### Cosmic rays in a turbulent interstellar medium: Recent progress and open questions

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What are the sources?







Abeysekara et al. (2019)



$$\psi(\mathbf{p}) = \psi_0(\mathbf{p})(1 + a\,\hat{\mathbf{a}}\cdot\hat{\mathbf{p}} + \ldots) \quad \text{with} \quad \mathbf{a} = rac{\lambda 
abla \psi}{\psi}$$



• Amplitude: 
$$a \equiv \frac{\phi_{\max} - \phi_{\min}}{\phi_{\max} + \phi_{\min}} \sim \frac{\psi(\mathbf{r} + \lambda \hat{\mathbf{a}}) - \psi(\mathbf{r} - \lambda \hat{\mathbf{a}})}{\psi(\mathbf{r} + \lambda \hat{\mathbf{a}}) + \psi(\mathbf{r} - \lambda \hat{\mathbf{a}})} = \frac{\lambda |\nabla \psi|}{\psi}$$
  
• Direction:  $\hat{\mathbf{a}} \sim \nabla \psi$ 

Mertsch & Funk (2014); also Ahlers (2016)

### Assumptions

- **1** No coherent magnetic field
- 2 No deviations from average over **B**-fields

$$\psi(\mathbf{p}) = \psi_0(p)(1+a\,\hat{\mathbf{a}}\cdot\hat{\mathbf{p}}+\ldots) \quad \text{with} \quad \mathbf{a} = rac{\lambda 
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$$\psi(\mathbf{p}) = \psi_0(\mathbf{p})(1+a \underbrace{\hat{\mathbf{b}} \cdot \hat{\mathbf{p}}}_{\mu=\cos \theta} + \ldots) \text{ with } a \simeq \frac{\lambda \hat{\mathbf{b}} \cdot \nabla \psi}{\psi}$$

The dipole does not point back to the source and its amplitude is reduced by the projection.

Mertsch & Funk (2014); also Ahlers (2016)

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$$\langle \psi(\mathbf{p}) 
angle = \psi_0(p)(1 + a \underbrace{\hat{\mathbf{b}} \cdot \hat{\mathbf{p}}}_{\mu = \cos \theta} + \ldots) \quad \text{with} \quad a \simeq \frac{\lambda \hat{\mathbf{b}} \cdot \nabla \psi}{\psi}$$

### Dipole anisotropies as a constraint

Amato & Blasi (2012); Evoli et al. (2012); Mertsch & Funk (2014); Evoli et al. (2022)



### Dipole data in CRDB

with M. Ahlers, H. Dembinski, D. Maurin



- CR database: https://lpsc.in2p3.fr/crdb/
- Python package crdb: https://github.com/crdb-project

# Outline

Motivation

#### **2** Small-scale anisotropies

**③** Test-particle simulations

O Suppressed diffusion

**6** Conclusions

### Small-scale anisotropies



- subtract off dipole and quadrupole
- smooth with  $5^{\circ}$  disk
- $\rightarrow$  small-scale features

Abeysekara et al., ApJ 871 (2019) 96

#### Energy-dependence

• No time-dependence

### Angular power spectrum



#### IceCube

Aartsen et al., ApJ 826 (2016)

#### 220 IceCube+HAWC

Abeysekara *et al.*, ApJ 871 (2019) 96



### Diffusion approximation

Jokipii (1968)

• Decompose  $\langle f \rangle$  into series in pitch-angle cosine  $\mu$ :

$$\langle f \rangle(p,\mu,t) = g(p,t) + h(p,\mu,t)$$

where

$$h(p,\mu,t) = -rac{v}{2}rac{\partial g}{\partial z}\int\mathrm{d}\mu rac{1-\mu^2}{D_{\mu\mu}} + \mathrm{const.}$$

• If scattering is isotropic,  $D_{\mu\mu}\propto (1-\mu^2) 
ightarrow h(\mu)\propto \mu$ : dipole

• Example: Second-order quasi-linear theory Shalchi (2005)





### Small-scale turbulence and ensemble averaging

• In standard diffusion, compute  $C_{\ell}$  from  $\langle f \rangle$ :

$$C_{\ell}^{\mathsf{std}} = \frac{1}{4\pi} \int \mathrm{d}\hat{\mathbf{p}}_1 \int \mathrm{d}\hat{\mathbf{p}}_2 \, P_{\ell}(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2) \langle f(\hat{\mathbf{p}}_1) \rangle \langle f(\hat{\mathbf{p}}_2) \rangle$$

