Hyperplane Parameterizations

Philipp Eller

Calibration Workshop

Discrete Systematics

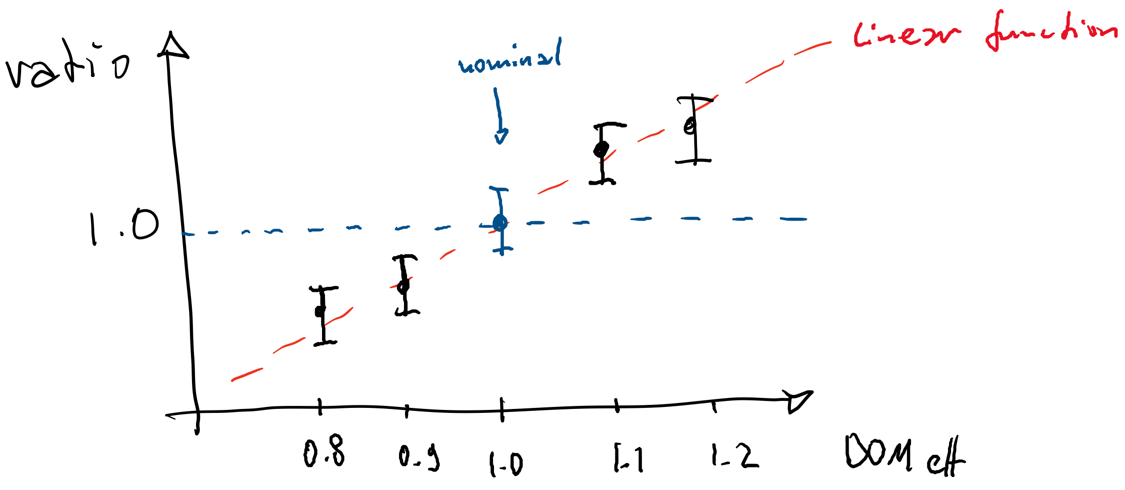
- When evaluating detector systematics, e.g. different DOM efficiencies, we need to re-simulate our MC
- That also entails re-running reconstruction
  - Very time consuming
  - Can only afford to do it for a handful of values
- However, in the analysis we usually want to be able to continuously change values as a nuisance parameters

1-d interpolation

- Established method in the LowEn group to deal with one discrete systematic at a time:
  - Given a number of MC sets for, say DOM eff values of [0.8, 0.9, 1.0, 1.1, 1.2], calculate the expected event distribution for your analysis
    - i.e. an event count for every bin in the analysis
  - For each analysis bin, calculate the ratio to the nominal value (e.g. define DOM eff = 1.0 as nominal)
  - Next slide....

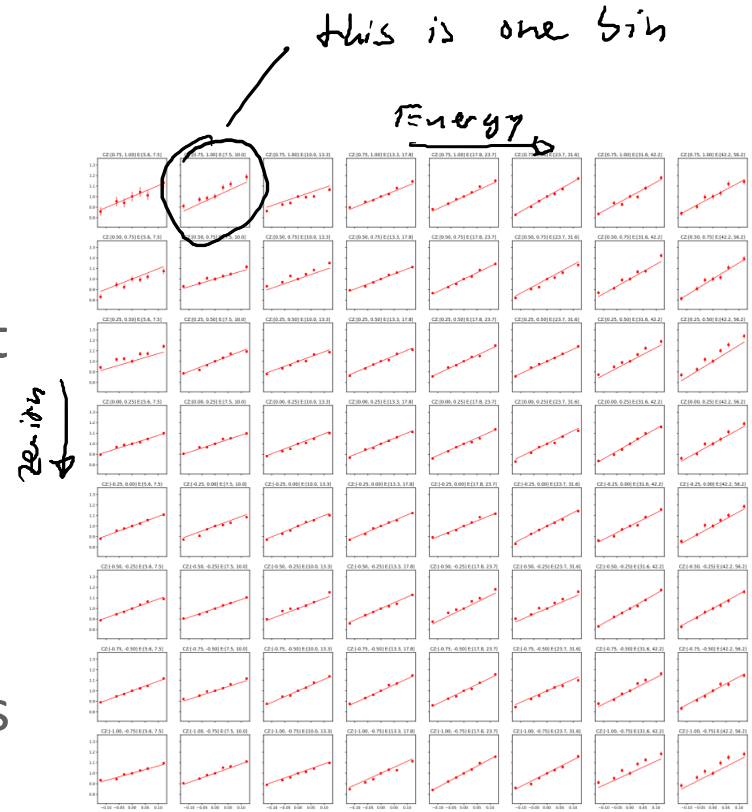
1-d interpolation cont'd

- Put ratio points vs. parameter values in a graph
- Fit a function to it for interpolation
  - Here linear, or could be higher order polynomial etc…



1-d interpolation cont'd

- Do this for every bin in the analysis
  - Here: obtain a slope and an offset per bin
- Then use this function with a global parameter in the analysis:
  - It means if you set DOM eff to 0.9345, the function in each bin is evaluated at that point to obtain the scaling factor to be applied to its event count

