Finite Difference Time-Domain Methods for Askaryan Propagation Modeling in IceCube-Gen2

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Outline

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1. Analytic ray-tracing in NuRadioMC
    - Index of refraction profile
    - Example of reconstruction
2. Beyond analytic ray-tracing: FDTD methods
    - What is FDTD, MIT Electromagnetic Equation Propagation (MEEP)
        - 2.1 RF antenna modeling in complex media
        - 2.2 Properties of phased arrays in complex media
        - 2.3 Analytic Askaryan emissions

Analytic ray-tracing in NuRadioMC

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The *index of refraction profile* $n(z)$ may be derived analytically with some basic assumptions about the volumetric compressibility of snow into ice:

$$n(z) = n_{\mathrm{ice}} - \Delta n \exp(z/z_0) \tag{1}$$

- n_{ice} is the established RF value for solid ice
- $\Delta n = n_{\mathrm{ice}} - n(0)$
- z_0 depends on compressibility, varies by location in Antarctica

Employing Fermat's Principle yields the solution:

$$S = \delta \int_A^B n ds \tag{2}$$

$$\delta S = 0 \tag{3}$$

Analytic ray-tracing in NuRadioMC

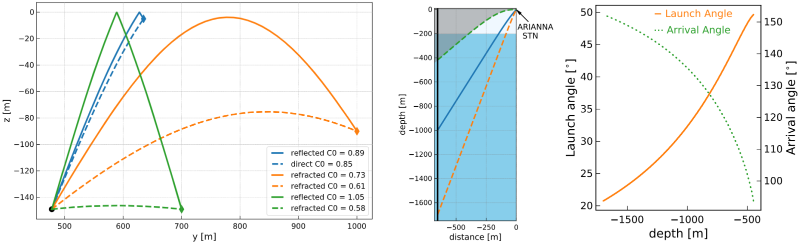


Figure 1: (Left) Example NuRadioMC analytic solutions. (Right) From SPICE core analysis (ARIANNA at SP). (C. Glaser *et al* (2020), G. Gaswint *et al* (2020)).

Analytic ray-tracing in NuRadioMC

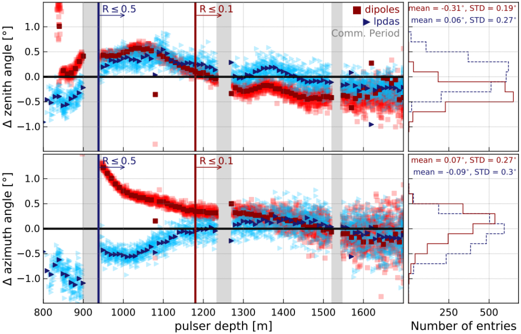


Figure 2: Example of predicted minus observed difference in arrival direction at SP from SPICE core transmitter data. (G. Gaswint *et al* (2020)).

Analytic ray-tracing in NuRadioMC

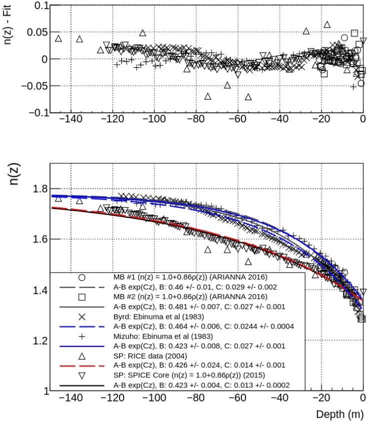


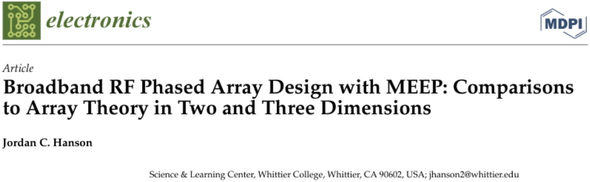
Figure 3: Index data from Antarctica. (S. W. Barwick (2018), also Prohira *et al* (2020)).

Beyond analytic ray-tracing: FDTD methods

Beyond analytic ray-tracing: FDTD methods

FDTD: Finite Difference Time Domain (Maxwell's Equations). MIT Electromagnetic Equation Propagation (MEEP): (Oskooi *et al* (2010)). Modeling Phased arrays (J. C. Hanson (2021)).

1. RF antenna modeling (radiation pattern, S-parameters)
2. Properties of antennas, phased-arrays in complex media
3. Analytic askaryan emissions



electronics

MDPI

Article

Broadband RF Phased Array Design with MEEP: Comparisons to Array Theory in Two and Three Dimensions

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Figure 4: (Reference above).

Beyond analytic ray-tracing: FDTD methods

The radiation patterns of antennas can be modeled with MEEP.

- Antenna systematics (comparisons with XFDTD)
- Open-source, python3, may be integrated with a version of NuRadioMC
- In the case of phased arrays, FDTD+MEEP has been shown to match array theory

$$P(\phi) = \left(\frac{\sin(\pi N (d_y/\lambda)(\sin(\phi) - \sin(\phi_0)))}{N \sin(\pi (d_y/\lambda)(\sin(\phi) - \sin(\phi_0)))} \right)^2$$

Figure 5: One formula for the E-plane radiation pattern of phased arrays in a *uniform media*. Note the importance of the d/λ factor.

