Particle Acceleration & Interactions in the Extragalactic Accelerators

Colby Haggerty
University of Chicago

Overview of Lecture

- **(Very Brief) Intro to Plasma Physics**
- **Collisionless Shocks**
- **Magnetic Reconnection**
- **Plasma Turbulence: Fermi II**

Intro to Plasma Physics Overview:

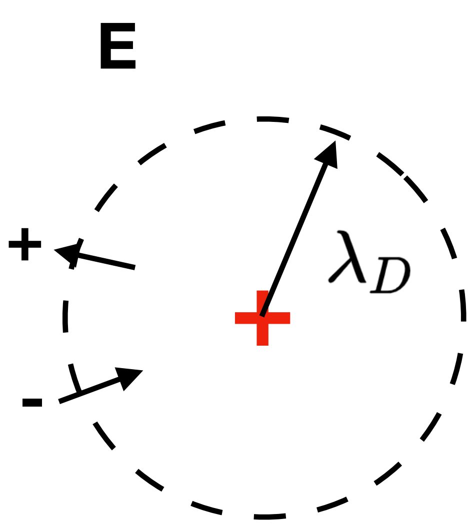
- What is a Plasma?
- Fluid Description: Magnetohydrodynamics (MHD)
- Alfvén Waves
- Fermi Acceleration Mechanism and the Power-Law
- The Hillas Criterion: The Maximum Energy Attainable

What is a Plasma?

- What makes a gas? Is 1 particle a gas? 2? 10?
- "Collection of charged particles that have a collective behavior"
- Debye Length: $$\lambda_D = (kT_e/4\pi ne^2)^{1/2}$$

Thermal/kinetic energy term

Electrostatic energy term



- Collective effects dominate when there are many particle in a Debye sphere:
- $$N_D \equiv \frac{4\pi}{3} n \lambda_D^3 \gg 1$$

Useful pages:
Plasma Constants: **https://en.wikipedia.org/wiki/Plasma_parameters**
NRL Plasma Formulary: **https://www.nrl.navy.mil/ppd/sites/www.nrl.navy.mil.ppd/files/pdfs/NRL_FORMULARY_18.pdf**

What is a Plasma?

- Collective effects dominate when there are many particle in a Debye sphere:

$$N_D \equiv \frac{4\pi}{3} n \lambda_D^3 \gg 1$$

Plasma	n_e (m^{-3})	T (K)	B (T)	λ_D (m)	N_D	ω_p (s^{-1})	ν_{ee} (s^{-1})	ω_c (s^{-1})	r_L (m)
Gas discharge	10^{16}	10^4	—	10^{-4}	10^4	10^{10}	10^5	—	—
Tokamak	10^{20}	10^8	10	10^{-4}	10^8	10^{12}	10^4	10^{12}	10^{-5}
Ionosphere	10^{12}	10^3	10^{-5}	10^{-3}	10^5	10^8	10^3	10^6	10^{-1}
Magnetosphere	10^7	10^7	10^{-8}	10^2	10^{10}	10^5	10^{-8}	10^3	10^4
Solar core	10^{32}	10^7	—	10^{-11}	1	10^{18}	10^{16}	—	—
Solar wind	10^6	10^5	10^{-9}	10	10^{11}	10^5	10^{-6}	10^2	10^4
Interstellar medium	10^5	10^4	10^{-10}	10	10^{10}	10^4	10^{-5}	10	10^4
Intergalactic medium	1	10^6	—	10^5	10^{15}	10^2	10^{-13}	—	—

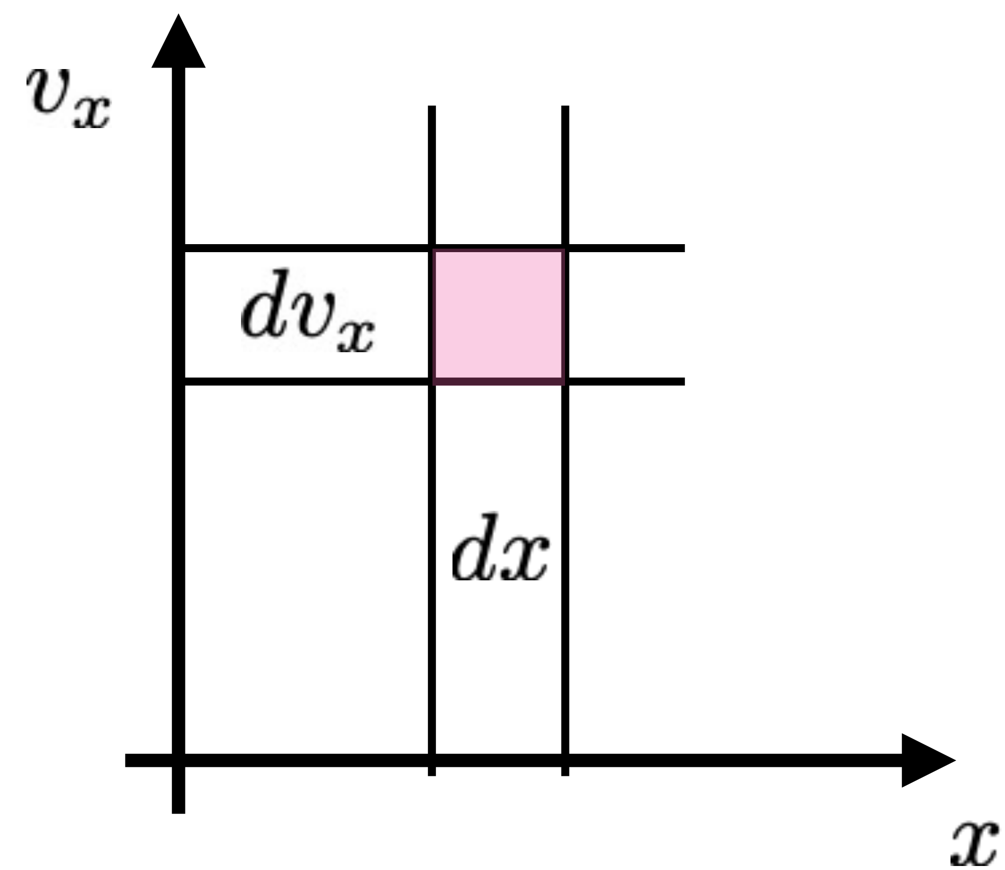
APPLICATIONS OF CLASSICAL PHYSICS: Blanford and Thorn: http://www.pmaweb.caltech.edu/Courses/ph136/yr2012/

The Distribution Function

- We can treat the plasma as a statistical collective fluid.

- Probability of find a particle in phase/real space

$$N = f(x, v_x, t)\ dx\, dv_x$$



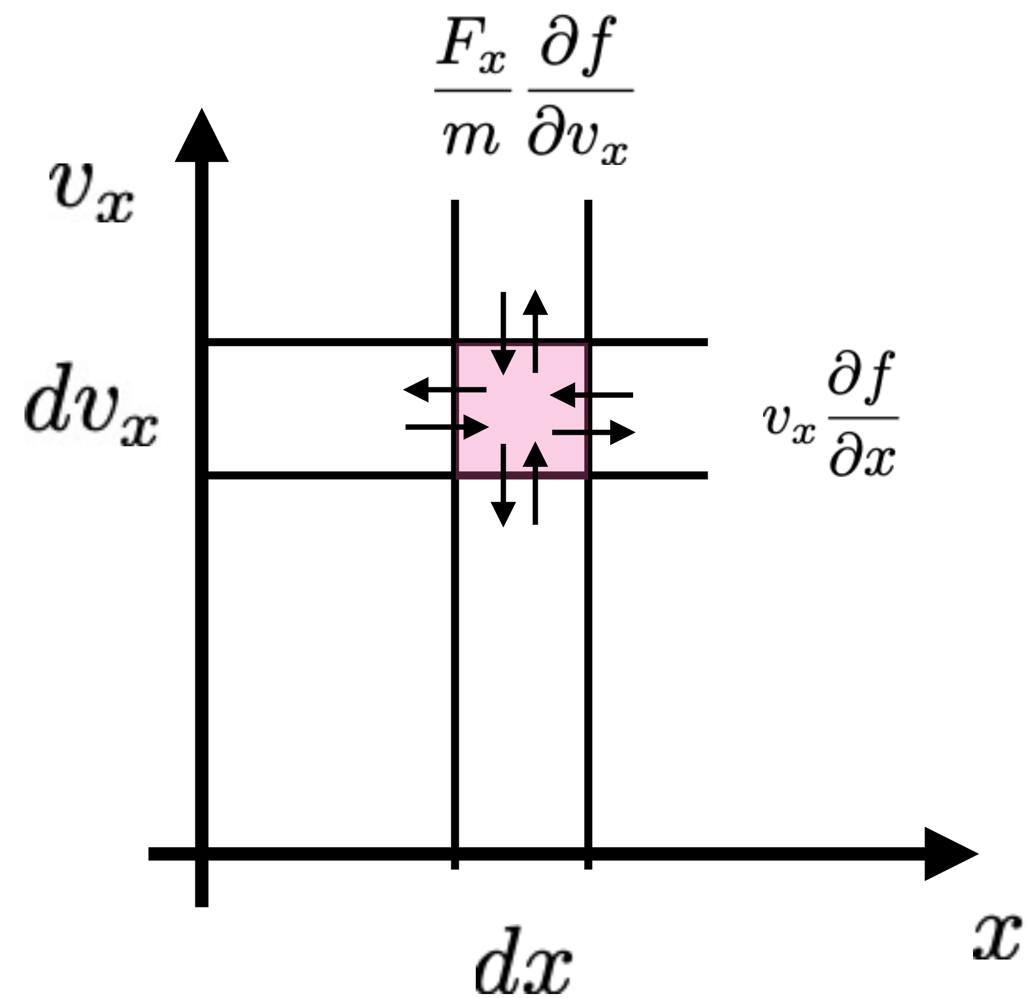
- Understanding how f evolves in time tells us everything we want to know about a plasma

- This is the **Kinetic** description of plasma

Boltzmann's Equation

- Phase space continuity equation for a given species

$$\frac{\partial f_s}{\partial t} + \vec{v} \cdot \vec{\nabla} f_s + \frac{\vec{F}_s}{m_s} \cdot \vec{\nabla}_v f_s = \delta_c f_s$$



- When the collision operator is neglected and the forces are E/M

- The Vlasov Equation:

$$\frac{\partial f_s}{\partial t} + \vec{v} \cdot \vec{\nabla} f_s + \frac{e_s}{m_s}\left[\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right] \cdot \vec{\nabla}_v f_s = 0$$

Moments of the Distribution Function

- So how is the distribution function useful:

0th moment (Density)
$$n_s = \int f_s d^3v$$

1st moment (Bulk Flow)
$$n_s u_{s,i} = \int v_i f_s d^3v$$

2nd moment (Pressure)
$$P_{s,i,j} = m_s \int (v_i - u_i)(v_j - u_j) f_s d^3v$$

- Integrating gives more "physical" fluid quantities
  And we can do the same thing to the Vlasov equation

$$\int d^3v \begin{pmatrix} 1 \\ v \\ v^2 \end{pmatrix} \left[\frac{\partial f_s}{\partial t} + \vec{v} \cdot \vec{\nabla} f_s + \frac{e_s}{m_s} \left[\vec{E} + \frac{\vec{v}}{c} \times \vec{B} \right] \cdot \vec{\nabla}_v f_s \right]$$

Magnetohydrodynamics (MHD)

- If we are only worried about *__large length and long time scales__* and we just consider ions and electrons. We can make the following approximations:

  - Quasi-neutrality $n \equiv n_i = n_e$

  - We can ignore light waves $\nabla \times B = \frac{4\pi}{c} J$

  - The electron mass is negligible compared to the proton. Electrons flow with ions to 0th order

Magnetohydrodynamics MHD Equations

Continuity:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{u} = 0$$

Force/ momentum:

$$\rho \left(\frac{\partial}{\partial t} + \vec{u} \cdot \nabla \right) \vec{u} = -\nabla \left(\frac{B^2}{8\pi} + P \right) + \frac{\vec{B} \cdot \nabla \vec{B}}{4\pi}$$

Magnetic Curvature

Thermal/Magnetic Pressure

Energy:

$$\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{p}{\rho^\gamma} \right) = 0$$

Faraday's/ Ohm's law

$$\nabla \times B = \frac{4\pi}{c} J$$

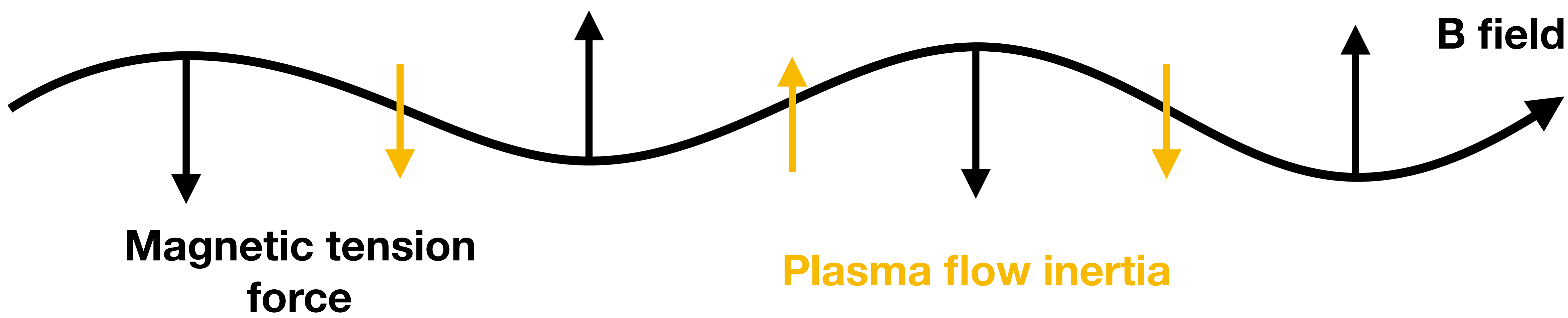
"Frozen-In"/ "Flux Freezing" Condition

$$\vec{E} = -\frac{\vec{u}}{c} \times \vec{B}$$

This is the **Fluid** description of plasma

The Alfvén wave

- MHD equations are like Naiver Stokes equations. Same term that makes sound waves. But extra curvature term creates wave like a wave on a string



- The Alfvén Speed: $$V_A = \frac{B}{\sqrt{4\pi m_i n}}$$

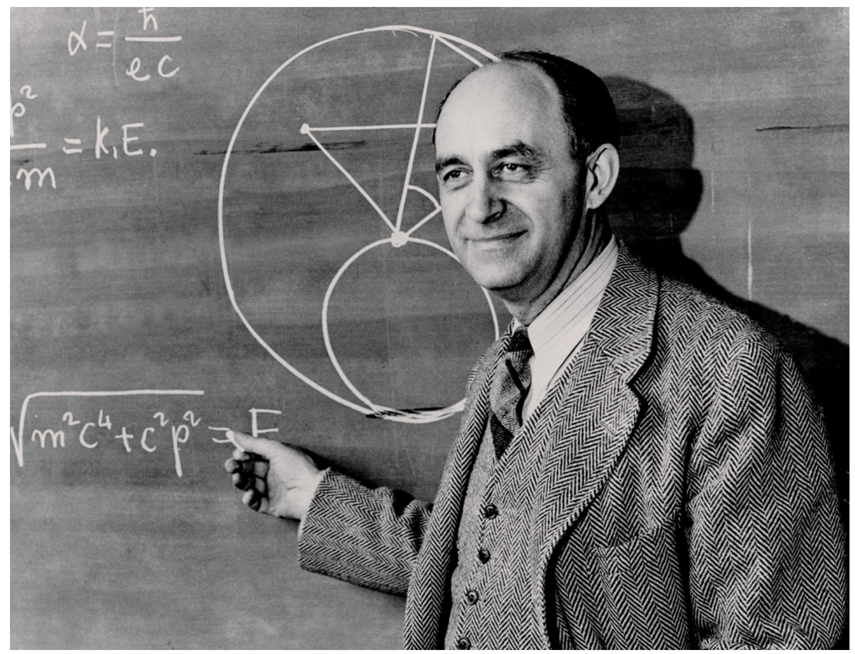
Why is this Important for Particle Acceleration?

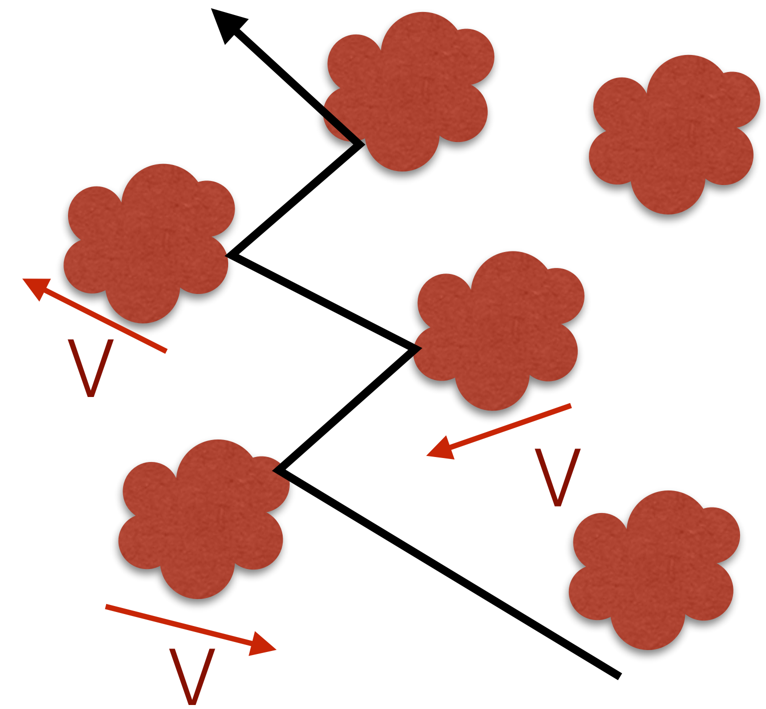
- The **MHD description** details the large scale systems: The energy that can be channeled into particle acceleration, and the conditions that allow for efficient acceleration

- The **Kinetic description** details the micro-physics of how charged particles are energized and accelerated.

How do Collisionless Plasmas Accelerate Charged Particles?

Fermi Acceleration!





PHYSICAL REVIEW VOLUME 75, NUMBER 8 APRIL 15, 1949

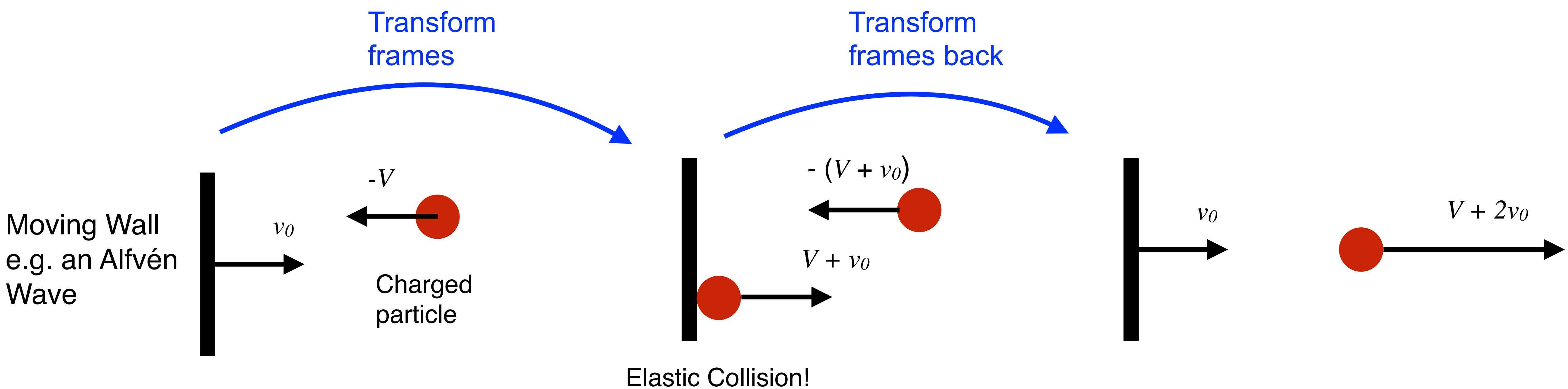
On the Origin of the Cosmic Radiation

ENRICO FERMI
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magmetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

Fermi Acceleration



In the relativistic limit with an arbitrary angle between the magnetic fluctuation and the particles we get:

$$\Delta E \sim E \left(\frac{v_0}{c} \cos(\theta) + \frac{v_0^2}{c^2} \right)$$

Energy gain is proportional to the Energy!

