

The Statistics of Survival

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The Cowboy Problem

A number of cowboys walk into a bar.

The **Infinite** Cowboy Problem

A **countably infinite** number of cowboys walk into the Infinity Bar.

RIP Cowboys

Suddenly and simultaneously, all cowboys draw their guns and shoot with **unerring accuracy**.

Each targets and shoots exactly **one cowboy**, chosen at random.

You are a cowboy.

RIP You?

What are the odds that you **survive**, given that you are one of the cowboys?

- No magic bullets
- Cowboys can shoot themselves
- A cowboy can be shot multiple times
- Countably infinite cowboys.
- Each and every cowboy gets to shoot

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- 3 Three Cowboys.

$$P_{survive}^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{4}{9} = \left(\frac{2}{3}\right)^3 = \left(1 - \frac{1}{3}\right)^3$$

Far more Cowboys

n -cowboys

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Citing the work of Jacob Bernoulli (not Euler!)

$$e^x = \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n$$

It's always 0, 1, e , or π ...

$$P_{\text{survive}} = 1/e$$