• However, in an individual realisation of  $\delta B$ ,  $\delta f = f - \langle f \rangle \neq 0$ 

$$\langle C_\ell \rangle = \frac{1}{4\pi} \int \mathrm{d} \hat{\mathbf{p}}_1 \int \mathrm{d} \hat{\mathbf{p}}_2 \, P_\ell(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2) \langle f(\hat{\mathbf{p}}_1) f(\hat{\mathbf{p}}_2) \rangle$$

• If  $f(\hat{\mathbf{p}}_1)$  and  $f(\hat{\mathbf{p}}_2)$  are correlated,

$$\langle f(\hat{\mathbf{p}}_1) f(\hat{\mathbf{p}}_2) \rangle \geq \langle f(\hat{\mathbf{p}}_1) \rangle \langle f(\hat{\mathbf{p}}_2) \rangle \quad \Rightarrow \quad \langle C_\ell \rangle \geq C_\ell^{\mathsf{std}}$$

#### Source of the small scale anisotropies?

Giacinti & Sigl, PRL 109 (2012) 071101

Ahlers, PRL 112 (2014) 021101, Ahlers & Mertsch, ApJL 815 (2015) L2, Pohl & Rettig, Proc. 36th ICRC (2016) 451,

López-Barquero et al., ApJ 830 (2016) 19, López-Barquero et al. ApJ 842 (2017) 54, Mertsch & Ahlers (2019), Kuhlen, Mertsch, Phan (2022)

### Gradient ansatz

Mertsch & Ahlers (2019), Kuhlen, Mertsch, Phan (2022)

• Vlasov equation:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \dot{\mathbf{r}} \cdot \nabla_{\mathbf{r}} f + \frac{q\mathbf{v}}{c} \times (\langle \mathbf{B} \rangle + \delta \mathbf{B}) \cdot \nabla_{\mathbf{p}} f$$
$$\simeq \frac{\partial f}{\partial t} + \underbrace{(c\hat{\mathbf{p}} \cdot \nabla_{\mathbf{r}} + q(\hat{\mathbf{p}} \times \langle \mathbf{B} \rangle) \cdot \nabla_{\mathbf{p}})}_{\mathcal{L}} f + \underbrace{(q(\hat{\mathbf{p}} \times \delta \mathbf{B}) \cdot \nabla_{\mathbf{p}})}_{\delta \mathcal{L}} f = 0$$

• Gradient ansatz:

$$f(\mathbf{r}, \mathbf{\hat{p}}) = f_{\oplus}(\mathbf{\hat{p}}) + (\mathbf{r}_{\oplus} - \mathbf{r}) \cdot \mathbf{G}$$

 $\rightarrow$  Dipolar source term in the Vlasov equation:

$$\frac{\partial f_{\oplus}}{\partial t} + \underbrace{\left(q(\hat{\mathbf{p}} \times \langle \mathbf{B} \rangle) \cdot \nabla_{\mathbf{p}}\right)}_{\mathcal{L}'} f_{\oplus} + \underbrace{\left(q(\hat{\mathbf{p}} \times \delta \mathbf{B}) \cdot \nabla_{\mathbf{p}}\right)}_{\delta \mathcal{L}} f_{\oplus} = c \, \hat{\mathbf{p}} \cdot \mathbf{G}$$

### Mixing matrices

Mertsch & Ahlers (2019), Kuhlen, Mertsch, Phan (2022)

• Define propagator:

$$U_{t,t_0} = \mathcal{T} \exp \left[ - \int_{t_0}^t \mathrm{d}t' \left( \mathcal{L}' + \delta \mathcal{L}(t') 
ight) 
ight]$$

• Formal solution of Vlasov equation:

$$f_{\oplus}(\mathbf{p},t) = U_{t,t_0}f_{\oplus}(\mathbf{p},t_0) + \int_{t_0}^t \mathrm{d}t' U_{t,t'} c\, \hat{\mathbf{p}}\cdot \mathbf{G}$$

