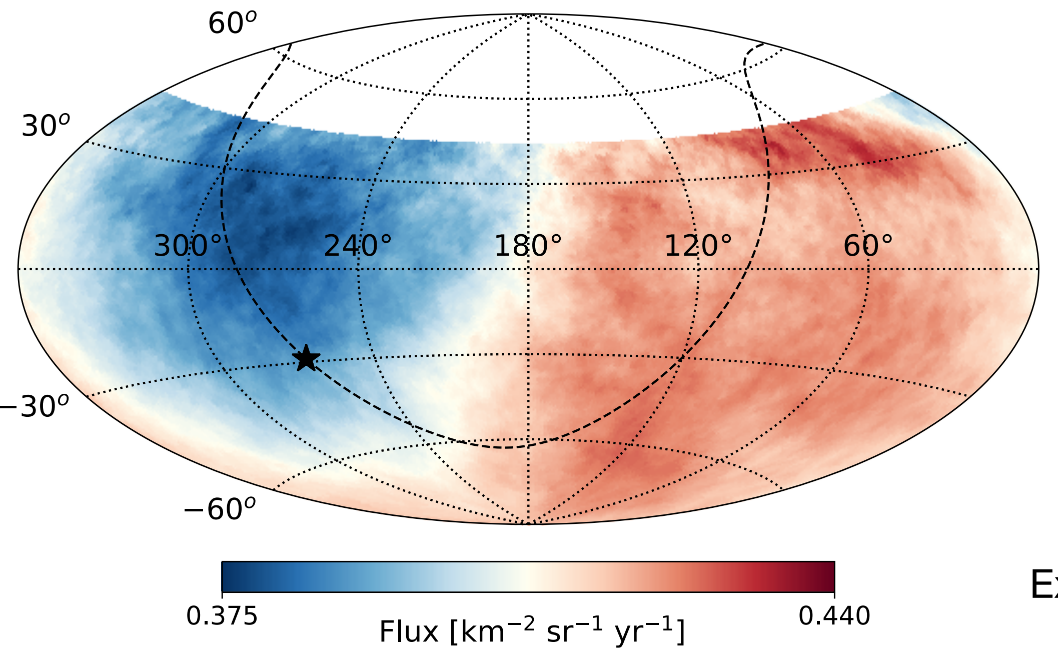
- The sources must produce a mixed mass composition (hard to imagine this may happen in the standard IGM)
- Fast transition between components
- The max energy cannot have a wide spread (F. Oikonomou talk)
- The sources must inject CR with very hard spectrum
- And yet protons should have a different spectrum…

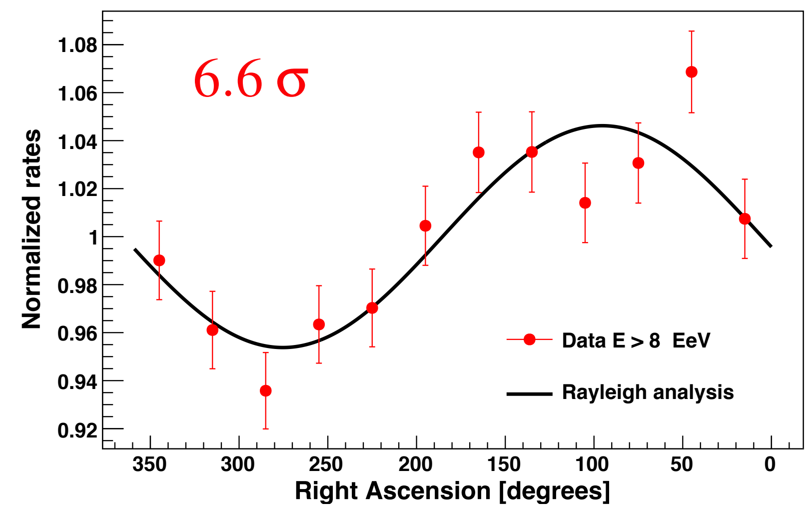
The hard spectra may result from acceleration in non-standard conditions, for instance like the ones in 3D reconnection (in GRB? in Radio Galaxies?)

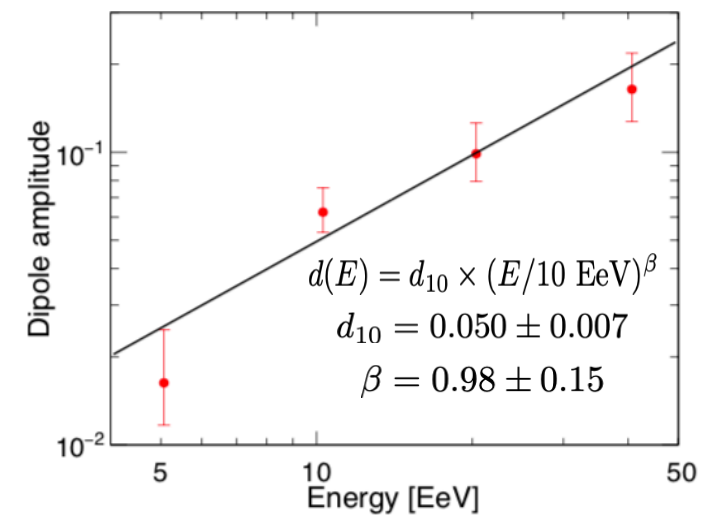
…but the hard spectra might reflect energy losses in the sources+energy dependent escape (model of Farrar, Unger…) or confinement effects due to magnetic fields

…or self-confinement around the sources (ask questions if you deem necessary)

Anisotropies

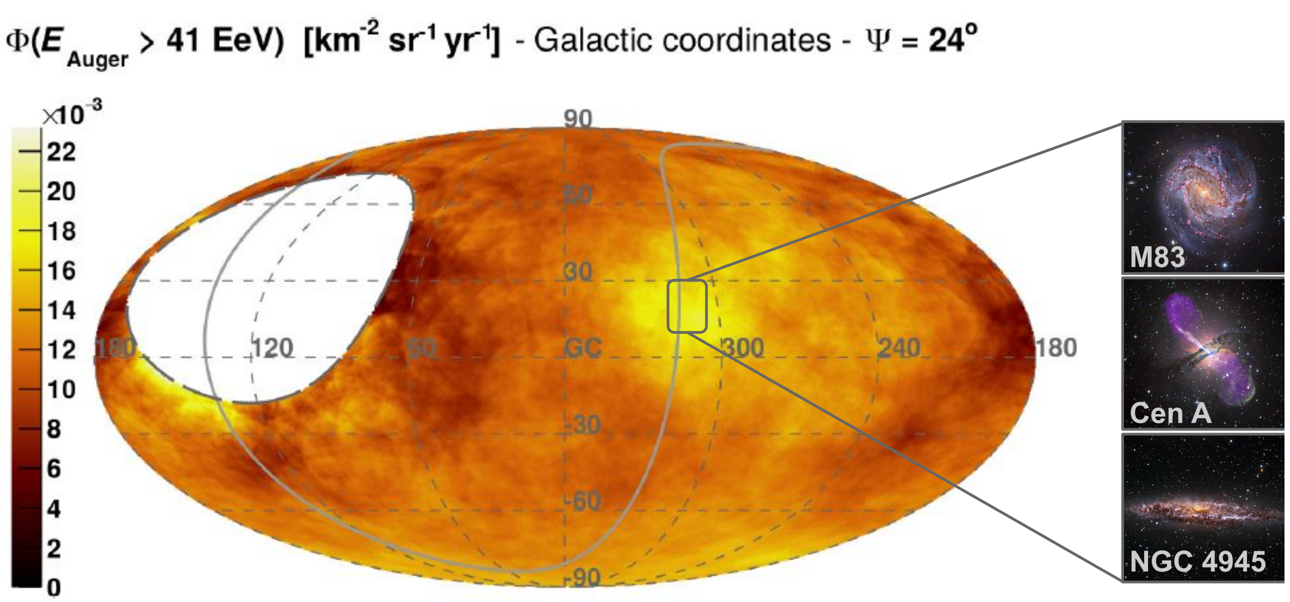






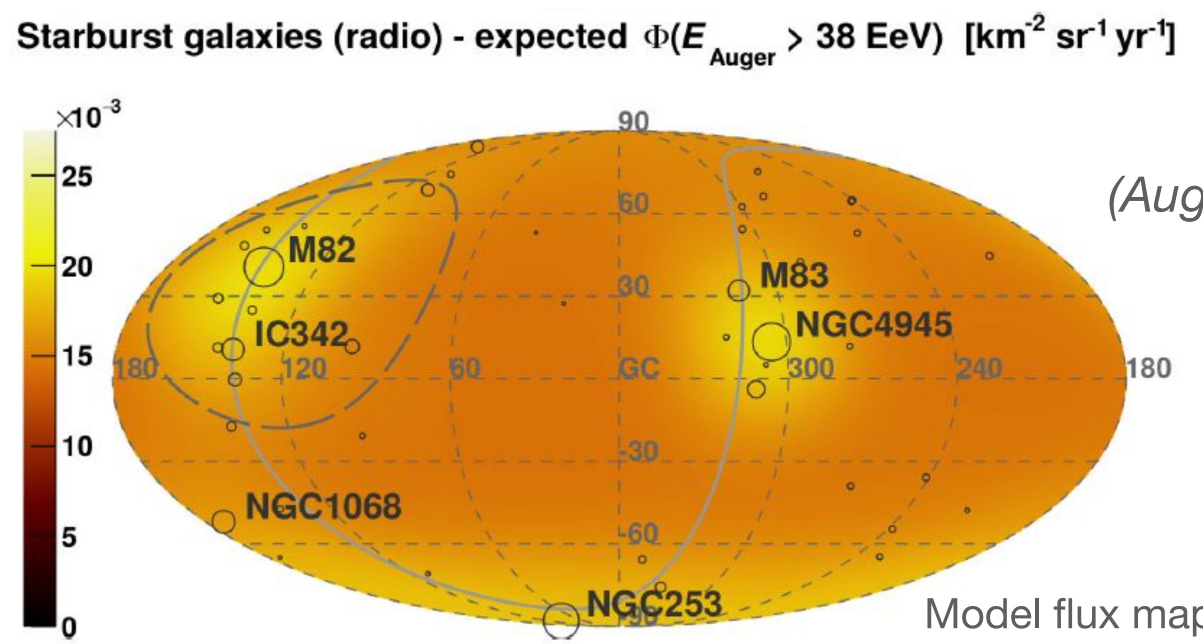
Exposure until end of 2020 (θ < 80°): 110,000 km² sr yr

$p \sim 5 \times 10^{-11}$



Direction fixed to that of Cen A, free E_{th} and Ψ

$E_{th} > 41$ EeV, $\Psi = 27°$: **3.9σ post-trial** deviation from isotropy (5% excess)



(Auger, ApJ 9

Notice that the possible correlation with starbursts does not mean that they are the sources of UHECR: in fact most SB galaxies do not have enough juice to even get close to UHE. This may happen for UFO (tail of SB), but then…

Acceleration/sources - UHE

One should appreciate how the situation changed in the last twenty years

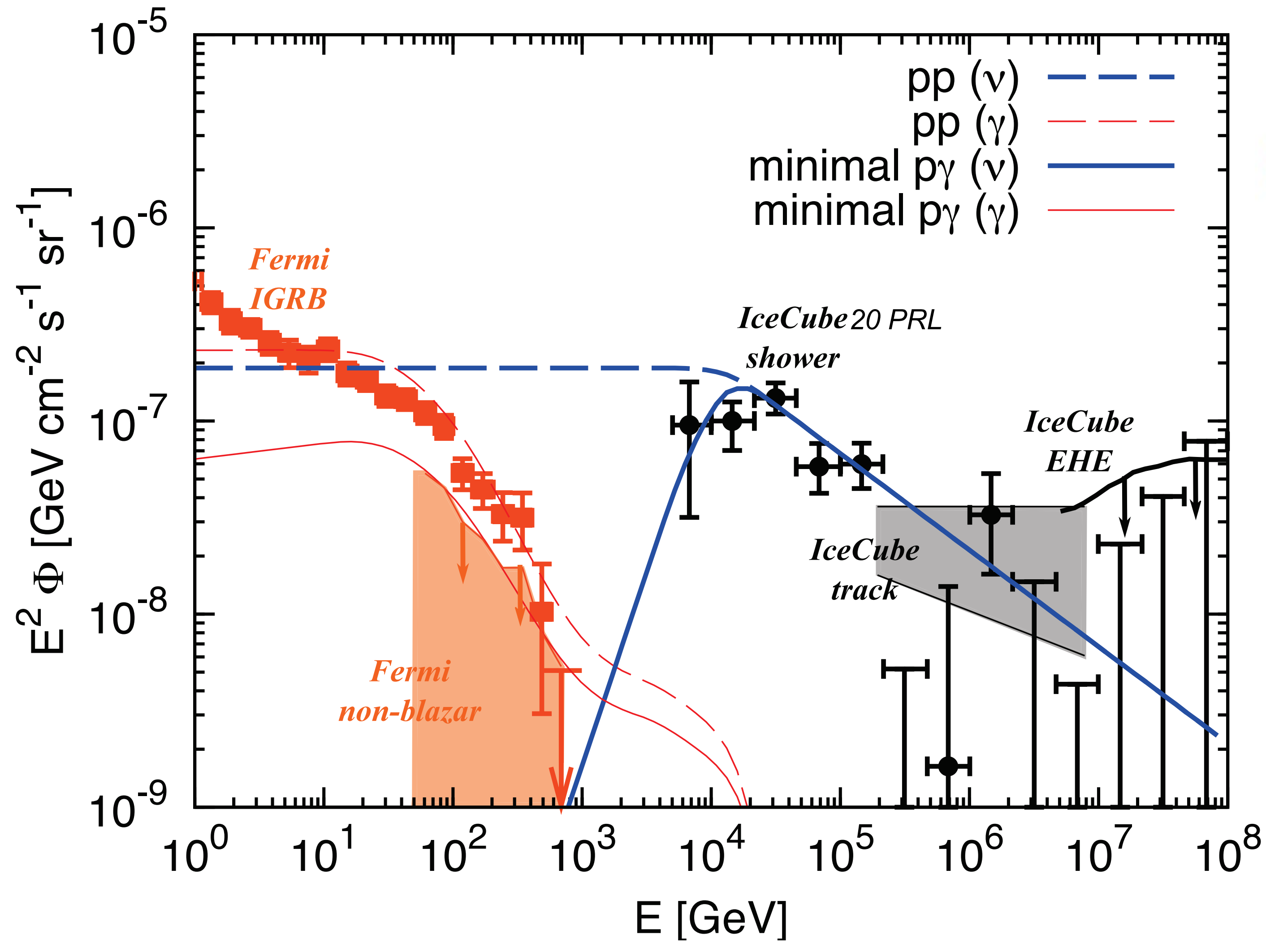
we went from a situation in which data showed that protons should be accelerated to ZeV energies, to a situation in which the max rigidity cannot be higher than ~2 EeV.

