BedMachine: mapping the bed under the Antarctic ice sheet by combining sparse radar data and mass conservation

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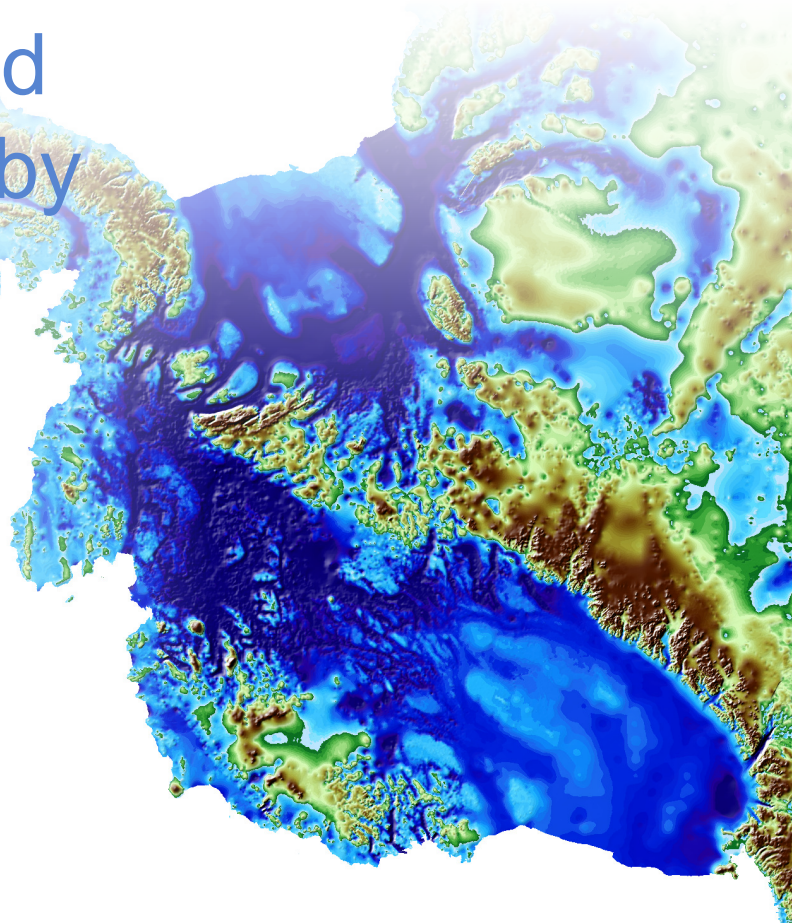
Department of Earth System Science



Conservation of mass

- Compressibility coefficient: $\chi_T = 1.2 \times 10^{-10}$ Pa^{-1}
- Linear thermal expansion of ice: 5×10^{-5} ºC^{-1}

→ Density varies by less than 0.5% in Antarctica and Greenland

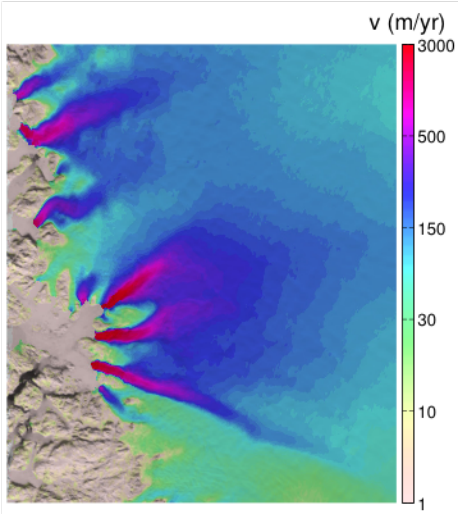
→ Ice can be considered an incompressible fluid

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = 0 \quad \rightarrow \quad \nabla \cdot \mathbf{v} = 0$$

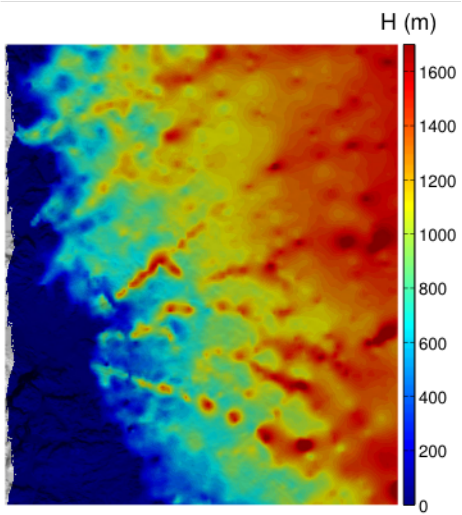
- Depth integrating between the bottom and the top surface of the ice:

$$\frac{\partial H}{\partial t} = -\underbrace{\nabla \cdot H\,\bar{\mathbf{v}}}_{\text{Flux divergence}} + \underbrace{\dot{M}_s}_{\text{SMB}} + \underbrace{\dot{M}_b}_{\text{Basal melt}}$$

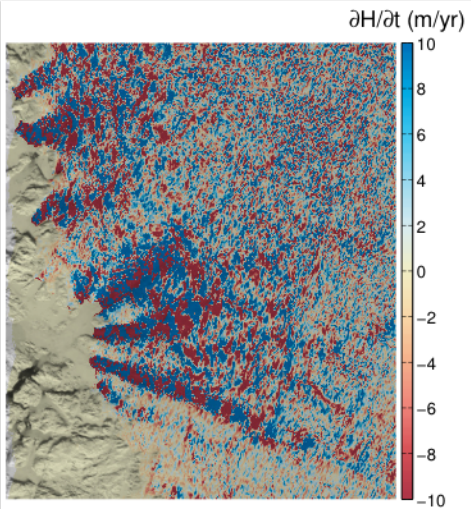
How about grounded ice?



(a) InSAR ice velocity
Rignot and Mouginot 2012



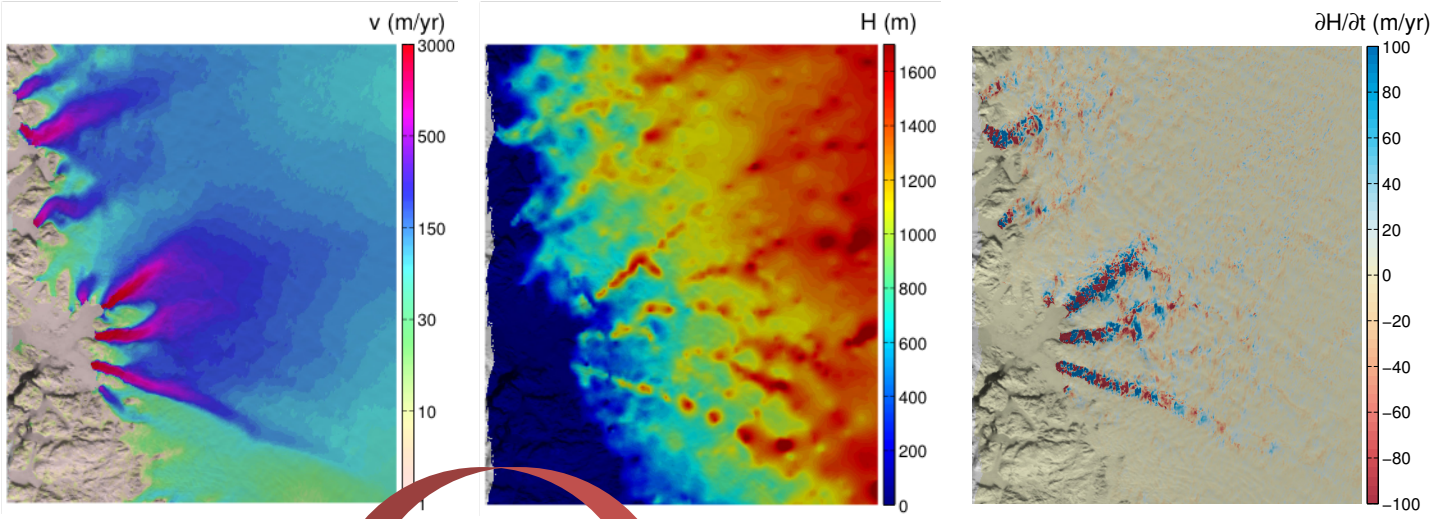
(b) Ice Thickness
From Kriging



(c) Calculated ∂H/∂t

$$\frac{\partial H}{\partial t} = -\underbrace{\nabla \cdot H\,\bar{\mathbf{v}}}_{\text{Flux divergence}} + \underbrace{\dot{M}_s}_{\text{SMB}} + \underbrace{\dot{M}_b}_{\text{Basal melt}}$$

How about grounded ice?