First order Acceleration

Second order Acceleration

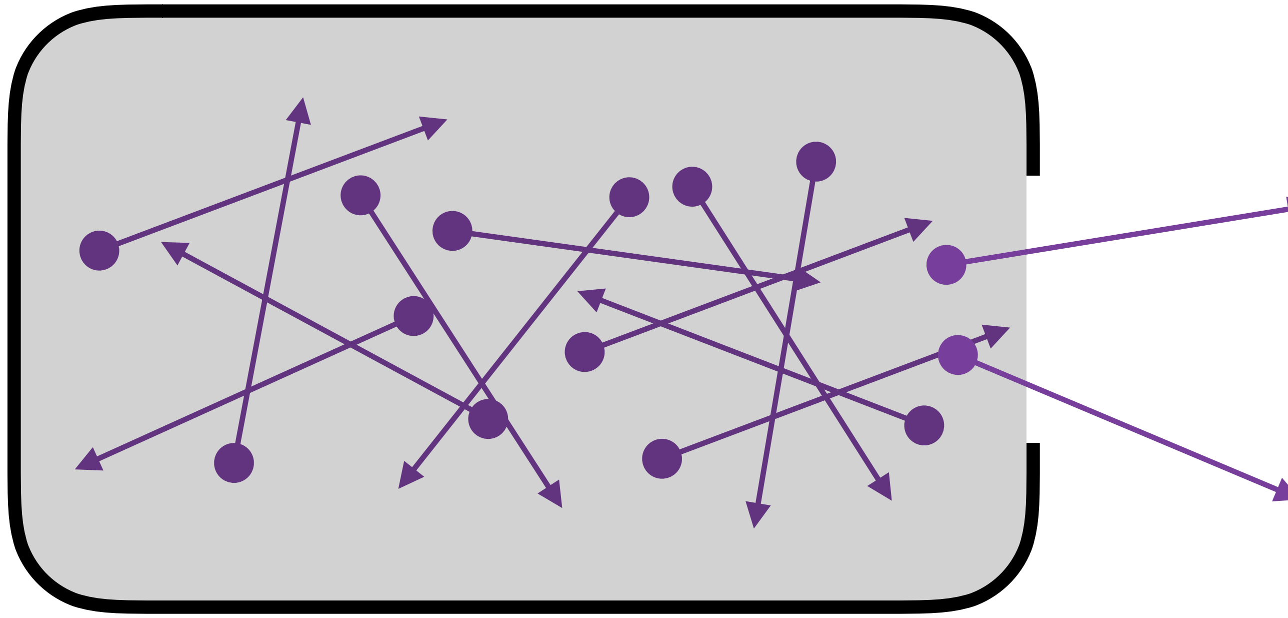
Assuming the flows and the particle velocity direction are uncorrelated and averaging over all directions you get:

$$\langle \Delta E \rangle = \frac{8}{3} \left(\frac{v_0}{c} \right)^2 E$$

Fermi Acceleration

- Fermi Acceleration (or any acceleration proportional to the total energy) leads to a **Power Law**

Energy Gain Per Interaction (G)



Probability of Particle Remaining (P)

Black Box Energetic Particle Accelerator

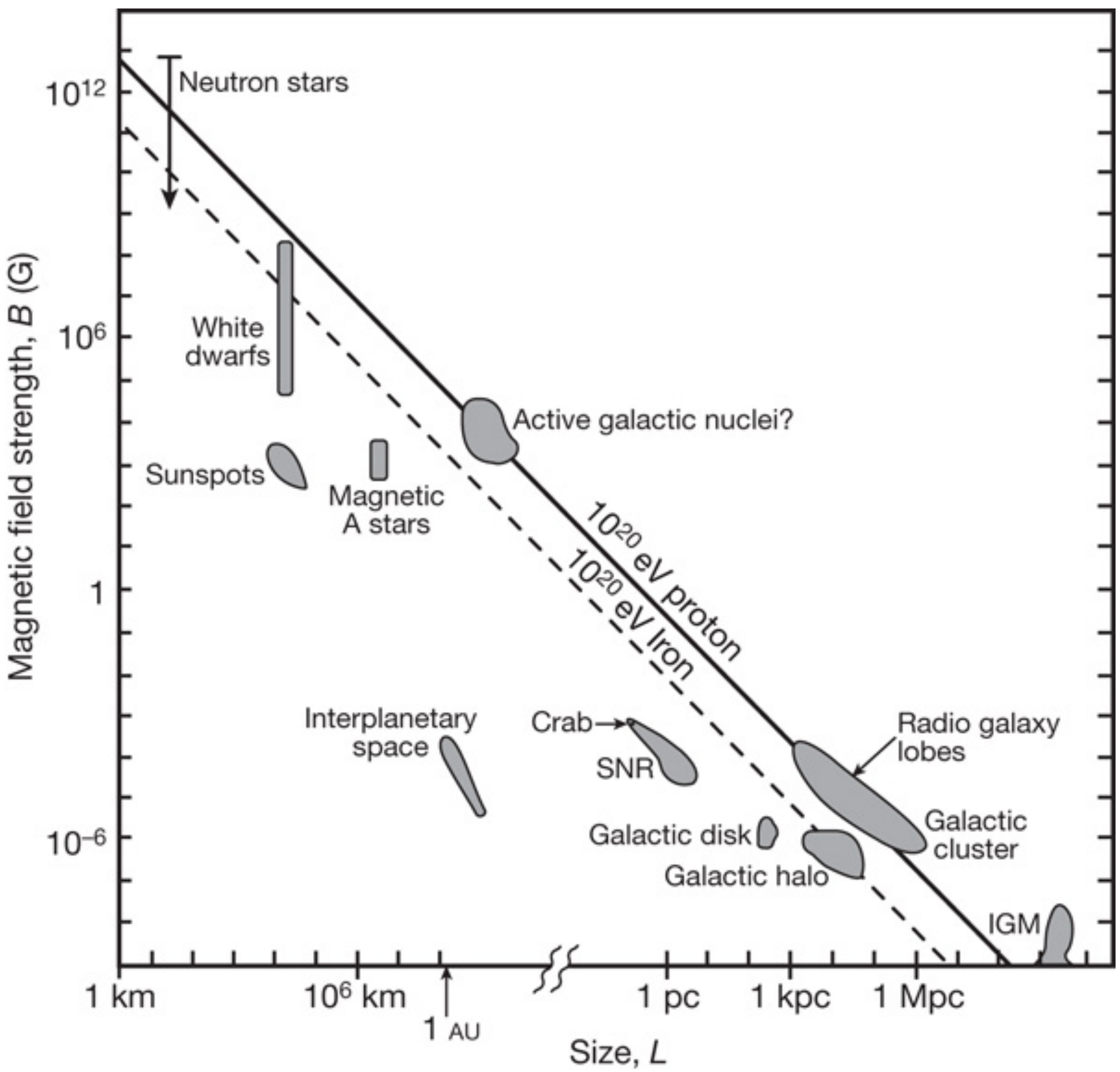
- The number of particles that remain in the box after "k" cycles, will have energy E_k

Number of Particles $N_k = P^k N_0$

Energy $E_k = G^k E_0$

$$\frac{N}{N_0} = \left(\frac{E}{E_0}\right)^{-\ln P / \ln G}$$

The Hilas Criterion



- From the basic idea of MHD, what is the Max energy attainable from a system with a given size *L* and magnetic field *B*

$$E_{\max} \sim ZeLE \sim ZeL\beta B$$

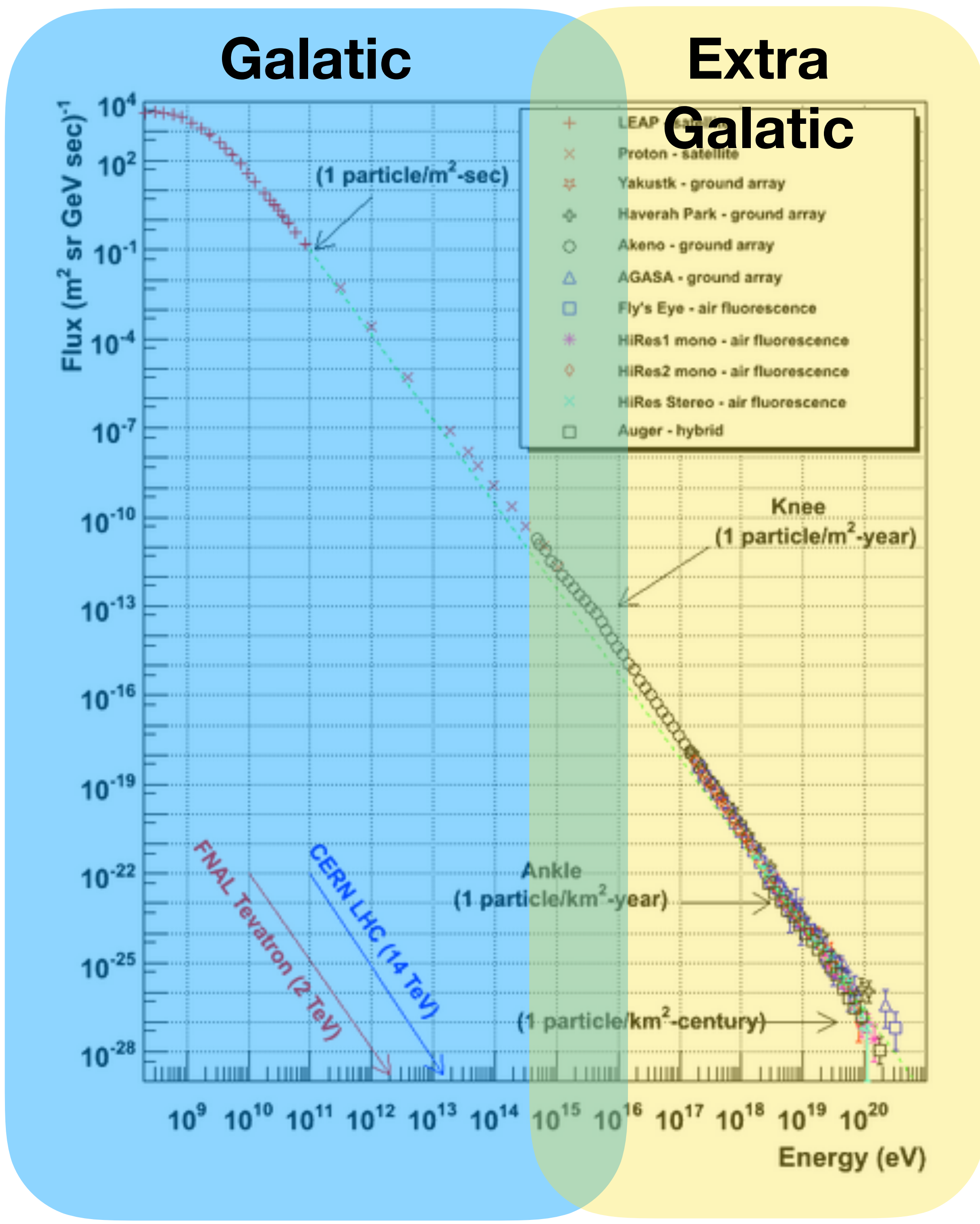
$$\frac{2E_{[\mathrm{PeV}]}}{Z\beta} < B_{[\mu\mathrm{G}]} L_{[\mathrm{pc}]}$$

Hillas diagram, from P. M. Bauleo & J. R. Martino *Nature* **458**, 847-851(16 April 2009)

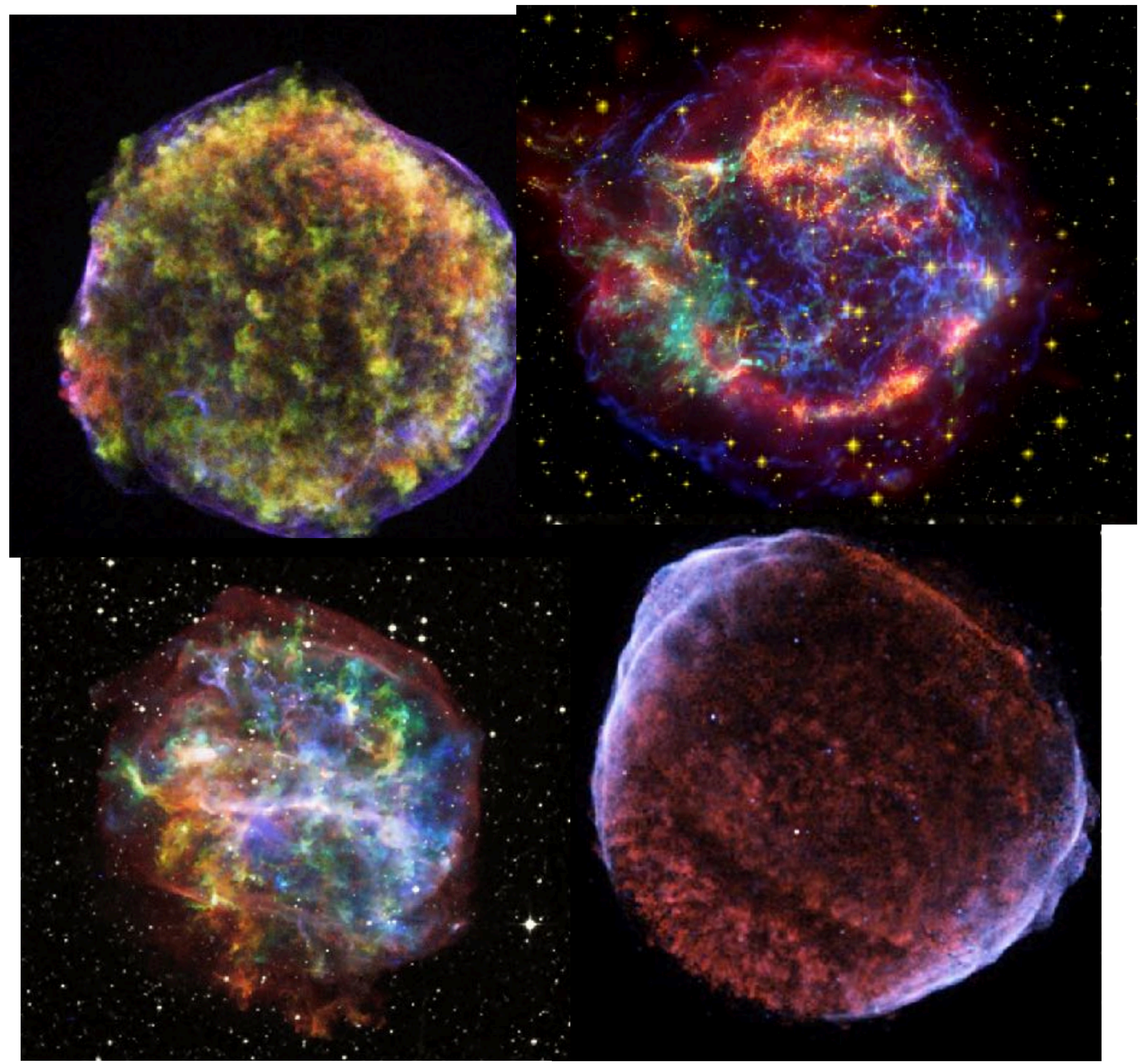
Collisionless Shocks

- Astrophysical Shocks
- Shock Hydrodynamics
- Diffusive Shock Acceleration (DSA)
- Simulations of Shocks

Galactic Cosmic Ray Spectrum

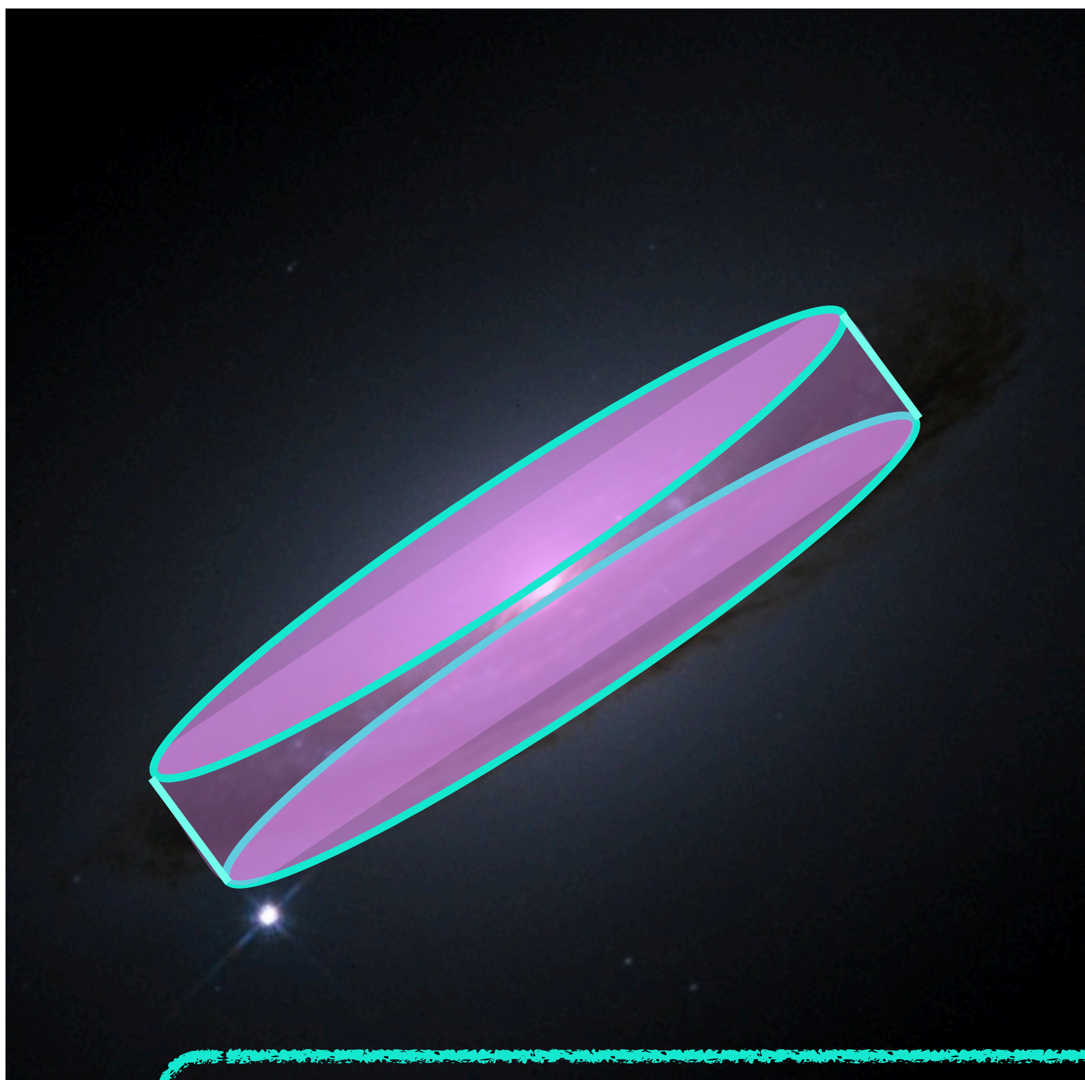


from http://www.physics.utah.edu/~whanlon/spectrum.html



Super Nova Remnant (SNR) Paradigm: Energetics

- Baade-Zwicky (1934): Energetic argument



$$\varepsilon_{CR} = .5 eV\,\mathrm{cm}^{-3}$$

$$V_{\mathrm{conf}} = \pi R^2 h = 2 \times 10^{67} \mathrm{cm}^3$$

$$W_{CR} = \varepsilon_{CR} V_{\mathrm{conf}} \approx 2 \times 10^{55} \mathrm{erg}$$

$$L_{CR} \approx \frac{W_{CR}}{\tau_{\mathrm{conf}}} \approx 5 \times 10^{40} \mathrm{erg}\ s^{-1}$$

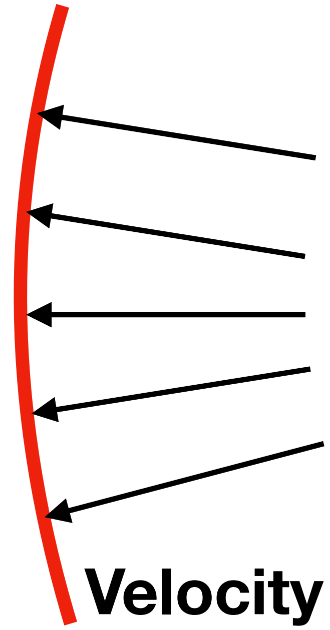
$$L_{SN} = R_{SN} E_{kin} \approx 3 \times 10^{41} \mathrm{erg} s^{-1}$$

~10% of SN ejecta kinetic energy can account for galactic CRs

(M)HD Shocks

Up-stream (subscript 1)

Smaller density,
smaller temperature,
flows are supersonic



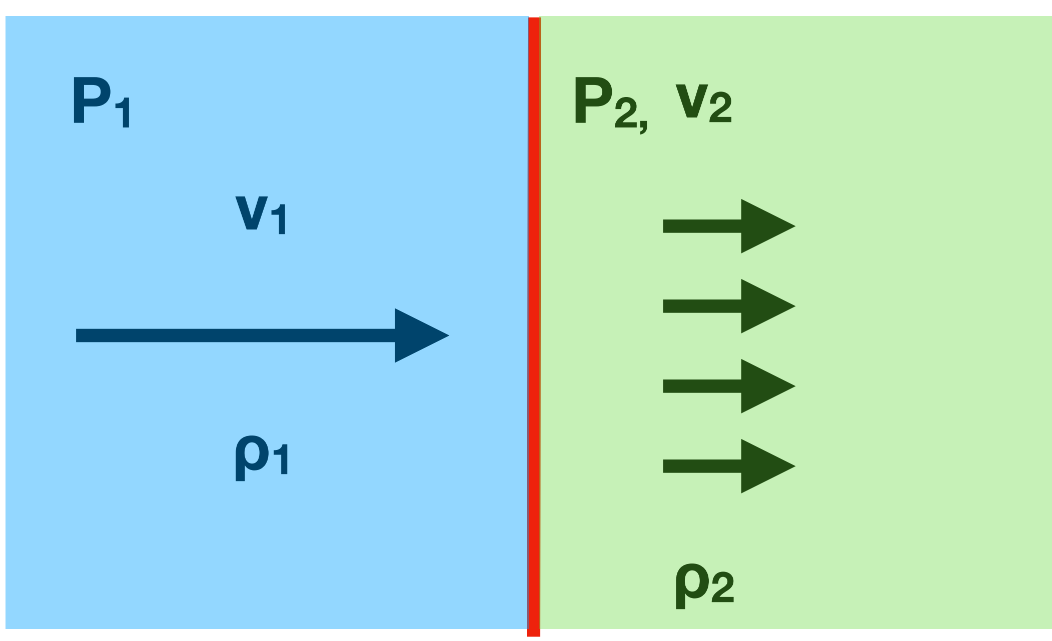
Down-stream (subscript 2)

Larger density,
larger temperature,
flows are subsonic

- In the frame moving with the shock everything looks stationary and quasi-1D. We can use a conservative form of the hydro equations

Rankine-Hugoniot Jump Conditions!