Clearly the problem of accelerating particles has become much less demanding

yet there are constraint: for instance the bulk of starbursts do not have enough potential to accelerate up to such rigidity — perhaps UFO (Ultra Fast Outflows) may be a rare exception

THE BEGINNING OF A HIGH ENERGY NEUTRINO ERA

The diffuse neutrino sky



Murase et al., 2016

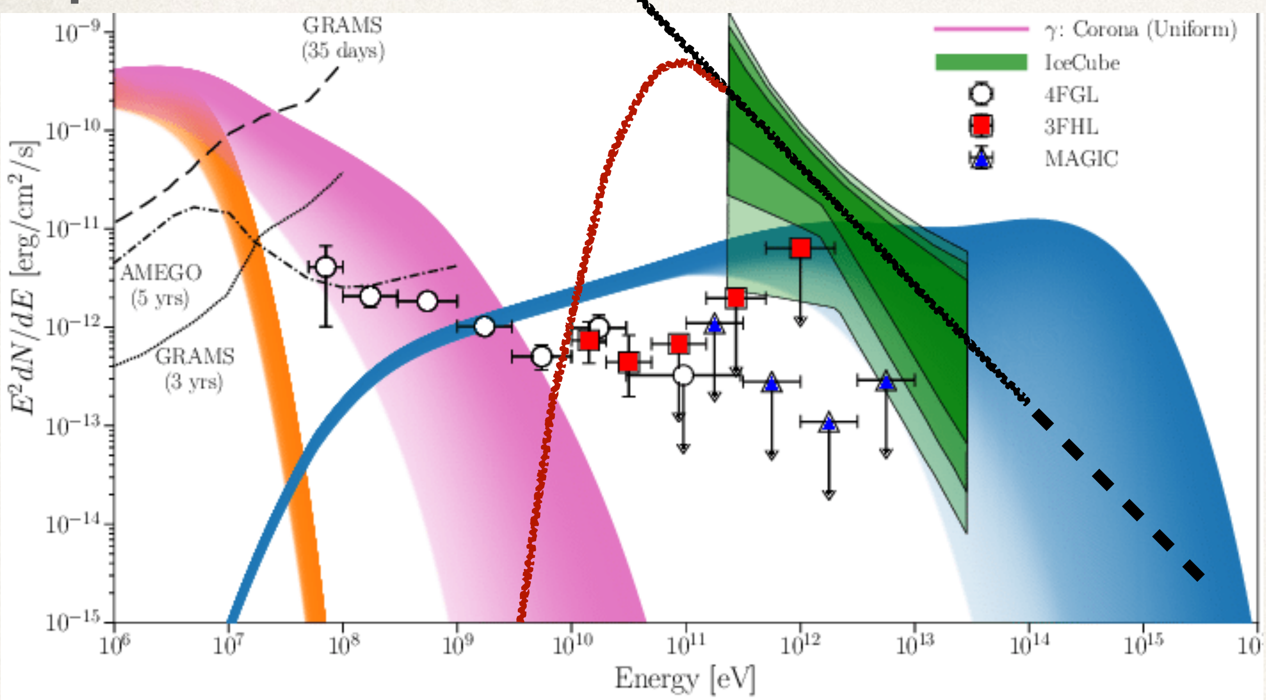
The diffuse neutrino flux by itself carries the seeds of a precious new piece of information: *most if not all the sources contributing to the diffuse flux must be obscured to gamma rays*

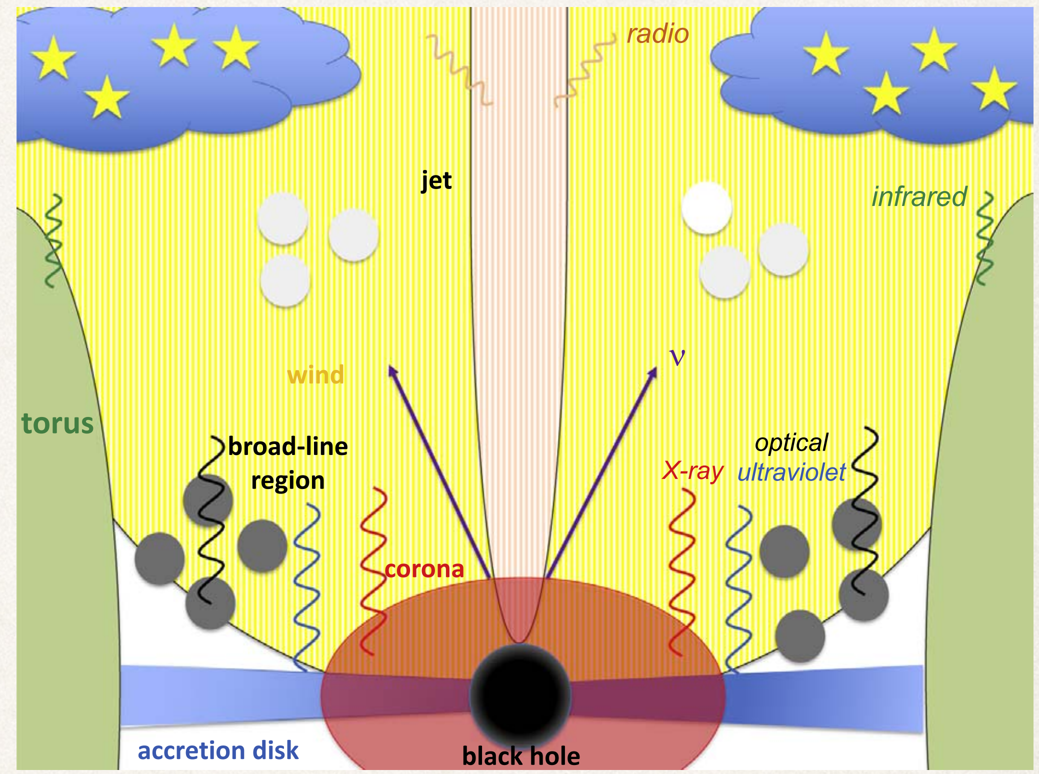
This suggests by itself that we are dealing with a new class of astrophysical objects, for which most photons and cosmic rays are trapped inside (neutrino cocoons)

The first clear source of neutrinos is a Seyfert 2

Adapted from Inoue et al. 2020

NGC1068

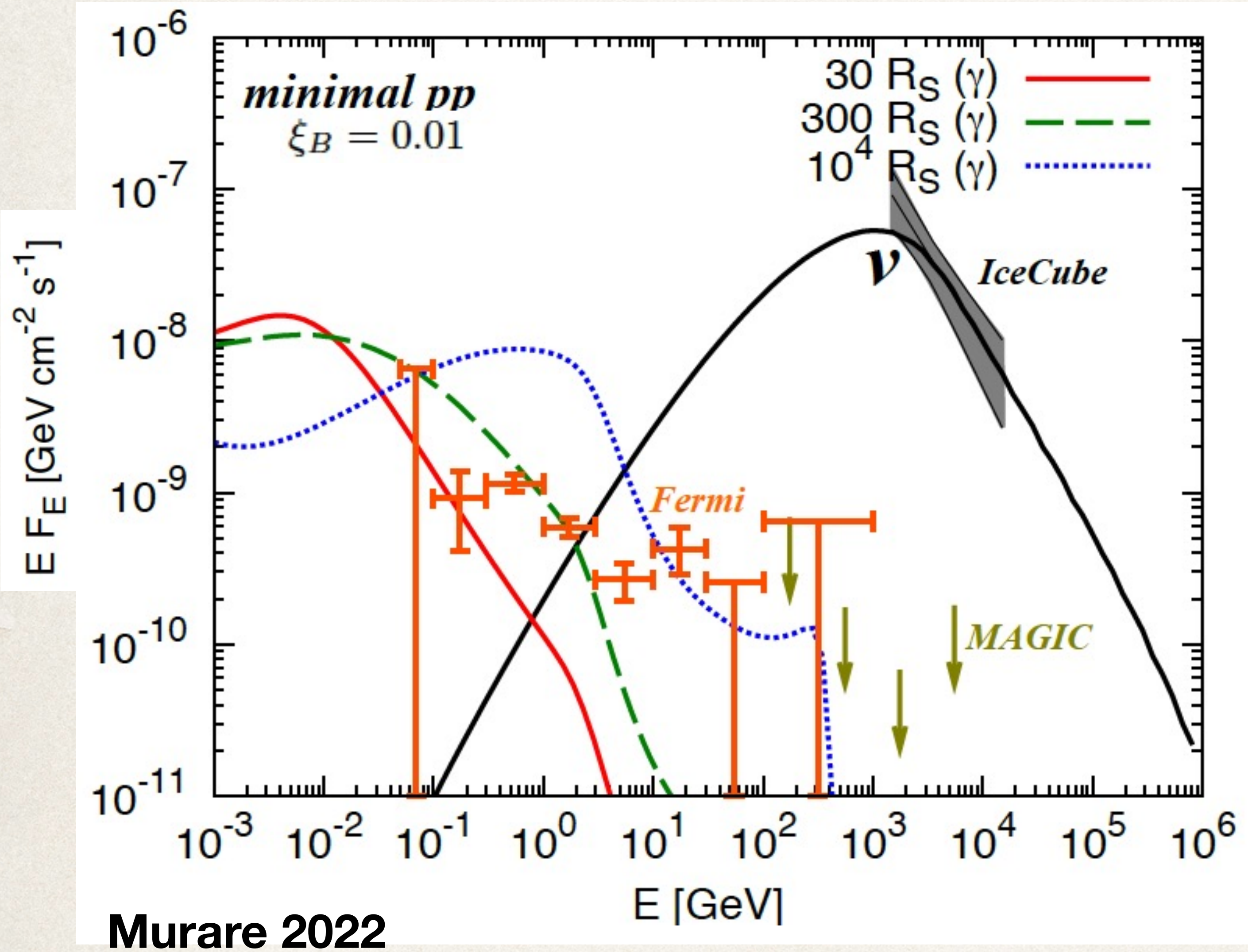




Murase, 2022

- *If to take the steep neutrino spectrum at face value, it would suggest a really wimpy accelerator (for instance a very weak shock)*
- *but shocks put most of their energy at low energies, which would violate energy conservation when the steep spectrum is extrapolated*
- *In this sense, the most natural explanation of the steep spectrum is that we are looking at a cutoff*
- *...and the only way of doing that is by inventing an effective accelerator with no escape (*second order turbulent acceleration!!!*) –* See also Talk by E. De Guveia-D'Alpino

How to look inside the BH corona...



Murare 2022

The gamma radiation produced together with neutrinos is eventually reprocessed inside the corona through E.M. cascade which buries the information in the form of gamma rays in the 1-10 MeV energy band, which can make it out!!!

It is of the utmost importance to investigate this region if we want to figure out what is going on inside the corona of massive BH

The fact that we do not see higher energy gamma rays constrains the size of the corona to be within a few tens of Schwarzschild radii!!!