Beyond analytic ray-tracing: FDTD methods

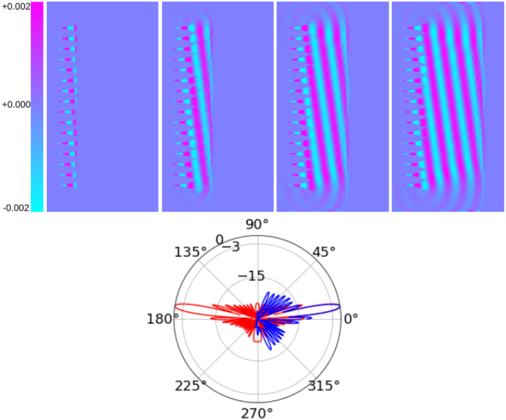

Figure 6: (Top) Electric field radiated from a $N = 16$ phased array with $f = 2.5$ GHz, $d/\lambda = 0.625$. (Bottom) Match between array theory (red) and simulation (blue) at a 9 degree angle from the main axis.

Beyond analytic ray-tracing: FDTD methods

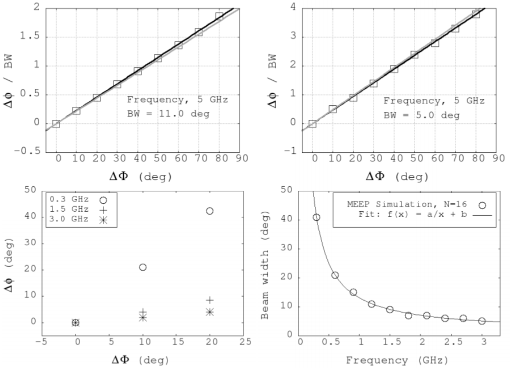

Figure 7: (Top left) The beam angle divided by the beam width for the N = 8 one-dimensional Yagi array versus the phase shift per element. (Top right) Same, for N = 16 array. (Bottom left) Same as top, for the N = 16 horn array, for several frequencies. (Bottom right) Frequency dependence of the beamwidth of the horn array.

Beyond analytic ray-tracing: FDTD methods

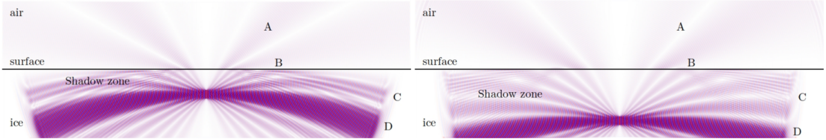

Figure 8: The magnitude of the z-component of z-polarized fat dipoles as they radiate horizontally as a phased array. (Left) The array depth is −15 m. (Bottom) The array depth is −35 m. (A) Radiation refracting through the surface into the air. (B) Grating lobes reflect from the surface into the shadow zone. (C) Grating lobes propagating through the shadow zone. (D) The main beam bent downward due to the gradient in n(z).

Beyond analytic ray-tracing: FDTD methods

We can derive E-field of Askaryan radiation analytically (paper forthcoming):

For $t_r > 0$, $\theta = \theta_C$:

$$\boxed{RE(t,\theta_{\mathrm{C}}) \approx \frac{1}{3}\hat{E}_0\omega_{\mathrm{CF}}^2\left(2\exp(-2\omega_{\mathrm{C}}t_r) - \left(1+\frac{1}{2}\epsilon\right)\exp(-\omega_0 t_r)\right)} \quad (4)$$

For $t_r < 0$, $\theta = \theta_C$:

$$\boxed{RE(t,\theta_{\mathrm{C}}) \approx \frac{1}{3}\hat{E}_0\omega_{\mathrm{CF}}^2\left(1-\frac{1}{2}\epsilon\right)\exp(\omega_0 t_r)} \quad (5)$$

For $\theta \neq \theta_C$:

$$\boxed{RE(t,\theta) = -\frac{E_0\omega_0\sin(\theta)}{8\pi p}t_r e^{-\frac{t_r^2}{4p}+p\omega_0^2}\operatorname{erfc}(\sqrt{p}\omega_0)} \quad (6)$$

Beyond analytic ray-tracing: FDTD methods

Analytic functions can be implemented as MEEP sources:

```
#custom source function
def f(phase):
    return lambda t: np.sin(2.0*3.4159*antennaFreq*t+phase)
...
#Add it as a source
sources.append(
    mp.Source(
        mp.CustomSource(src_func=f(this_phase),start_time=t1),
        component=mp.Ey,
        center=antenna_location,
        size=antenna_size,
        amplitude=1
    )
)
this_phase+=d_phase
```

Beyond analytic ray-tracing: FDTD methods

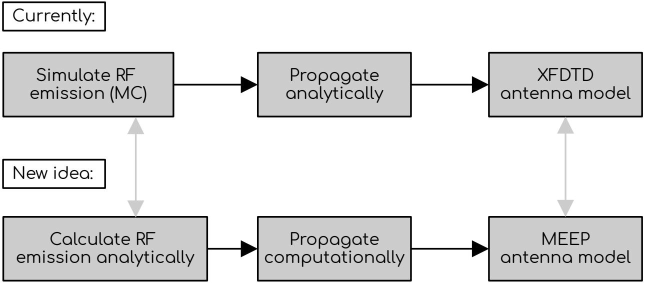

Figure 9: Comparison of computational strategies for signal cross-comparison to reveal systematics introduced by ice perturbations.

Conclusion

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