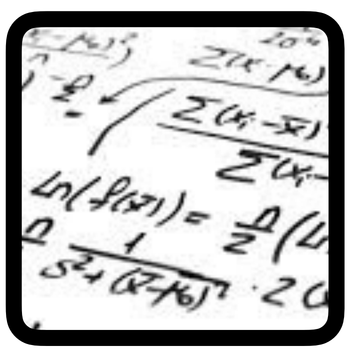
Point Source Likelihood Technique

“Finding needles in haystacks”

Josh Wood

Overview

Define the problem



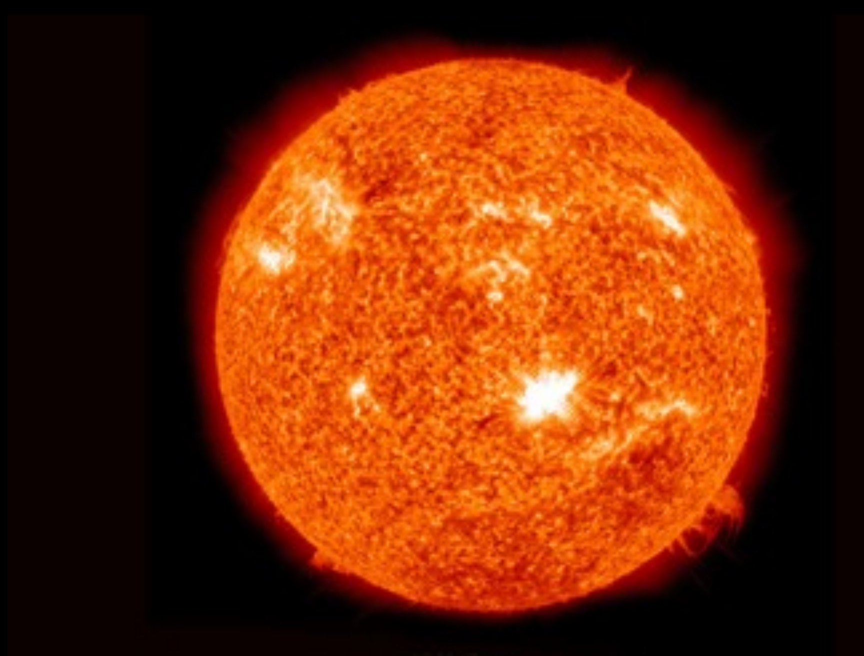
Develop formalism

Python examples

What is a point source?

- Source = object emitting light, neutrinos, ponies… etc
- Point = emission comes from an object that appears smaller than the spatial resolution of your instrument

The Sun



The Moon

Stars

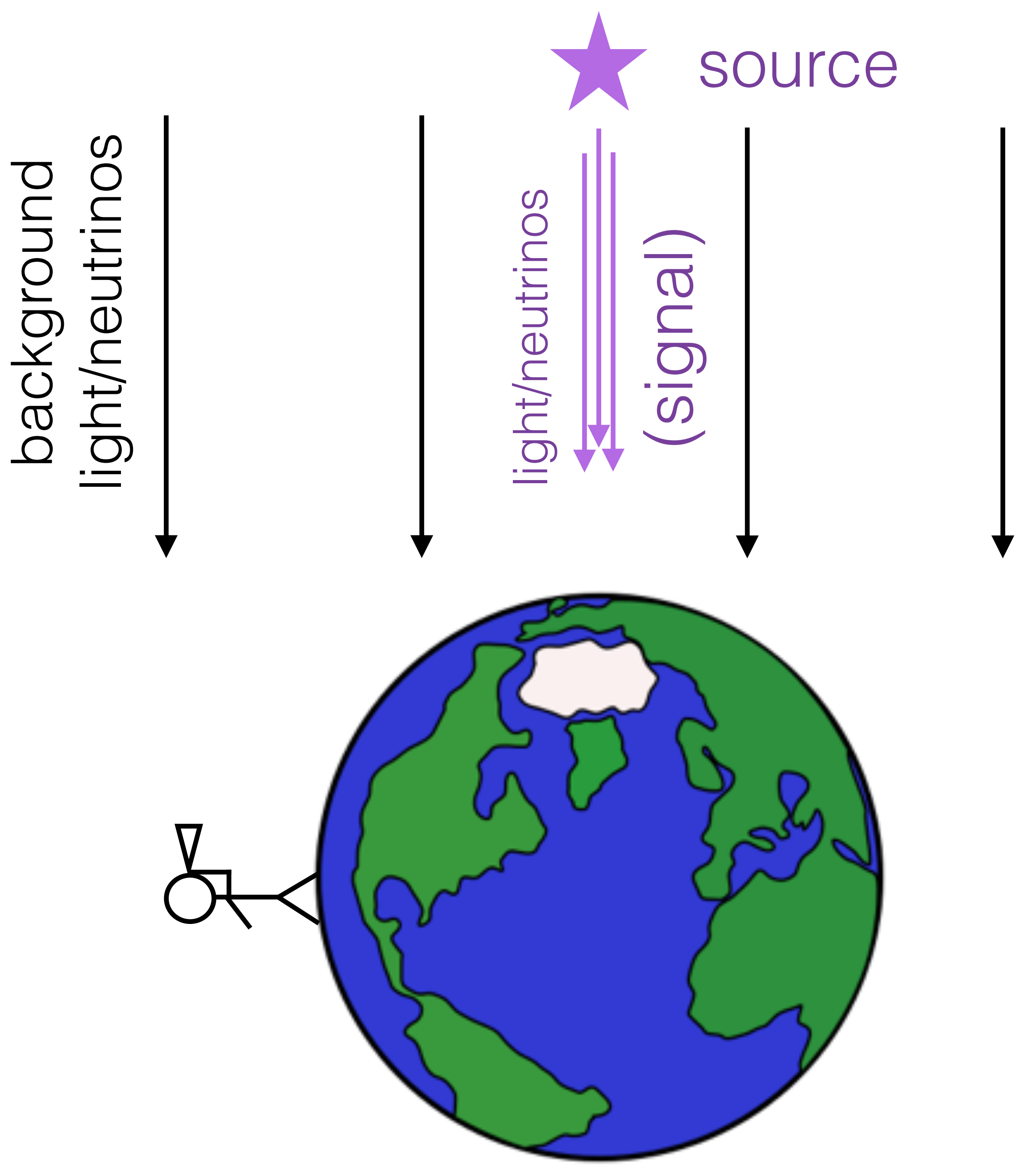
The Problem

- Astronomy is easy when you don't have background

- For some messengers (high energy photons, neutrinos) we can't turn backgrounds off, but we still want to find sources. <u>How to find sources on top of background?</u>

What to do? Think about it!