General Remarks

- Experiments got so sensitive that statistics is rarely a problem, but systematics often limiting factor (think of C and O spectra)

- A topic that here was basically uncovered but it is probably one of the hottest topics is the existence of TeV halos and suppressed diffusion near sources

- The self-generation of turbulence is central to acceleration, to escape from sources and to transport on galactic scales, as well as likely for escape of UHECR from their sources - not discussed here

General Remarks

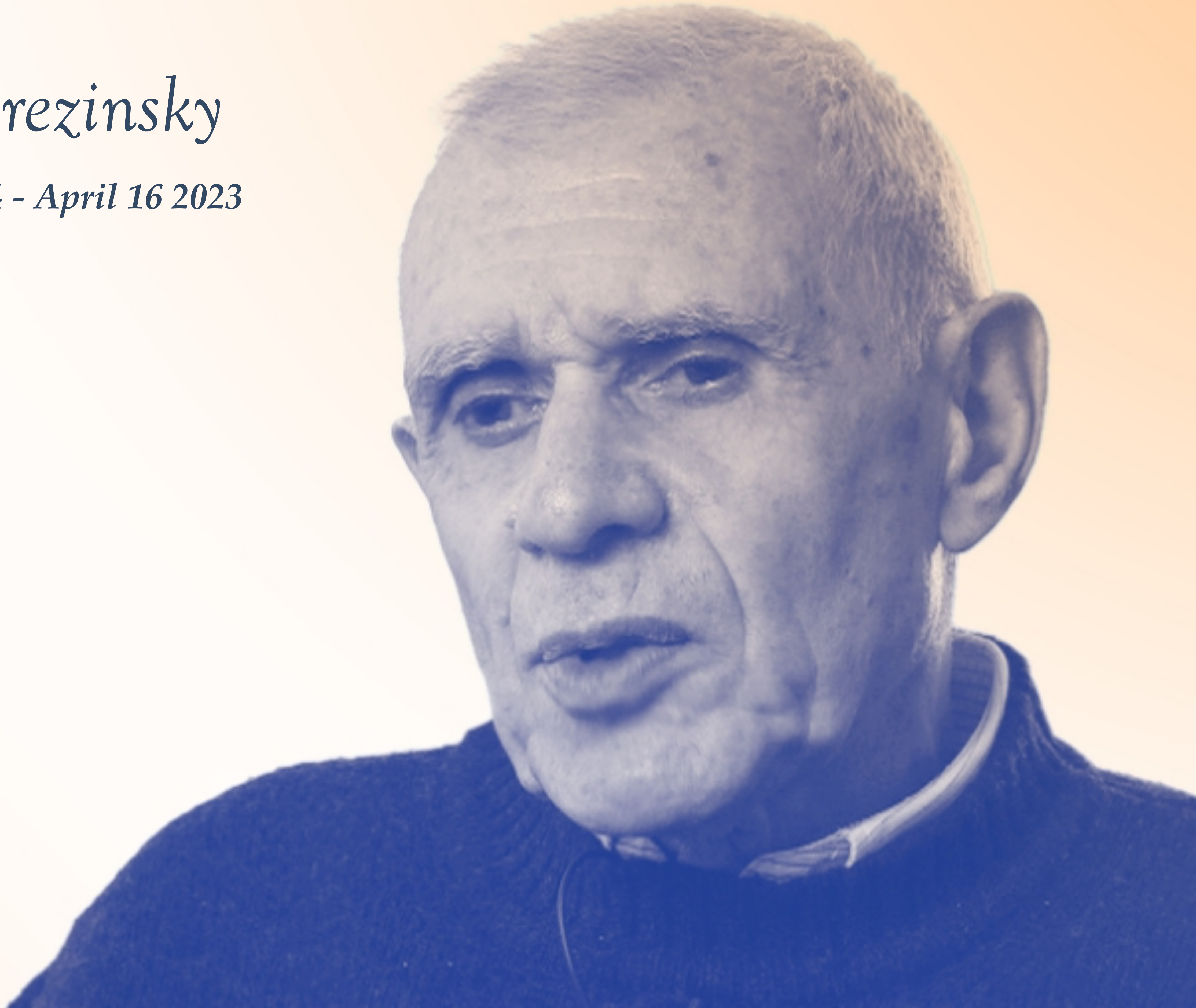
- These are considerations that play a crucial role not only for theory but observation (think of the distinction between diffuse flux and near-source interactions, or UHECR suppression at low E, or grammage experienced by CRs)
- On Galactic scales self-generation ceases to be important at few hundred GV, and at higher energies we still lack a satisfactory theory of CR scattering. We do not even know if we need one…
- it is clear that B-fields on cosmological scales may play a crucial role in shaping the UHECR spectrum (magnetic horizon)…yet there is currently no clear indication that there is any decent B in voids

General Remarks

- Seeking PeVatrons remains a priority but at present normal SNRs have a hard time, and star clusters are only now being investigated, but it doesn't look good
- Very luminous trans-relativistic SNRs are the only exception, but very rare, hard to see in gamma rays

V.S. Berezinsky

April 17 1934 - April 16 2023



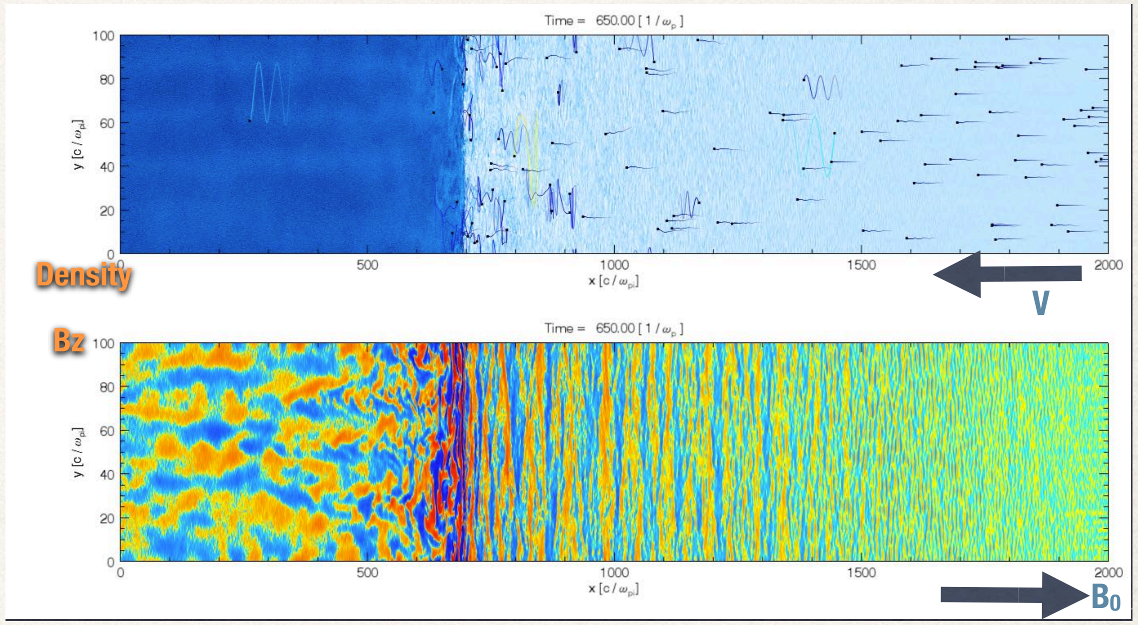
Additional Material for Discussion

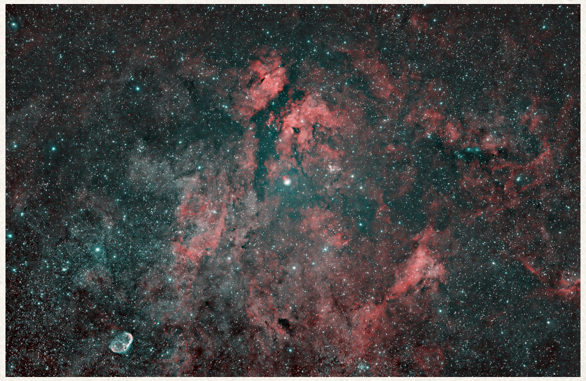
Galactic PeVatrons

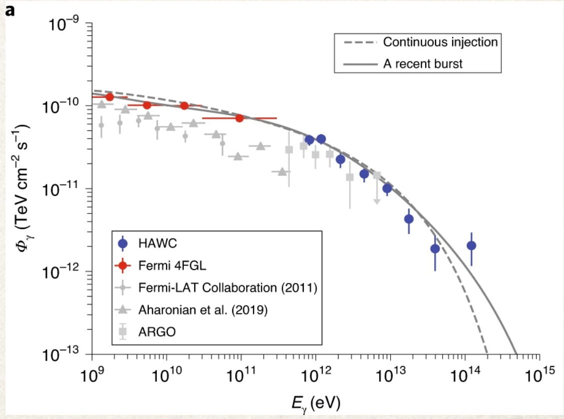
THE PROBLEM OF ACCELERATING COSMIC RAYS TO PeV ENERGIES REMAINS AS SERIOUS AS EVER, EVEN IN THE AFTERMATH OF THE DISCOVERY OF FAST CR INDUCED INSTABILITIES

MUCH INVESTIGATION IS TAKING PLACE IN THE DIRECTION OF HIGH PERFORMANCE COMPUTATION OF THE MICROPHYSICS OF PARTICLE ACCELERATION

...NOT ONLY IN SUPERNOVA REMNANTS BUT ALSO IN OTHER CLASSES OF ASTROPHYSICAL OBJECTS, ESPECIALLY STAR CLUSTERS WHERE VHE GAMMA RAYS HAVE BEEN DETECTED BY HAWC AND LHASSO