 $\rightarrow\,$  Differential equation for  $\langle {\it C}_\ell \rangle$  ,

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle C_{\ell}\rangle(t) + \left(\lim_{t_0 \to t} \frac{\delta_{\ell\ell_0} - M_{\ell\ell_0}(t, t_0)}{t - t_0}\right) \langle C_{\ell_0}\rangle(t) = \frac{8\pi}{9} \mathcal{K}|\mathbf{G}|^2 \delta_{\ell 1}$$

where

$$M_{\ell\ell_0}(t,t_0) = rac{1}{4\pi} \int \mathrm{d}\mathbf{\hat{p}}_A \int \mathrm{d}\mathbf{\hat{p}}_B \mathrm{P}_\ell(\mathbf{\hat{p}}_A \cdot \mathbf{\hat{p}}_B) \langle U^A_{t,t_0} U^{B*}_{t,t_0} 
angle rac{2\ell_0 + 1}{4\pi} \mathrm{P}_{\ell_0}(\mathbf{\hat{p}}_A \cdot \mathbf{\hat{p}}_B) \langle U^A_{t,t_0} U^{B*}_{t,t_0} \rangle$$

### Ignoring correlations

Mertsch & Ahlers (2019), Kuhlen, Mertsch, Phan (2022)

• Without "interactions":

$$\langle U^A_{t,t_0} U^{B*}_{t,t_0} \rangle \quad \simeq \quad \underbrace{-}_{} + \underbrace{-}_{} \underbrace{-}_{} + \underbrace{-}_{} \underbrace{-}_{} + \underbrace{-}_{} \underbrace{-}_{} + \underbrace{-}_{} \underbrace{-}_{} \underbrace{-}_{} + \underbrace{-}_{} \underbrace{-}_$$

• Mixing matrix diagonal:

 $M_{\ell\ell_0}(t,t_0)\sim \delta_{\ell\ell_0}$ 

$$\frac{\mathrm{d}}{\mathrm{d}t} \langle C_{\ell} \rangle(t) + \left( \lim_{t_0 \to t} \frac{\delta_{\ell \ell_0} - M_{\ell \ell_0}(t, t_0)}{t - t_0} \right) \langle C_{\ell_0} \rangle(t) = \frac{8\pi}{9} \mathcal{K} |\mathbf{G}|^2 \delta_{\ell 1} \,,$$

 $\rightarrow$  Only dipolar anisotropy:

 $\langle C_\ell \rangle \propto \delta_{\ell 1} \,,$ 

### With correlations

Mertsch & Ahlers (2019), Kuhlen, Mertsch, Phan (2022)

• With "interactions"

• Mixing matrix **not** diagonal:

$$M_{\ell\ell_0}(t,t_0) \sim \, \delta_{\ell\ell_0} + \sum_{\ell_A} \kappa_{\ell_A}(t-t_0) \left( egin{array}{cc} \ell & \ell_A & \ell_0 \ 0 & 0 & 0 \end{array} 
ight)^2 (2\ell_0+1)\ell_0(\ell_0+1)$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \langle C_{\ell} \rangle(t) + \left( \lim_{t_0 \to t} \frac{\delta_{\ell \ell_0} - M_{\ell \ell_0}(t, t_0)}{t - t_0} \right) \langle C_{\ell_0} \rangle(t) = \frac{8\pi}{9} K |\mathbf{G}|^2 \delta_{\ell 1} \,,$$

### With correlations

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 $\rightarrow\,$  Gradient source term is mixing into higher harmonics!

### Test particle simulations

Kuhlen, Mertsch, Phan (2022)

- Need to check analytical results with simulations
- Set up turbulent magnetic field on computer
- Solve the equations of motion for  $\mathcal{O}(10^7)$  particles numerically



H.264 avi

- Rinse and repeat
- Compute diffusion coefficients, angular power spectra, ...