Imagine you're an astronomer looking for a point source in the presence of uniform background.



<u>def</u>: **signal** is a particle that came from the source you're looking for

<u>def</u>: **background** is a particle that *did not* come from the source but looks identical to a particle emitted by the source

Ex. photon/neutrino with same energy as one from the source

<u>def</u>. **event** a detected particle.
Can be photon, neutrino etc.

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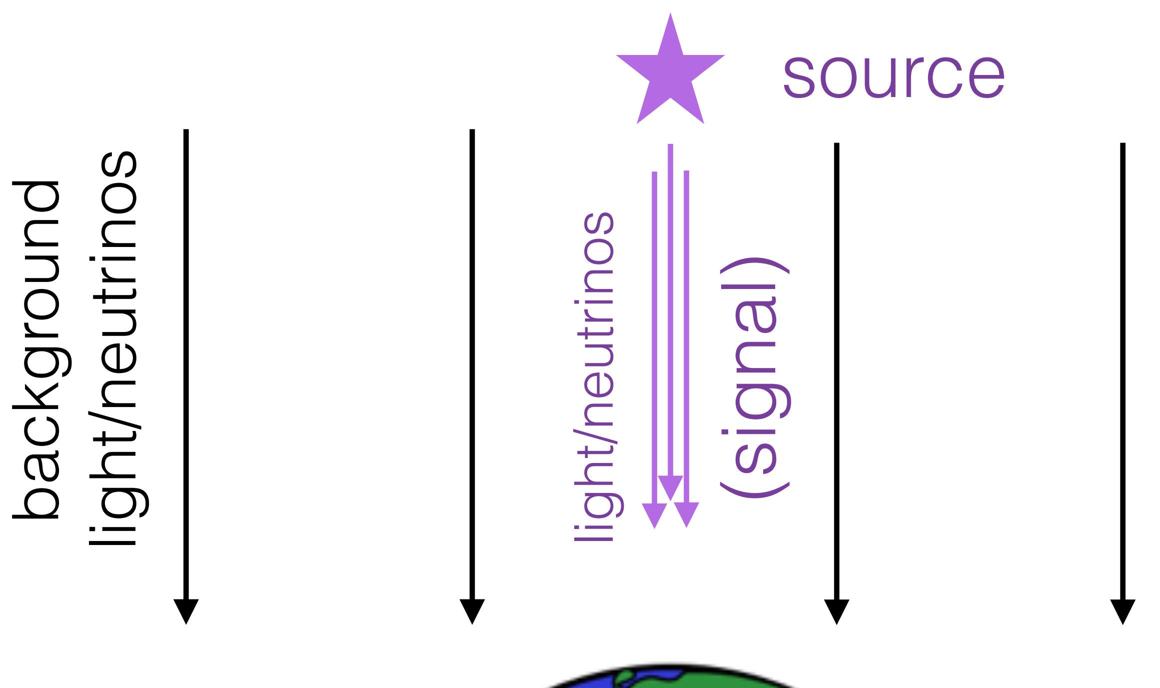


def: **signal** is a particle that came from the source you're looking for

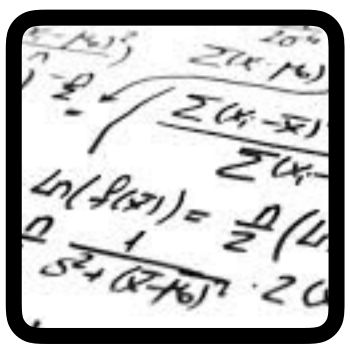
def: **background** is a particle that *did not* come from the source but looks identical to a particle emitted by the source

Q: What's different about photons/neutrinos from the source vs the uniform background?
(note: color doesn't count)

signal is clustered together in one spot!