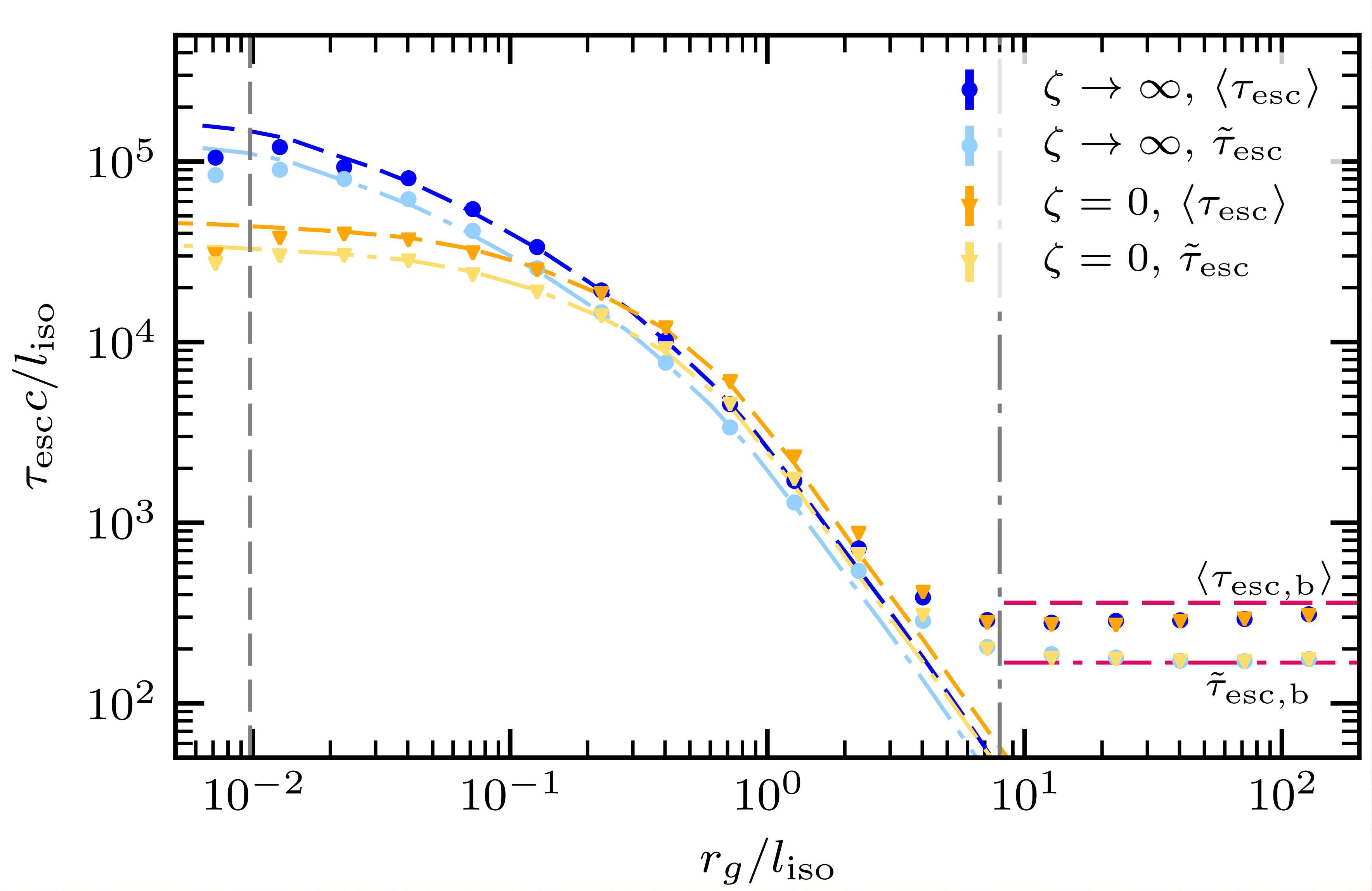




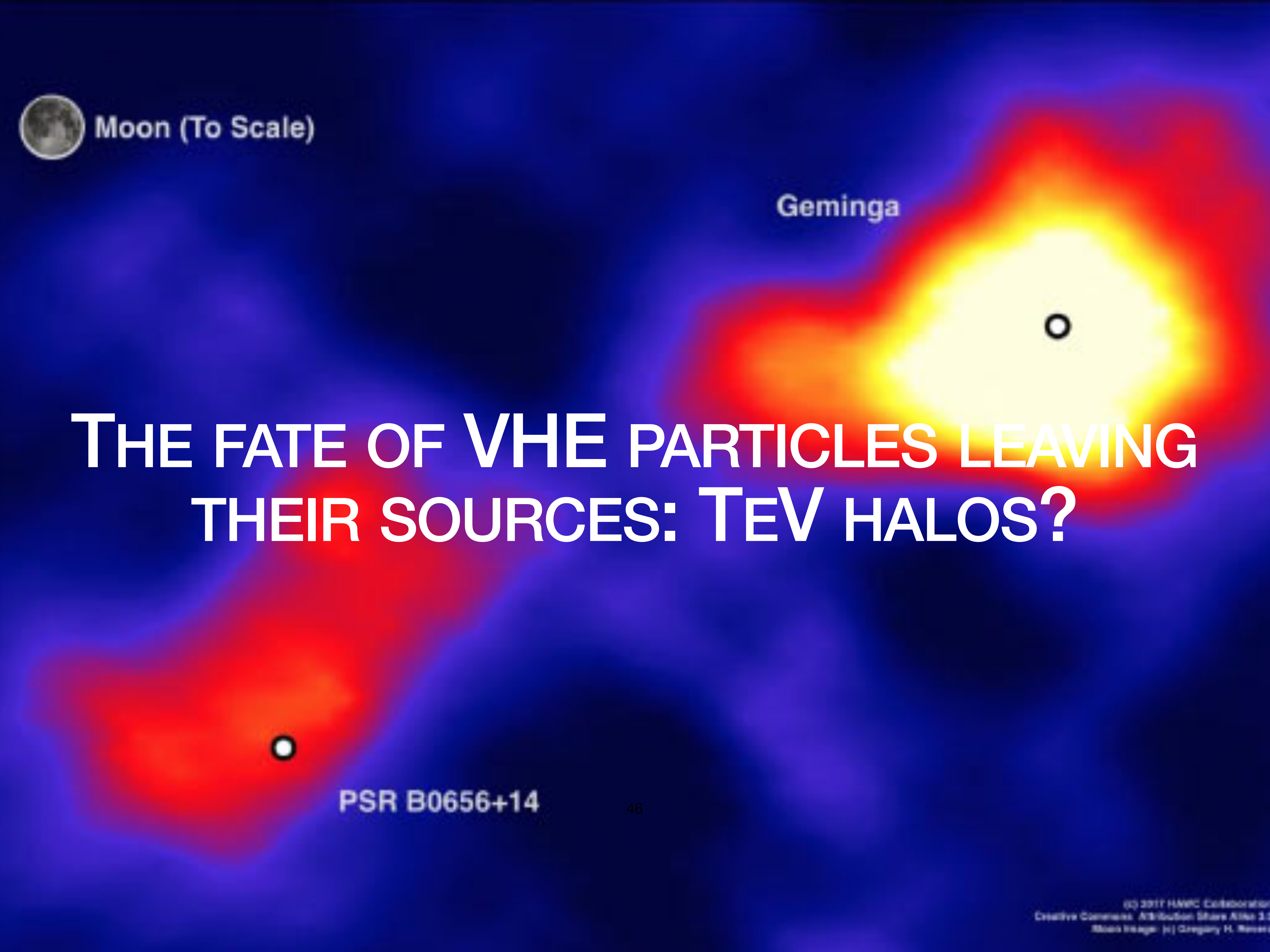


Confinement time with no resonances

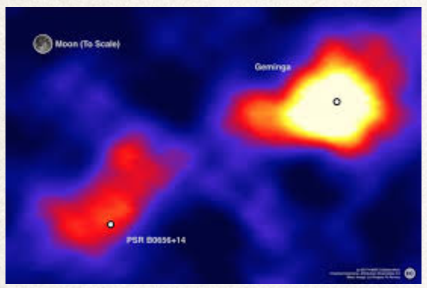
Pezzi&PB 2023

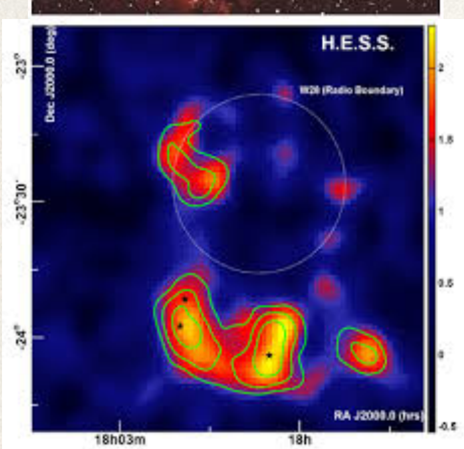


THE FATE OF VHE PARTICLES LEAVING THEIR SOURCES: TEV HALOS?



Reduced diffusivity around sources: WHY???





HAWC has recently detected regions of extended gamma ray emission around selected PWNe, in the >TeV energy region, suggesting that the diffusion coefficient in these regions is ~1/100 of the Galactic one [Abeysekara+ 2017]

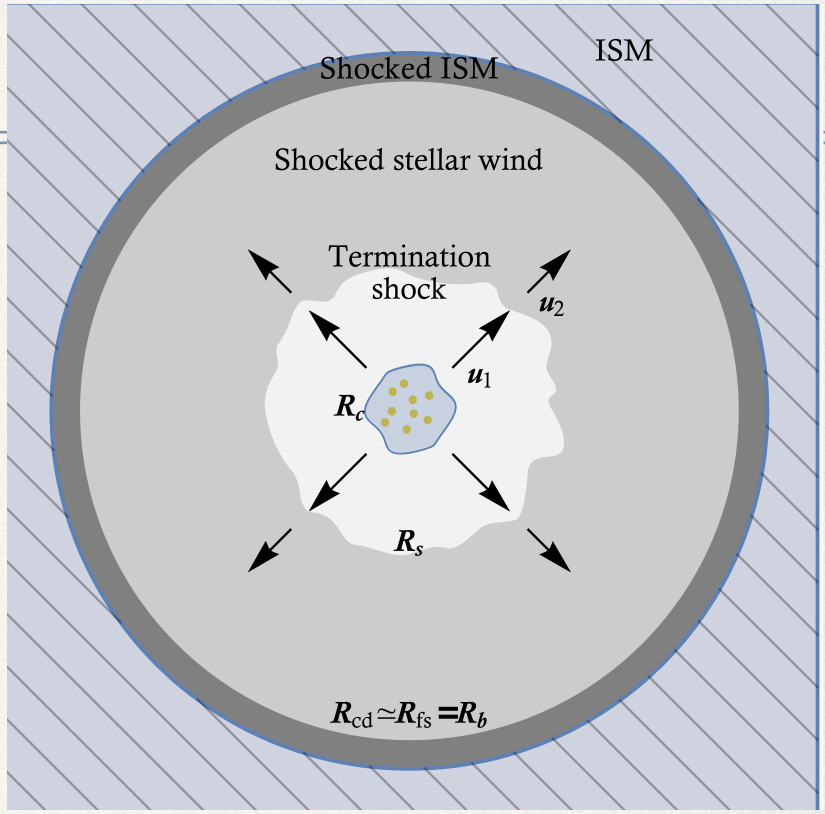
HESS observations of several star clusters have also shown extended regions (~100 pc) with TeV gamma ray emission, with inferred D(E)<< than the Galactic one [Aharonian+ 2018]

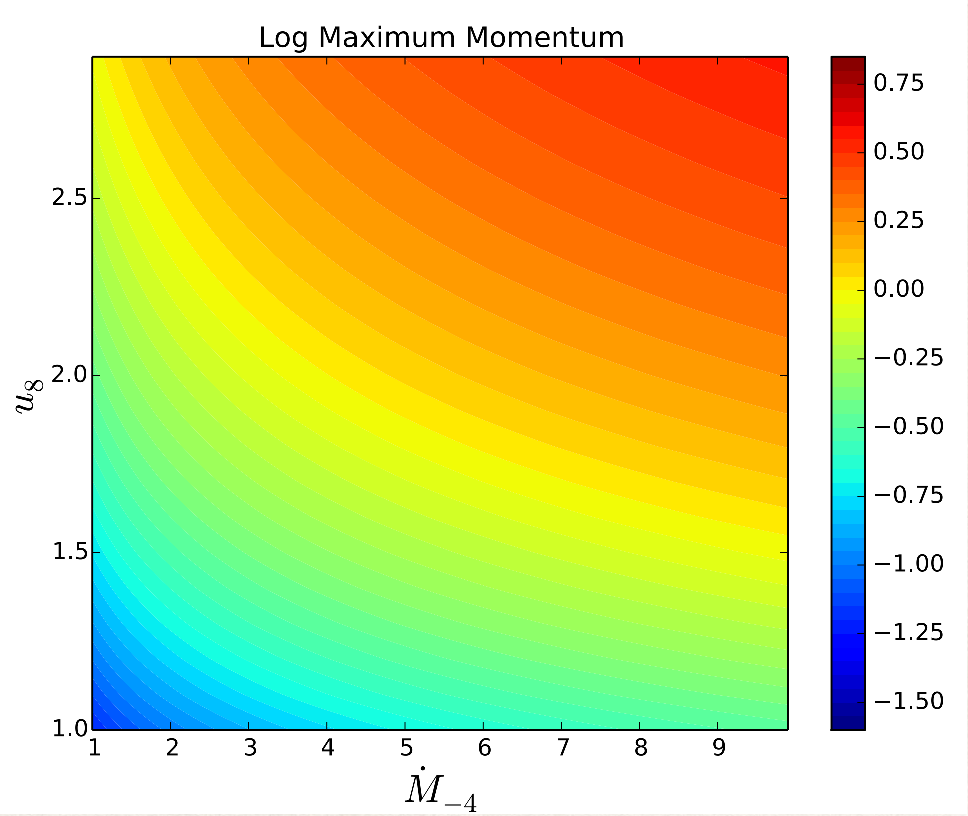
Evidence from gamma ray observations of gamma ray emission from molecular clouds positioned at different distances from SNRs (for instance W28) that the diffusion coefficient is ~1/40 of the Galactic one [Gabici+ 2010]

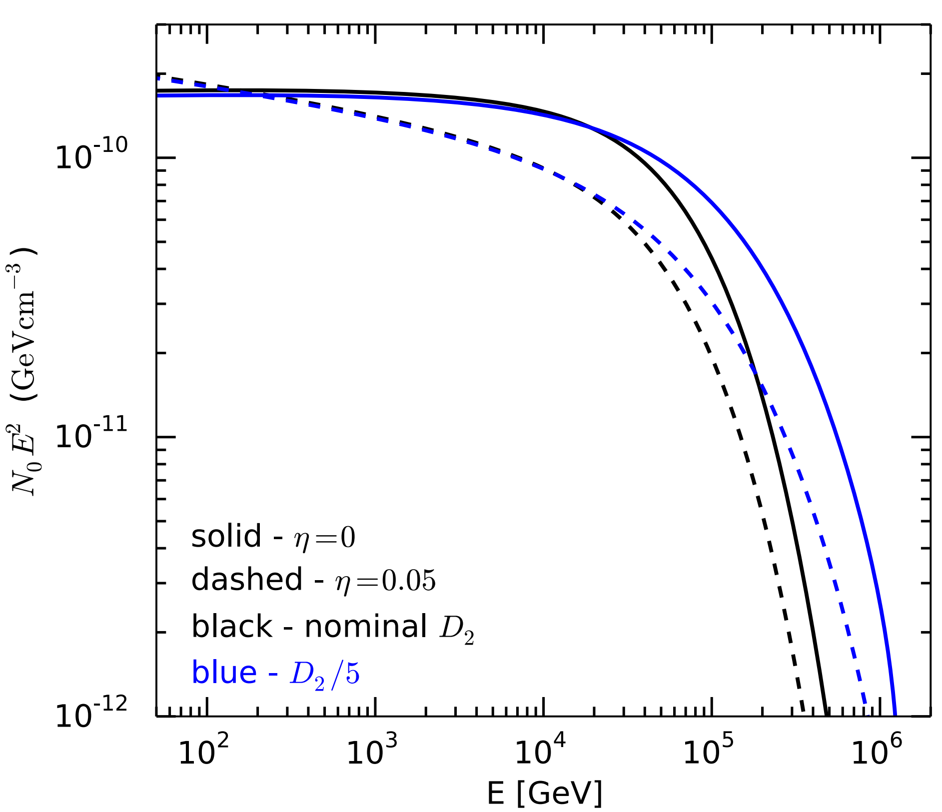
Acceleration in Star Clusters

- Star clusters/superbubbles may in principle be efficient accelerators — DSA but in spherical symmetry (Talk by E. Peretti)
- As discussed by V. Tatischeff these structures would address and probably solve the ^{22}Ne problem
- The E_{max} can be estimated as:

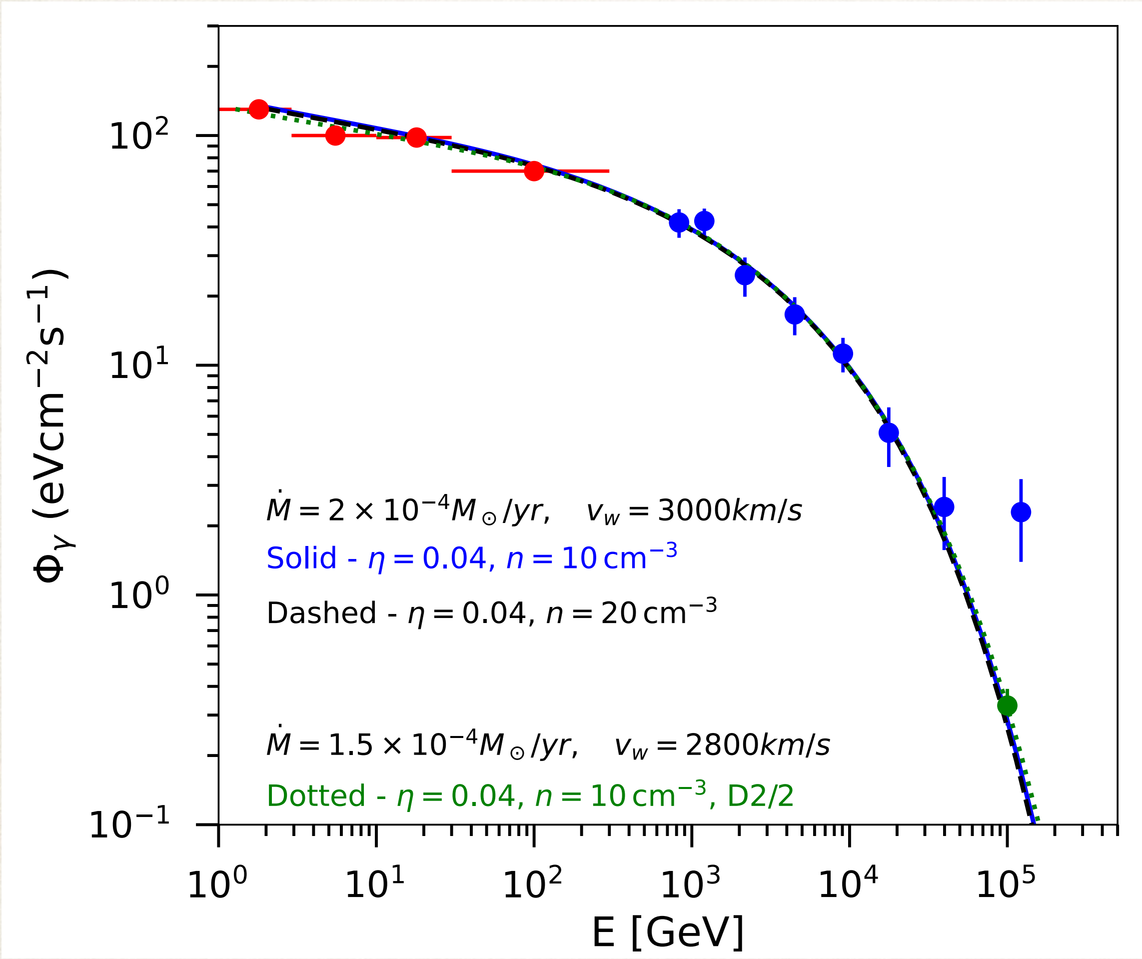
$$E_{\rm max} \approx 4\times 10^{14}\, \eta_B^{1/2} \dot{M}_{-4}^{4/5} v_8^{13/5} \rho_1^{-3/10} t_{10}^{2/5} \left(\frac{L_c}{2\rm pc}\right)^{-1} \ {\rm eV}$$

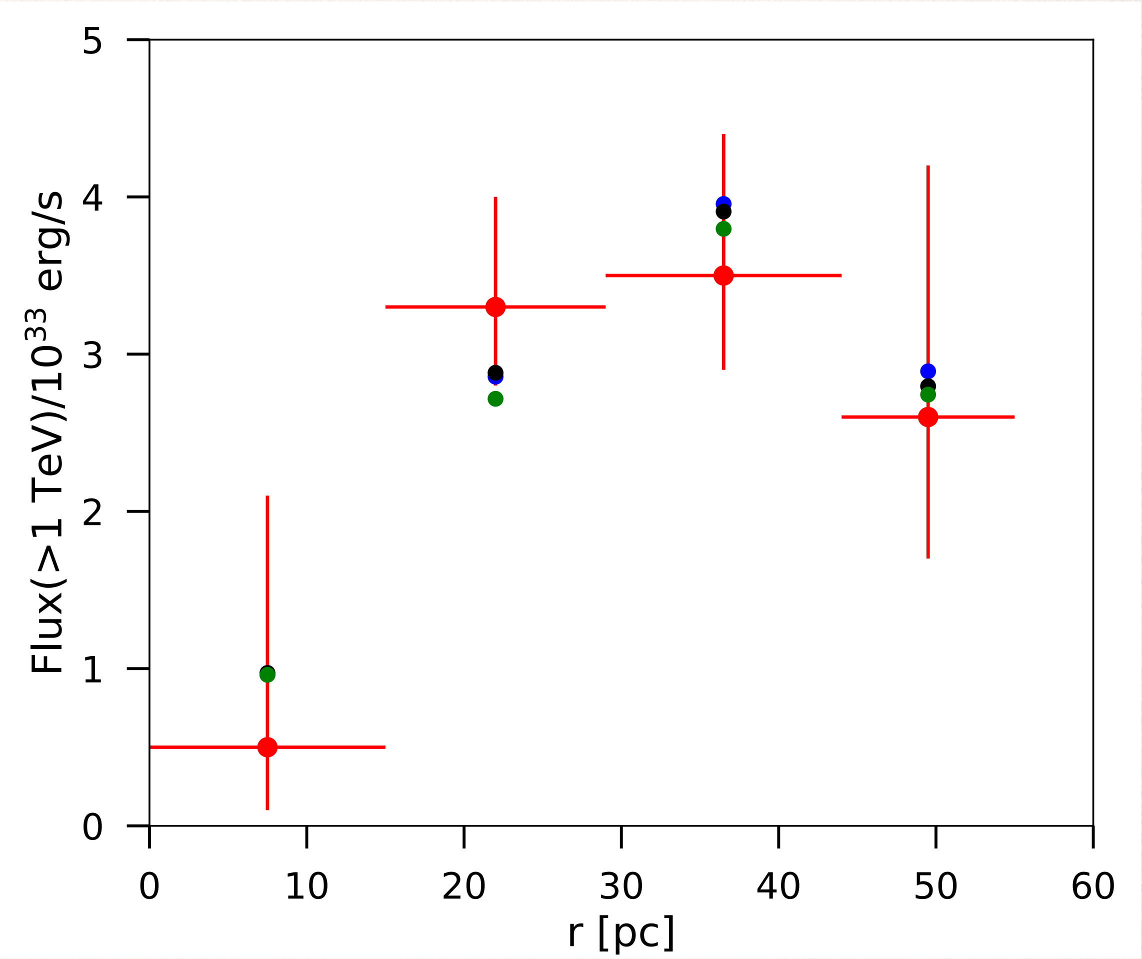




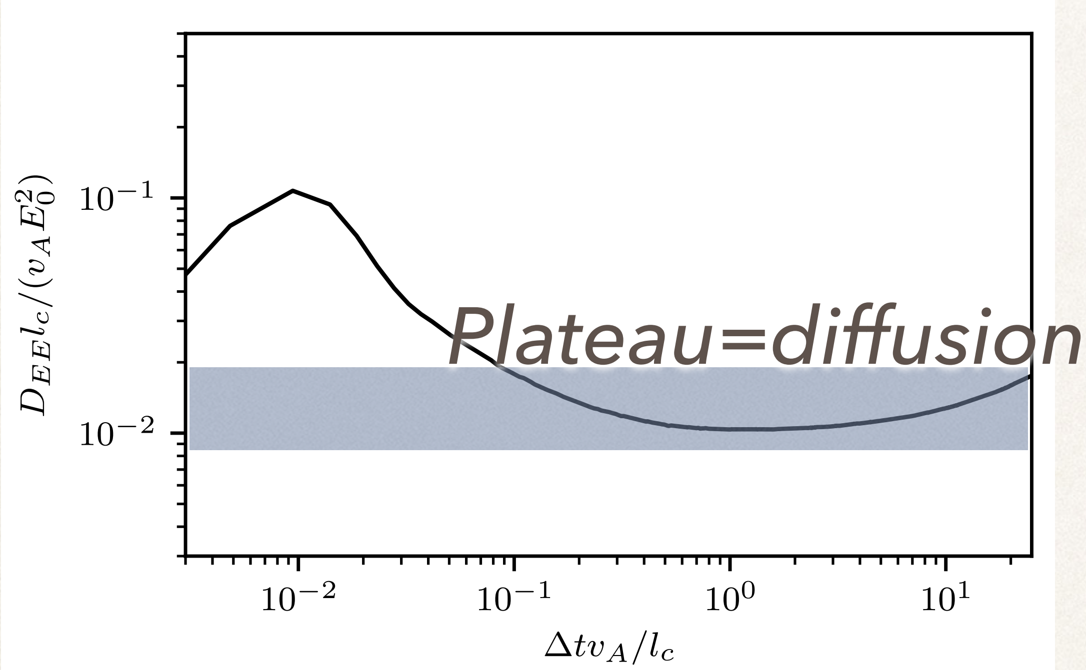


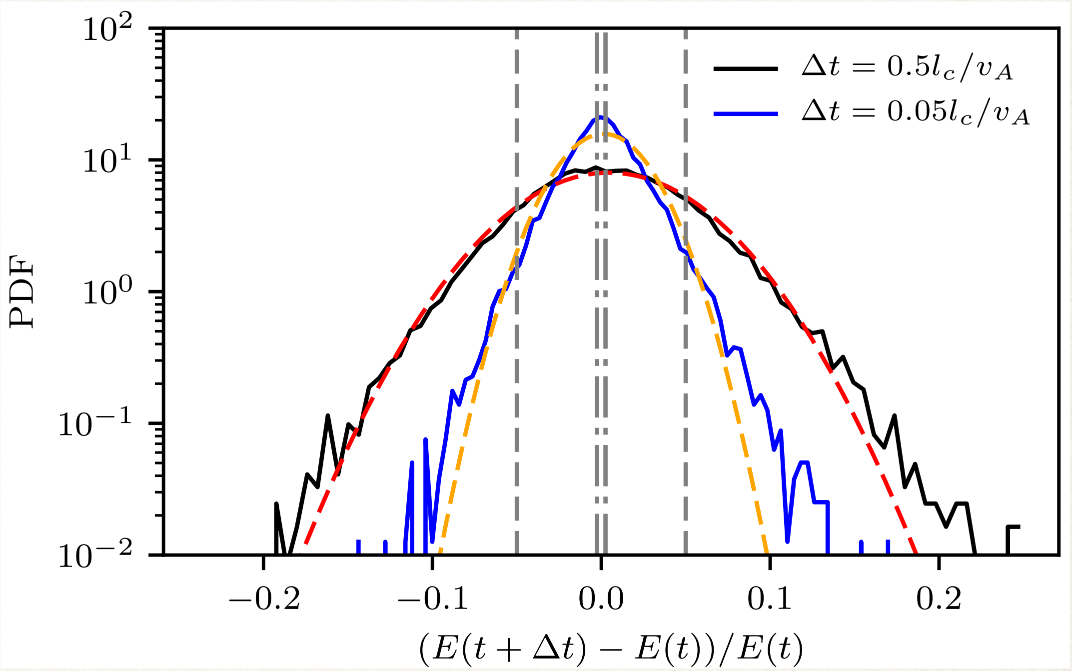
Acceleration in Star Clusters: the case of Cygnus OB2