Up **Down**



$$[\rho v]_2^1$$

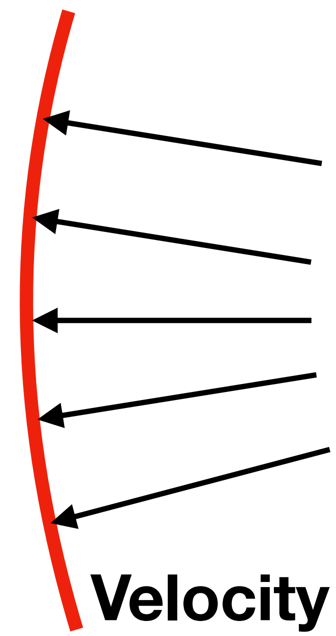
$$\left[\rho v^2 + P\right]_2^1$$

$$\left[\frac{1}{2}\rho v^3 + \frac{\gamma}{\gamma - 1}Pv\right]_2^1$$

(M)HD Shocks

Up-stream (subscript 1)

Smaller density,
smaller temperature,
flows are supersonic

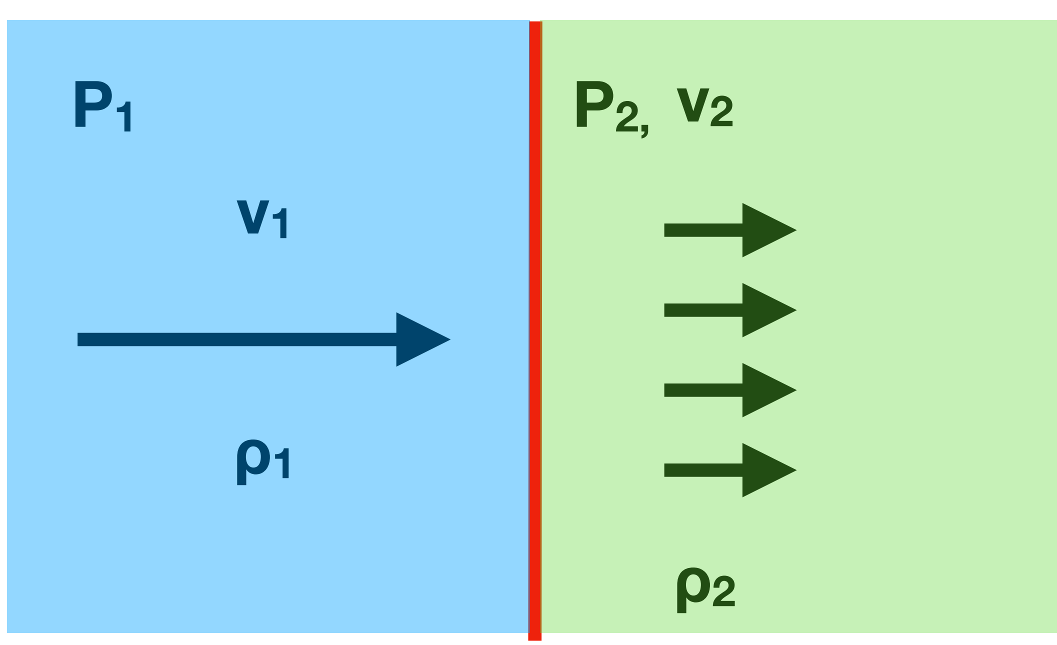


Down-stream (subscript 2)

Larger density,
larger temperature,
flows are subsonic

- In the frame moving with the shock everything looks stationary and quasi-1D. We can use a conservative form of

Up **Down**



$$r = \frac{v_1}{v_2} = \frac{\rho_2}{\rho_1}, \quad M = \sqrt{\frac{\rho_1 v_1^2}{\gamma P_1}}$$

Compression Ratio — **Sonic/Alfvénic Mach Number**

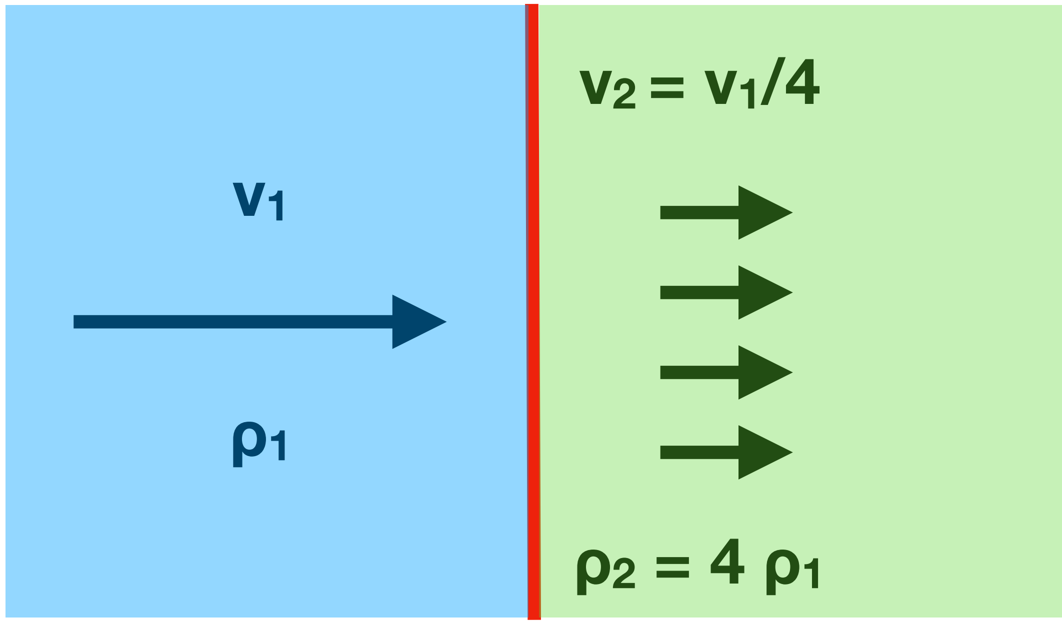
$$M \gg 1 \qquad r \approx \frac{\gamma + 1}{\gamma - 1} = 4$$

Solving the RH Jump Conditions

- Let $r = \frac{v_1}{v_2} = \frac{\rho_2}{\rho_1}, \; M = \frac{\rho_1 v_1^2}{\gamma P_1}$
- Then:

$$\frac{\rho_1}{\rho_2} = \frac{\gamma - 1}{\gamma + 1} + \frac{2}{(\gamma + 1)M^2}, \qquad \frac{P_2}{P_1} = \frac{2\gamma M^2}{\gamma + 1} - \frac{\gamma - 1}{\gamma + 1}$$

- For typical SNR $M \gg 1$ and so

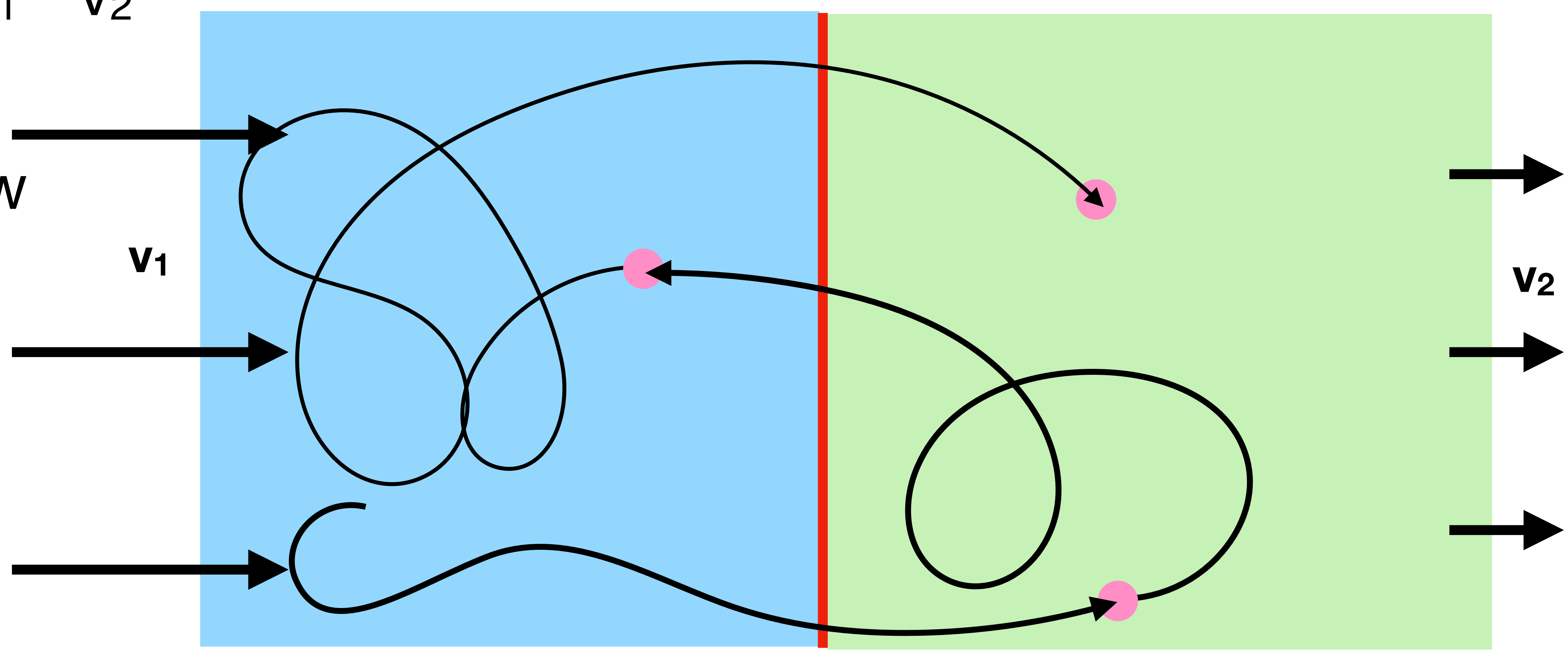


$$r \approx \frac{\gamma + 1}{\gamma - 1} = 4$$

Diffusive Shock Acceleration (DSA)

- Imagine that we have an energetic particle with a gyro radius larger than the shocks width. This particle can pass both up and down stream of the shock. Each time the particle crosses the shock it will experience a Fermi kick proportional to the difference in velocity v_1 - v_2

- Think back to power-law idea. We need: probability to remain and energy gain



$$P = 1 - 4\frac{v_2}{c}$$

$$G = 1 + \frac{4(v_1 - v_2)}{3c}$$

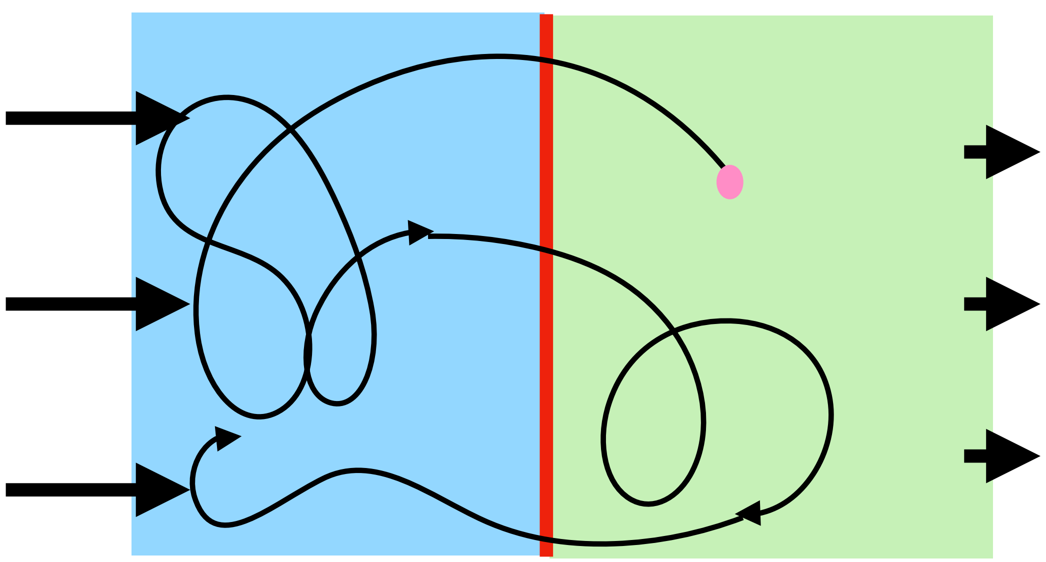
$$q = 1 - \frac{\ln P}{\ln G} \approx 1 + \frac{3v_3}{v_1 - v_2} = \frac{r+2}{r-1}$$

$$f(E) \propto E^{-2}$$

DSA Energy Gain

- Every time a particle goes through one of these cycles it will have an energy gain associated with a fermi reflection

$$E' = \gamma_V(E + p_x V) \approx E(1 + \frac{V}{c}\cos\theta)$$



where

$$\mathrm{V} = v_1 - v_2$$

- Averaging over all angles:

$$\frac{\Delta E}{E} = \int_0^{\pi/2} \frac{V}{c}\cos\theta \cdot 2\cos\theta\sin\theta d\theta = \frac{2V}{3c} \rightarrow \frac{4V}{3c}$$

What order is this?

Power Law from DSA

- Consider the energy gain/probability of a particle escaping per cycle.

- Number of remaining Particles/Energy after the k^th cycle

$$N_k = N_0 P^k, \; E_k = E_0 \Gamma^k$$

Where P is the probability of remaining and Γ comes from Fermi

- From this we can get an equation for the number of particles as a function of E

$$N(E) \propto E^{\frac{\ln P}{\ln \Gamma}}$$

$$f(E) = \frac{dN}{dE} \propto E^{\frac{\ln P}{\ln \Gamma} - 1}$$

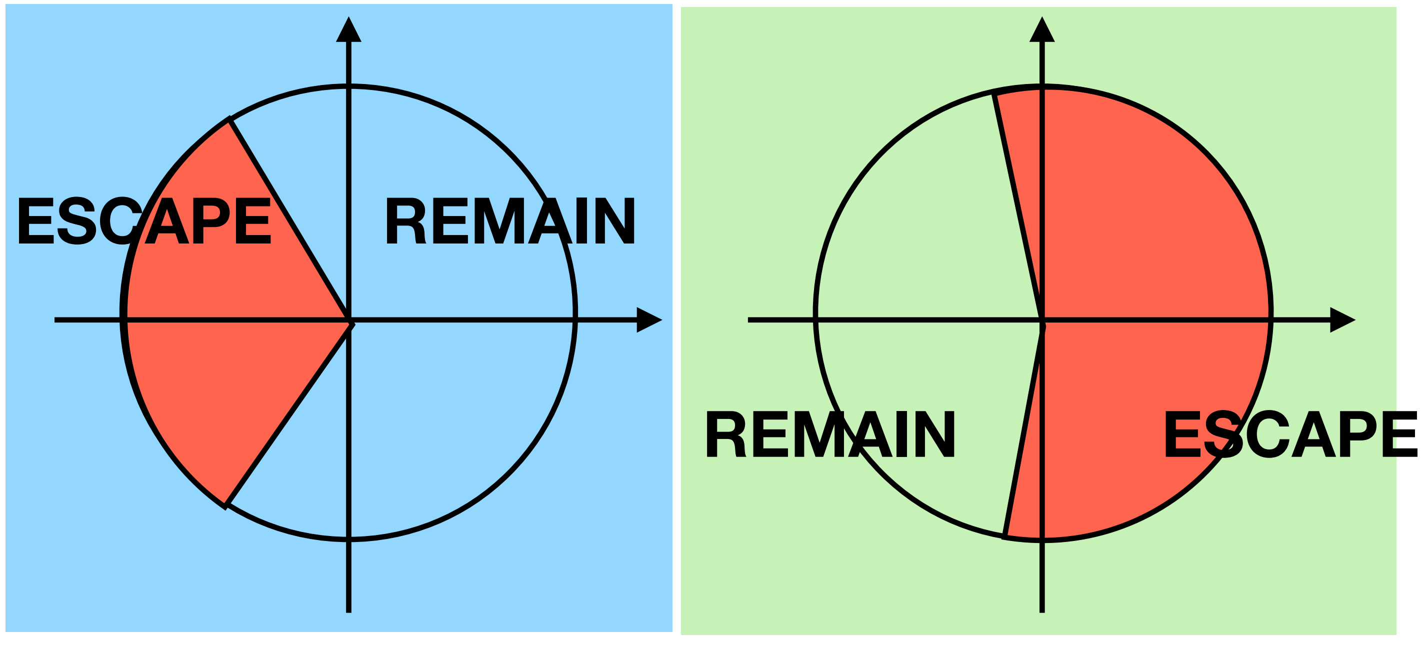
- Since P < 1 and Γ > 1 f(E) will be a power law with and index

$$f(E) \propto E^{-q}; \; q = 1 - \frac{\ln P}{\ln \Gamma}$$

Power Law from DSA

- Finally we just need to calculate the probability of escape over one cycle:

Velocity space up and down stream



- Integrating over the particles with pitch angles that will cross the shock again gives:

$$P = 1 - 4\frac{v_2}{c}$$

Power Law from DSA

- Finally we just need to calculate the probability of escape over one cycle:

- Integrating over the particles with pitch angles that will cross the shock again gives: $P = 1 - 4\frac{v_2}{c}$

- Then we can calculate the DSA predicted power law index:

$$q = 1 - \frac{\ln P}{\ln \Gamma} = 1 - \frac{\ln(1 - 4v_2/c)}{\ln(1 + 4(v_1 - v_2)/3c)} \approx 1 + \frac{3v_2}{v_1 - v_2} = \frac{r+2}{r-1}$$

- For strong shock recall r -> 4 => $f(E) \propto E^{-2}$

or $f(p) \propto p^{-4}$

Particle Acceleration Mechanisms (#2)