Overview

Define the problem

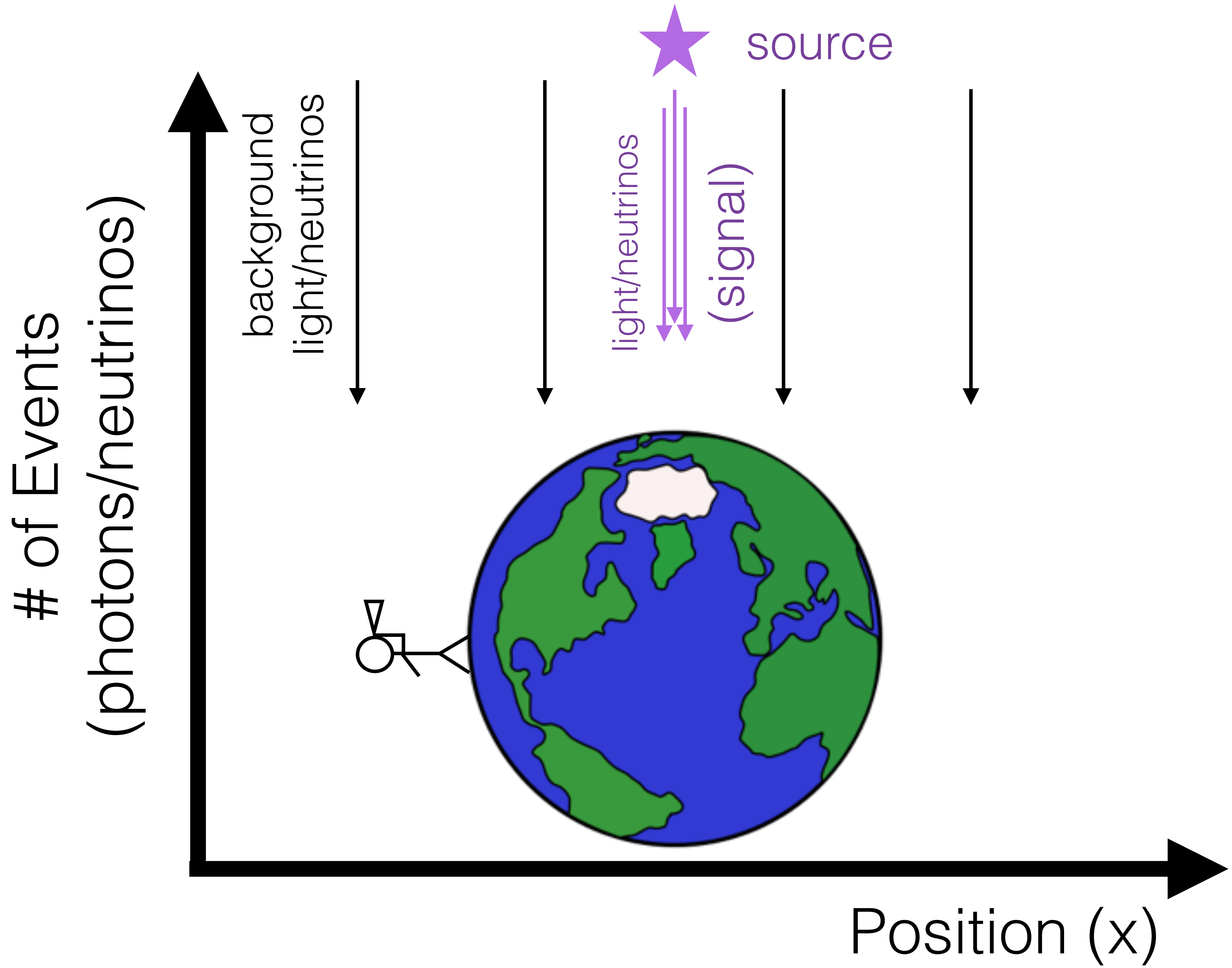


Develop formalism

Python examples

Formalism Part I: Spatial Distributions

Let's start by adding some axes to our example



Formalism Part I: Spatial Distributions

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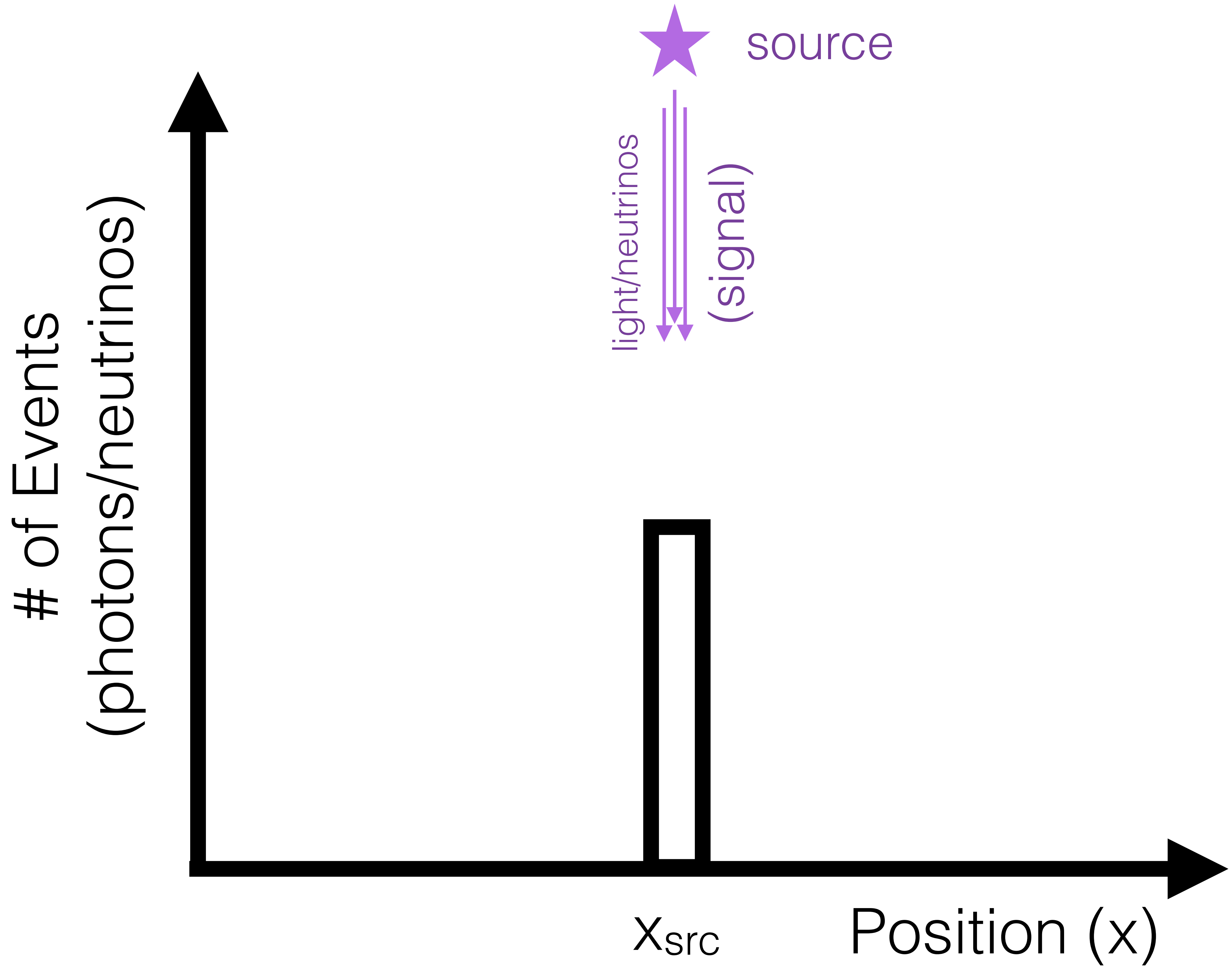