(a) InSAR ice veloci… Rignot and Mouginot …012

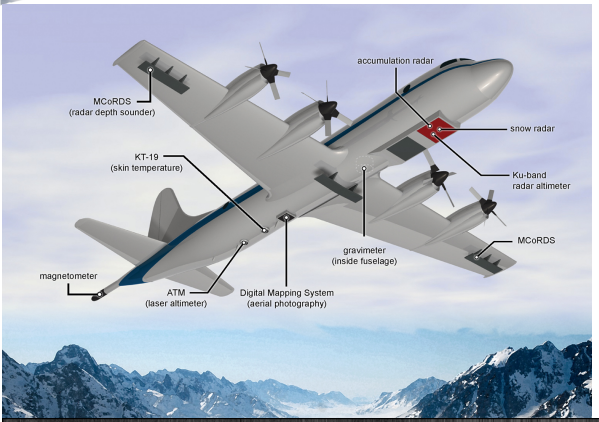
(b) …e Thickness Fr… Kriging

(c) Calculated ∂H/∂t

$$\frac{\partial H}{\partial t} = \underbrace{-\nabla \cdot H\,\bar{\mathbf{v}}}_{\text{Flux divergence}} + \underbrace{\dot{M}_s}_{\text{SMB}} + \underbrace{\dot{M}_b}_{\text{Basal melt}}$$

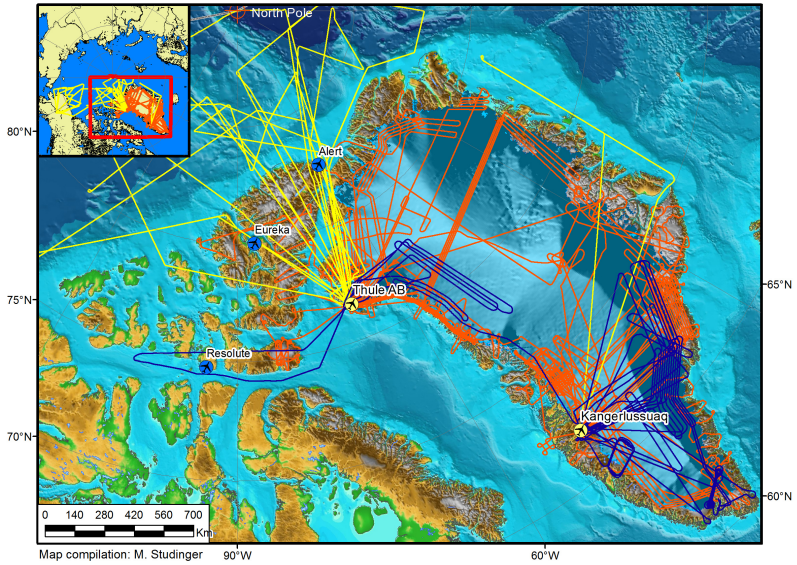
- Initializati… of ice sheet mod… difficult
- Strong need… for “physics-d…ived” bedrock elevation maps

Ice penetrating radar

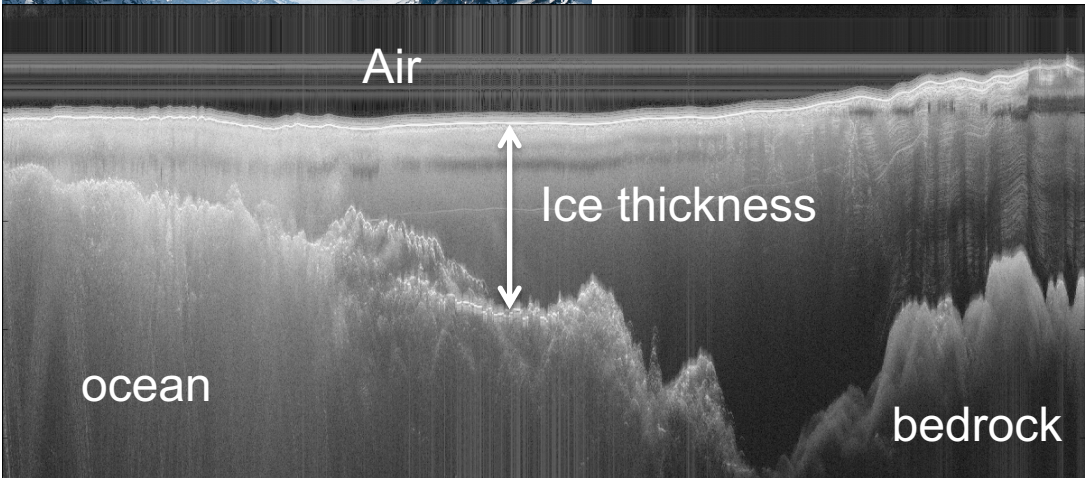


NASA P-3

Flight lines OIB Arctic 2012



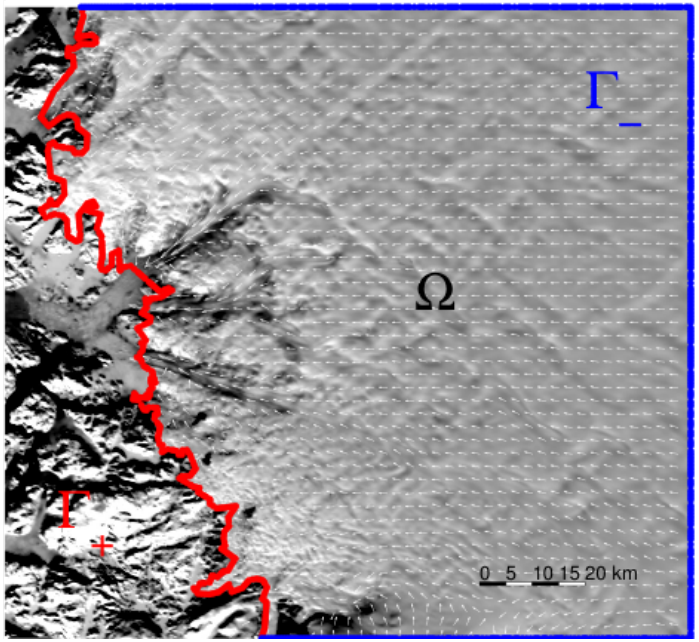
Radar echogram



Mass Conservation Approach

Find *H* such that:

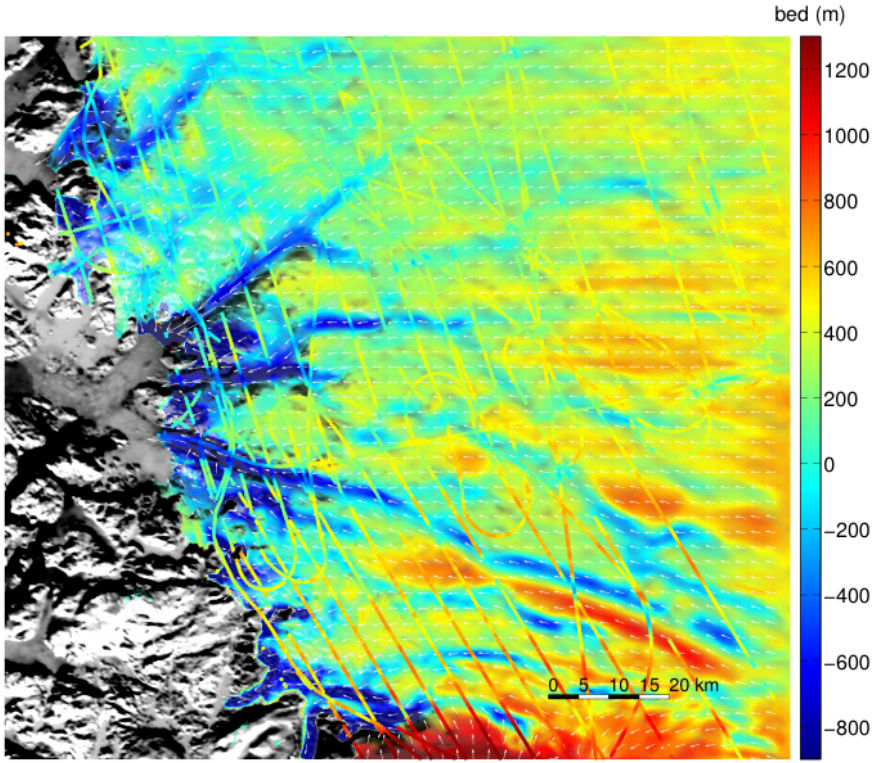
$$\begin{cases} \nabla \cdot H\,\bar{\mathbf{v}} = \underbrace{\dot{M}_s + \dot{M}_b - \dfrac{\partial H}{\partial t}}_{\dot{a}} & \text{in } \Omega \\ H = H_{obs} & \text{on } \Gamma_- \end{cases}$$



- ∂H / ∂t from ICESat/ATM
- SMB from regional climate model
- H_{obs} from OIB flight line
- Velocity ≈ InSAR surface velocities or modeled

Mass Conservation Approach

- Thickness only constrained on the inflow boundary
- We want the *modeled* thickness to match the measurements



PDE-Constrained optimization

Minimize the cost function:

$$\mathcal{J}\left(\bar{\mathbf{v}}, \dot{a}\right) = \int_{\text{Tracks}} \frac{1}{2}\left(H - H_{obs}\right)^2 dl$$