Colby Haggerty
University of Chicago

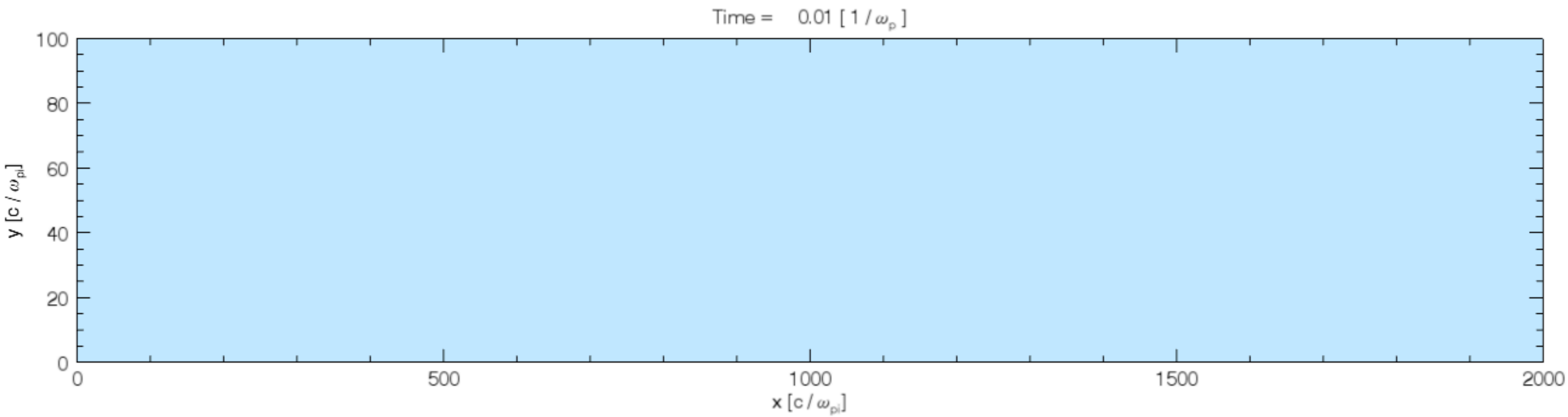
Overview of Lecture

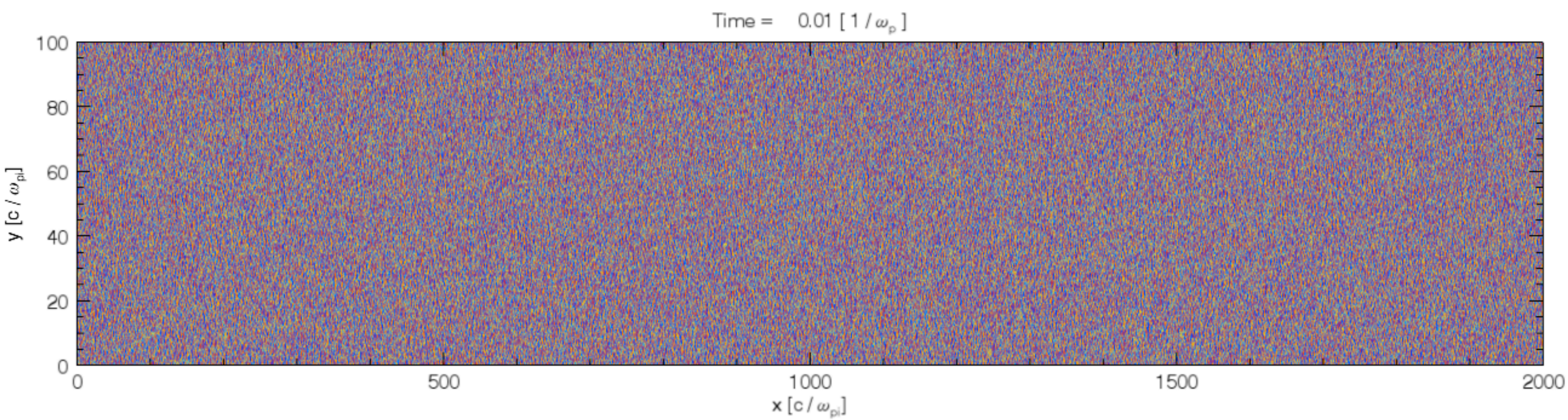
- Intro to Plasma Physics:
  From MagnetoHydroDynamics (MHD) to Kinetic Physics
- Collisionless Shocks:
  Diffusive Shock Acceleration (DSA)
- Magnetic Reconnection: **Lesson #2**
  Magnetic Energy Conversion Engine
- MHD/Kinetic Plasma Turbulence:
  The Actual context for all of these systems

Missing Pieces

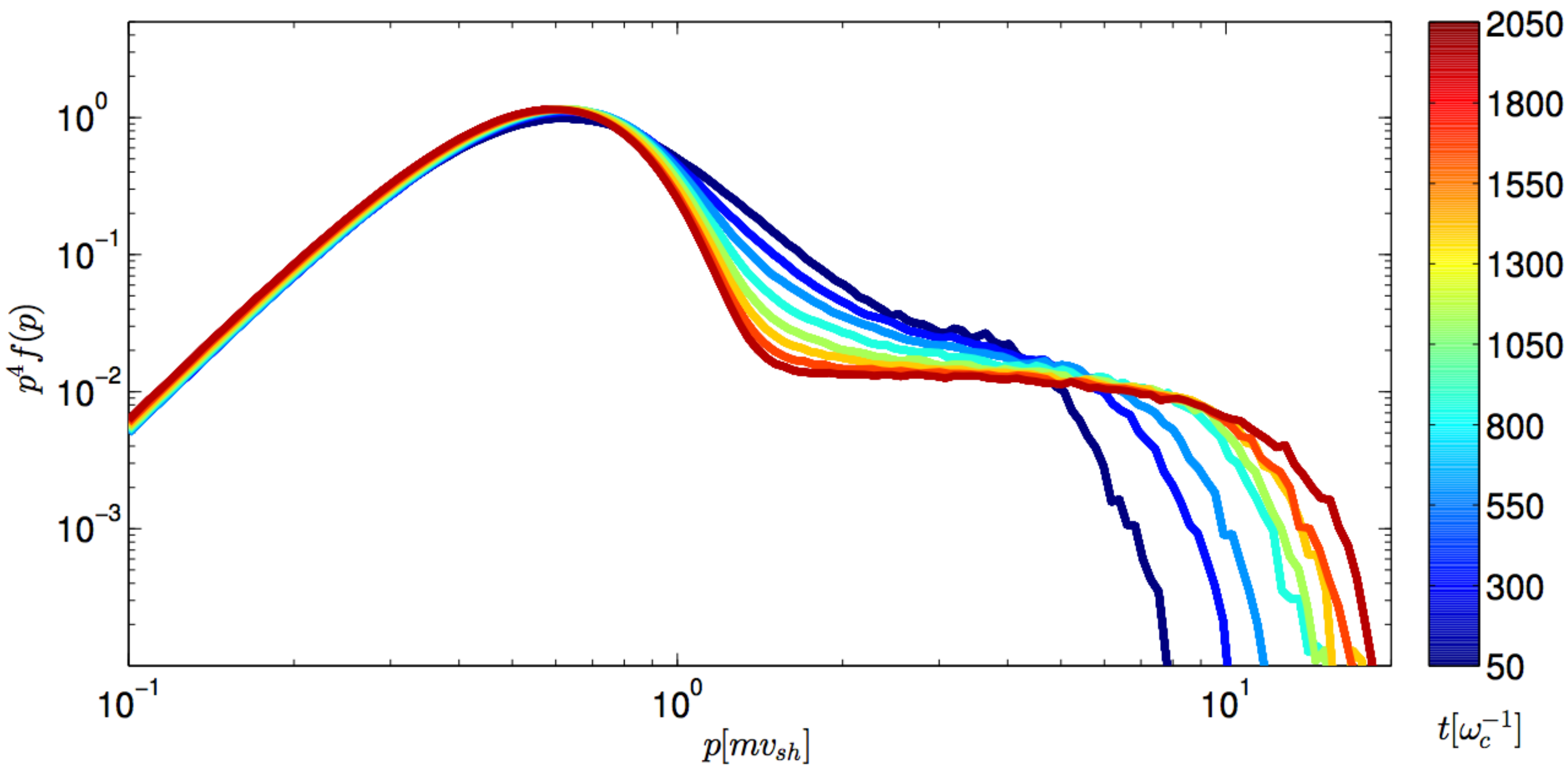
- Where are the magnetic fields that are scattering the particles coming from?
- How do you get energetic particles in the first place? (Some times referred to as the injection problem)
- How does the magnetic field change DSA?
- The answer to these questions have been studied with **Kinetic** Self-Consistent simulations!

Hybrid Simulation of Parallel Shocks



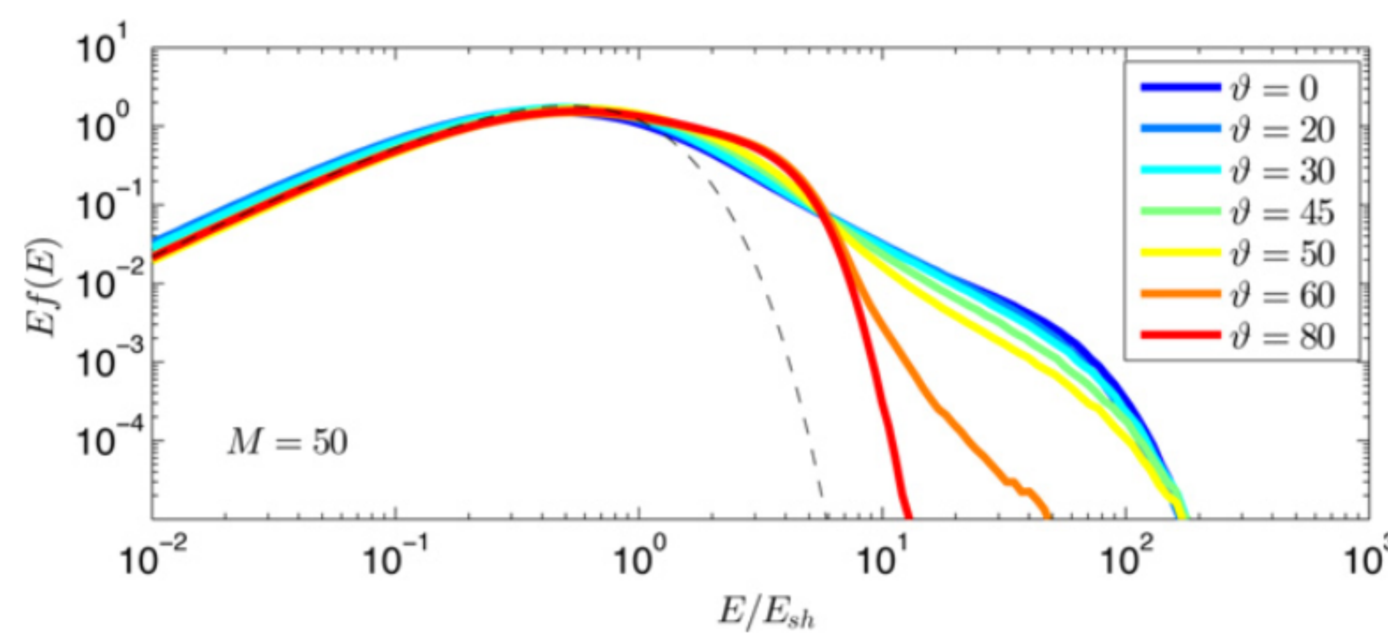


Power Law Spectrum



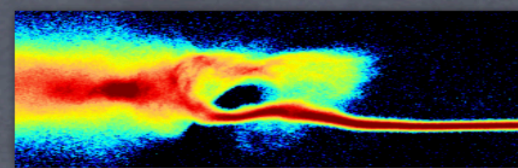
- Hybrid (kinetic ions/fluid electrons simulations show power law develop!

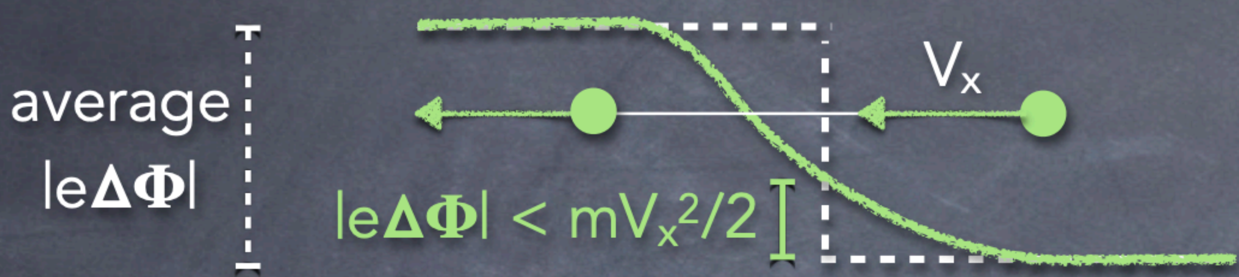
- However power law only develops for quasi-parallel shocks. No extended power law for quasi perp



Thermal Injection of CR

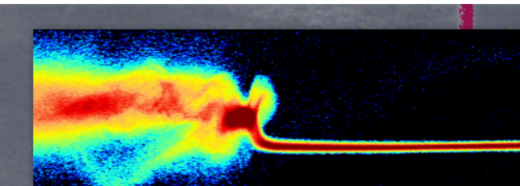
- Low barrier (reformation)

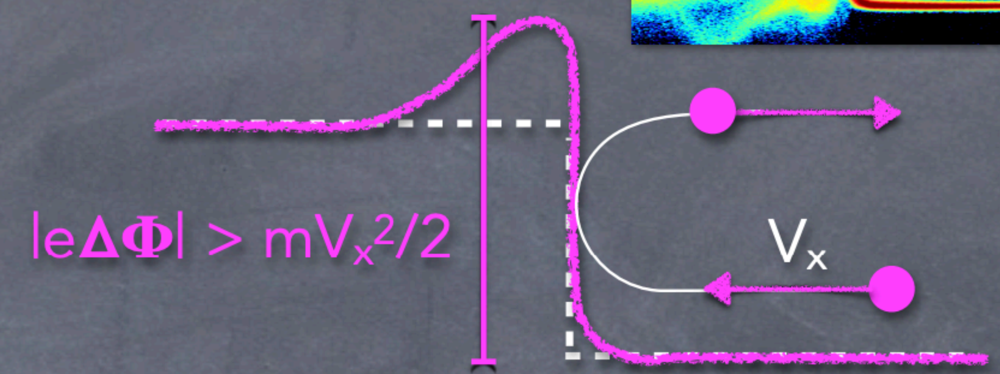




Ions advected downstream, and thermalized

- High barrier (overshoot)





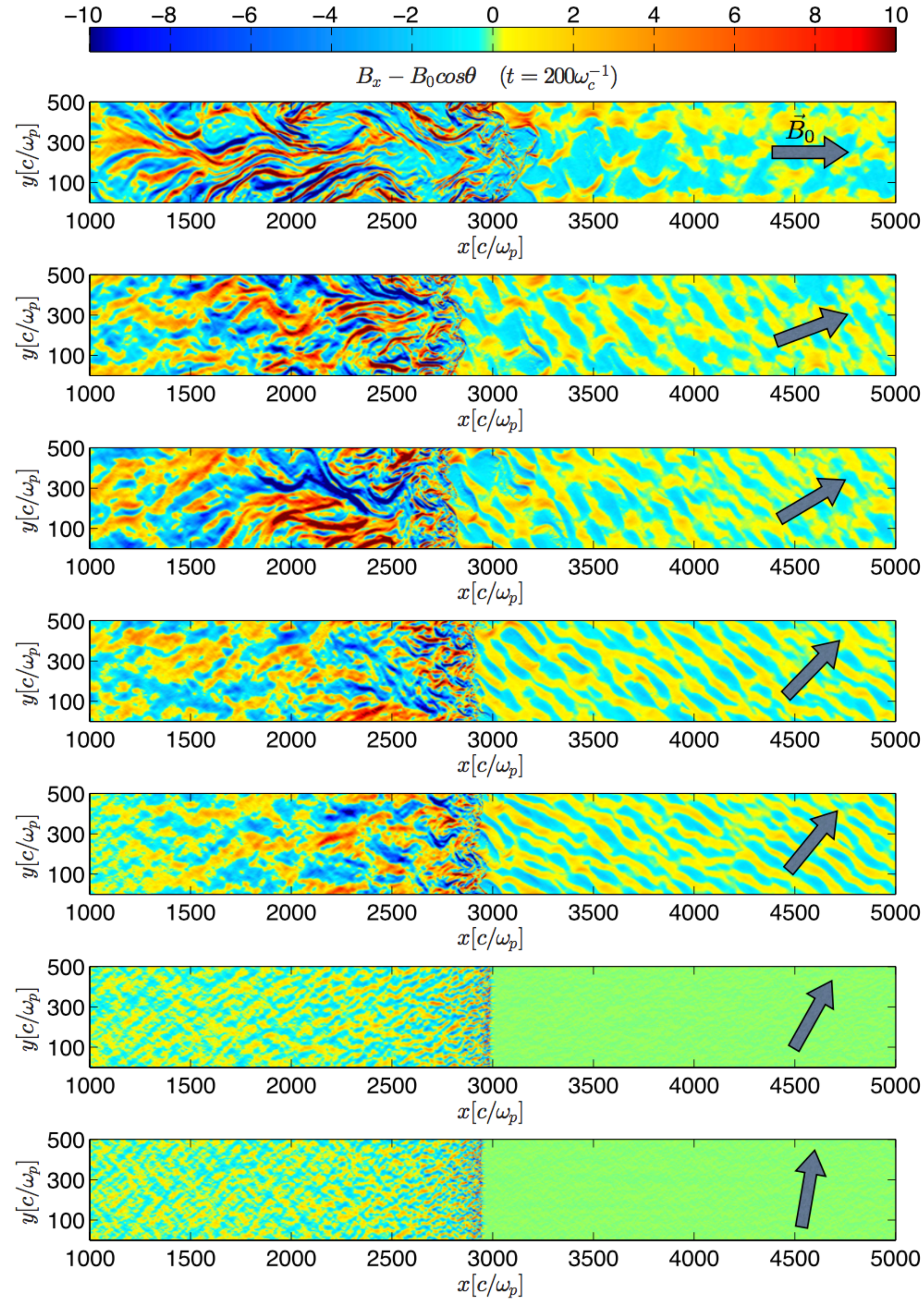
Ions reflected upstream, and energized via Shock Drift Acceleration

- To overrun the shock, ions need a minimum E_{inj}, increasing with ϑ (DC, Pop & Spitkovsky 1
  - Ion fate determined by barrier duty cycle (~25%) and shock inclination
- After N SDA cycles, only a fraction $\eta \sim 0.25^N$ has not been advected
  - For $\vartheta=45^\circ$, $E_{inj}\sim 10E_0$, which requires N~3 -> $\eta\sim 1\%$
  - For $\vartheta>45^\circ$, $E_{inj}>10E_0$, hence N>3 and $\eta<<1\%$

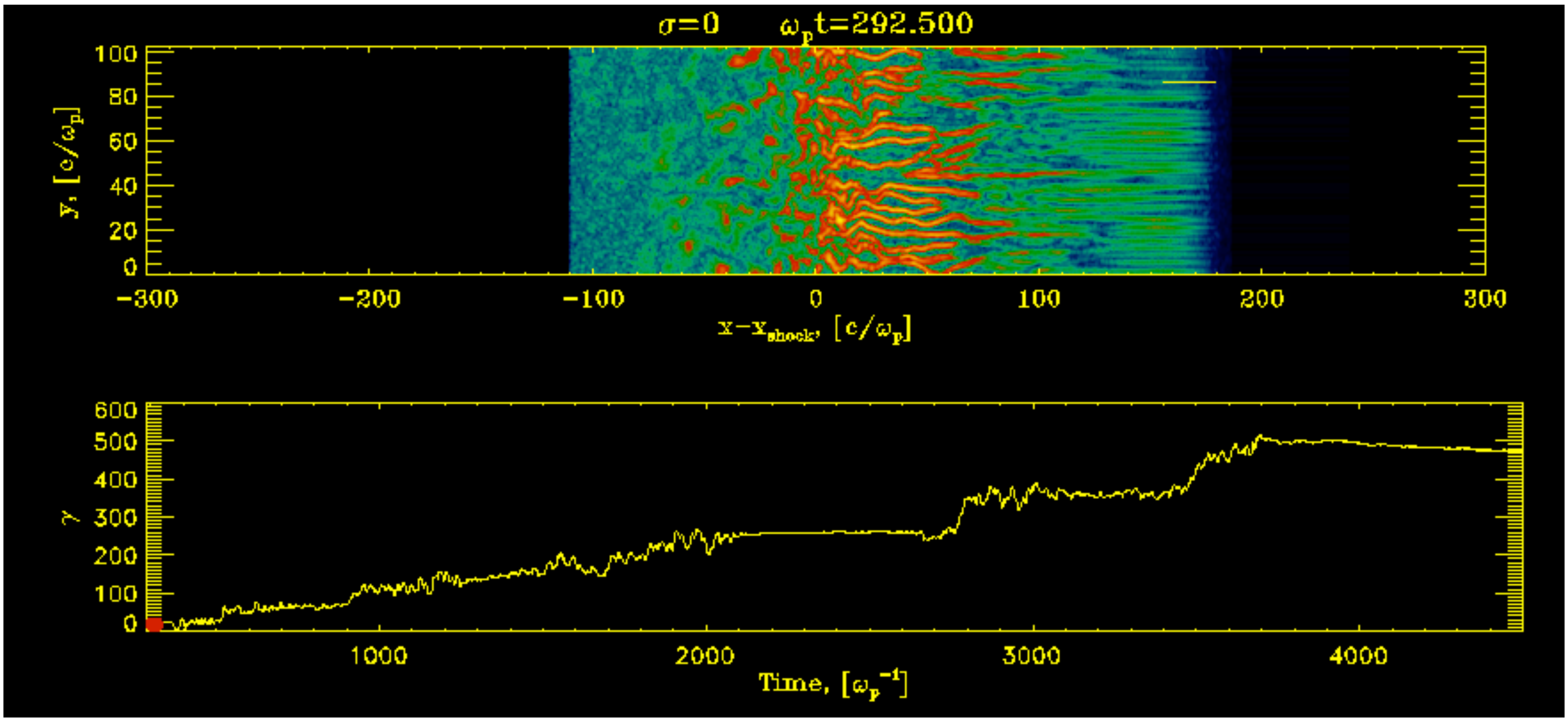
Slide courtesy of D. Caprioli

Self Generated Magnetic Turbulence

- For parallel shocks, energetic particles travel upstream and form an instability:
- Bell or Non-Resonant instability! Like two stream
- Large magnetic field amplification
- Creates B field to reflect CRs