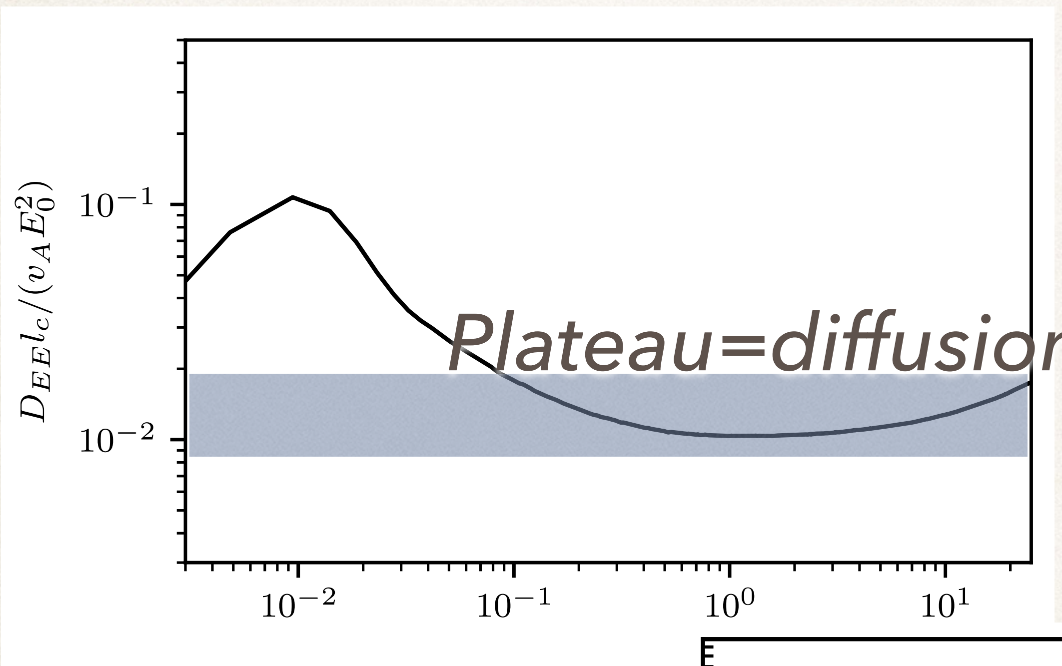
DIFFUSION IN MOMENTUM SPACE
A.K.A. SECOND ORDER FERMI ACCELERATION

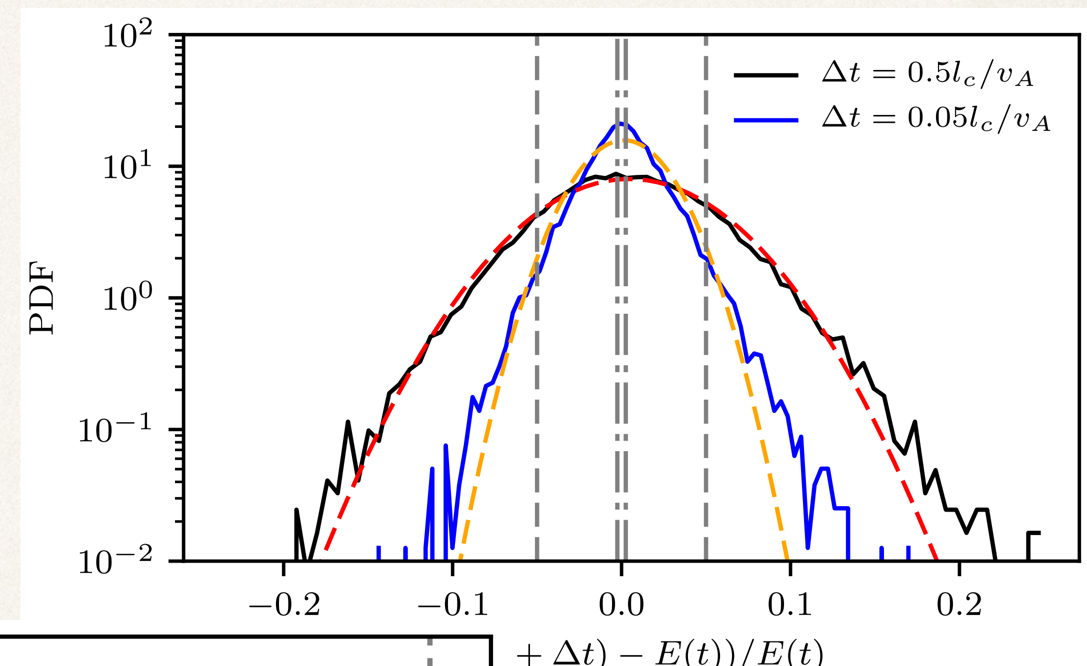


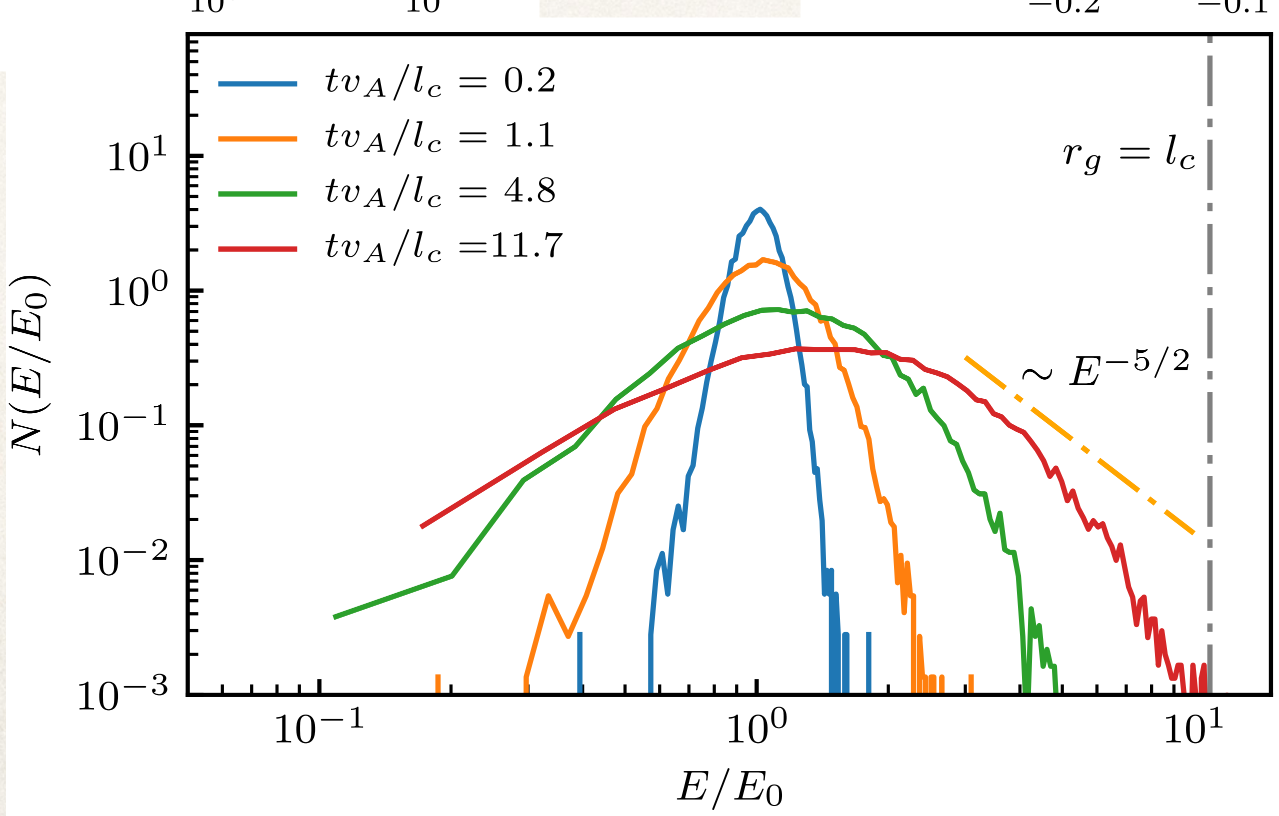


DIFFUSION IN MOMENTUM SPACE

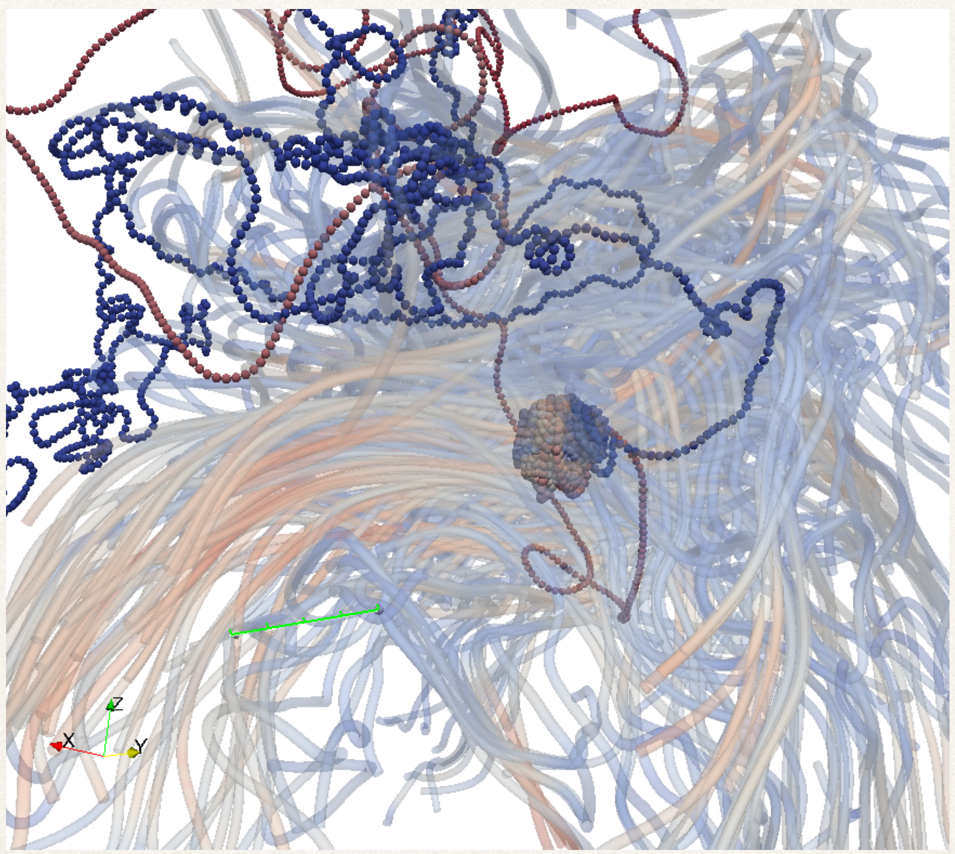
A.K.A. SECOND ORDER FERMI ACCELERATION

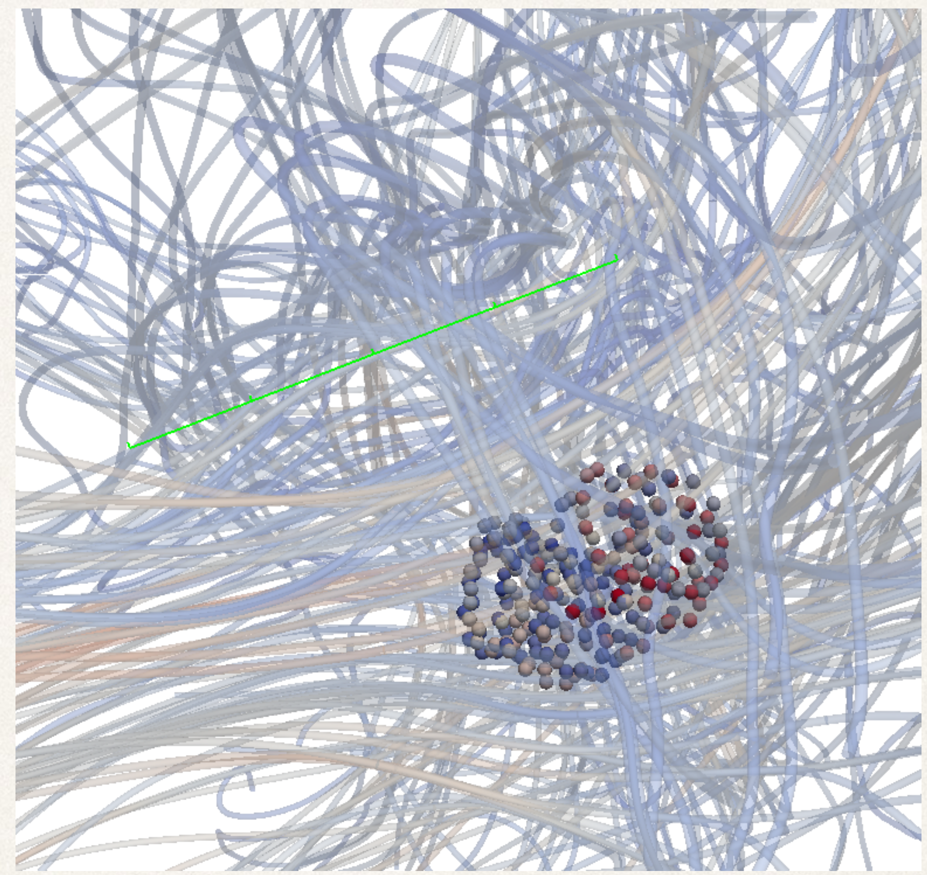






SO FAR SO GOOD…BUT SOMETHING NEW POPPED OUT — PARTICLE TRAPPING



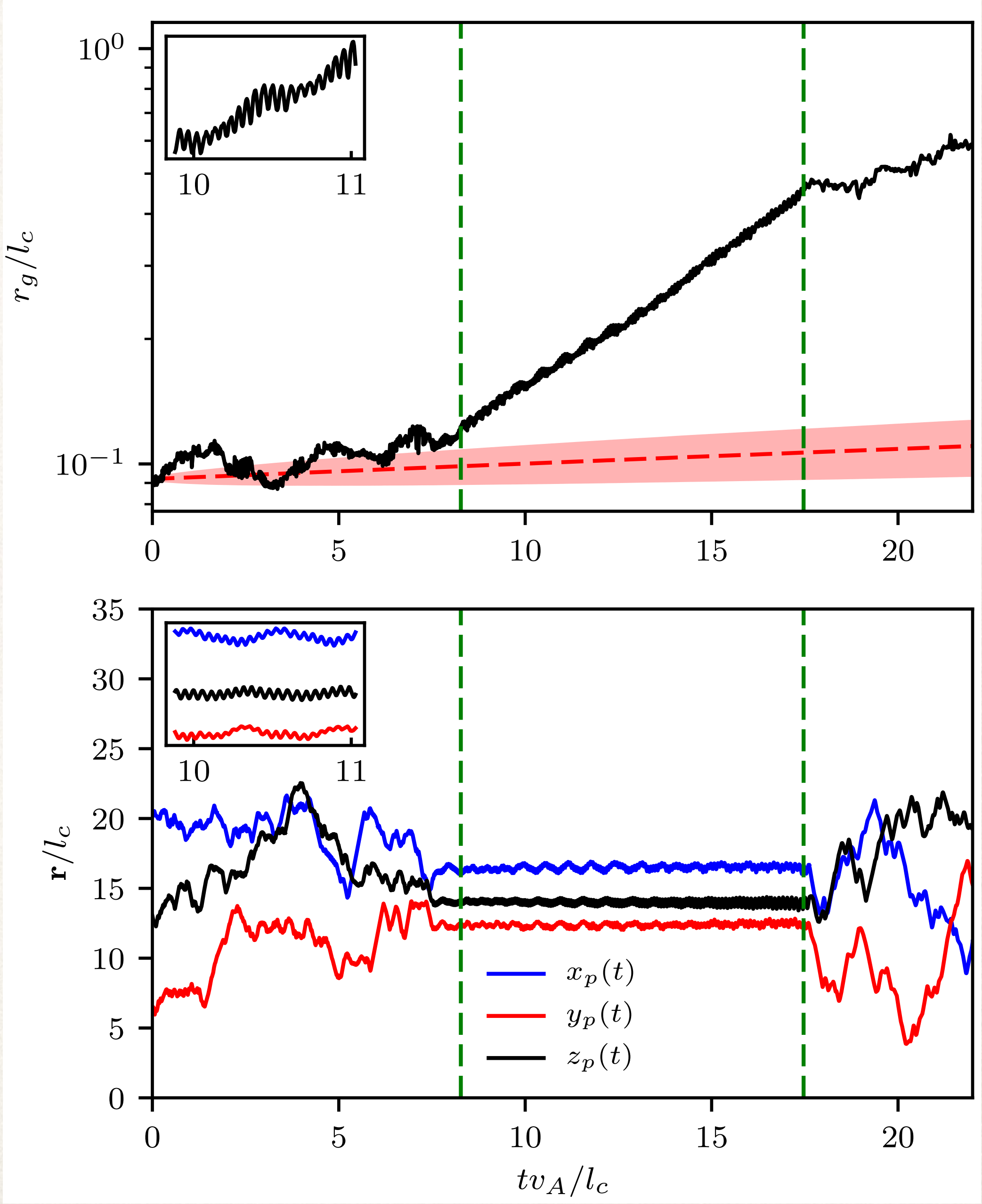


A FEW OUT OF 100,000 PARTICLES SEEM TO EXPERIENCE THIS PHENOMENON–

BUT THOSE FEW PARTICLES BEHAVE IN VERY PECULIAR MANNER

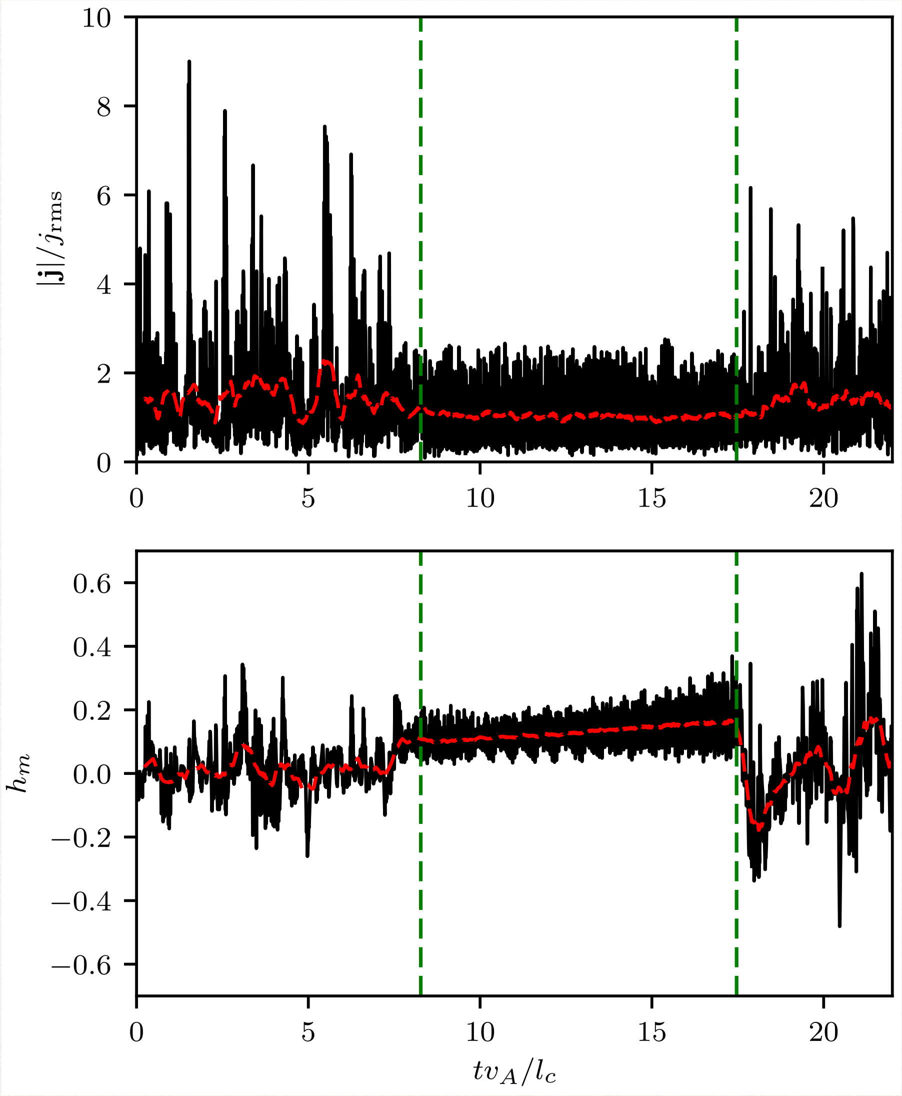
PARTICLE TRAPPING — EXPONENTIAL ENERGY INCREASE

- FOR THE LONGEST TIME PARTICLES SIMPLY DIFFUSE IN SPACE (AND ENERGY)
- THEN EVENTUALLY A FEW OF THEM GET TRAPPED SOMEWHERE