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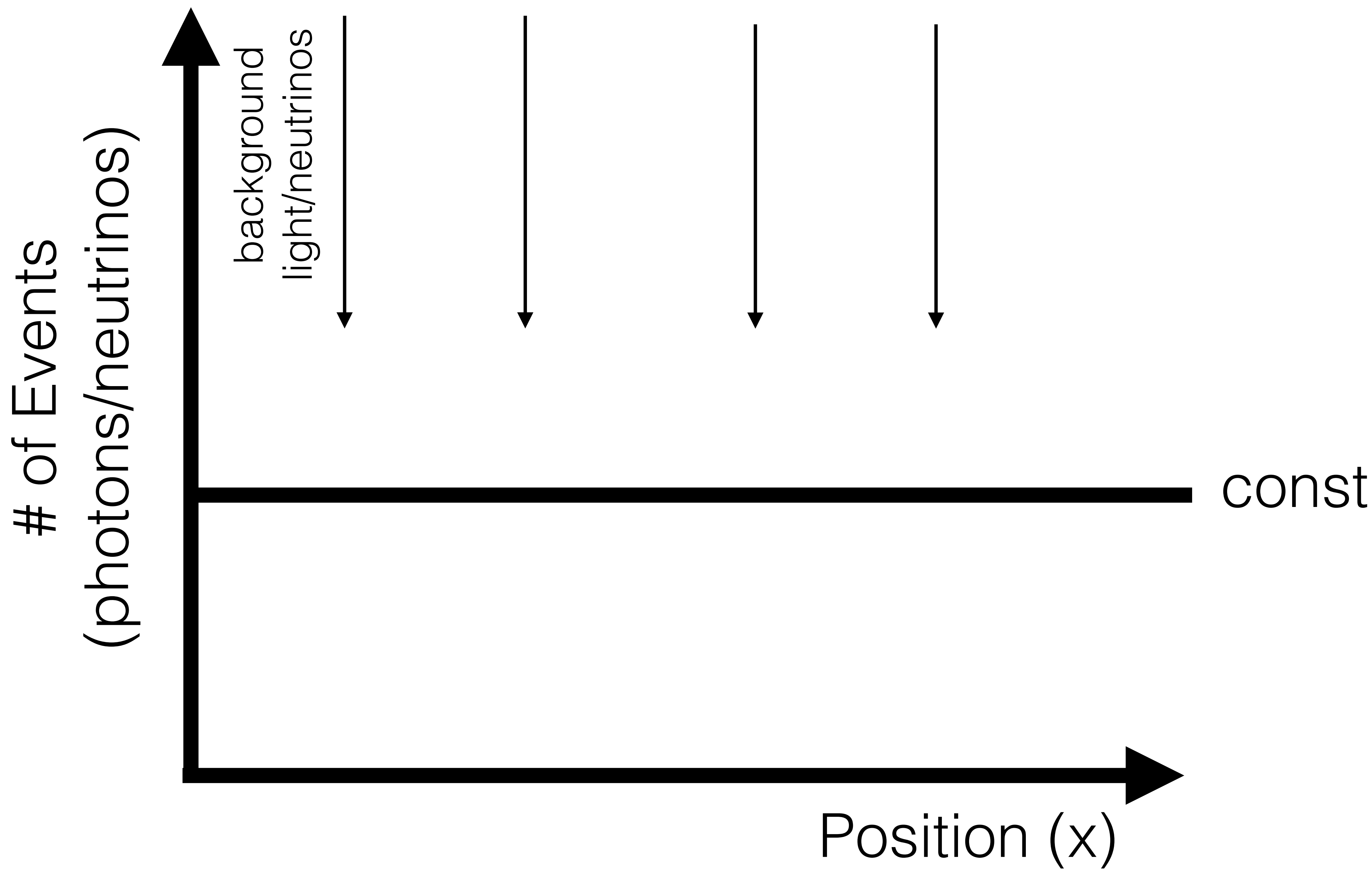