DSA Example from PIC

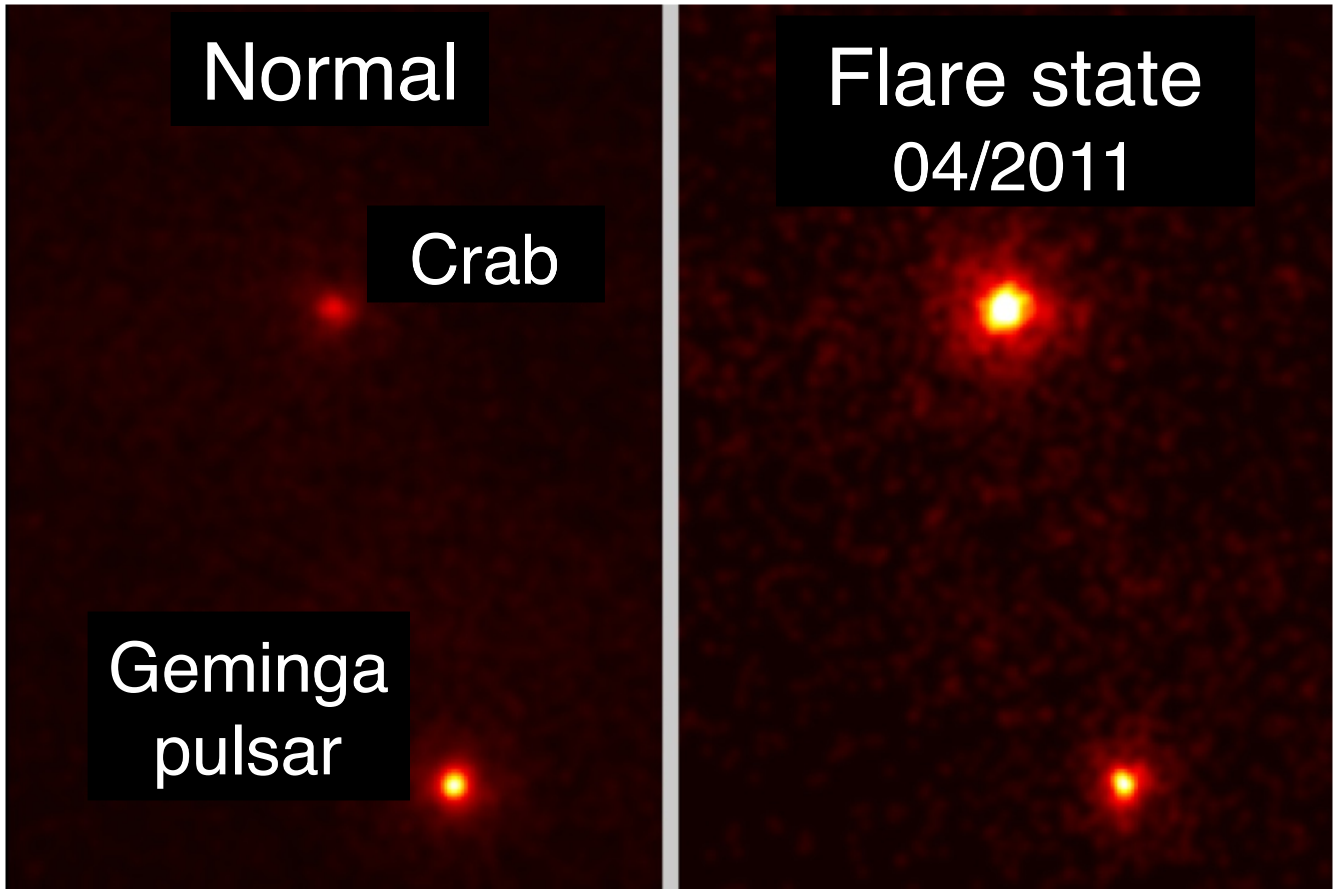


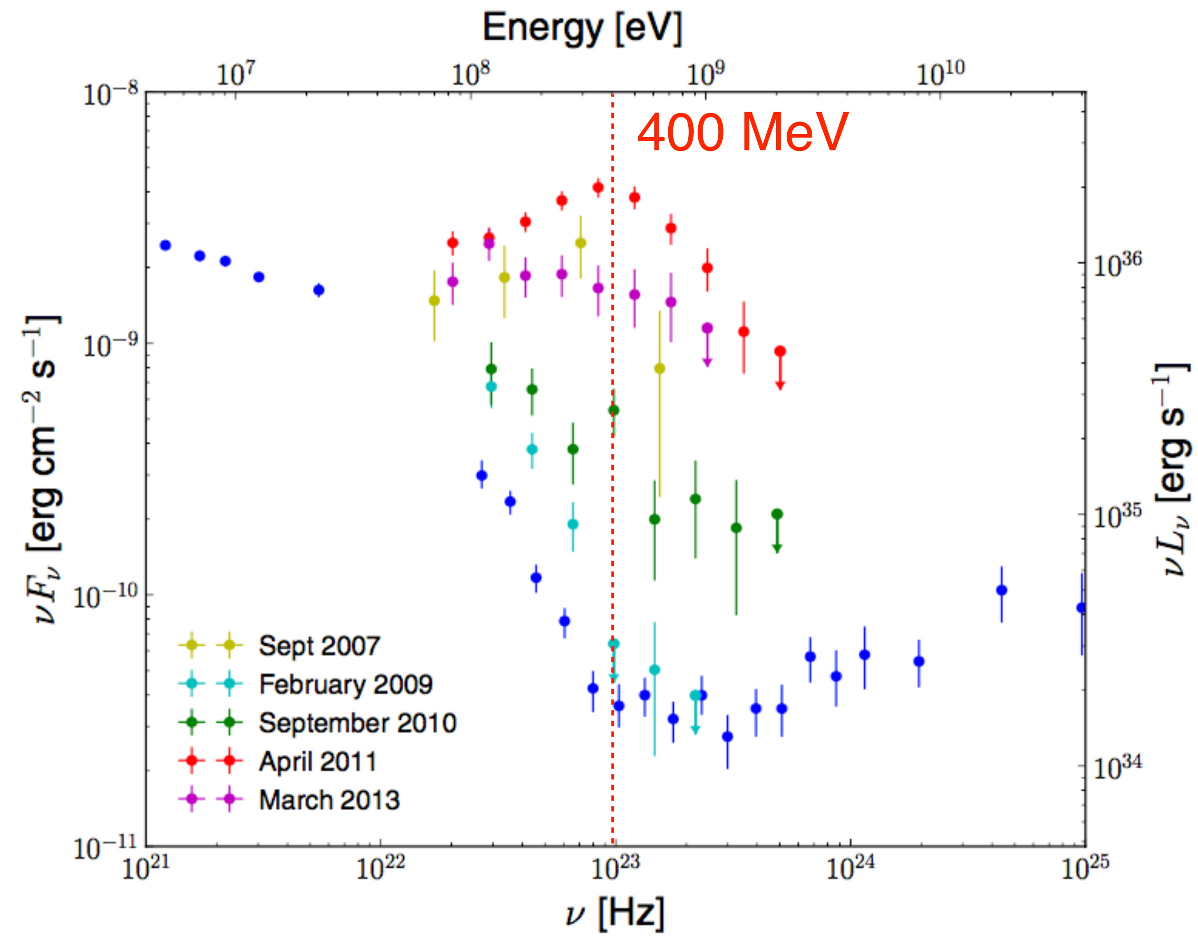
PIC simulation of unmagnetized, relativistic pair shock, Spitkovsky 2008

Magnetic Reconnection Overview:

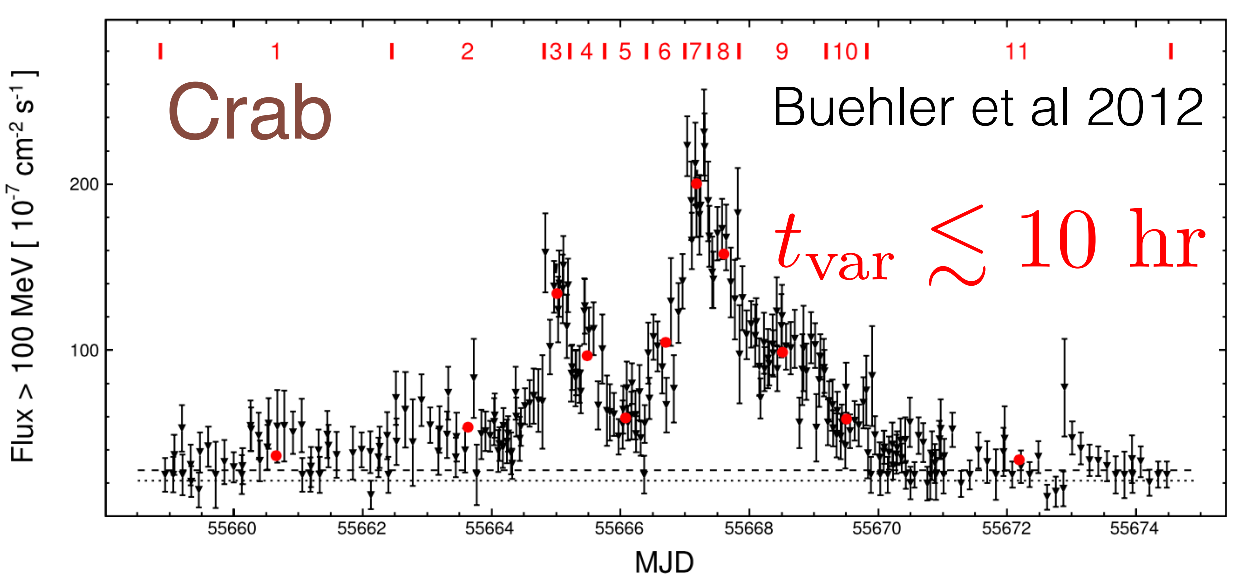
- Rapid onset, short duration variability
- Basics of Reconnection
- Reconnection in the Heliosphere
- How Fast is Reconnection
- Relativistic Reconnection
- Particle Acceleration

Time Variable Observations





Buehler & Blandford 2014



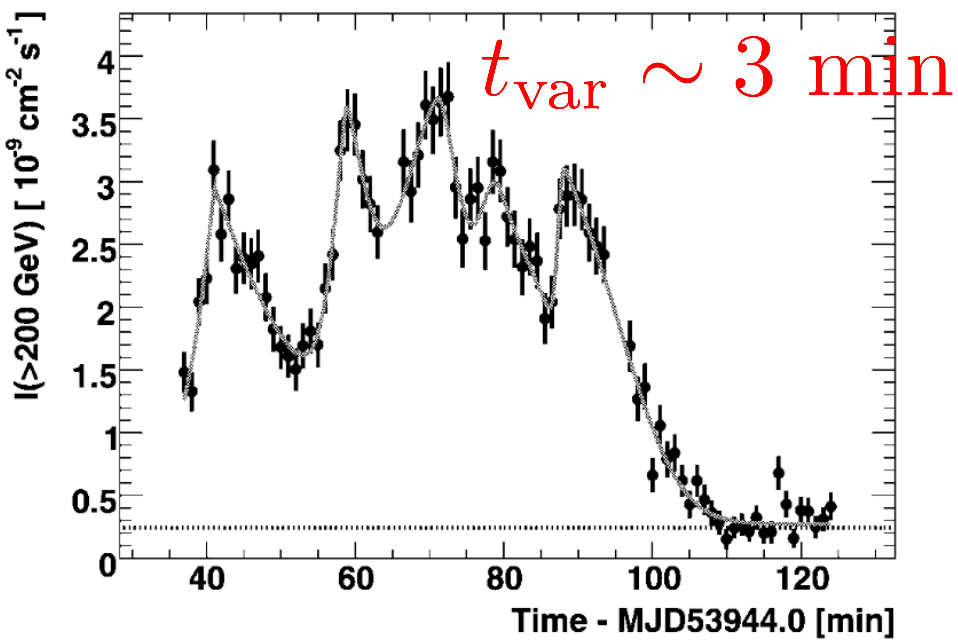
- Astro variability on very short time scales
- Energy above synchrotron limit

$$E_{\rm syn,lim} = \frac{m_e c^2}{\alpha_F} = 160\ \mathrm{MeV}$$

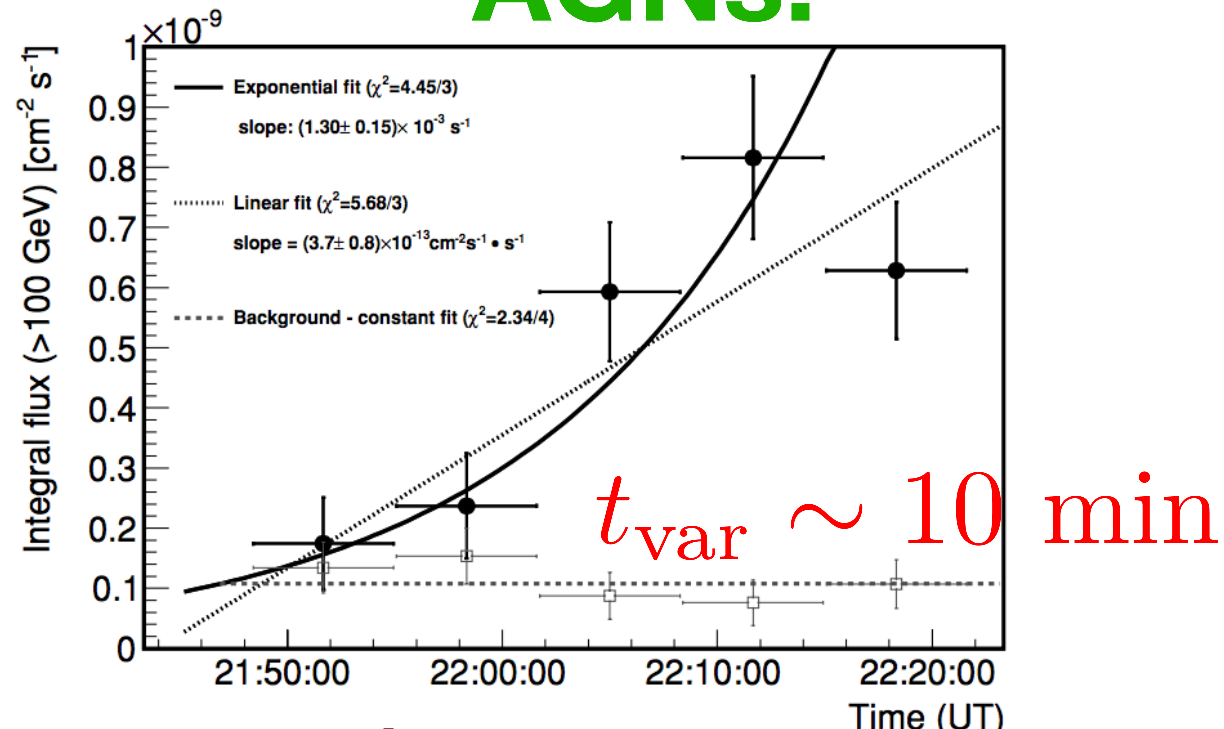
Other Variable Sources

Blazars!

Aharonian et al 2007, PKS 2155-304



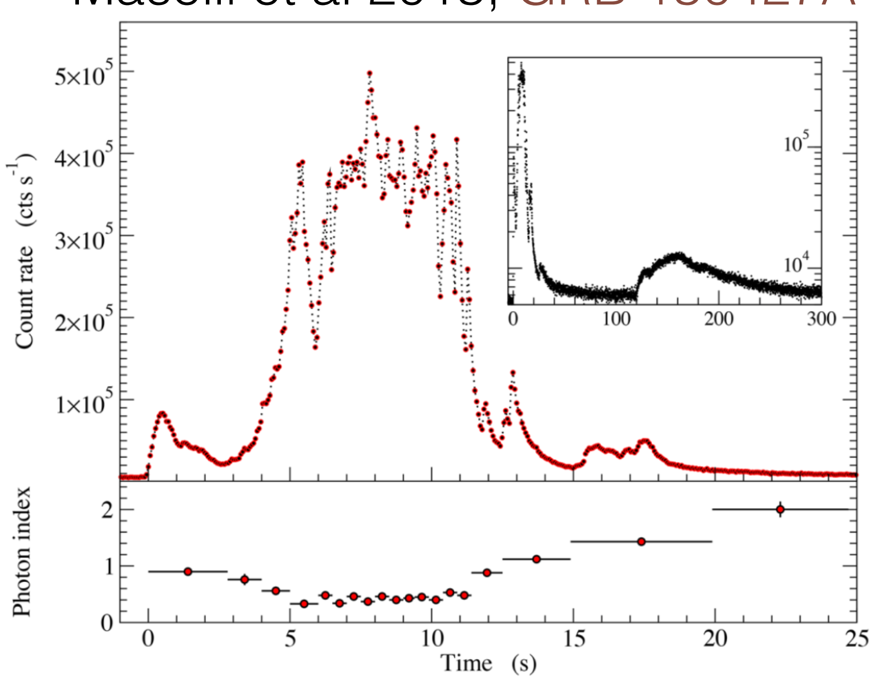
AGNs!



Aleksic et al 2011, PKS 1222+21

GRBs!

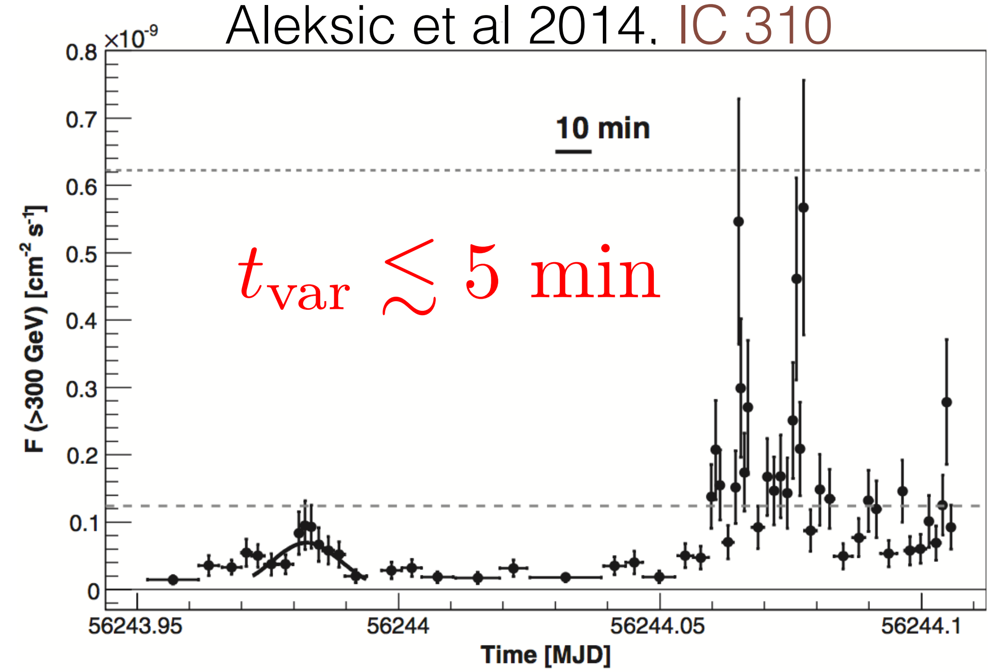
Maselli et al 2013, GRB 130427A



$t_{\rm var} \sim 10$ ms

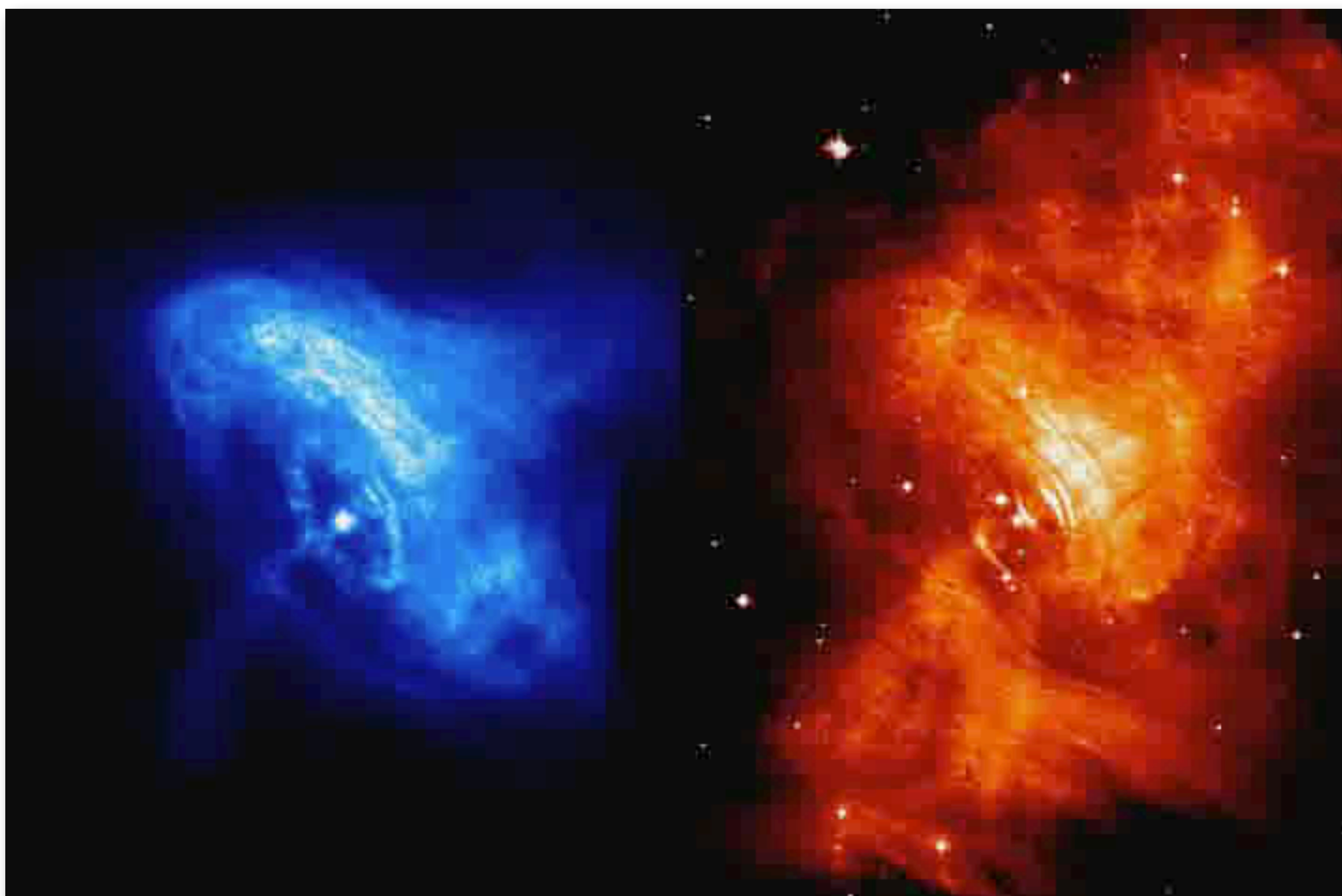
Micro-Quasars?

