Fractional Calculus

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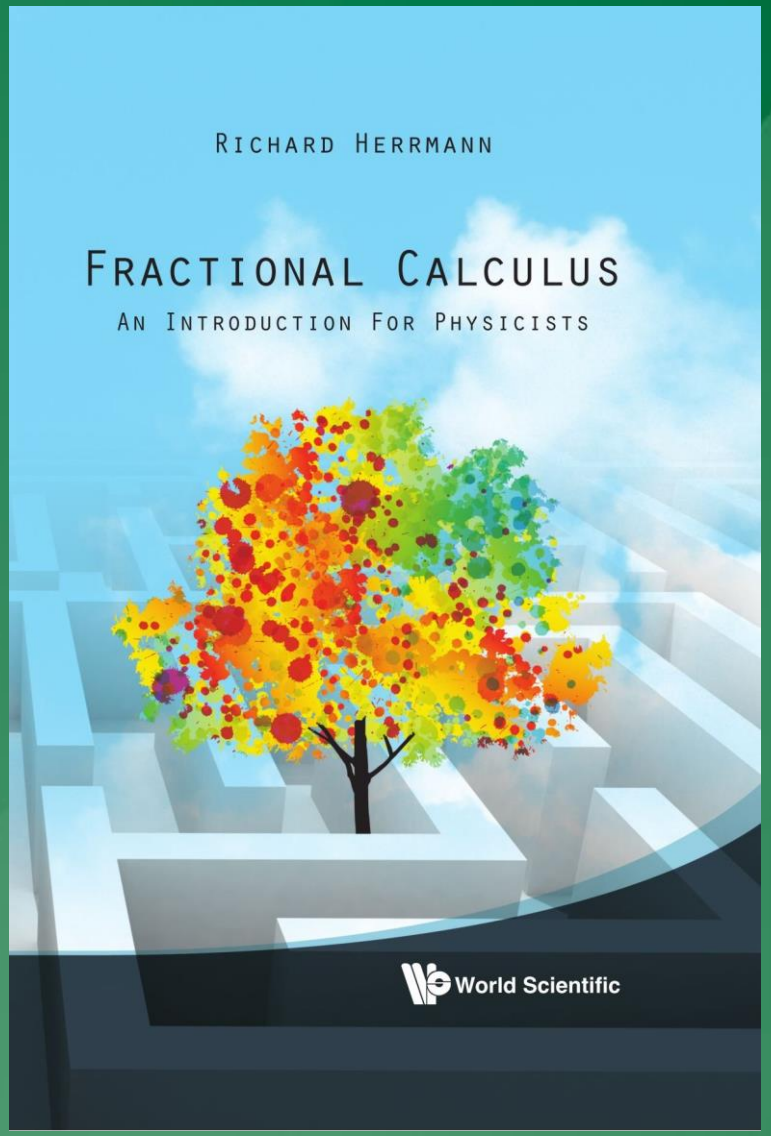
Goals

1. **Introduce you to fractional calculus**
2. **Show you were it's been used**
3. **Show you were it might be useful**



Origins and Results

Start with a letter from l'Hospital to Leibniz

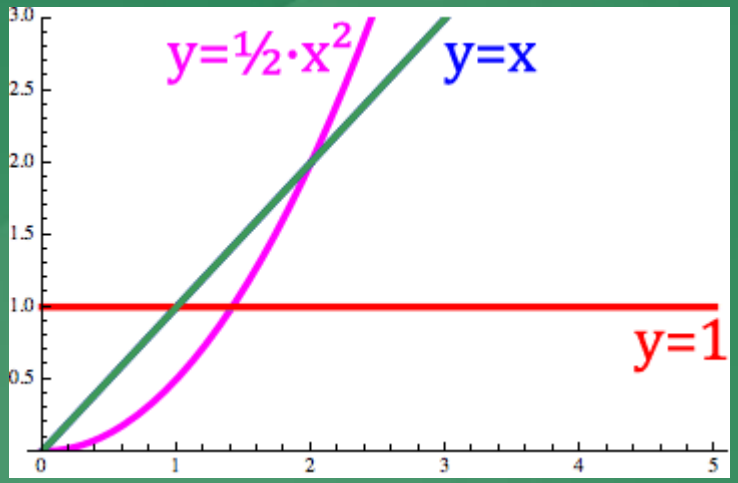
- If $\frac{d^n y}{dx^n}$ for $n \in \mathbb{Z}^+$ why not $n \in \mathbb{R}$ or $n \in \mathbb{C}$

$$\frac{d^{1/2} f}{dx^{1/2}} = ?$$

Proposing extensions

$$\frac{d^n}{dx^n} x^k = \frac{k!}{(k-n)!} x^{k-n}$$

$$\frac{d^\alpha}{dx^\alpha} x^k = \frac{\Gamma(k+1)}{\Gamma(k-\alpha+1)} x^{k-\alpha}$$



Different Derivatives

- Riemann-Liouville derivative
- Caputo derivative
- Riesz derivative
- Erdélyi–Kober derivative

Ex:

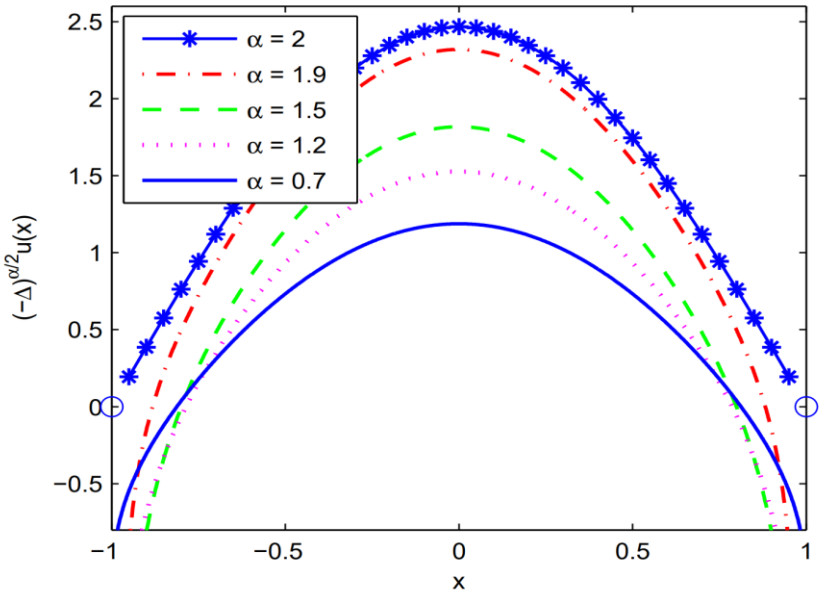
$$ {}_a^C D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau $$

INTERGRAL OPERATORS!

Applications

- Fractional quantum mechanics, first developed by Nick Laskin
- Fractional diffusion processes in anomalous materials
- A New paper discussing fractional effects in material for electrodynamics

$$i\partial_t \psi(x,t) = (-\Delta)^{\alpha/2}\,\psi + V(x)\,\psi + \beta|\psi|^2|\psi$$



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THANK YOU