With the constraint:

$$\begin{cases} \nabla \cdot H\,\bar{\mathbf{v}} &= \dot{a} & \text{in } \Omega \\ H &= H_{obs} & \text{on } \Gamma_{-} \end{cases}$$

Controls:

$$\bar{\mathbf{v}} \in \left[0.95\left(\mathbf{v}_{obs} - 50\right) \quad \mathbf{v}_{obs} + 50\right] \text{ m/yr}$$

$$\dot{a} = \dot{a}_{obs} \pm 1 \text{ m/yr}$$

Variational data assimilation

Lagrangian:

$$\mathcal{L}\left(H, \lambda, \bar{\mathbf{v}}, \dot{a}\right) = J\left(H\right) + \int_{\Omega} \lambda \left(\nabla \cdot H\bar{\mathbf{v}} - \dot{a}\right) d\Omega$$

The adjoint state is defined by:

$$\forall \varphi \in V_H \quad \int_T \varphi \left(H - H_{obs}\right) dx - \int_{\Omega} \varphi \, \bar{\mathbf{v}} \cdot \nabla \lambda \, d\Omega + \int_{\Gamma+} \varphi \, \lambda \, \bar{\mathbf{v}} \cdot \mathbf{n} \, dS = 0$$

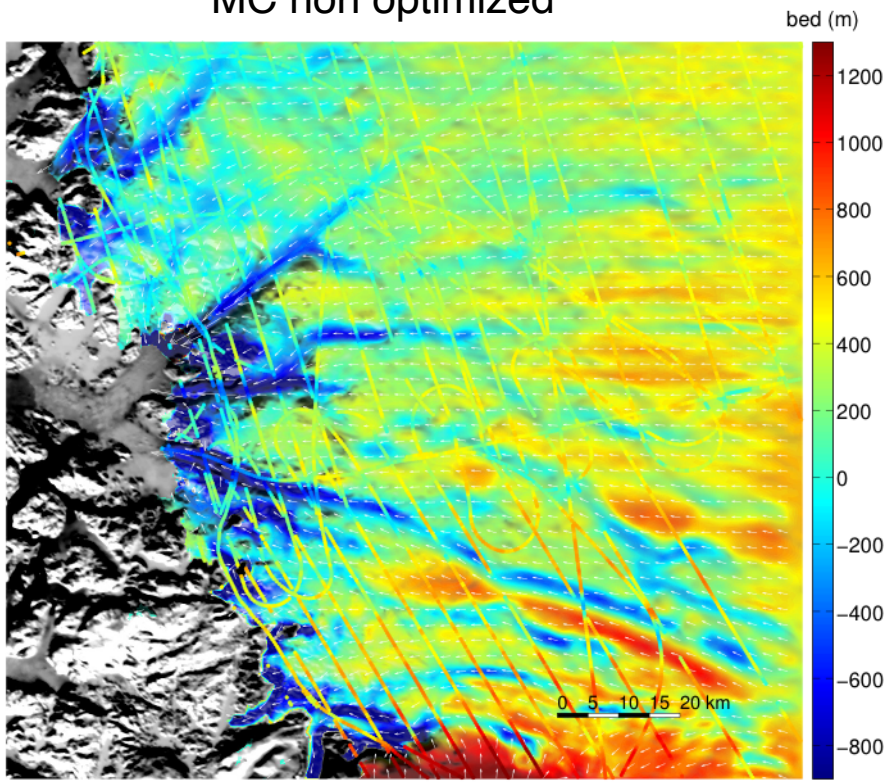
Gradients:

$$\mathcal{J}'\left(\bar{\mathbf{v}}\right) \simeq -H\nabla\lambda$$

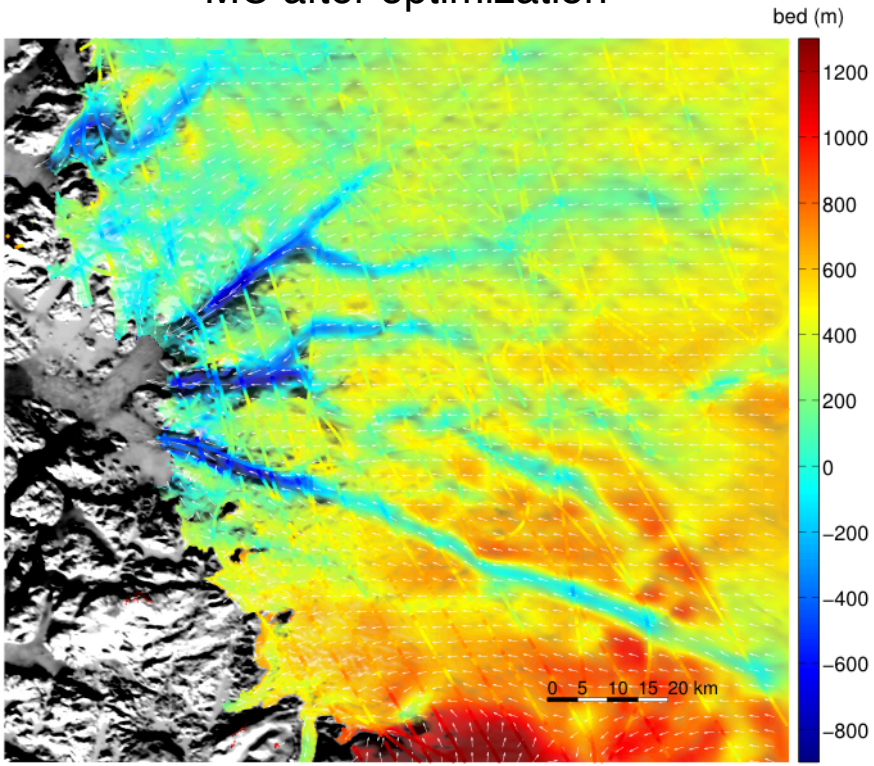
$$\mathcal{J}'\left(\dot{a}\right) \simeq \lambda$$