Aleksic et al 2014, IC 310



Rotating Sources with Large Magnetic Fields

- Rotating plasma (conductor) can amplify and twist and contort magnetic field leading to highly magnetized plasma



Chandra and Hubble of Crab

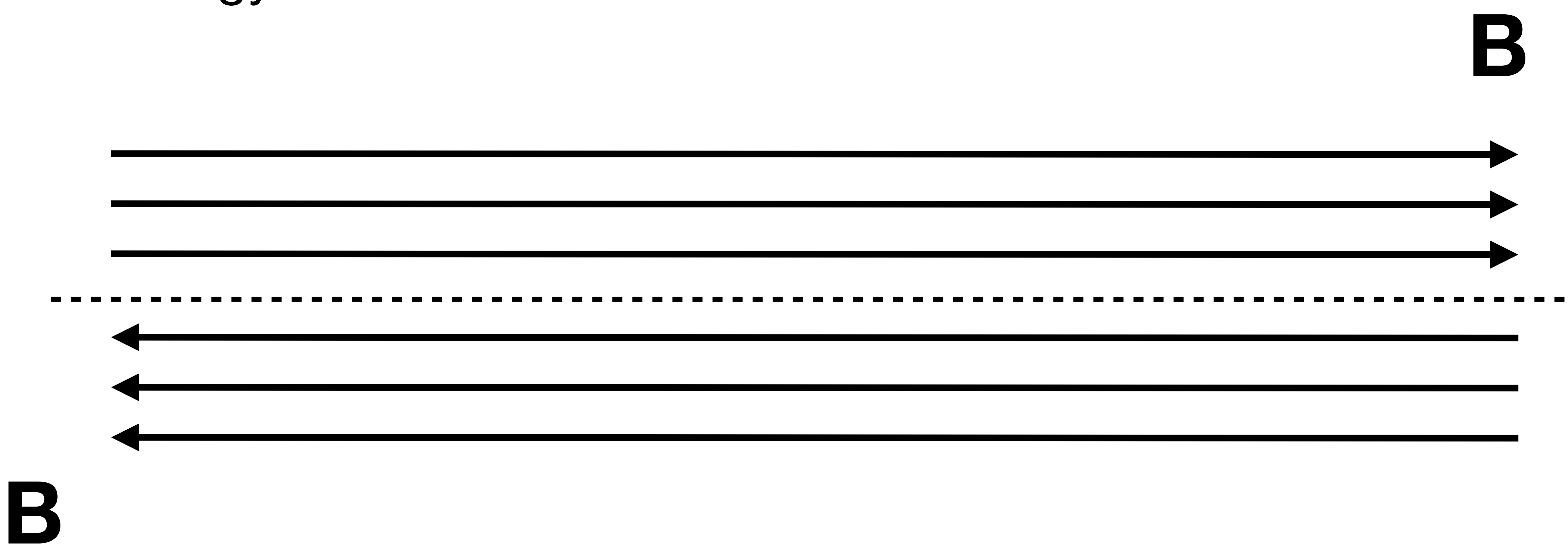
What Mechanism is Responsible?

- Fast variability → small emitting region
- Large Magnetic Fields
- Above radiation reaction limit

- DSA takes too long (Many gyro-periods)
- Idea: **Magnetic reconnection!**

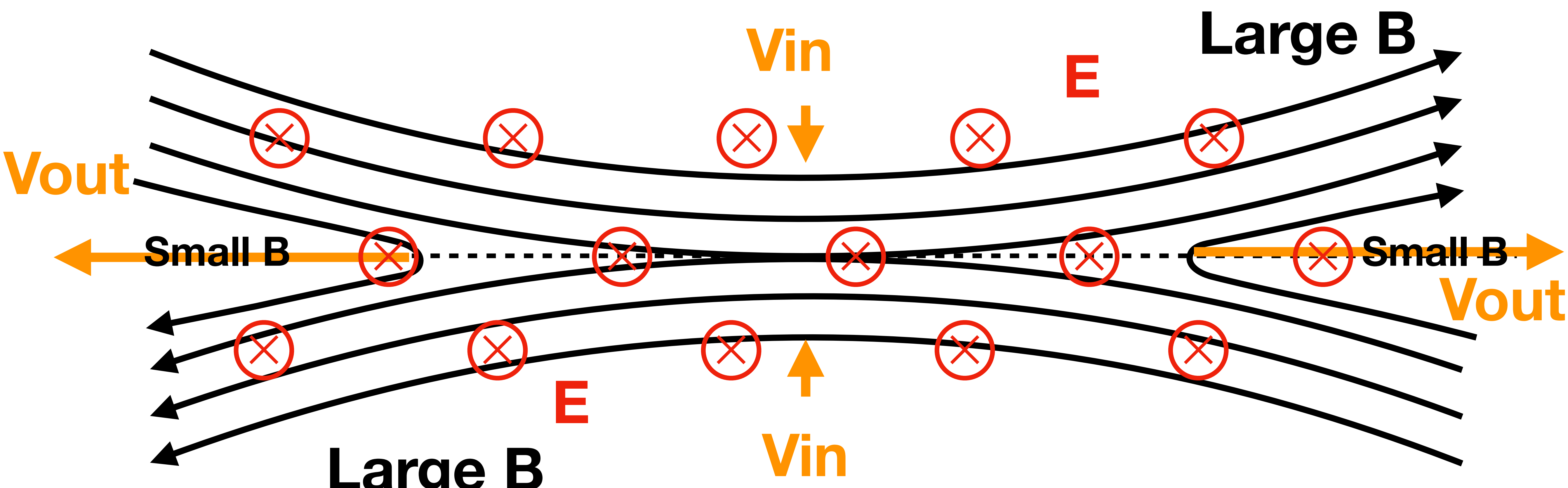
Magnetic Reconnection

- Magnetic reconnection is a process that converts magnetic energy to thermal and bulk flow.
- Long straight magnetic field lines at a current boundary layer field lines “break” and “reconnect”
- Energy released on Alfvénic time scales

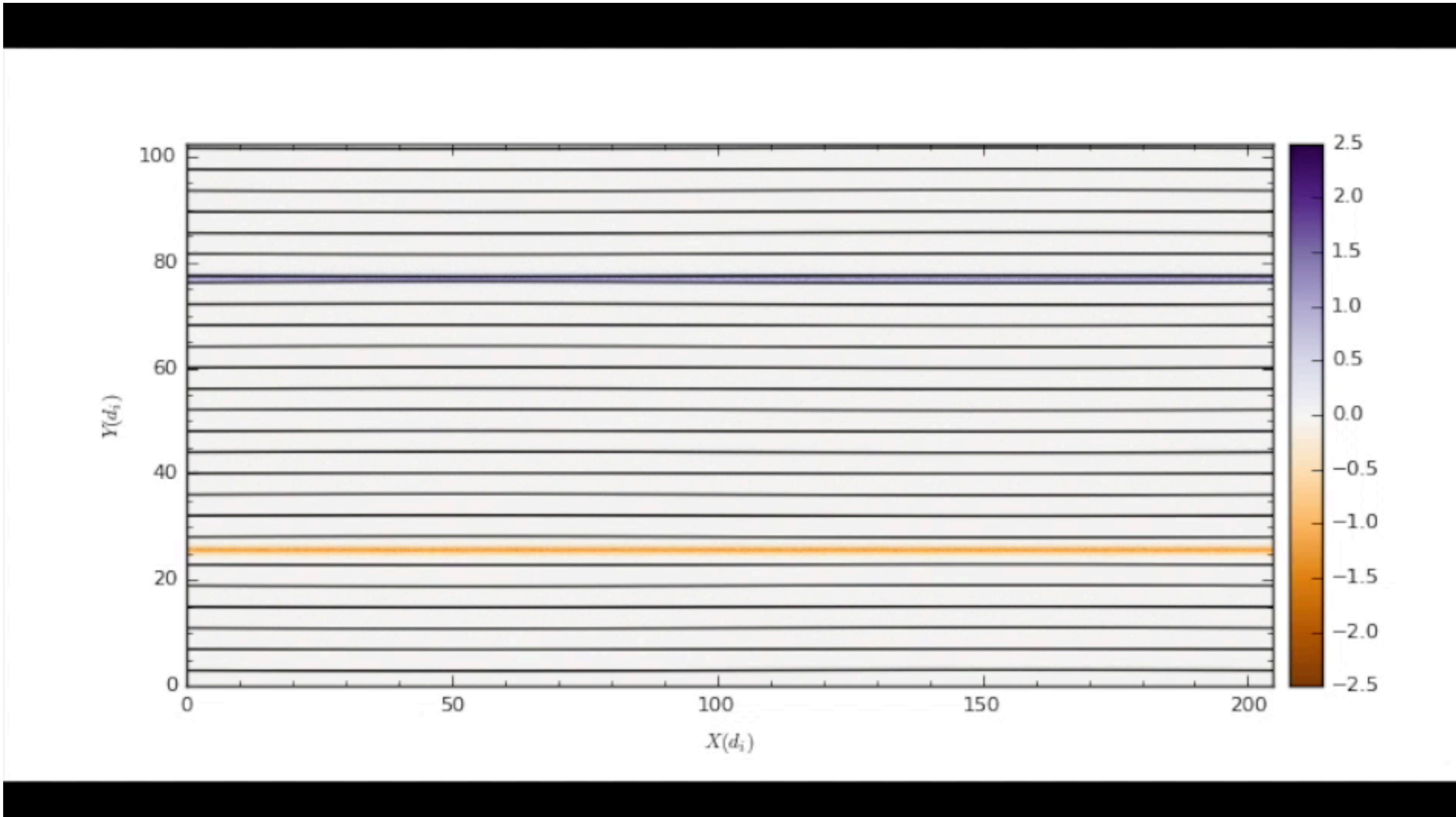


Magnetic Reconnection

- Initiated by tearing mode instability
- Reconnected Field lines: large curvature $\vec{B} \cdot \vec{\nabla} \vec{B}$
- Outflow induces E field that draws in more plasma
- Rapid conversion of energy



Magnetic Reconnection



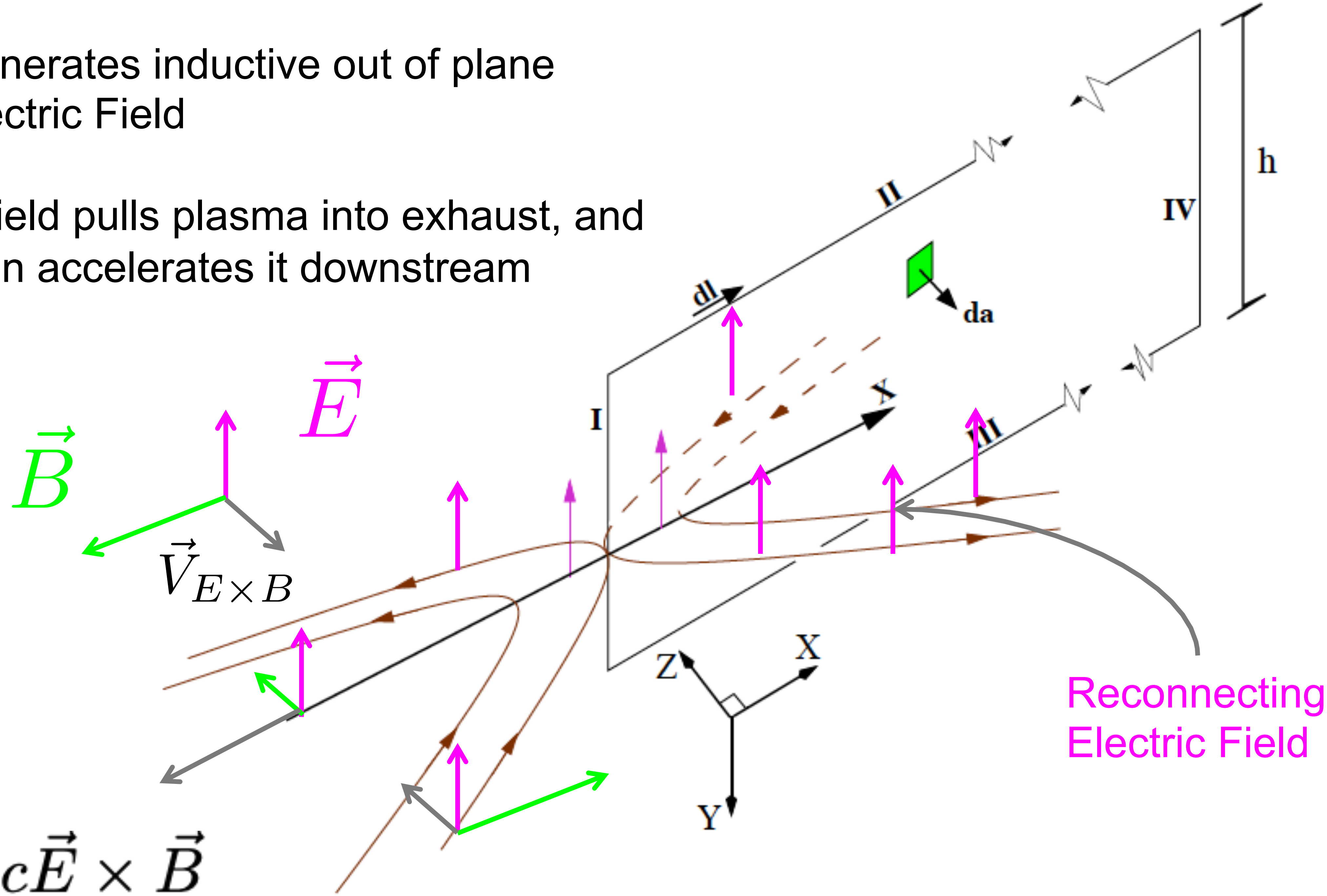
Magnetic Reconnection Vocabulary

- Anti-parallel: when the two reconnecting B fields are at 180 deg of each other
- Guide field: The component of the B filed that is non anti-parallel and is constant across the current sheet
- X-point/line: The “exact” point where reconnection occurs
- Asymmetric: When the B field, density or temperature is different on either side of the current sheet
- Plasmoid/flux tube/island: the topologically separated regions generated from two x-points forming along a current sheet

Amperian Loop of Reconnection (Online Only)

Generates inductive out of plane Electric Field

E field pulls plasma into exhaust, and then accelerates it downstream



$$\vec{V}_{E\times B} = \frac{c\vec{E}\times\vec{B}}{B^2}$$

Magnetic Reconnection in the Heliosphere

- Reconnection has been believed to cause and observed in many impulsive heliospheric processes including:

- Solar Flares

- Coronal Mass Ejections (CME)

- Plasma Turbulence

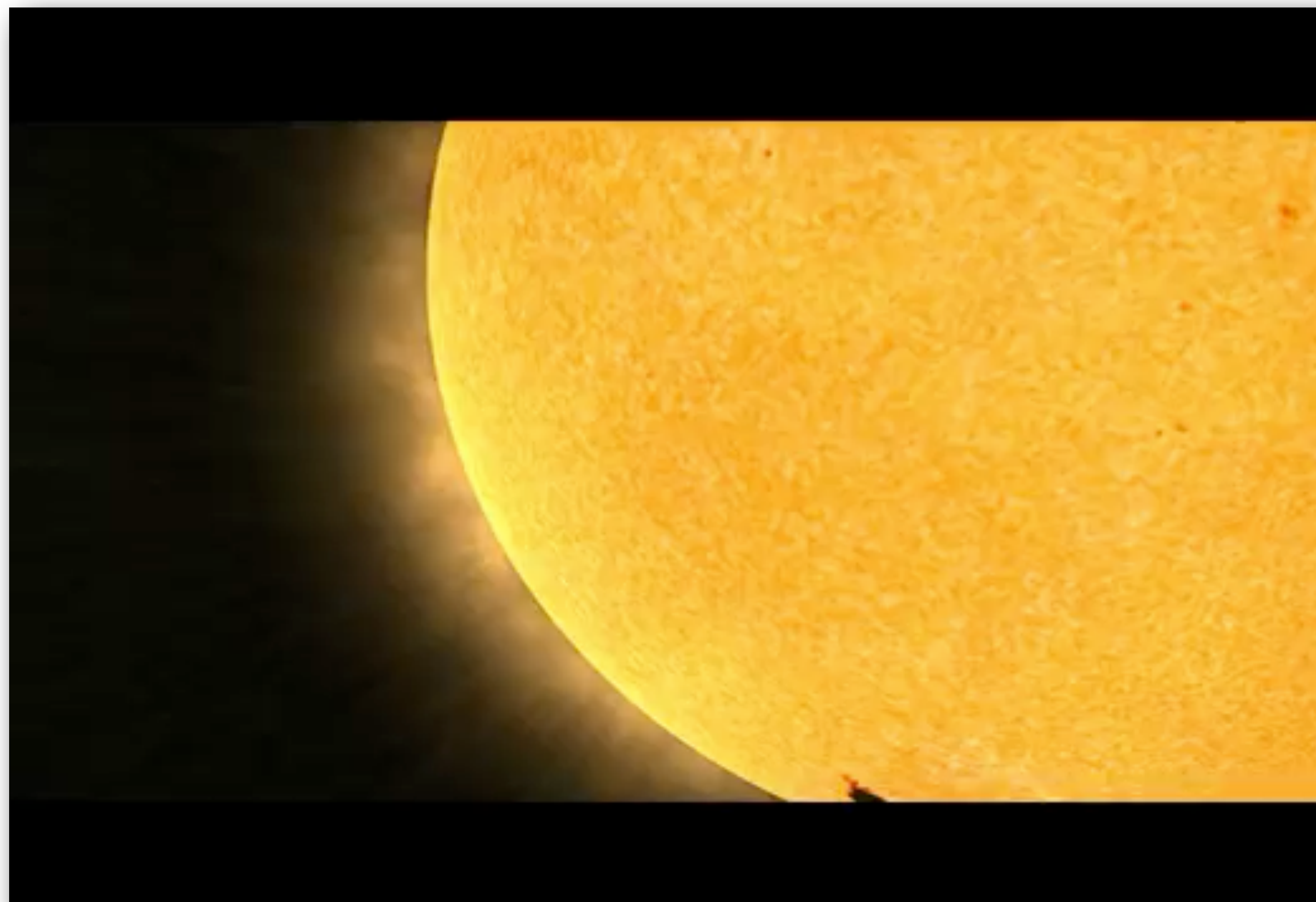
- Geomagnetic Storms

- The Aurora

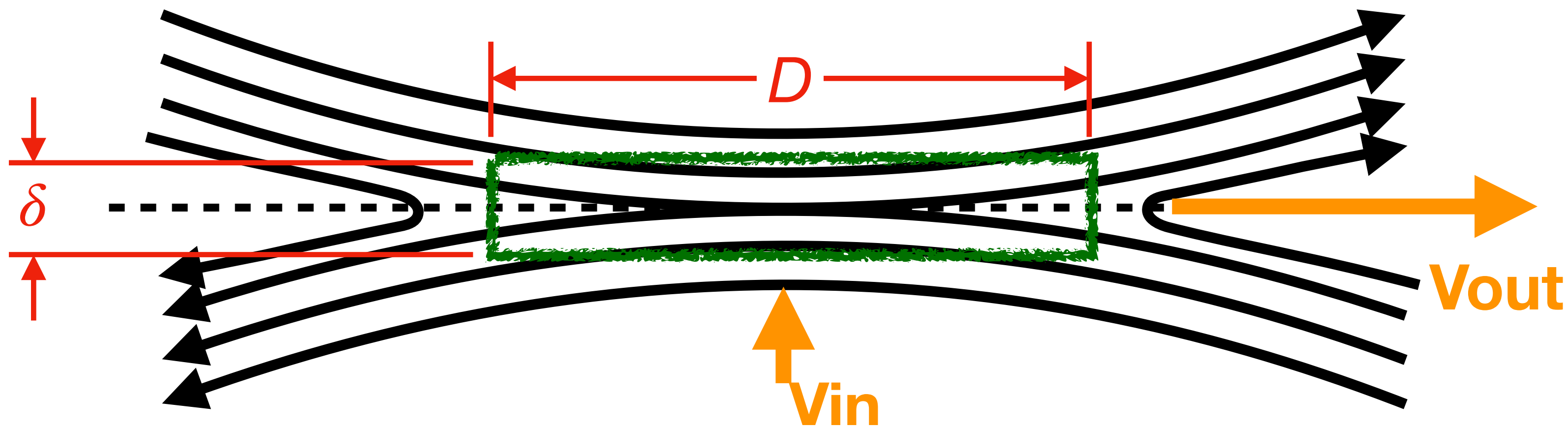
Magnetic Reconnection in the Heliosphere

- Reconnection has been believed to cause and observed in many impulsive heliospheric processes including:

- Solar Flares
- Coronal Mass Ejections (CME)
- Plasma Turbulence
- Geomagnetic Storms
- The Aurora



How Fast is Reconnection?