- DURING THOSE PERIODS THE ENERGY GROWS EXPONENTIALLY
- …UNTIL THEY EVENTUALLY ESCAPE THE TRAPPING REGION



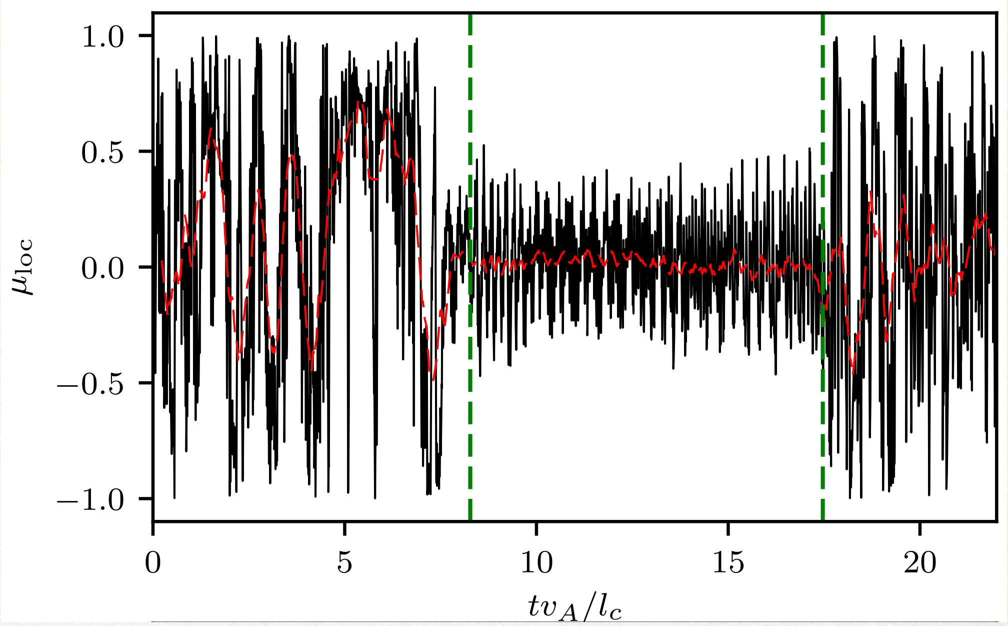
WHAT ARE THESE TRAPPING REGIONS?

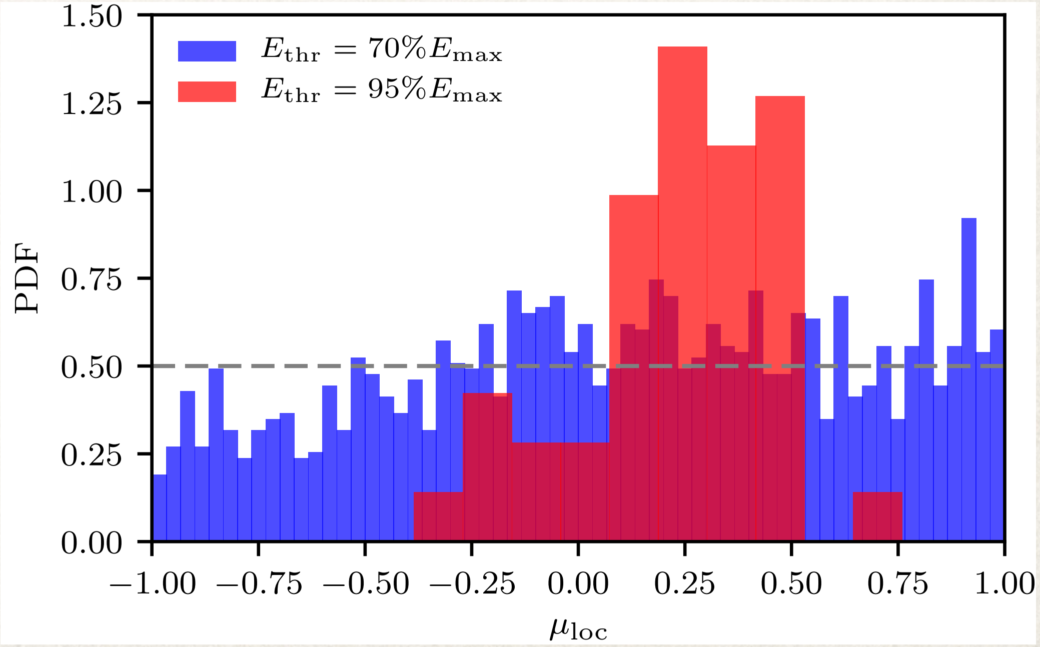
- IN THE TRAPPING REGIONS THE CURRENT IS NOT LARGE!
- THESE ARE NOT RECONNECTION REGIONS, WHERE USUALLY PEOPLE ASSUME INTERESTING THINGS SHOULD HAPPEN
- THIS IS CONFIRMED BY THE HELICITY WHICH ALSO IS NOT



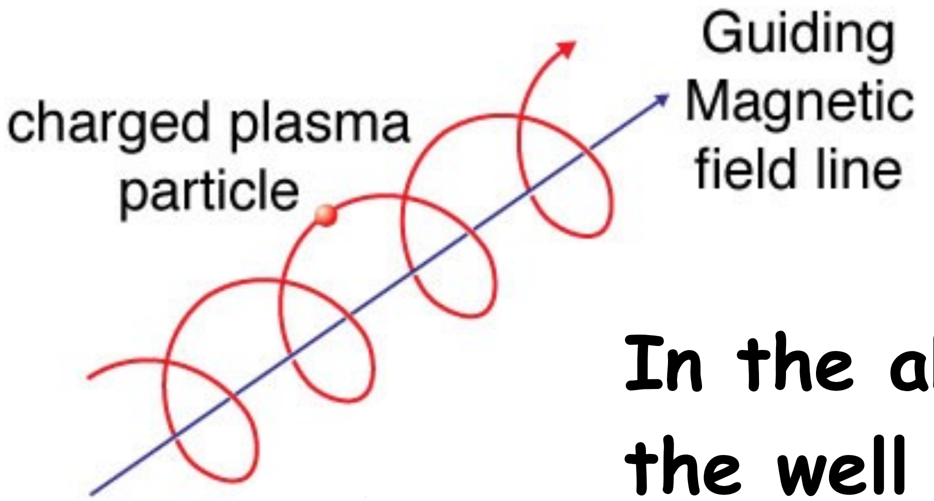
Pitch angle selection

- THE PARTICLES THAT ARE TRAPPED IN THE REGION HAVE COSINE OF THE PITCH ANGLE VERY CLOSE TO ZERO
- THE COSINE SHOWS FLUCTUATIONS ON A SCALE OF Δμ~0.2 (conservation of adiabatic invariant?)
- BUT THE FLUCTUATIONS ARE MUCH LARGER OUTSIDE THE REGION
- PARTICLES ARE TRAPPED IN THERE!!!