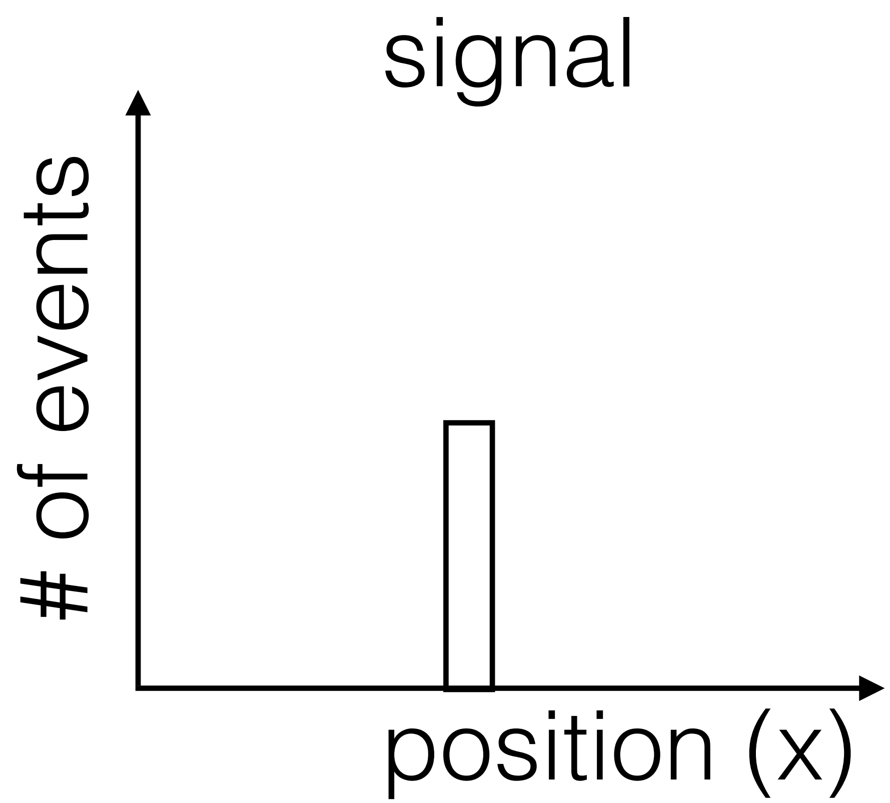
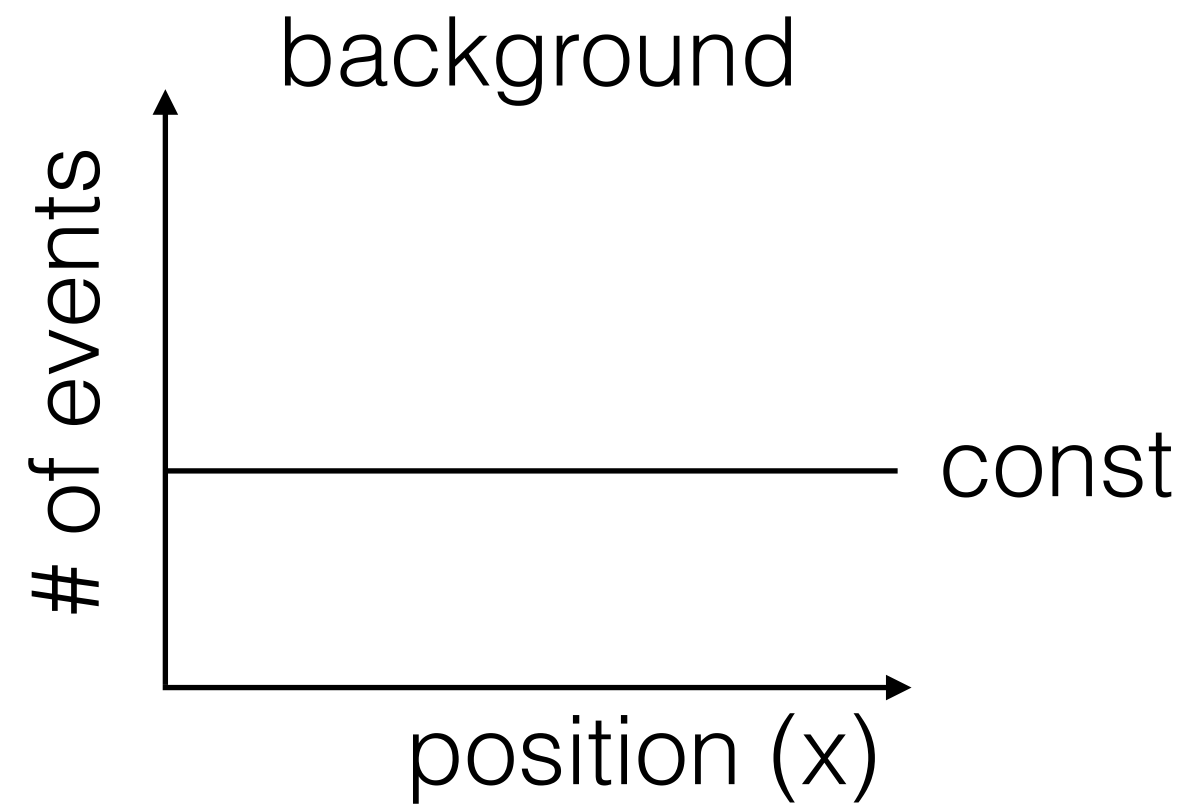
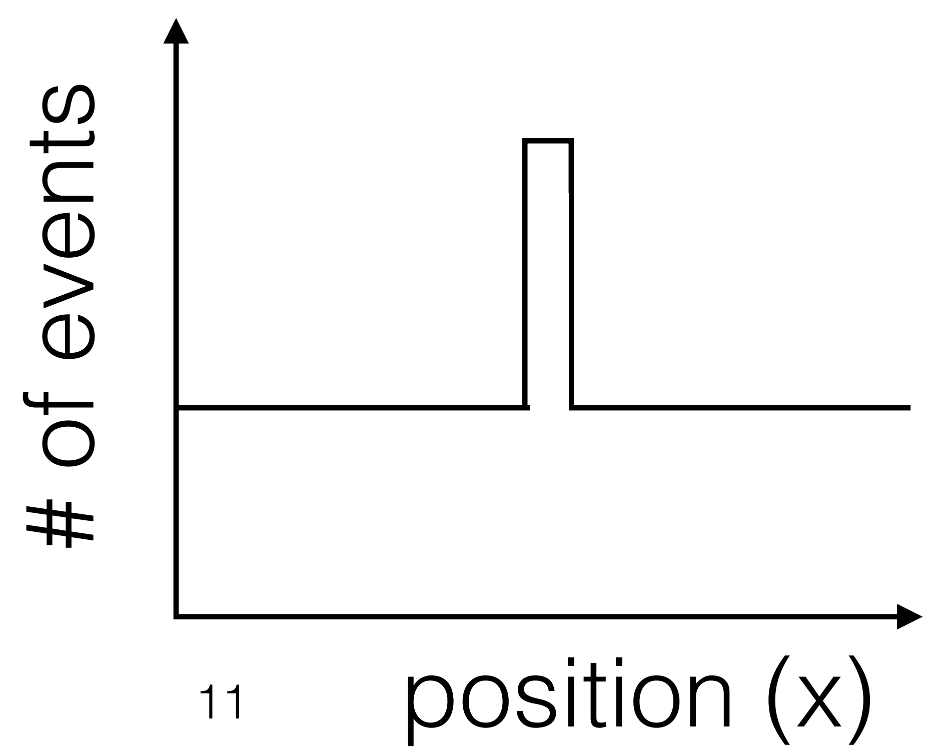