# Results

Kuhlen, Mertsch, Phan (2022)





### Results

- Numerical and analytic angular power spectra become steeper for smaller energies
- Observed angular power spectra become flatter for smaller energies

### Possible reasons for different scaling

- A feature in the power spectrum
- A very small outer scale
- Slab turbulence does not describe data well
- Finite energy resolution

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Ø Small-scale anisotropies

**3** Test-particle simulations

O Suppressed diffusion

**6** Conclusions

# Different scaling of $\kappa_{\parallel}$ and $\kappa_{\perp}$

- Regular and turbulent field:  $\mathbf{B}(\mathbf{r}) = \mathbf{B}_0 + \delta \mathbf{B}(\mathbf{r})$
- Isotropic turbulence (!)
- Kolmogorov power spectrum with largest turbulent scale L<sub>c</sub>

• Naive expectation:

$$\begin{split} \kappa_{\parallel} &\sim \left(\frac{r_g}{L_c}\right) \quad \left(\frac{\partial B}{B_0^2}\right) \\ \kappa_{\perp} &\sim \left(\frac{r_g}{L_c}\right)^{1/3} \left(\frac{\delta B^2}{B_0^2}\right) \end{split}$$

 $( - )^{1/3} ( S P^2 )^{-1}$ 



### Test particle simulations

Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05881]; also Mertsch (2019)



- **1** Set up realisation of  $\delta \mathbf{B}$  on computer
- 2 Propagate a large number of particles for long times
- 3 Rinse and repeat
- 4 Running diffusion coefficients:

$$egin{aligned} d_{\parallel}(t) &\equiv rac{1}{2}rac{\mathrm{d}}{\mathrm{d}t}\langle (\Delta z)^2 
angle \ d_{\perp}(t) &\equiv rac{1}{2}rac{\mathrm{d}}{\mathrm{d}t} \left( \langle (\Delta r_{\perp})^2 
angle 
ight) \end{aligned}$$

Results depend on:

• Reduced time: 
$$\Omega t$$
  
• Reduced rigidity:  $\frac{r_{\rm g}}{L_{\rm c}}$   
• Turbulence level:  $\eta = \frac{\delta B^2}{B_0^2 + \delta B^2}$ 

## Running parallel diffusion coefficient

Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05881]



- Initially ballistic:  $\langle (\Delta z)^2 
  angle \propto t^2$
- Ultimately diffusive:  $\langle (\Delta z)^2 
  angle \propto t$
- Dependence on turbulence level
- Suppression at intermediate times

## Running perpendicular diffusion coefficient

Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05881]



• Initially ballistic:  $\langle (\Delta r_{\perp})^2 
angle \propto t^2$ 

- Ultimately diffusive:  $\langle (\Delta r_{\perp})^2 \rangle \propto t$
- Suppression at intermediate times

• Subdiffusion:  $\langle (\Delta r_{\perp})^2 
angle \propto t^{0.5...0.7}$ 

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Why subdiffusion?

### Mean-free paths

Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05881]



### Ratio of mean-free paths

Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05881]



 $\kappa_{\parallel}$  and  $\kappa_{\perp}$  scale differently at medium rigidites, but they scale the same at low rigidities

### Perpendicular transport





- (a) Straight field line and gyration
- (b) Wandering field line and gyration
- (c) Wandering field line and diffusion



$$d_{\mathsf{FL}}(z) = rac{1}{2} rac{\mathrm{d}\langle (\Delta r_{\perp}^{\mathsf{FL}})^2 
angle}{\mathrm{d}z}$$

### Heuristic model

Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05882]

 $\label{eq:perpendicular transport} Perpendicular transport = particle transport along field line + transport of field line$ 

Start from 
$$d_{FL}(z) = \frac{1}{2} \frac{d\langle (\Delta r_{\perp}^{FL})^2 \rangle}{dz}$$
Integrate:  $\langle (\Delta r_{\perp}^{FL})^2 \rangle(z) = 2 \int_0^z dz' d_{FL}(z')$ 
Assume that particles follow field lines:  $\langle (\Delta r_{\perp}^{CR})^2 \rangle(z) = \langle (\Delta r_{\perp}^{FL})^2 \rangle(z)$ 
Subsitute into  $d_{\perp}(t) \equiv \frac{1}{2} \frac{d}{dt} \left( \langle (\Delta r_{\perp}^{CR})^2 \rangle \right) = \frac{d}{dt} \int_0^{z(t)} dz' d_{FL}(z')$ 
Evaluate  $z(t)$  as  $\sqrt{\langle z^2 \rangle(t)} \Rightarrow d_{\perp}(t) = \frac{d}{dt} \int_0^{\sqrt{\langle z^2 \rangle}} dz' d_{FL}(z') = \frac{d_{FL}(\sqrt{\langle (\Delta z)^2 \rangle})}{\sqrt{\langle (\Delta z)^2 \rangle}} d_{\parallel}(t)$ 

N.B.: This can also be derived from a microscopic model of particle transport.