CHARGED PARTICLES IN A REGULAR B FIELD



$$\frac{d\vec{p}}{dt} = q\left[\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right]$$

In the absence of an electric field one obtains the well known solution:

$$p_z = \text{Constant}$$

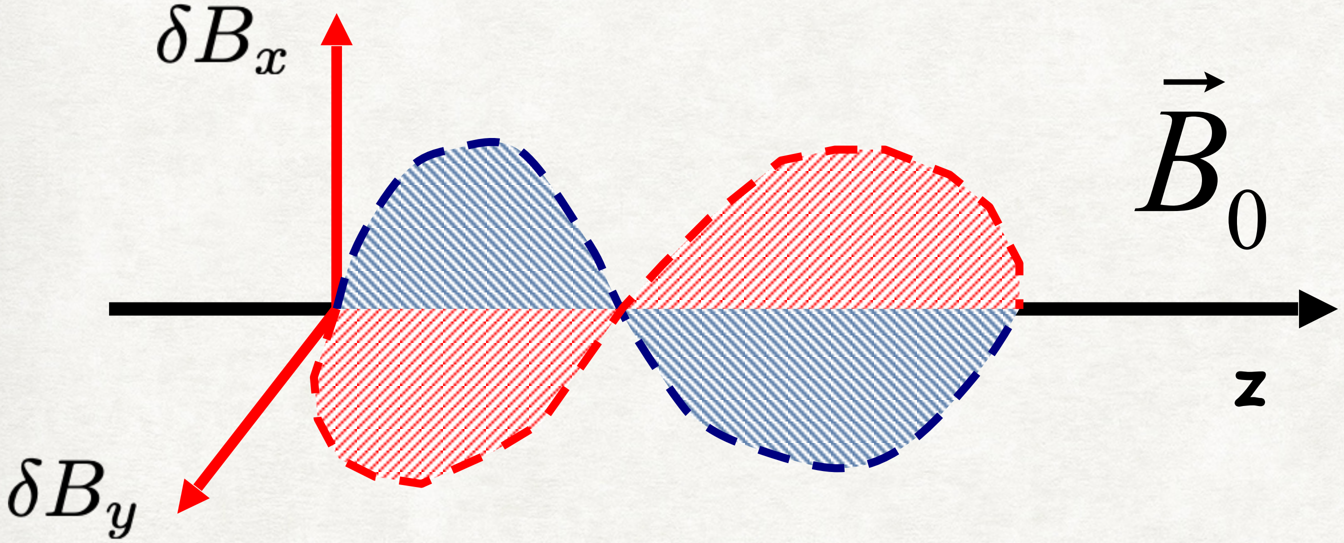
$$v_x = V_0 \cos[\Omega\, t]$$

$$v_y = V_0 \sin[\Omega\, t]$$

LARMOR FREQUENCY

$$\Omega = \frac{q\, B_0}{m\, c\, \gamma}$$

MOTION OF A PARTICLE IN A WAVY FIELD



Let us consider an Alfven wave propagating in the z direction:

$$\delta B << B_0 \qquad \delta\vec{B} \perp \vec{B}_0$$

We can neglect (for now) the electric field associated with the wave, or in other words we can sit in the reference frame of the wave:

$$\frac{d\vec{p}}{dt} = q\,\frac{\vec{v}}{c} \times (\vec{B}_0 + \delta\vec{B})$$

THIS CHANGES ONLY THE X AND Y COMPONENTS OF THE MOMENTUM

THIS TERM CHANGES ONLY THE DIRECTION OF P_Z=Pμ

Remember that the wave typically moves with the Alfven speed:

$$v_a = \frac{B}{(4\pi\rho)^{1/2}} = 2 \times 10^6 B_\mu n_1^{-1/2} \ cm/s$$

Alfven waves have frequencies << ion gyration frequency $\Omega_p = qB/m_p c$

It is therefore clear that for a relativistic particle these waves, in first approximation, look like static waves.

The equation of motion can be written as:

$$\frac{d\vec{p}}{dt} = \frac{q}{c}\vec{v} \times (\vec{B}_0 + \vec{\delta}B)$$

If to split the momentum in parallel and perpendicular, the perpendicular component cannot change in modulus, while the parallel momentum is described by

$$\frac{dp \parallel}{dt} = \frac{q}{c}|\vec{v}_\perp \times \delta\vec{B}| \qquad p_{||} = p \ \mu$$

$$\frac{d\mu}{dt} = \frac{q}{pc} v(1-\mu^2)^{1/2} \delta B cos(\Omega t - kx + \psi)$$

Wave form of the magnetic field with a random phase and frequency

$$\Omega = qB_0/mc\gamma$$

Larmor frequency

In the frame in which the wave is at rest we can write

$$x = v\mu t$$

$$\frac{d\mu}{dt} = \frac{q}{pc} v(1-\mu^2)^{1/2} \delta B cos\left[(\Omega - kv\mu)t + \psi\right]$$

It is clear that the mean value of the pitch angle variation over a long enough time vanishes

$$\langle \Delta\mu \rangle_t = 0$$

We want to see now what happens to

$$\langle \Delta\mu \Delta\mu \rangle$$

Let us first average upon the random phase of the waves:

$$\langle \Delta\mu(t')\Delta\mu(t")\rangle_\psi = \frac{q^2 v^2 (1-\mu^2)\delta B^2}{2c^2p^2} cos\left[(\Omega - kv\mu)(t' - t")\right]$$

And integrating over time:

$$\langle \Delta\mu\Delta\mu\rangle_t = \frac{q^2 v^2 (1-\mu^2)\delta B^2}{2c^2p^2} \int dt' \int dt" \, cos\left[(\Omega - kv\mu)(t' - t")\right]$$

$$= \frac{q^2 v (1-\mu^2)\delta B^2}{c^2p^2\mu} \delta(k - \Omega/v\mu)\Delta t$$

RESONANCE

Many waves

IN GENERAL ONE DOES NOT HAVE A SINGLE WAVE BUT RATHER A POWER SPECTRUM:

$$P(k) = B_k^2/4\pi$$

THEREFORE INTEGRATING OVER ALL OF THEM:

$$\langle \frac{\Delta\mu\Delta\mu}{\Delta t} \rangle = \frac{q^2(1-\mu^2)\pi}{m^2c^2\gamma^2}\frac{1}{v\mu}4\pi \int dk \frac{\delta B(k)^2}{4\pi}\delta(k-\Omega/v\mu)$$

OR IN A MORE IMMEDIATE FORMALISM:

$$\left\langle \frac{\Delta\mu\Delta\mu}{\Delta t} \right\rangle = \frac{\pi}{2}\Omega(1-\mu^2)k_{res}F(k_{res})$$

$$k_{res} = \frac{\Omega}{v\mu}$$

RESONANCE!!!

DIFFUSION COEFFICIENT

THE RANDOM CHANGE OF THE PITCH ANGLE IS DESCRIBED BY A DIFFUSION COEFFICIENT

$$D_{\mu\mu} = \left\langle \frac{\Delta\theta\Delta\theta}{\Delta t} \right\rangle = \frac{\pi}{4}\Omega k_{res} F(k_{res})$$

FRACTIONAL POWER $(\delta B/B_0)^2$ $= G(k_{res})$

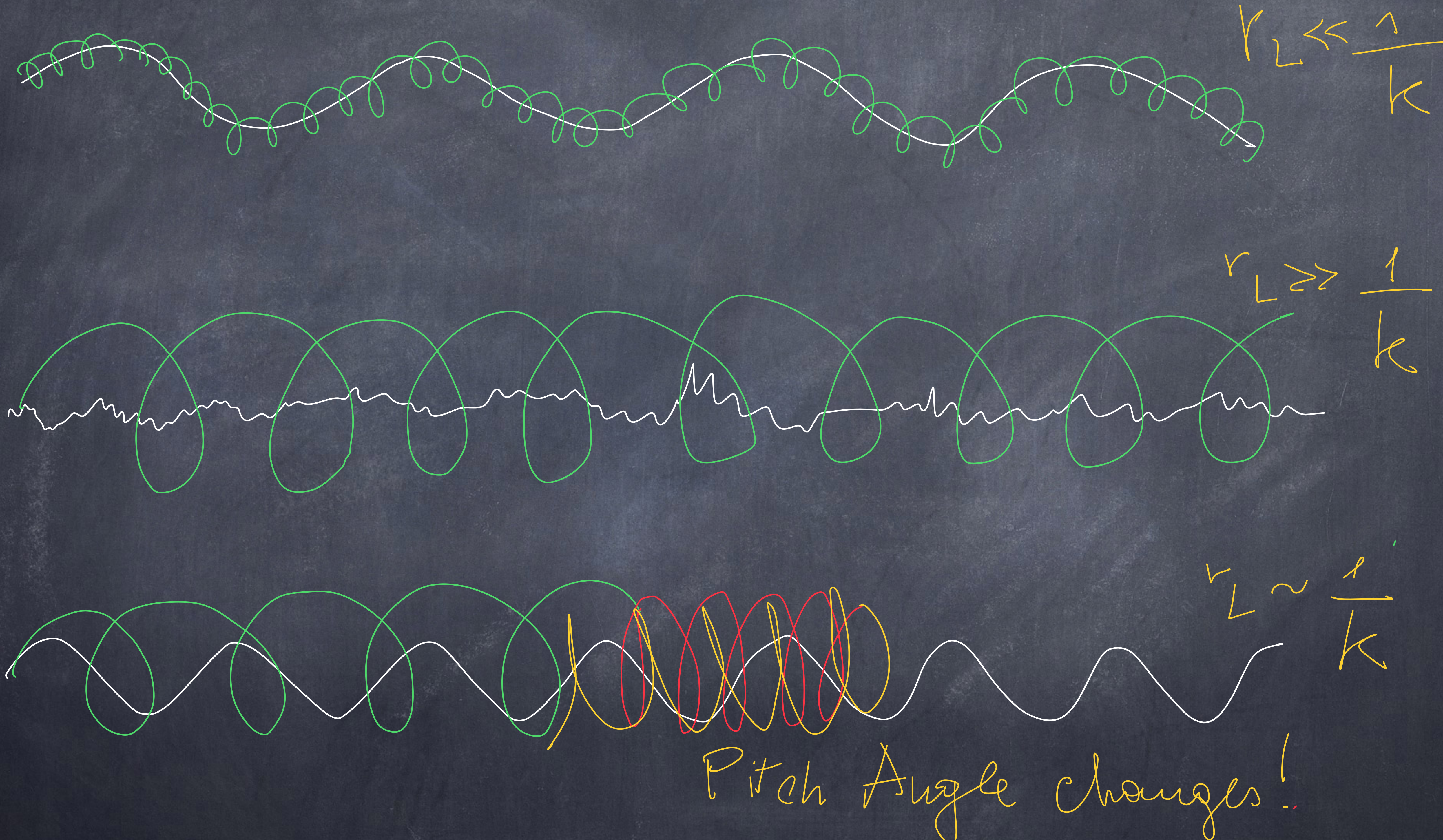
THE DEFLECTION ANGLE CHANGES BY ORDER UNITY IN A TIME:

$$\tau \approx \frac{1}{\Omega\, G(k_{res})}$$

PATHLENGTH FOR DIFFUSION ~ VT

$$\left\langle \frac{\Delta z\Delta z}{\Delta t} \right\rangle \approx v^2\,\tau = \frac{v^2}{\Omega\, G(k_{res})}$$

SPATIAL DIFFUSION COEFF.


$r_L \ll \frac{1}{k}$
$r_L \gg \frac{1}{k}$
$r_L \sim \frac{1}{k}$
Pitch Angle changes!