- Green Box is the "Diffusion Region", place where fluid (MHD) approximations break down

- Conservation of mass/momentum argument gives Vout and Vin

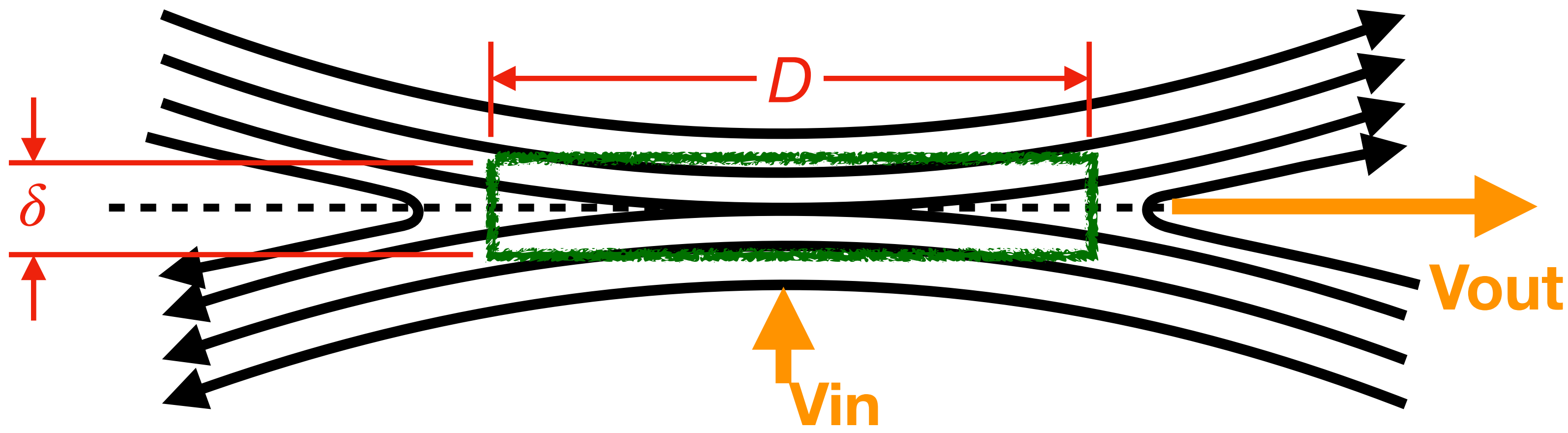
$$V_{\rm in} \sim \frac{\delta}{D} V_{\rm out}\,, \qquad V_{\rm out} = \frac{B}{\sqrt{4\pi m n}}$$

- Reconnection outflow is Alfvénic

"Reconnection Rate"

the rate at which magnetic energy is converted/dissipated

How Fast is Reconnection?



- Historically first calculated using MHD and magnetic dissipation depended on resistivity. This meant $\delta/D \ll 1$ Called “Slow”/“Sweet-Parker” reconnection

- Kinetic theory and simulations show that for a collisionless plasma, the reconnection rate universally tends to $\delta/D \sim .1$ Called “Fast” reconnection

How Fast is Reconnection?

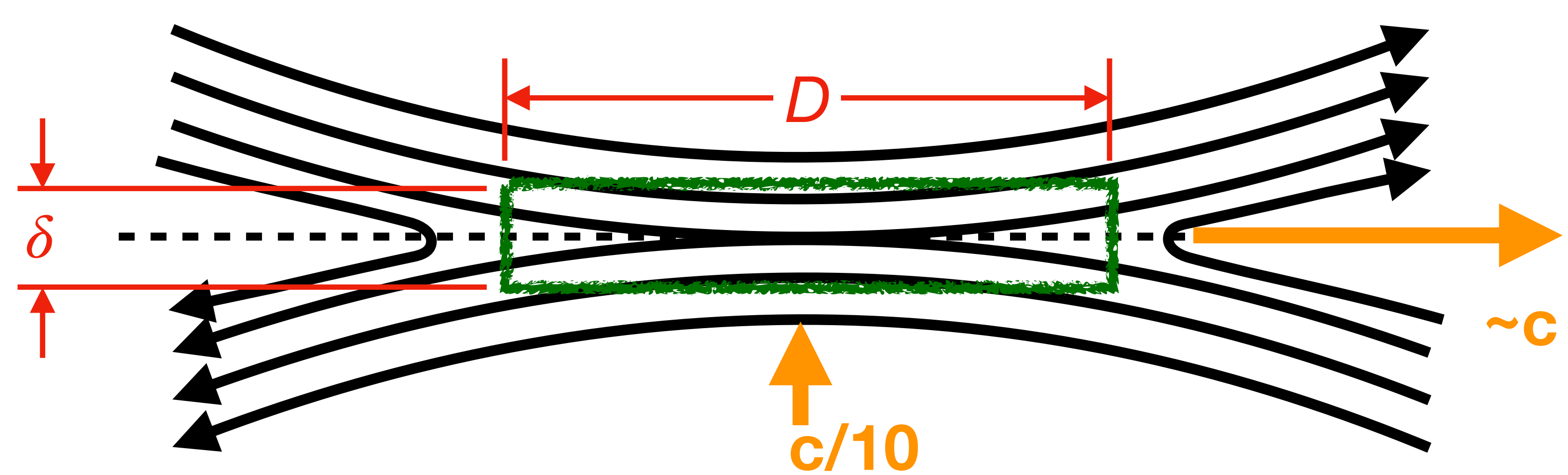
- What happens when the magnetic field gets really large?

$$\frac{B}{\sqrt{4\pi m n}} > c \qquad\qquad V_A \to c$$

- sigma is a better quantity to characterize the system:

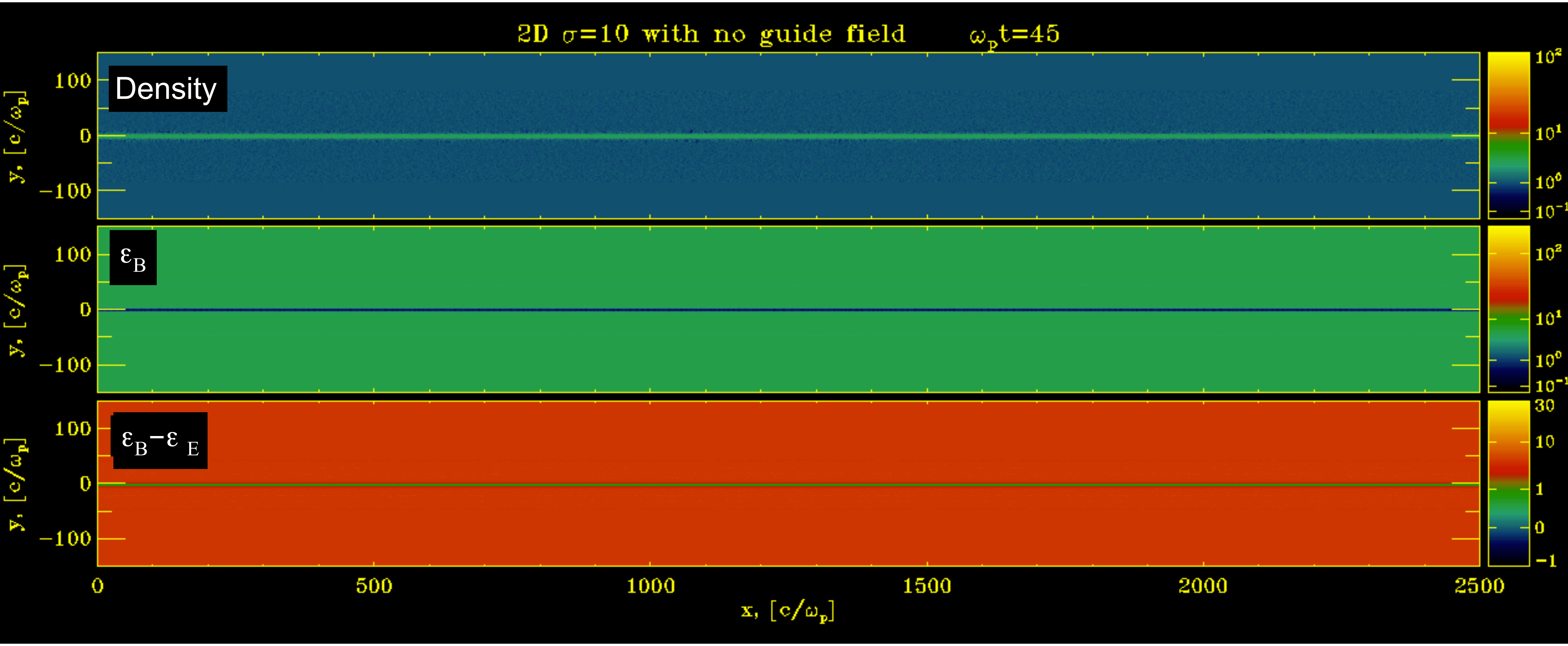
$$\sigma = \frac{B^2}{4\pi n m c^2} = \frac{\text{Magnetic Energy per particle}}{\text{Rest Mass Energy}}$$

- How relativistic reconnection will be
- The outflow -> c

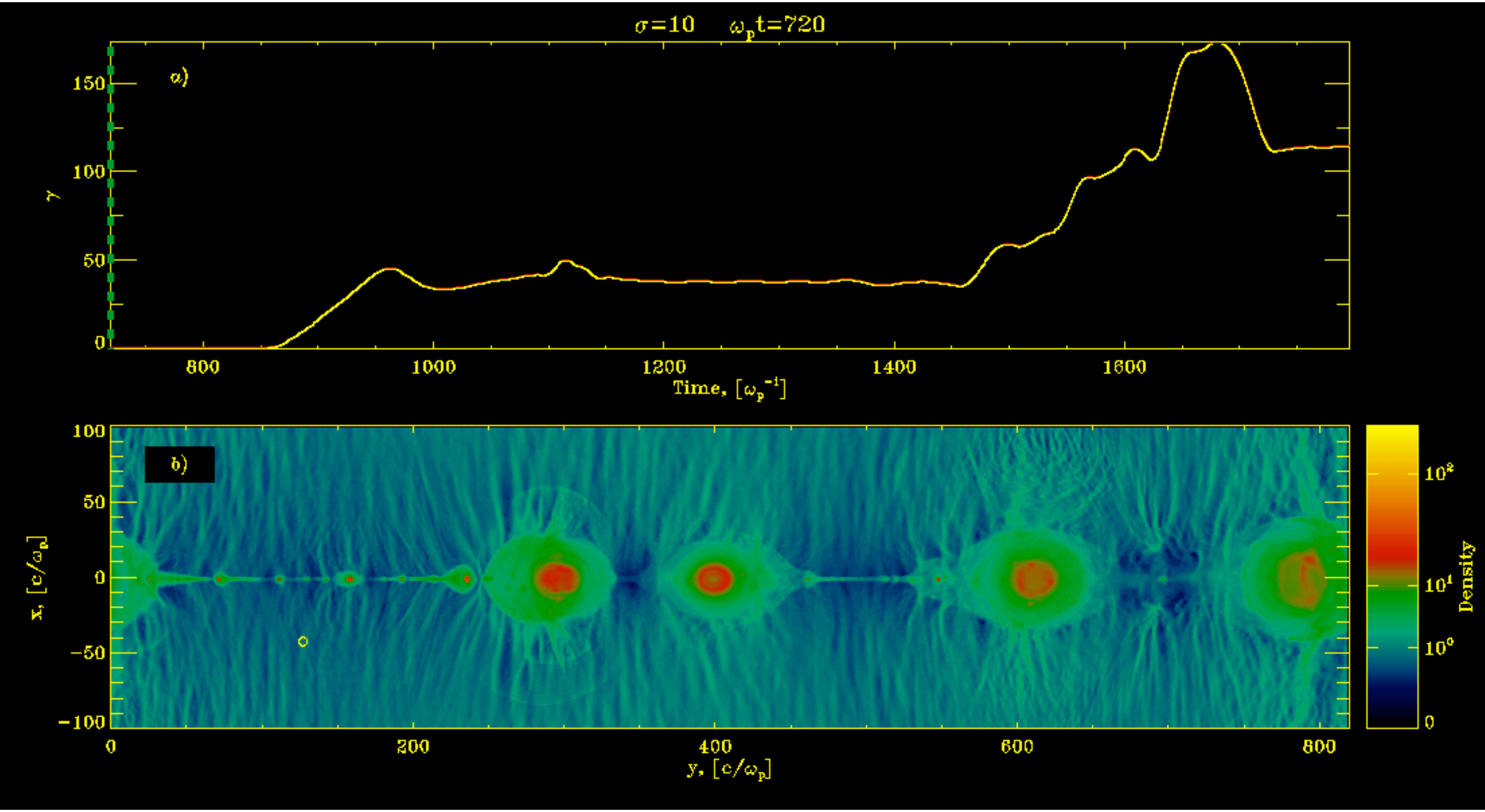


How Does Reconnection Produce Non-thermal Acceleration?

- Simulations of relativistic pair-plasma

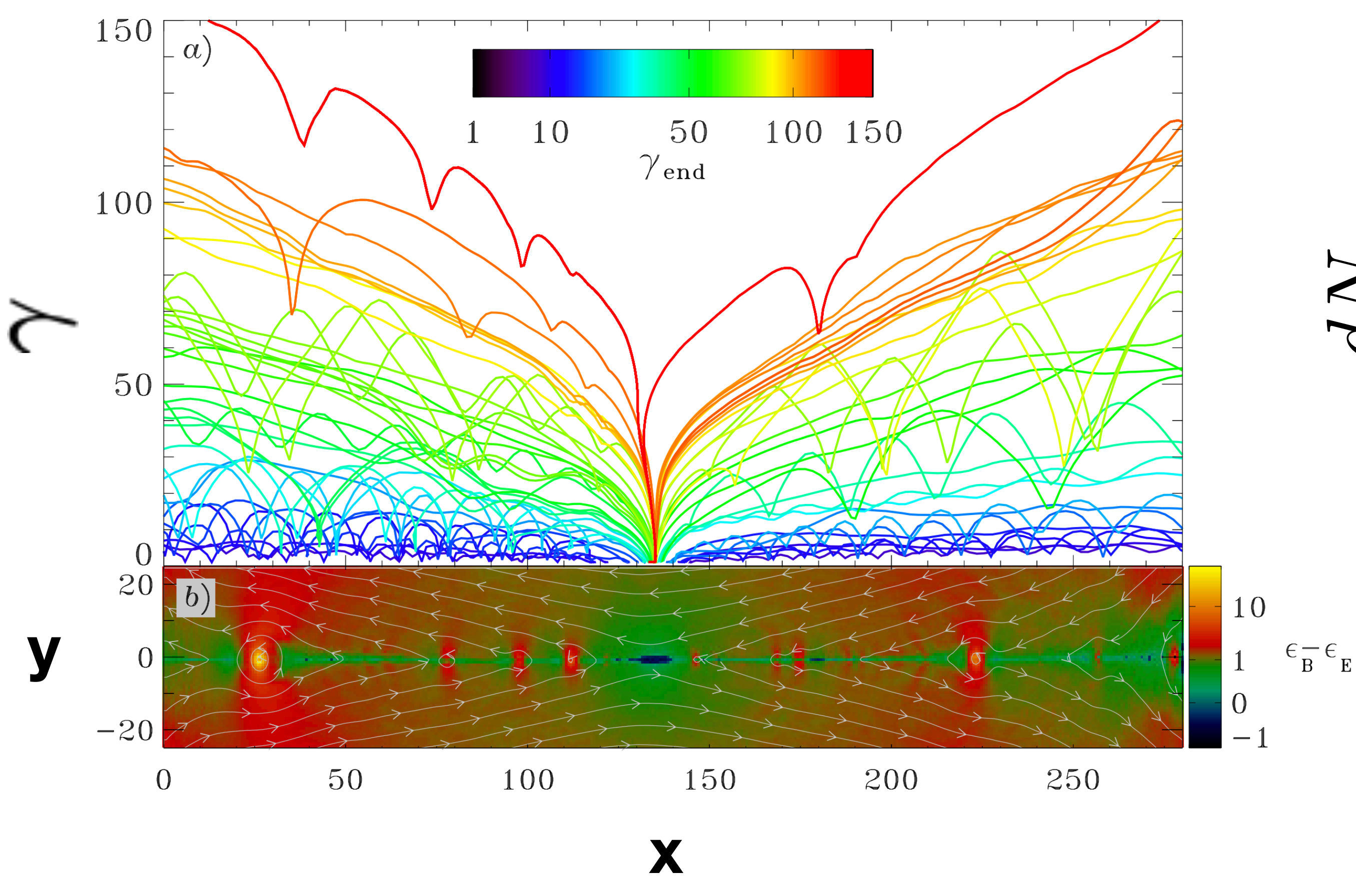


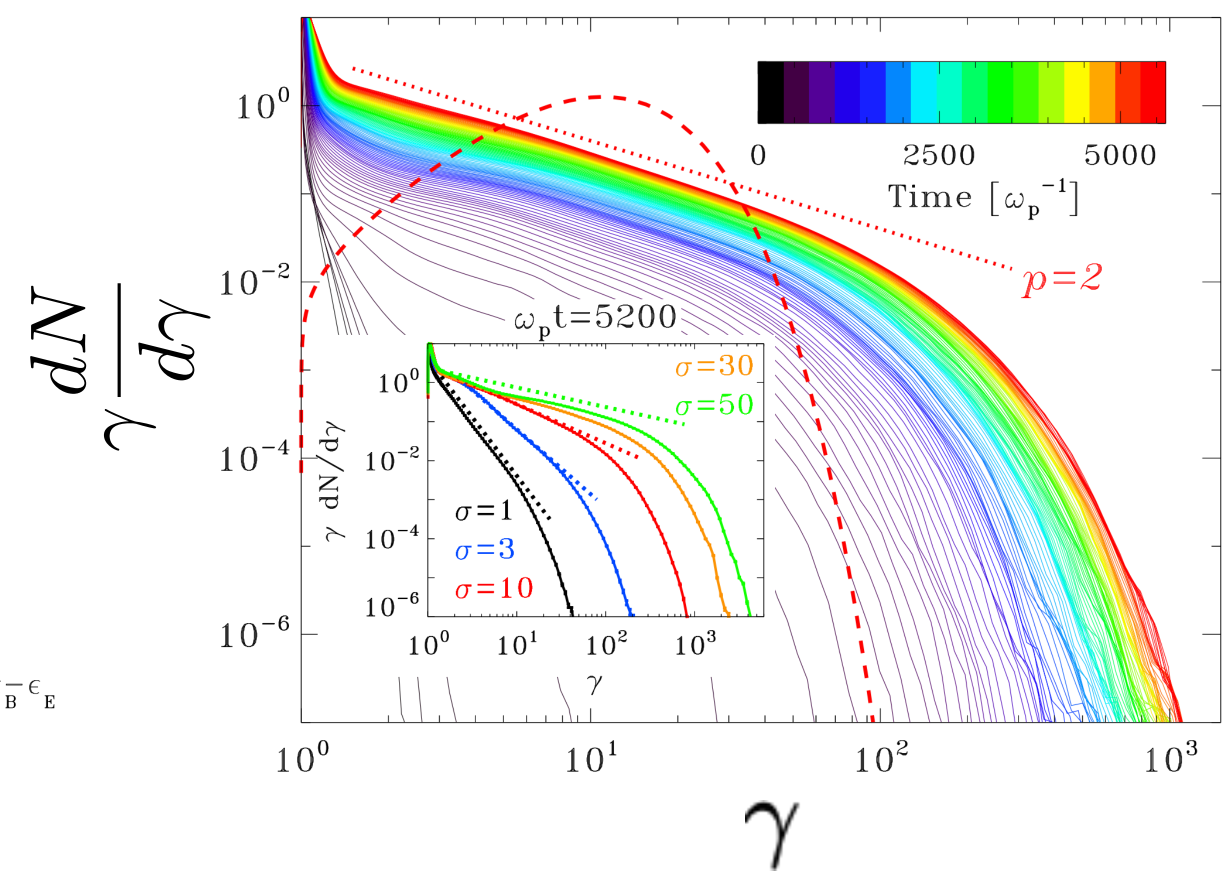
Particle Acceleration in Magnetic Reconnection



Particle Acceleration in Magnetic Reconnection

- Particles that can pass through the x-point can be free accelerated by the reconnection electric field!
- Continue to be accelerated by curvature drift are pre-energized





Turbulence

- Real astrophysical systems are complex and messy. All of the simulations that we have looked at are basically idealized test cases to try to understand the underlying physics.