Formalism Part I: Spatial Distributions





data = signal + background



needle in a haystack!

Formalism Part II: Probabilities

Now that we know what signal & background distributions look like, we can formulate them in terms of **probabilities**

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def. **probability** is the chance of getting a given result out of the total number of outcomes.

—> ranges 0 to 1 (never to always)

—> sum of all outcomes must be 1

This way we can ask the question: *what is the probability* that our data are consistent with background + signal versus the case of background only?

—> quantify if a point source is present in data

Formalism Part II: Probabilities

Ok, let’s turn our distributions of events into probability densities —> scale such that integral of distribution is 1

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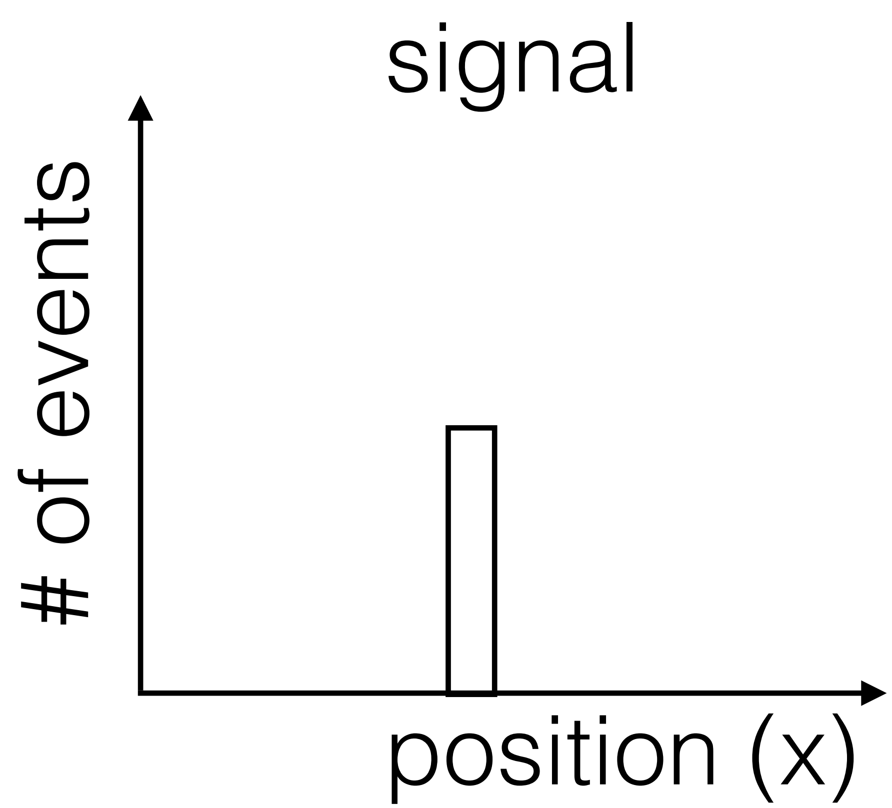

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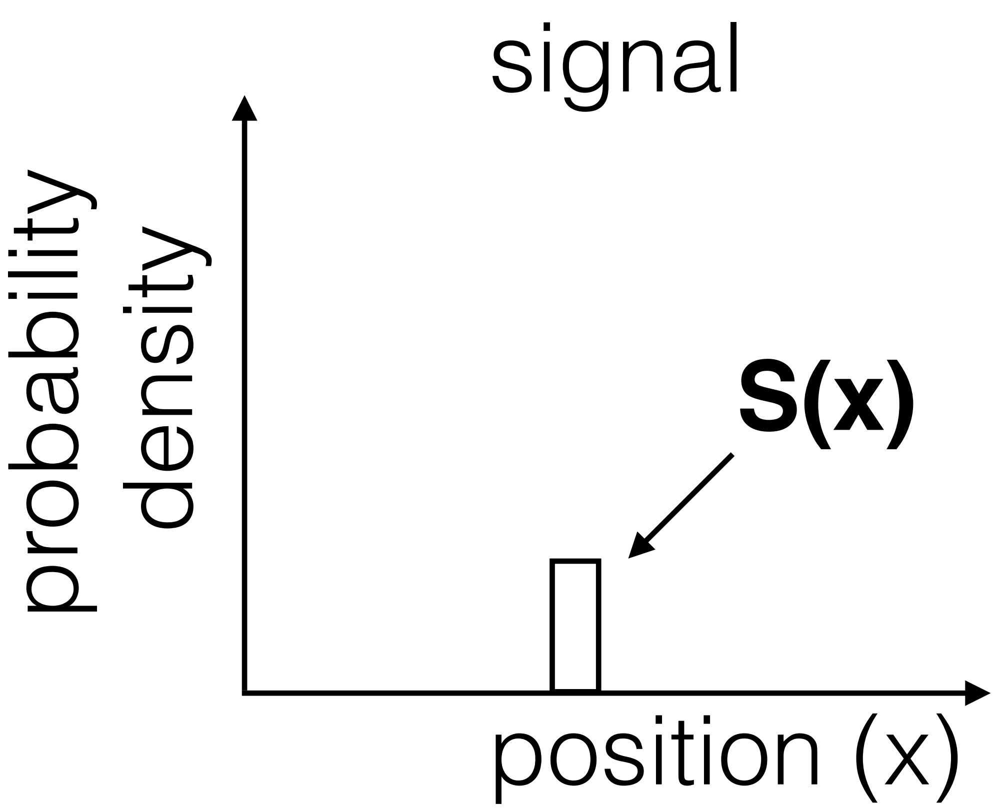