PDE-Constrained optimization

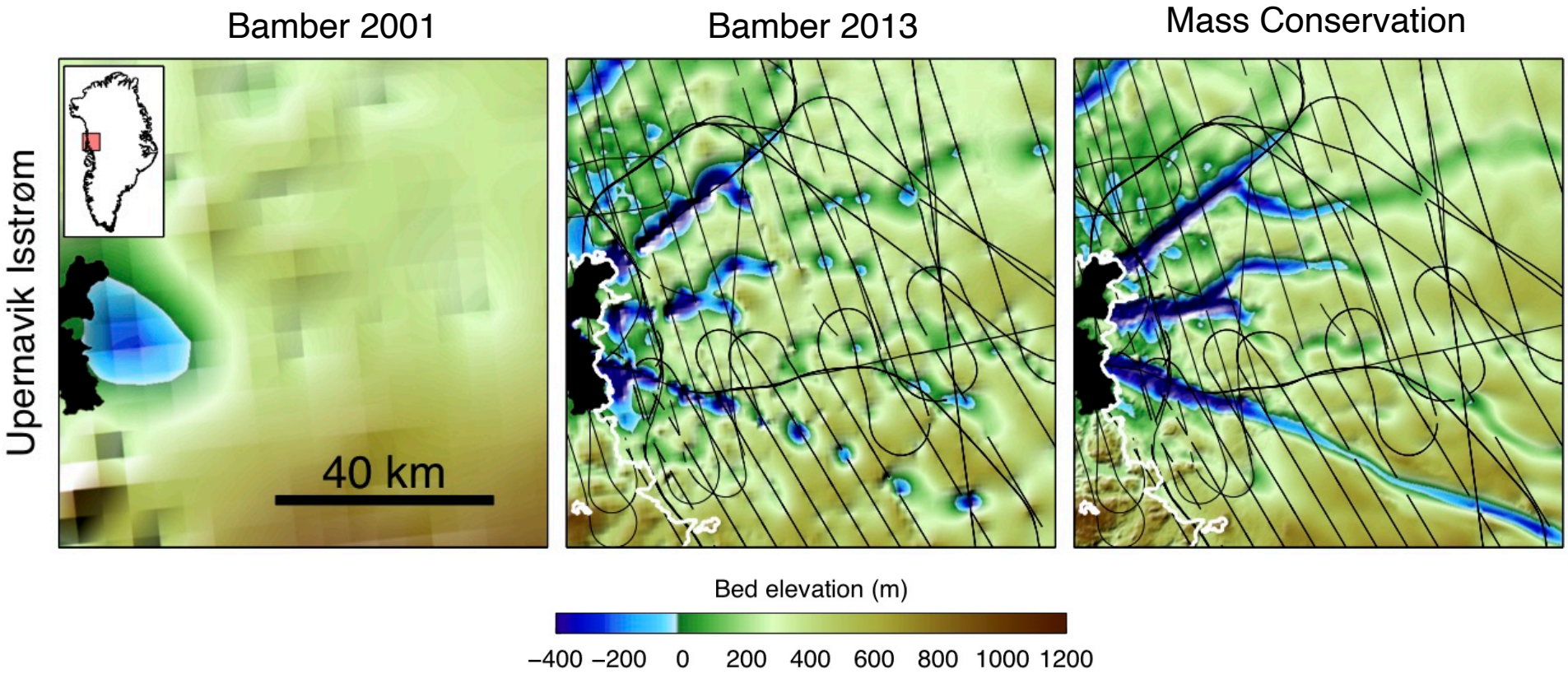
MC non optimized



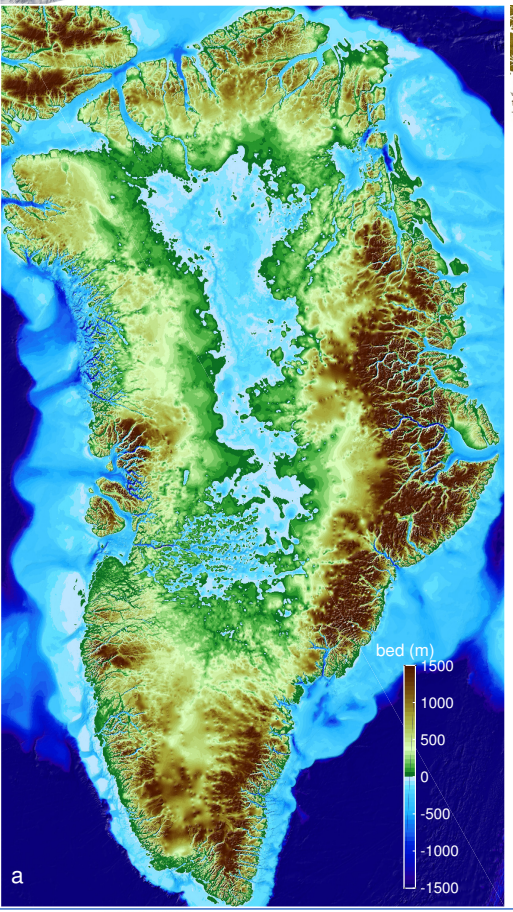
MC after optimization

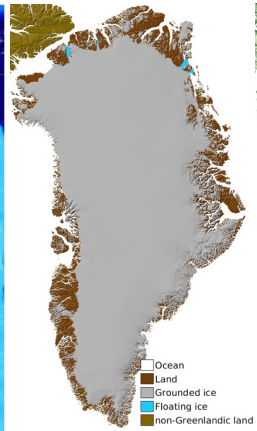


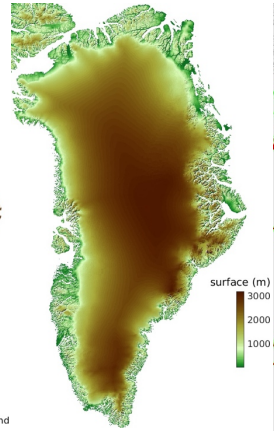
Comparison with other datasets

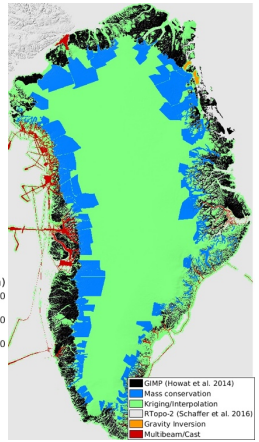


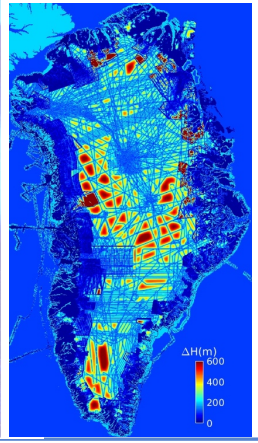
BedMachine Greenland v3

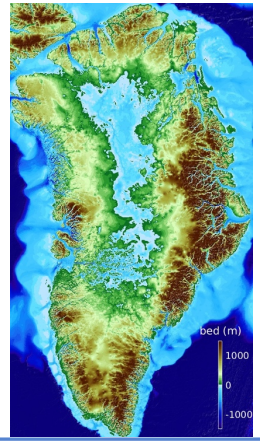


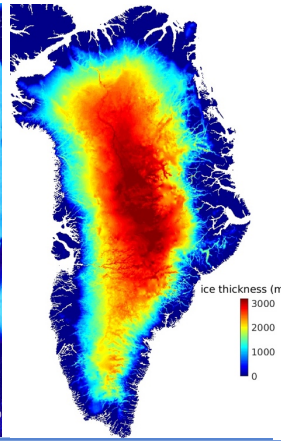






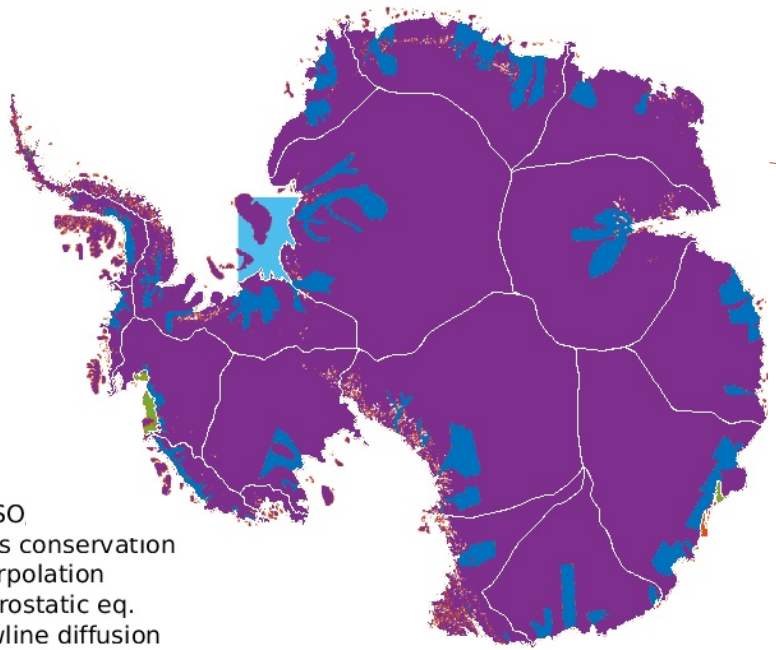


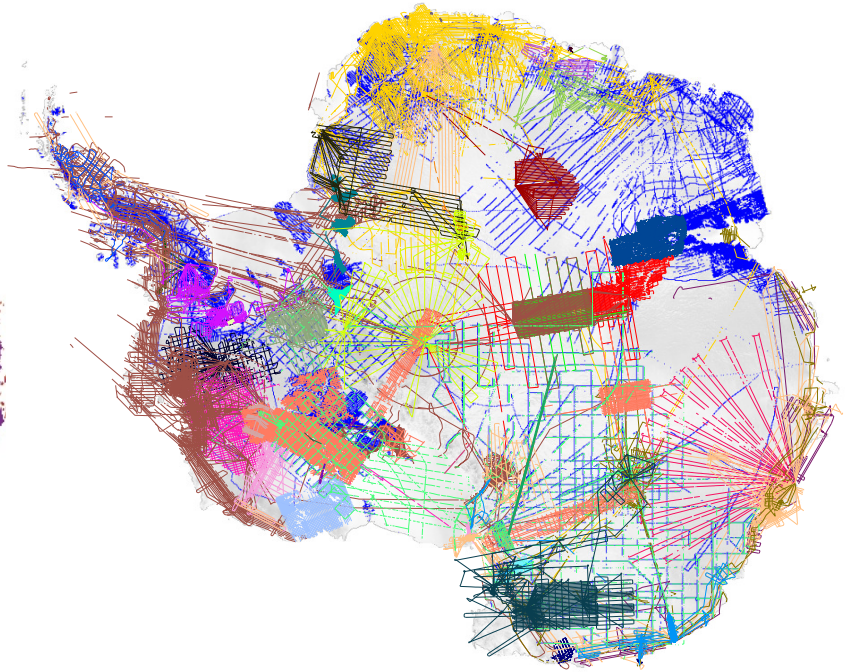




- Self-consistent dataset of the topography of Greenland
  - mask
  - surface elevation
  - ice thickness
  - bed elevation
  - error map
  - source map
  - Geoid height (WGS84)
- 150-m grids (true resolution depends on the region)
- V1: Posted at NSIDC since 2014
- V3: Available since Sept. 2017
- V4: under development…

BedMachine Antarctica





Bedmap1
AGAP-BAS
AGAP-USAP
AGASEA-BAS
AGASEA-UT
AWI
BASEC
CASERTZ
CReSIS
FISS
FISS2
GANOVEX
GEA
GIMBLE
GRADES
ICECAP-EAGLE
ICECAP-IPY
ICECAP-OIB
ICEGRAV
IMAFI
IPY-traverse
ItaRES
KRT1
Luyendyk
NARE-IceRises
OIR
PARIS
PCMEGA
PolarGAP
Ragnhild
RDrews
Rutford
SOAR
SPRI
UTIG-DCS
UTIG-DVD
UTIG-RBG
UTIG-STI
UTIG-WAG
WISE
WISE-ISODYN

Credit: P Fretwell (BAS)