- Many of these phenomena occur in turbulent mediums and the physics of what is actually occurring as well as the frequency will quite likely depend on the turbulence.

- Turbulence is quite difficult and I have most likely run out of time for this lecture so here are a few pretty movies of kinetic plasma turbulence.

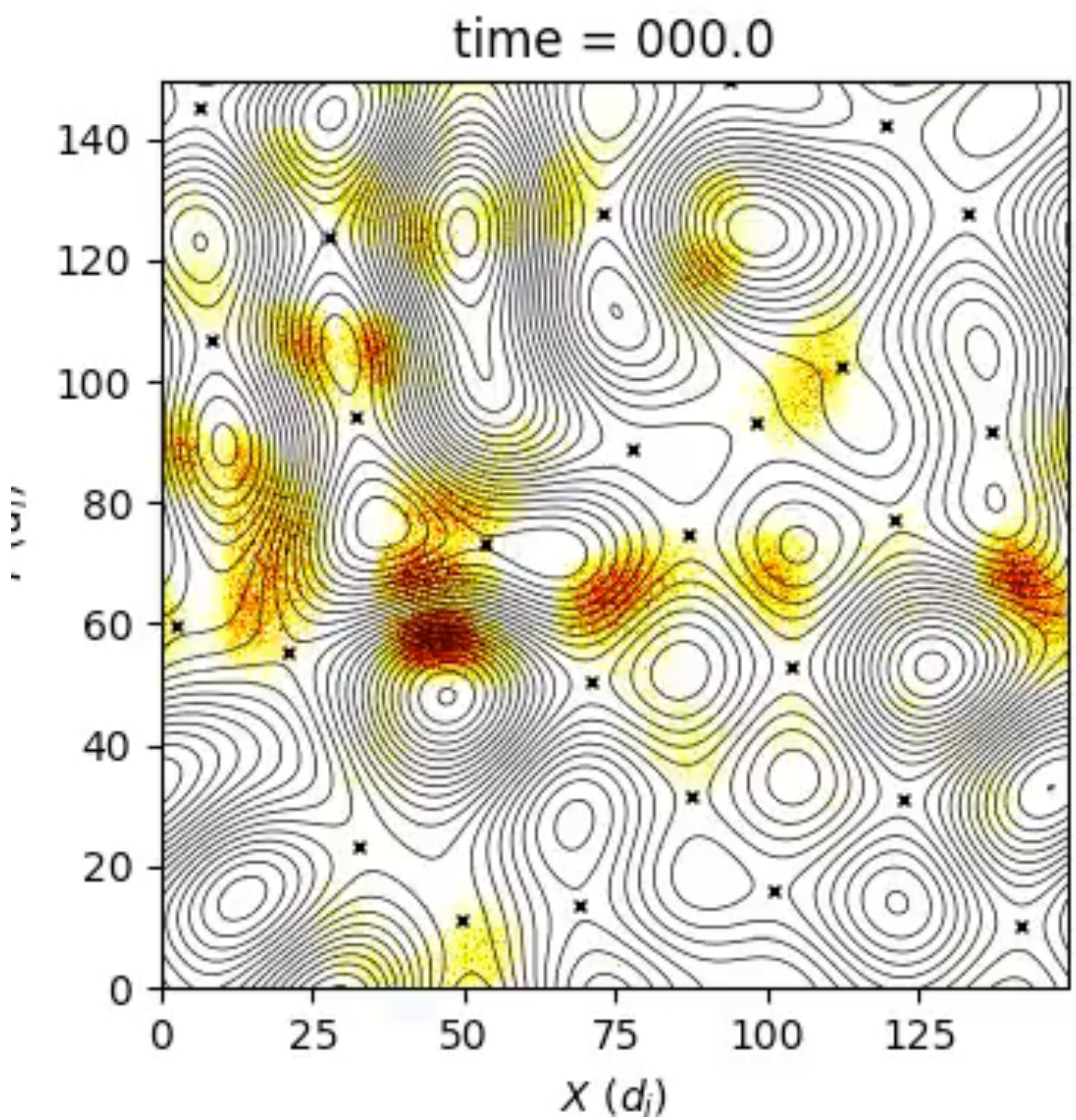
Plasma Turbulence

- Transfer of energy from bigger scales to smaller scales

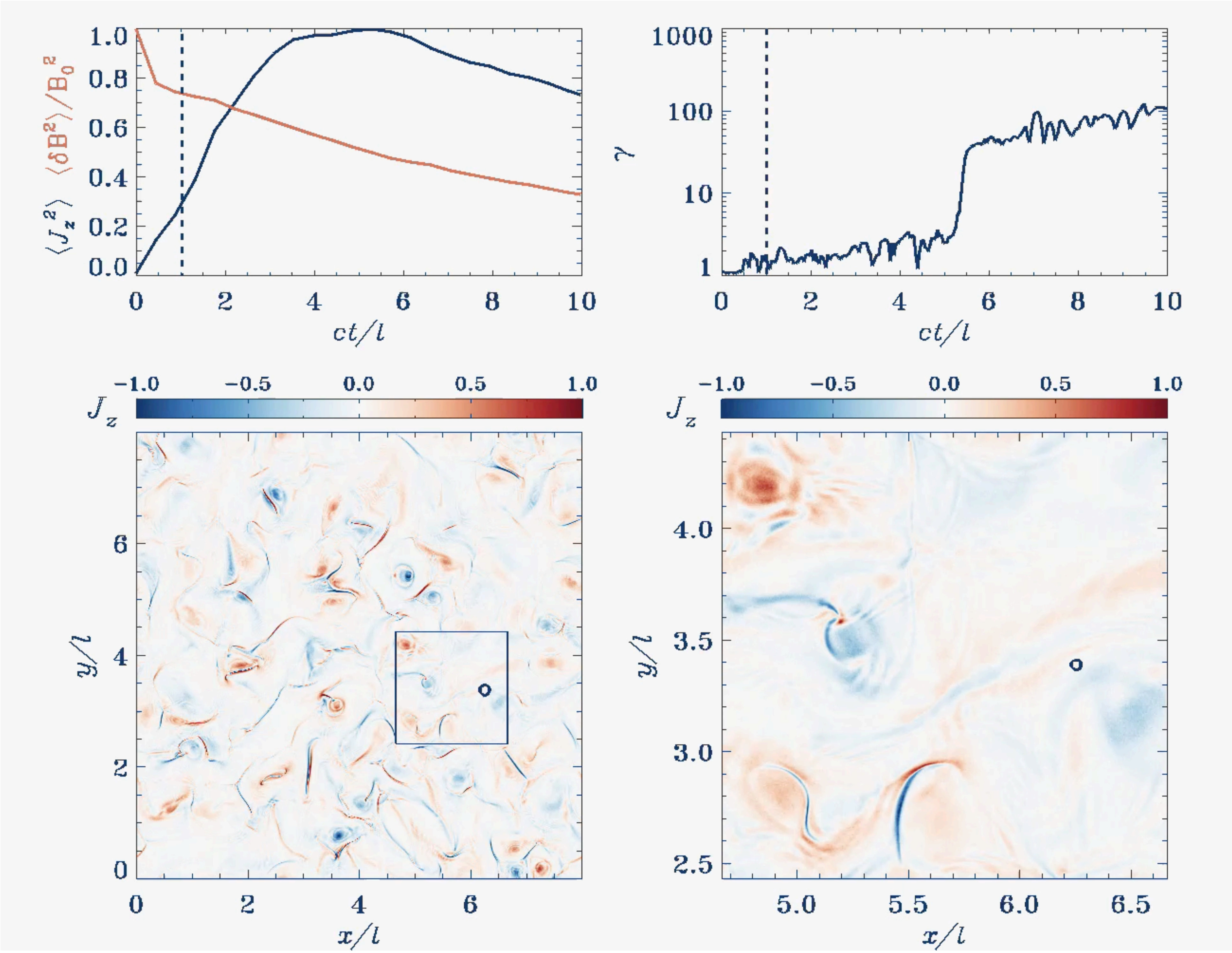
- Many magnetic and velocity fluctuations mean Fermi II acceleration.

- Reconnection in turbulence

- Turbulence in Shocks



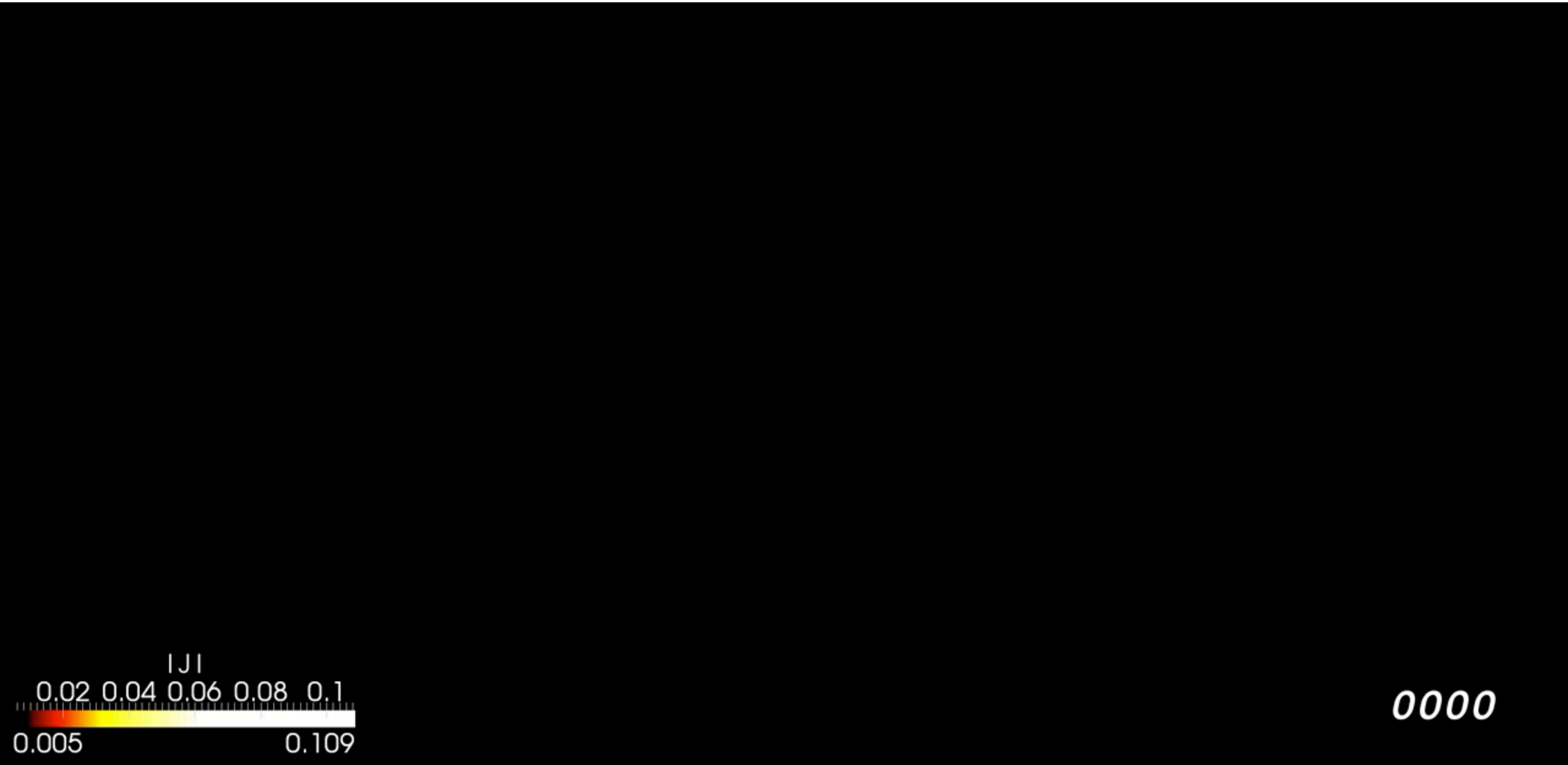
Plasma Turbulence



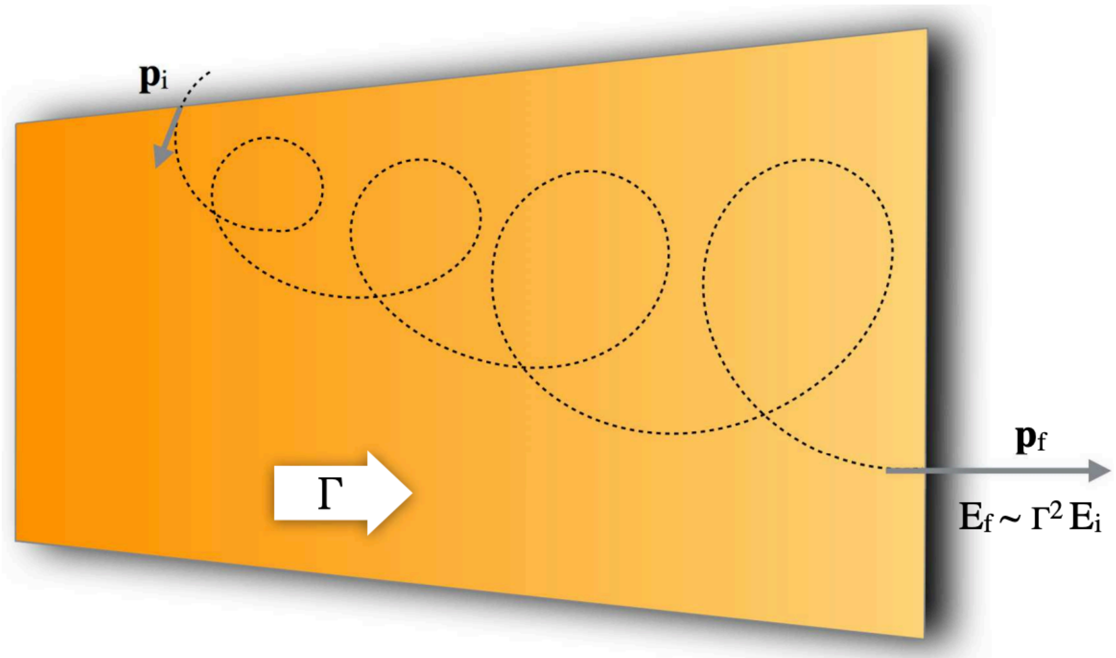
Movies

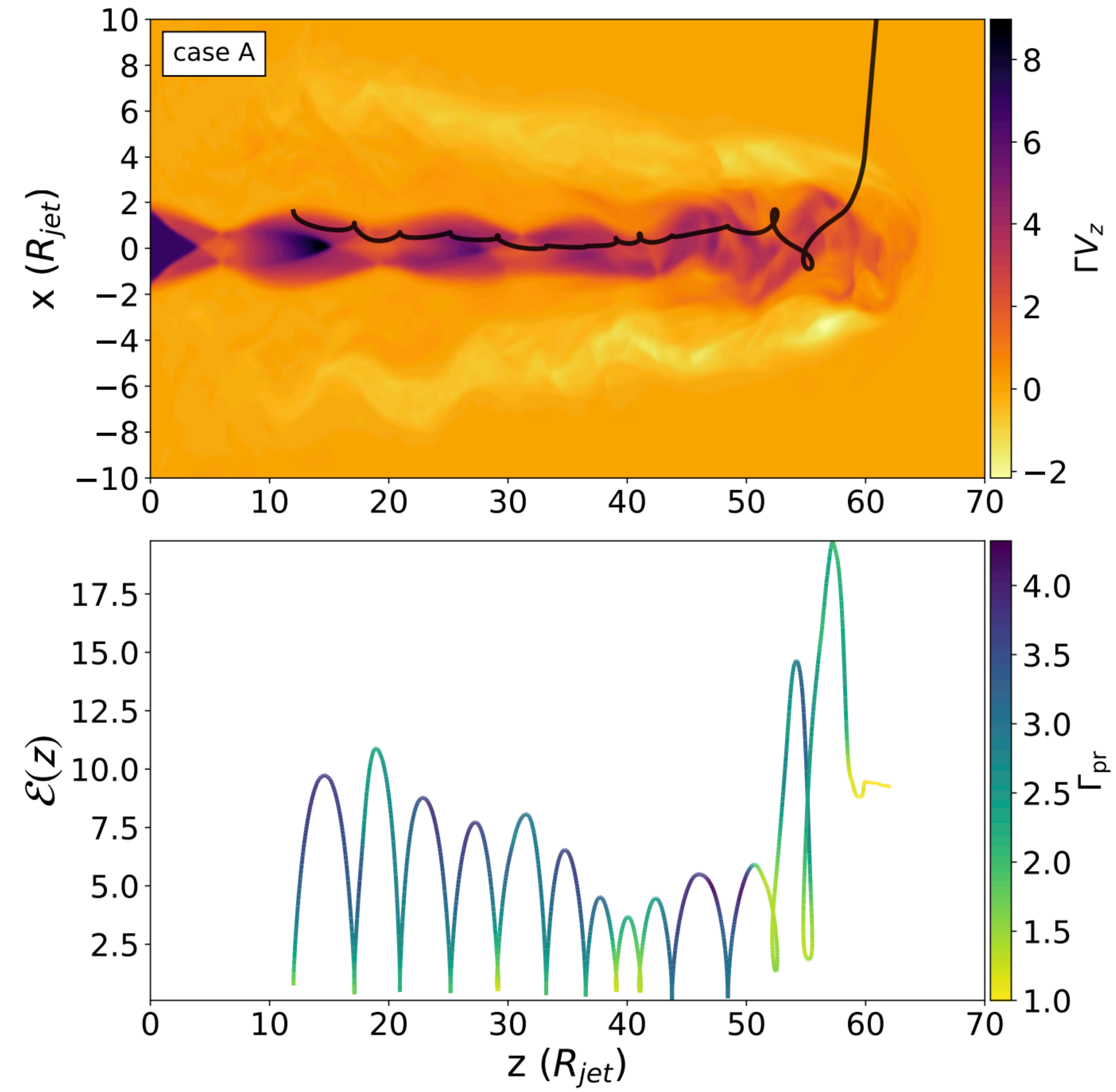
- Turbulence generated by shear flow through the Kelvin Helmholtz instability, where the color is out of plane current

- Turbulence eddies form along with reconnection sites



Espresso Mechanism





- High energy CR passes through a large Gamma jet
- Gets single energy kick from electric field $\propto \Gamma^2$
- Boost CR energy by 10^3 for strong jets