S(x) = probability density of finding signal at x

S(x) dx = probability of finding signal within dx of x

Formalism Part II: Probabilities

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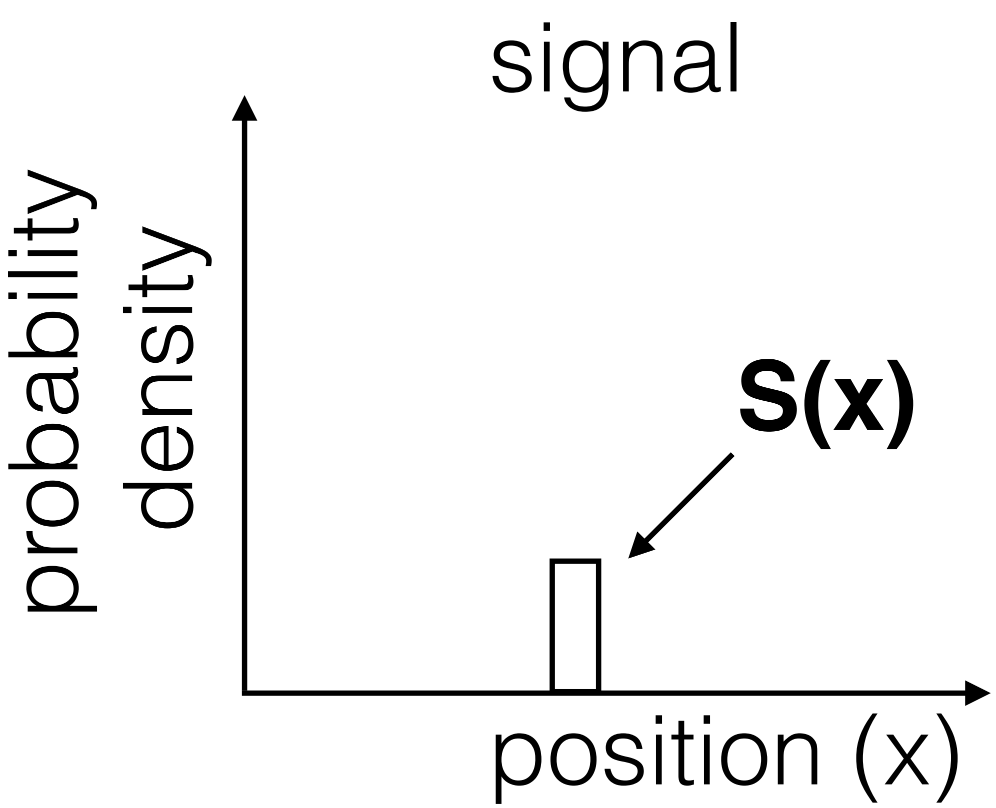

Don't worry about exact form of S(x) today, we'll provide it.

Form depends on detector, typically modeled as Gaussian

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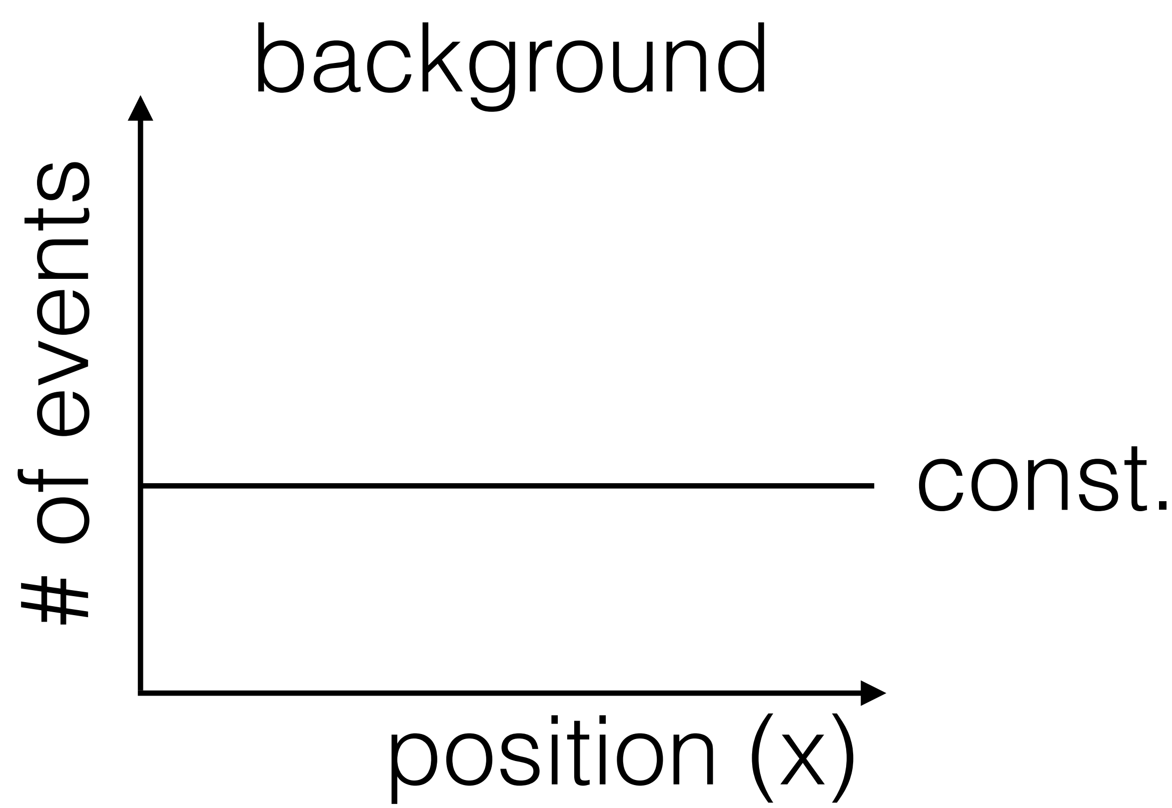