Flowline diffusion

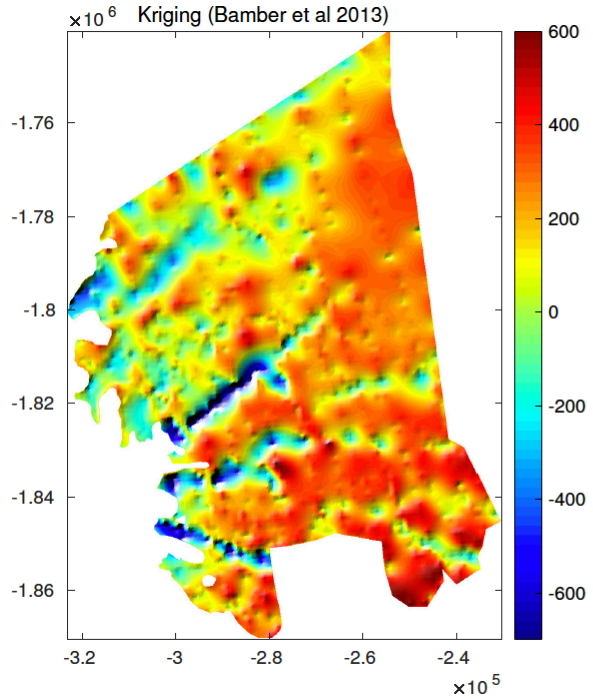
Find the ice thickness, H, such that

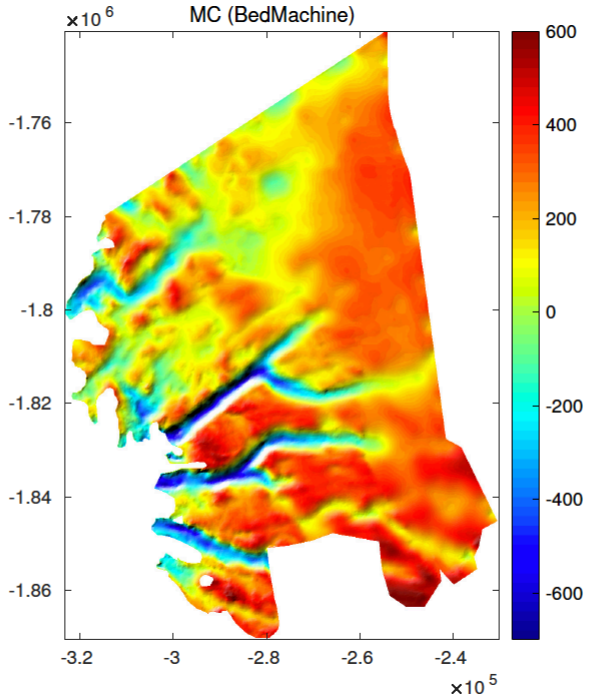
$$\begin{cases} \nabla \cdot \mathbf{D}\, \nabla H &= 0 & \text{in } \Omega \\ H &= H_{obs} & \text{on } T \end{cases}$$

Anisotropic diffusion tensor:

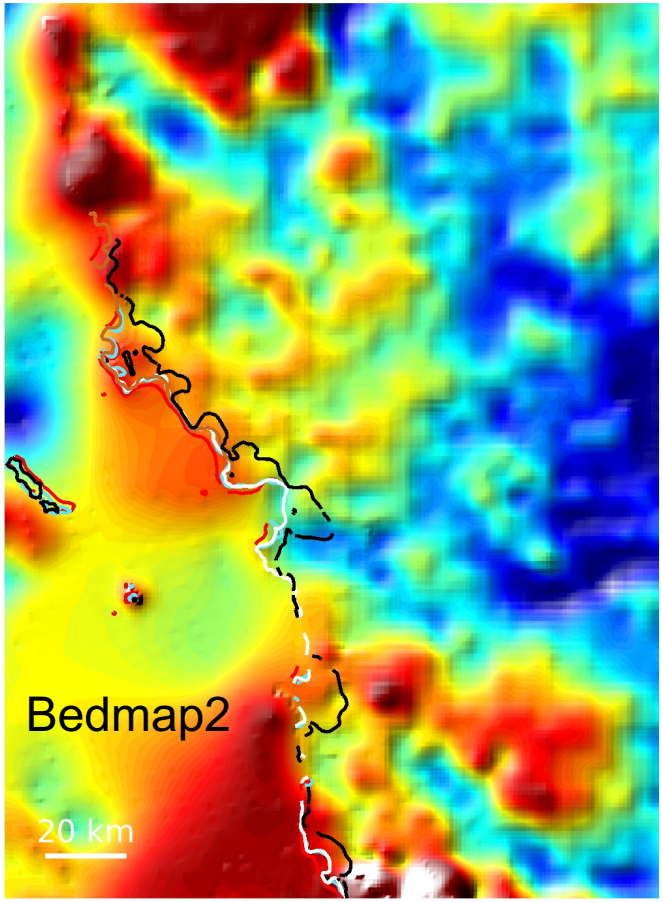
$$\mathbf{D} = \mathbf{v}_s \otimes \mathbf{v}_s + \tau \mathbf{I}$$