### Heuristic model

Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05882]

$$d_{\perp}(t) = rac{d_{\mathsf{FL}}(\sqrt{\langle (\Delta z)^2 
angle})}{\sqrt{\langle (\Delta z)^2 
angle}} d_{\parallel}(t)$$

Parametrise  $d_{FL}$  and  $d_{\parallel}$  by broken power laws



Field-line transport is subdiffusive at intermediate distances!

See also Sonsrettee et al. (2016)

## Mean-free paths

Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05882]



### Ratio of mean-free paths

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(More details  $\rightarrow$  Appendix)

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# Gamma-ray halos

Abeysekara et al. (2017)



- Gamma-ray emission around two nearby pulsars
- Emission from  $e^{\pm}$  much more confined than expected
- $\rightarrow\,$  Ambient diffusion coefficient suppressed by factor  $\sim 100$
- Also evidence of suppressed diffusion around some supernova remnants

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# NOT the conflict

Evoli, Linden, Morlino (2018), Linden & Mukhopadhyay (2022)



- Growth rate:  $\Gamma_{1D} \propto \frac{\partial f}{\partial z}$
- Volume occupied:  $V_{1D} \sim A \left< (\Delta z)^2 \right>^{1/2}$



## NOT the conflict

Evoli, Linden, Morlino (2018), Linden & Mukhopadhyay (2022)



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- Volume occupied:  $V_{1D} \sim A \left< (\Delta z)^2 \right>^{1/2}$



#### Locally, transport is **not** isotropic

See Lopez-Coto and Giacinti (2019) though

### The conflict



 $\rightarrow$ 

 $\rightarrow$ 

Suppression of diffusion



 $\kappa_{\perp} \ll \kappa_{\parallel} \quad \Leftrightarrow \quad \delta B^2 \ll B_0^2$ 

### Spherical distribution





### A swiss cheese Galaxy



- Diffusion suppressed in bubbles around sources
- $\rightarrow$  Transport of cosmic rays affected on large scales?
- Different diffusion coefficient in disk and halo:  $\kappa_{disk} = \alpha \kappa_{halo}$

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### A swiss cheese Galaxy





- Diffusion suppressed in bubbles around sources
- $\rightarrow$  Transport of cosmic rays affected on large scales?
- Different diffusion coefficient in disk and halo:  $\kappa_{disk} = \alpha \kappa_{halo}$



- Filling fraction f ≤ (a few) % → negligible?
- Difficult to model numerically
- $\rightarrow\,$  Adopt course-grainined  $\kappa_{\rm disk}=\alpha\kappa_{\rm high}$  and  $\kappa_{\rm halo}=\kappa_{\rm high}$



- Study impact of  $\alpha < 1$  on cosmic ray observables
- The coarse-grained  $\kappa_{\rm disk}$  can only depend on  $\kappa_{\rm high}$ ,  $\kappa_{\rm low}$  and the filling fraction
- $\rightarrow\,$  Can infer filling fraction from data?

# $^{10}\mathrm{Be}/^{9}\mathrm{Be}$

Jacobs, Mertsch, Phan, in prep.



- $\bullet$  Unstable  $^{10}\mathrm{Be}$  created in disk
- At high energies: essentially stable
- At low energies: decays while diffusing

If diffusion is suppressed,  $\kappa_{\rm disk} < \kappa_{\rm halo},$   $^{10}{\rm Be}/^{9}{\rm Be}$  is increased at low energies

## Posterior distributions

Jacobs, Mertsch, Phan, in prep.





#### With prelim. AMS-02 data

- Best fit for  $\alpha\simeq$  0.2
- lpha=1 excluded at  $\sim 4\sigma$
- Implies very large filling fraction  $f \sim 0.5$

### Summary



Small-scale anisotropies formed in transition from ballistic to diffusive motion



Perpendicular diffusion due to parallel diffusion and field-line subdiffusion



Evidence for suppressed diffusion from  ${
m ^{10}Be}/{
m ^9Be}$