Formalism Part II: Probabilities

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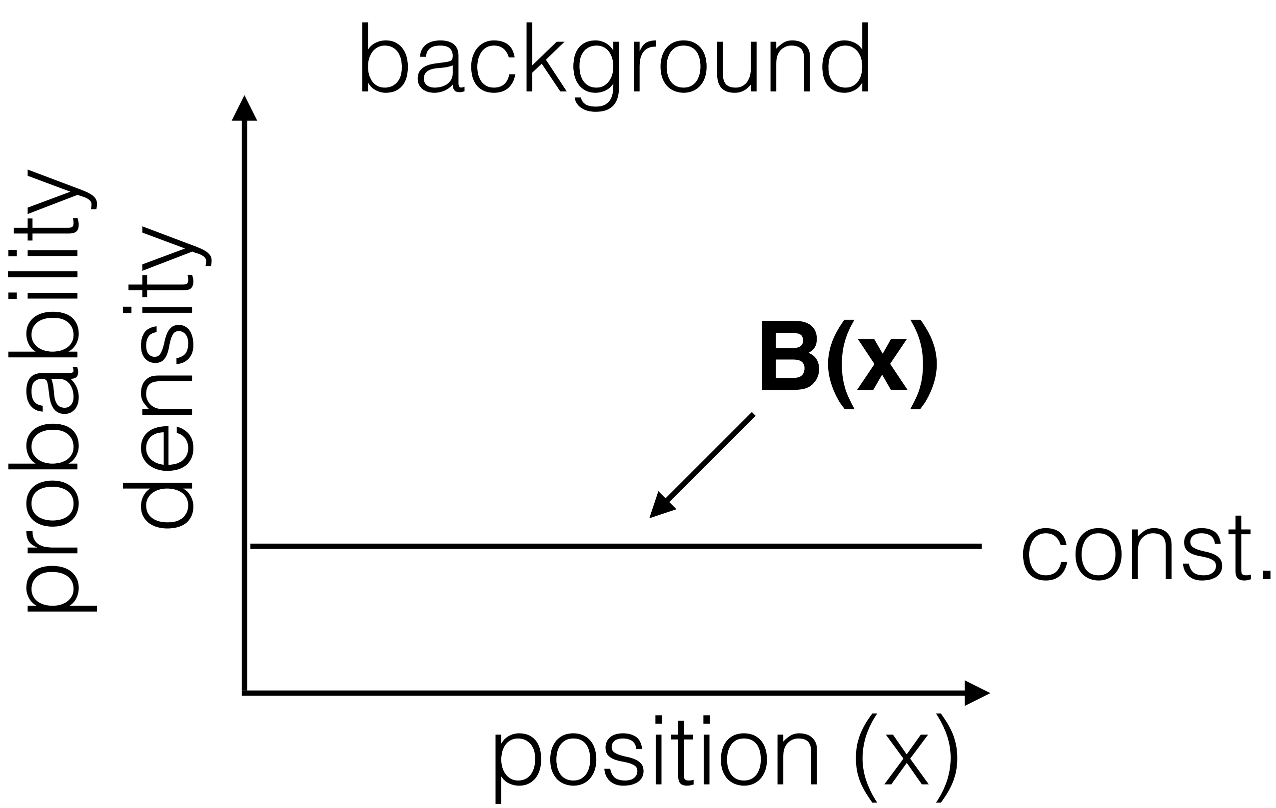


B(x) = probability density of finding background at x

In astronomy, we typically work on surface of a sphere ⟶ uniform **B(x)** = 1/4π

Formalism Part II: Probabilities

The story so far:

→ **S(x)** = probability density of finding a signal event at x

Provided in example code: S(event, source)

→ **B(x)** = probability density of finding a background event at x

const = 1/4π

Both functions describe probability density for finding a *single event* at position **x**, for signal and background.

What about a data set with *multiple events*?

Formalism Part II: Probabilities

For a dataset with:

→ **N** total events

→ n_s signal events

→ x_i is the position where we detect the i^{th} event ($i \in [1, N]$)

total i^{th} signal prob. total i^{th} background prob.

$$\frac{n_s}{N} * S(x_i) + \left(1 - \frac{n_s}{N}\right) * B(x_i)$$

$S(x_i)$: probability density of signal at x_i

$B(x_i)$: probability density of background at x_i

$\frac{n_s}{N}$: probability i^{th} event is a signal event

$\left(1 - \frac{n_s}{N}\right)$: probability i^{th} event is a background event

Formalism Part II: Probabilities

For a dataset with:

- → **N** total events
- → $\mathbf{n_s}$ signal events
- → $\mathbf{x_i}$ is the position where we detect the i^{th} event ($i \in [1, N]$)

total i^{th} signal prob. total i^{th} background prob.

$$\prod_{i=1}^{N} \left[\frac{n_s}{N} * S(x_i) + \left(1 - \frac{n_s}{N}\right) * B(x_i) \right]$$

Total probability of i^{th} event

How to combine probabilities of all events? Product!

Formalism Part II: Probabilities

For a dataset with:

→ **N** total events

→ $\mathbf{n_s}$ signal events

→ $\mathbf{x_i}$ is the position where we detect the i^{th} event ($i \in [1, N]$)

$$\mathbf{L(n_s)} = \prod_{i=1}^{N} \frac{\mathbf{n_s}}{\mathbf{N}} * \mathbf{S(x_i)} + \left(\mathbf{1} - \frac{\mathbf{n_s}}{\mathbf{N}}\right) * \mathbf{B(x_i)}$$

$\mathbf{L(n_s)}$ is the total probability of the dataset containing $\mathbf{n_s}$ signal events. It is called a **likelihood function**.

The best estimate for the true value of $\mathbf{n_s}$ is the value which **maximizes the likelihood function**.

now for some math voodoo…

Formalism Part III: Voodoo

Working with **ratios of likelihoods** has some nice statistical properties.

Also, addition is easier and faster than multiplication so we define a **test statistic (TS):**

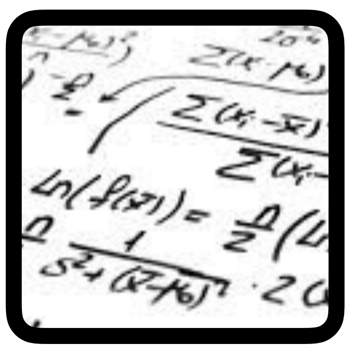
$$\mathbf{TS = 2\log(L(n_s) / L(n_s = 0))}$$

$$\mathbf{TS = 2 \sum_{i=1}^{N} \log\left[\frac{n_s}{N} * \frac{S(x_i)}{B(x_i)} + \left(1 - \frac{n_s}{N}\right) \right]}$$

Finding the value of $\mathbf{n_s}$ which maximizes TS is equivalent to maximizing the likelihood. TS = 0 means consistent with background only. TS ~ 25 typically proof of a point source.

Overview

Define the problem



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https://icecube.wisc.edu/~jwood/bootcamp2018/