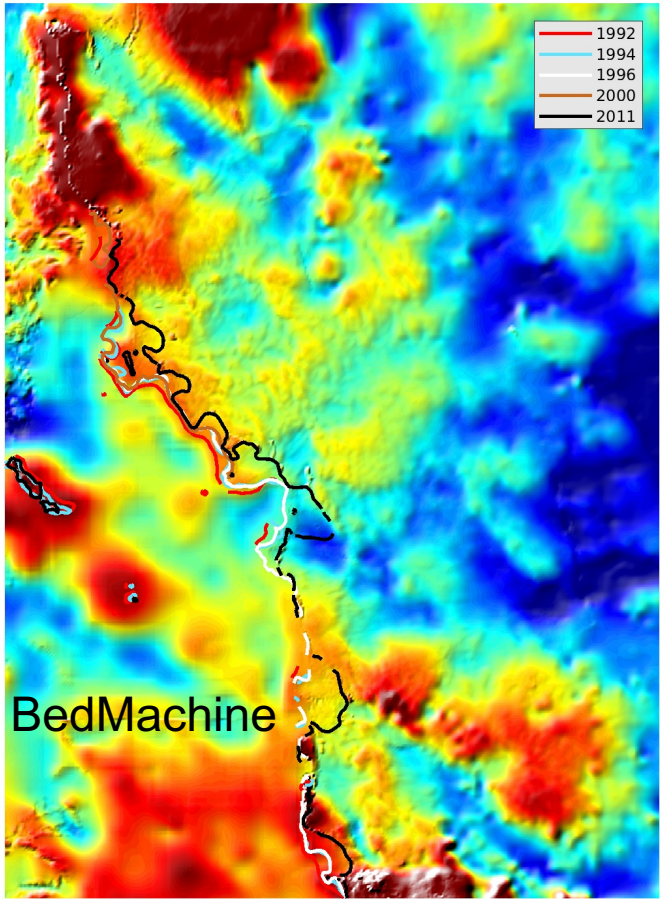
Flowline diffusion

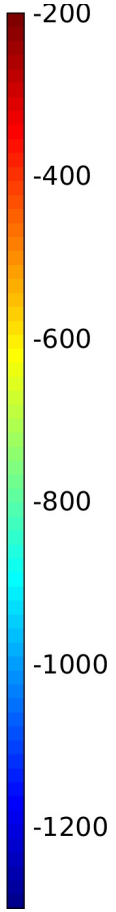




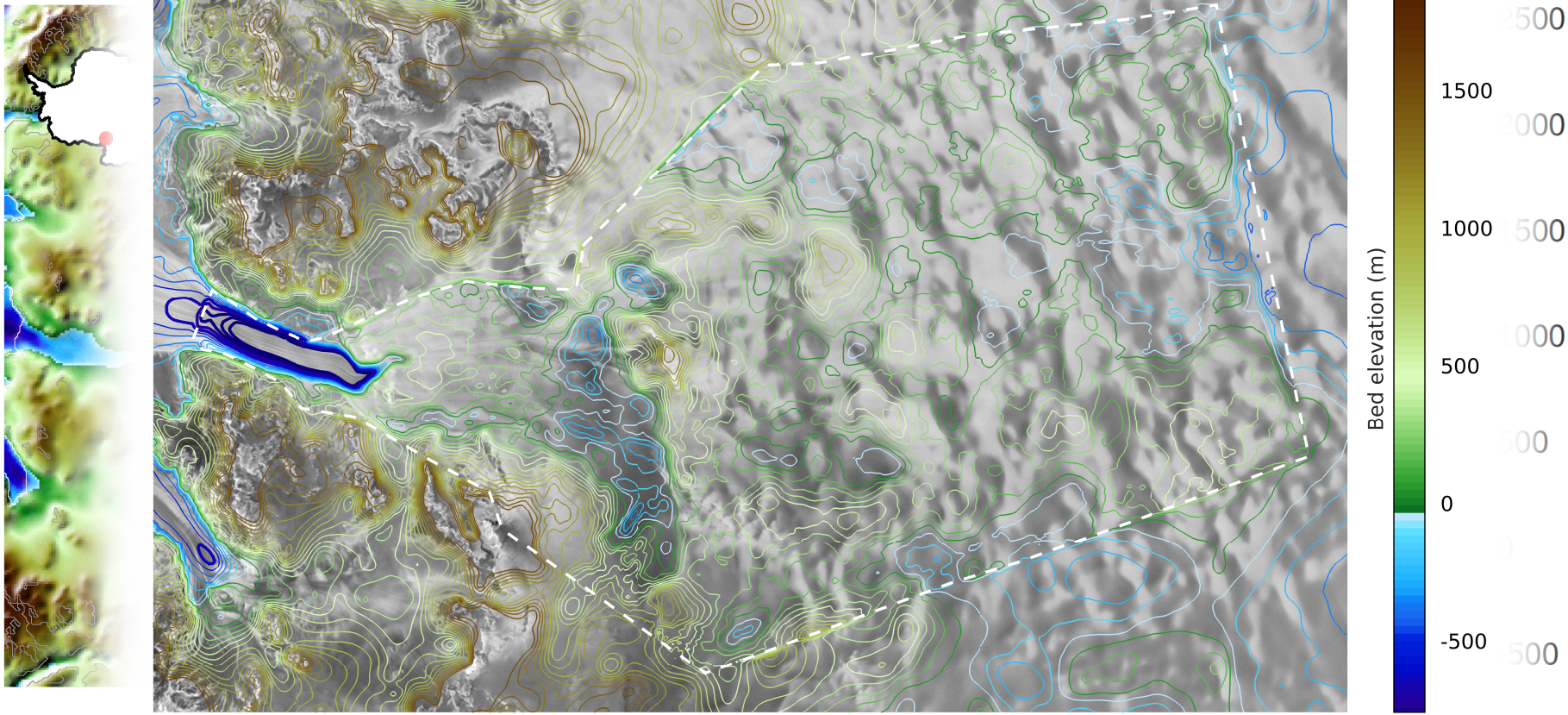
Thwaites Glacier



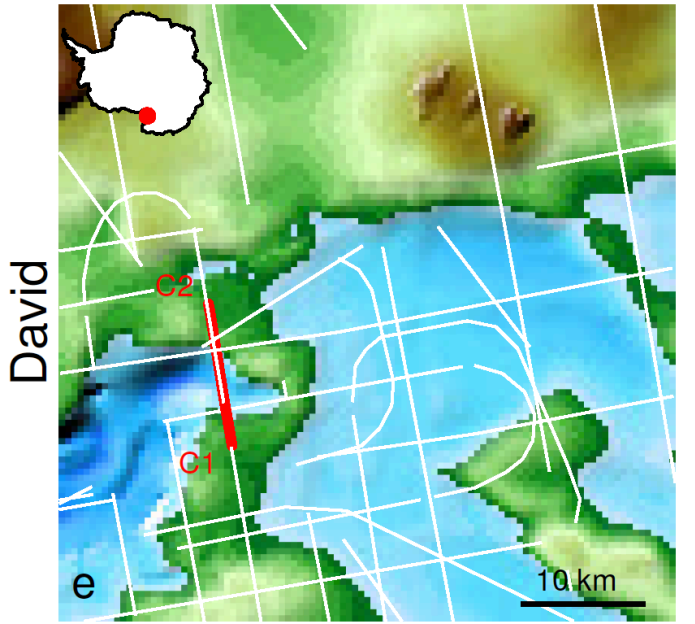


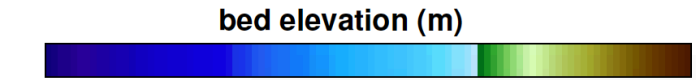


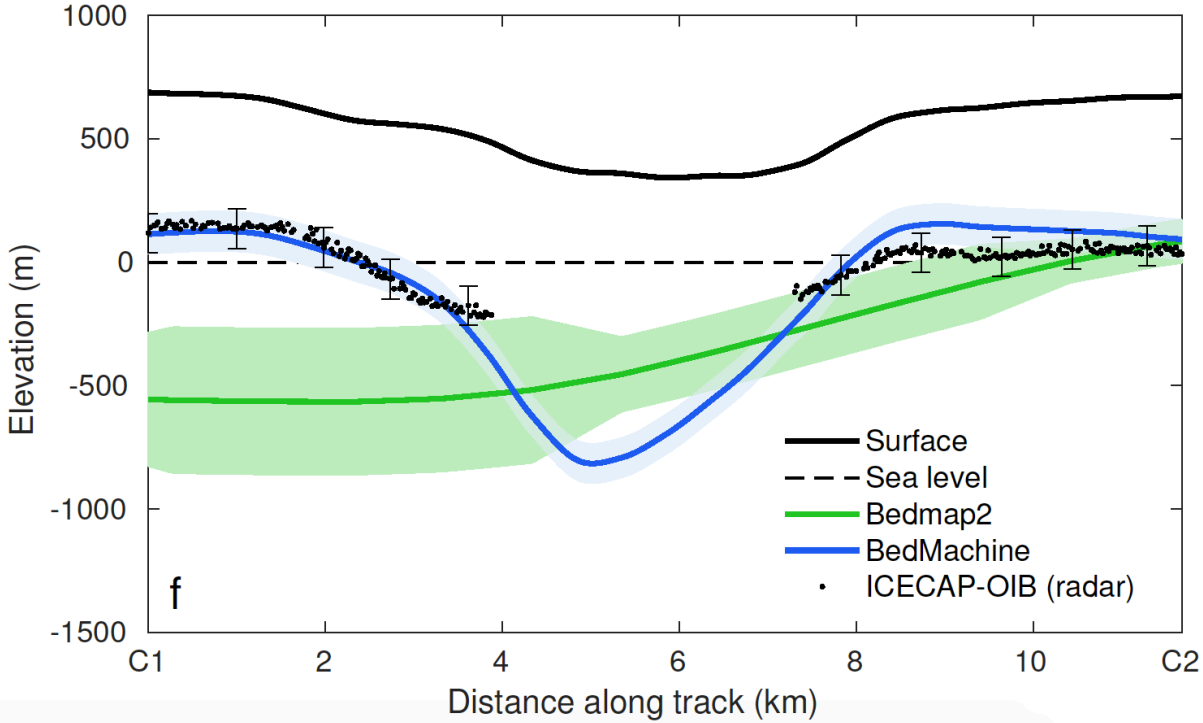
Mulock Glacier



David Glacier

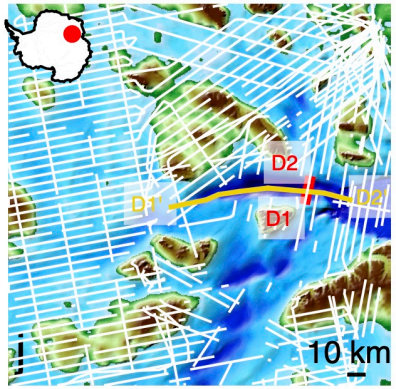


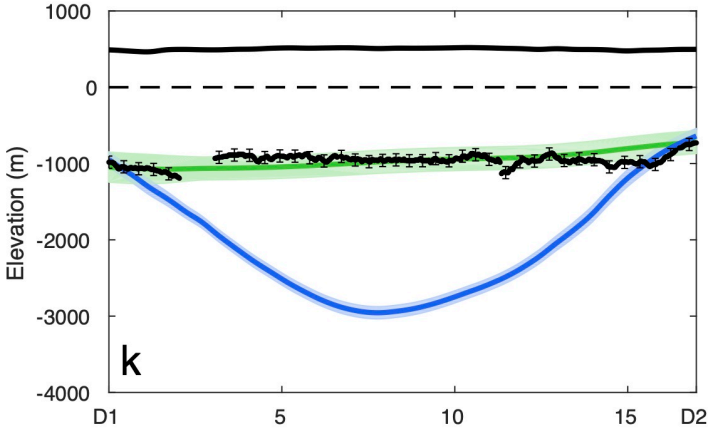


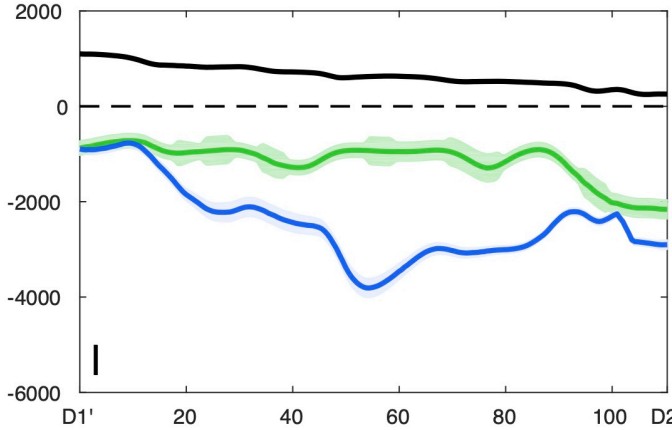


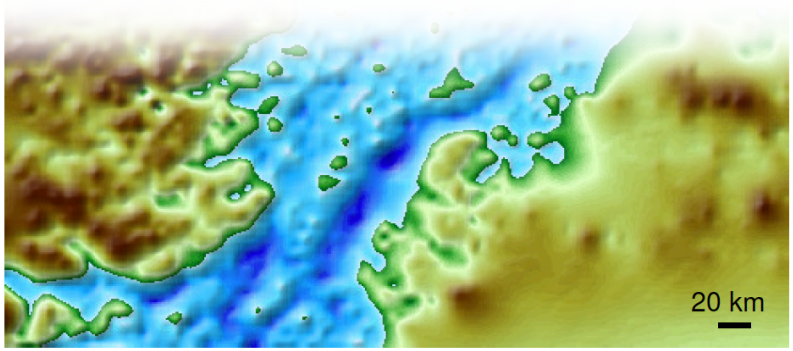
Mellor/Amery

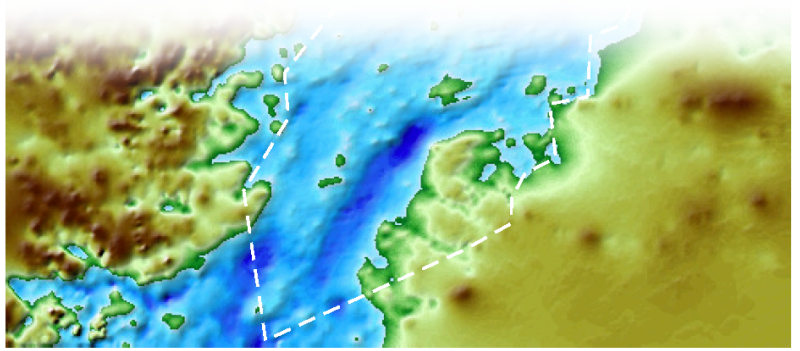
Mellor

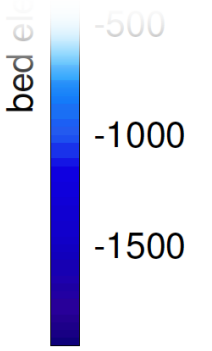








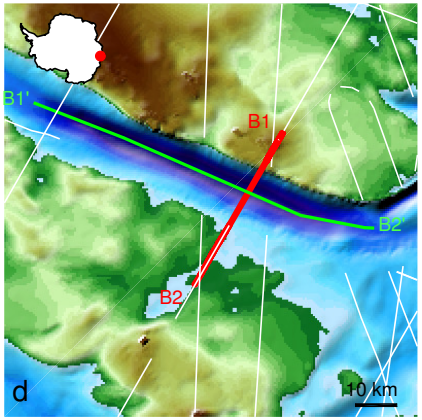


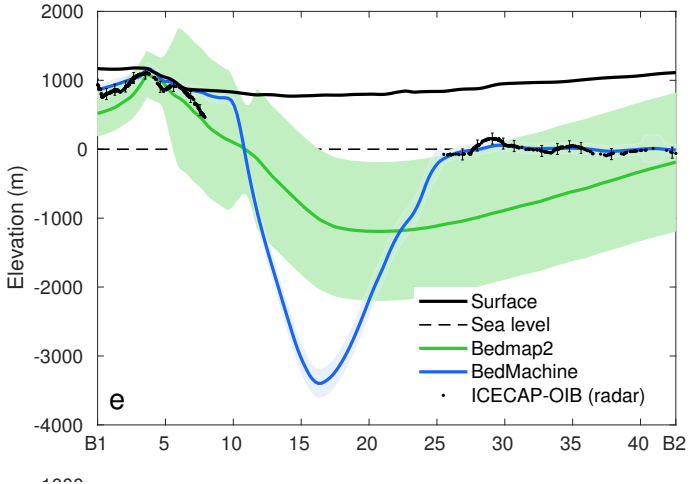


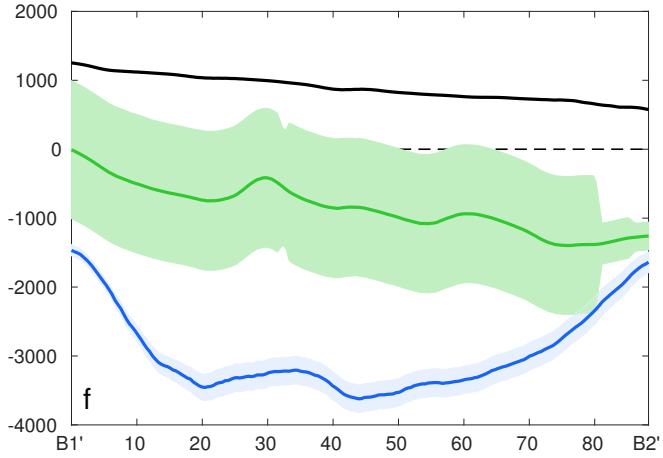
Denman

Bedmap2

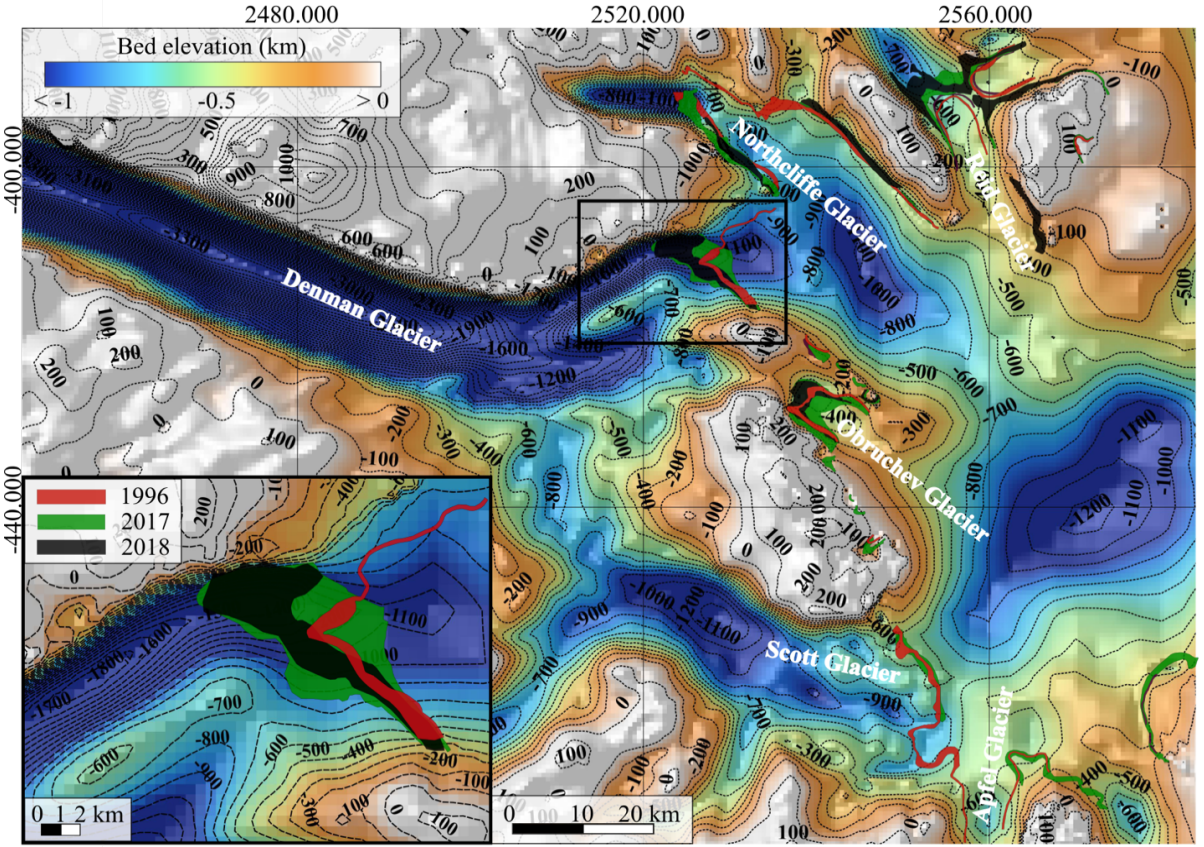
Denman





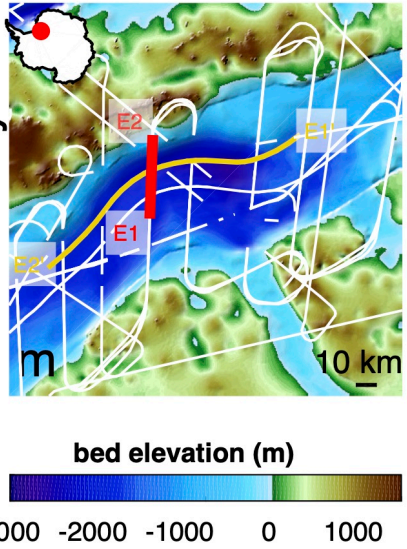


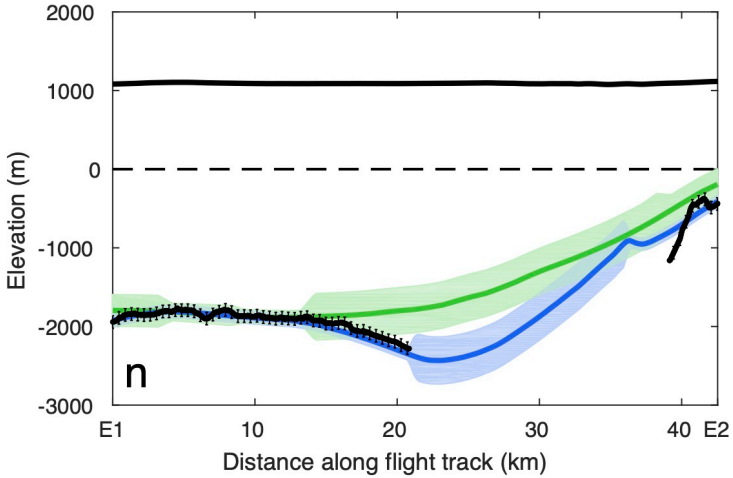
Denman

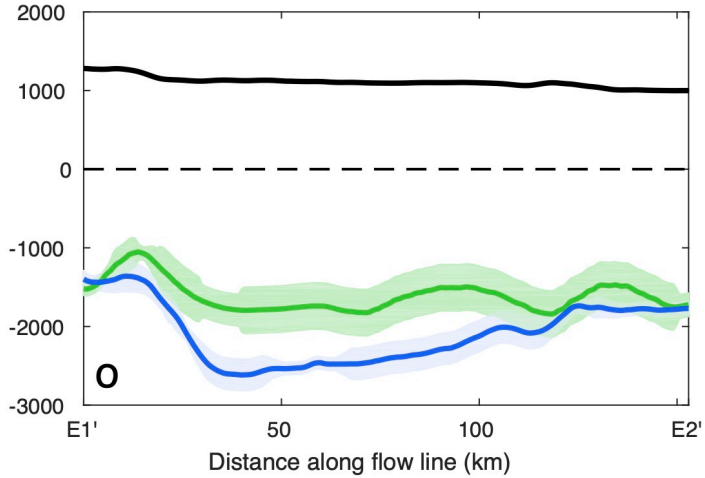


Brancato et al. 2019, submitted

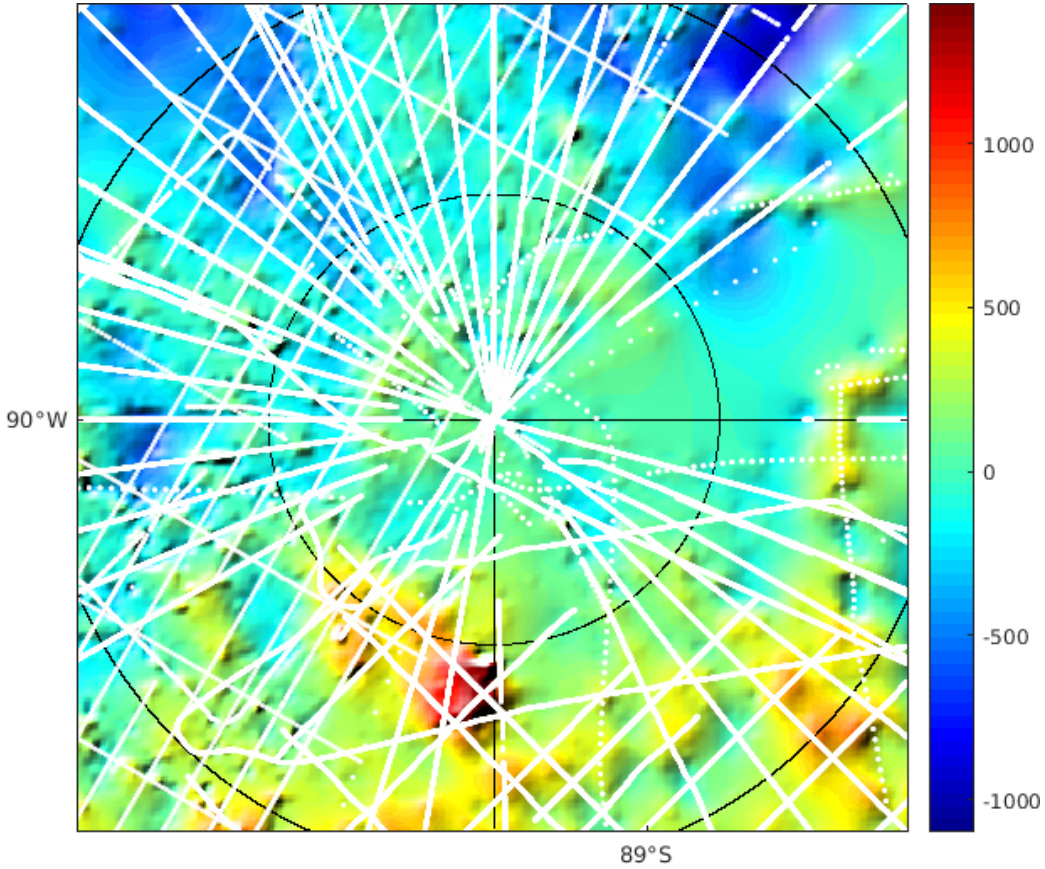
Recovery Ice Stream



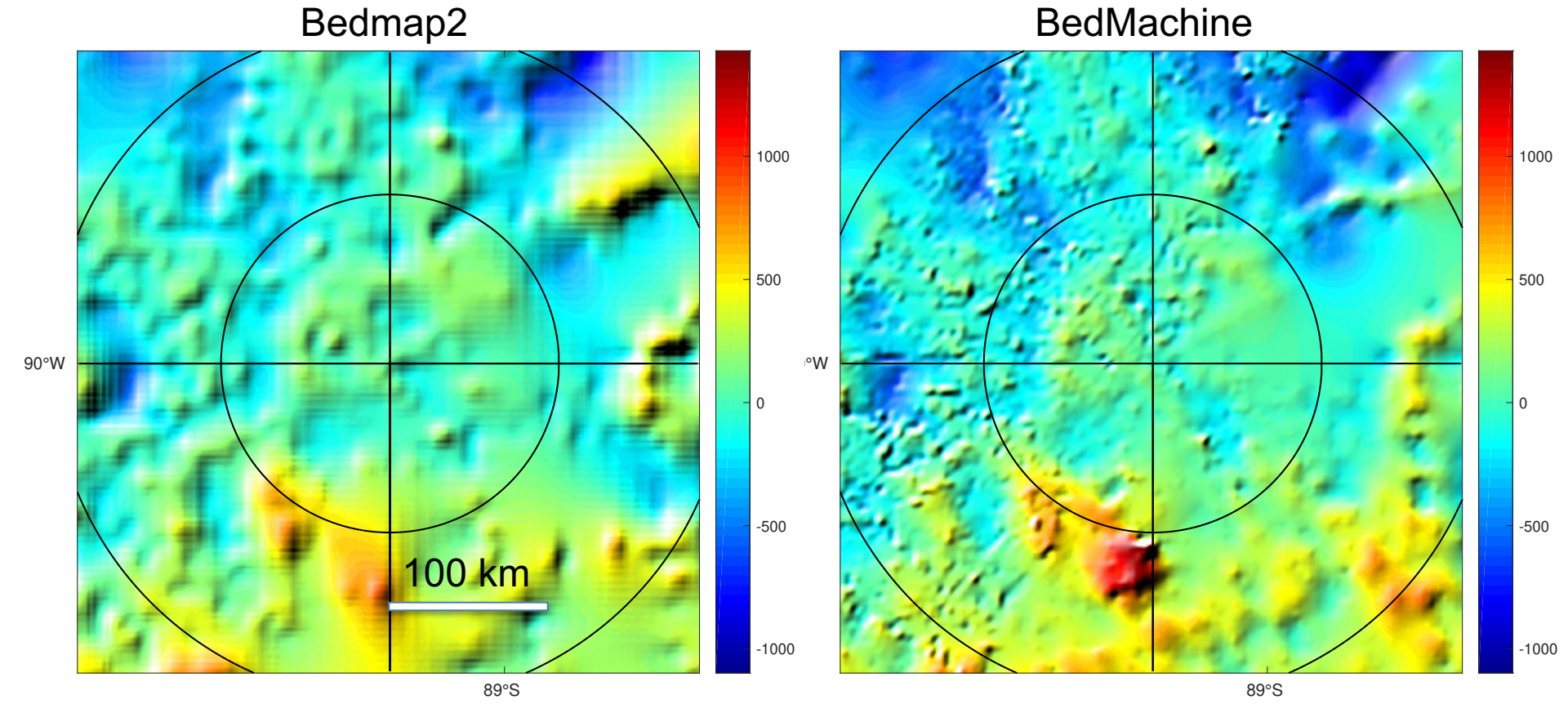




Bed topography around South Pole



Bed topography around South Pole



Thank you !
Questions ?