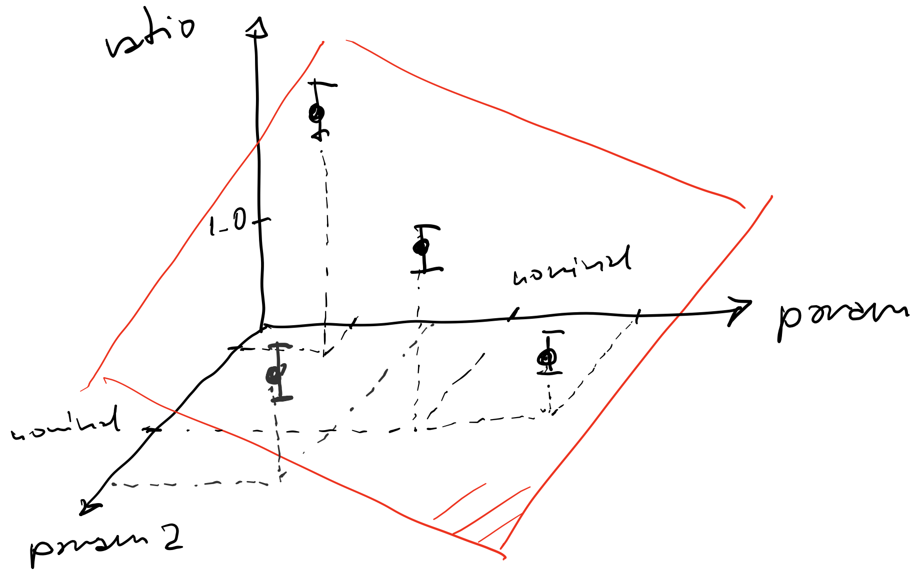


Multiple Systematics

- The before described approach works well if only one systematic needs to be described
- It can be use subsequently for several systematics, but this brings some problems with it
  - Sets must be produced only changing one systematic at a time while the others are at nominal
  - Cannot handle correlations
  - The nominal MC set is used in every parameterization and this results in over-corrections

Introducing: **Hyperplanes**

- Instead, we can do the parameterization of multiple systematics at the same time using a single function



Hyperplanes

- Simplest (linear) equation for n-dimensions

$$f(p_1, p_2, \ldots, p_n) = a + \sum_{i=1}^{n} p_i \cdot s_i$$

a ↑ offset, s_i ↑ slopes

- Still done for every bin of the analysis
- Allows to use arbitrary points
- Fit more stable (less parameters), example:
  - 3 systematics the old way = 6 parameters
  - 3d Hyperplanes = 4 parameters

Flavour/interaction dependence

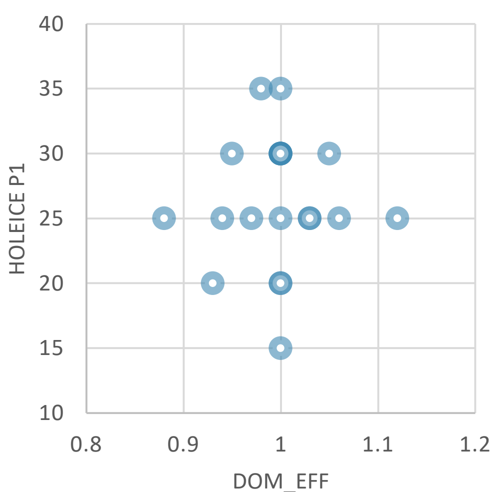
- Muon tracks, hadronic and electromagnetic showers can behave differently under changing detector systematics
- Therefore we now parameterize it separately for
  - Charge Current (CC) nue
  - CC numu
  - CC nutau
  - Neutral Current (NC) for all flavours combined

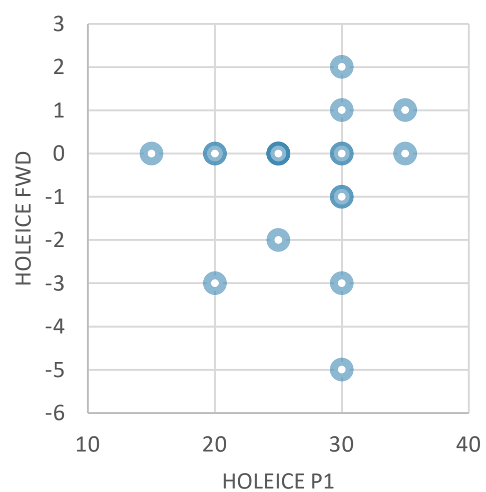
Analysis example

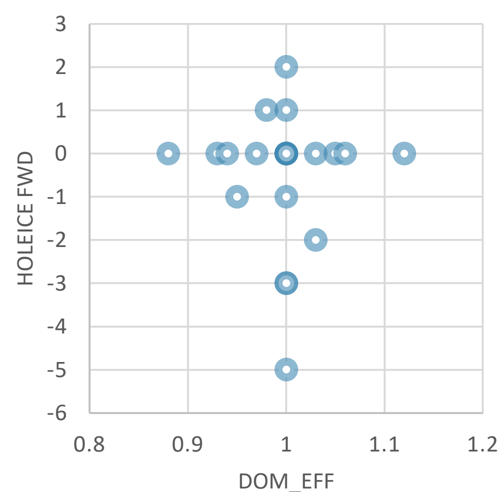
- For the DRAGON tau neutrino analysis
- Detector systematics, apart from flux uncertainties, are the most important ones in our analysis
- Hyperplanes allowed to
  - include more MC sets
  - Interpolate between the angular acceptance and direct propagation of holeice models
  - Get more stable results

Analysis example

- We use 28 discrete MC sets
- And 4 systematics (DOM eff + 3 Hole-ice sys)
- (4^{th} dim is spiciness, which would be another talk)

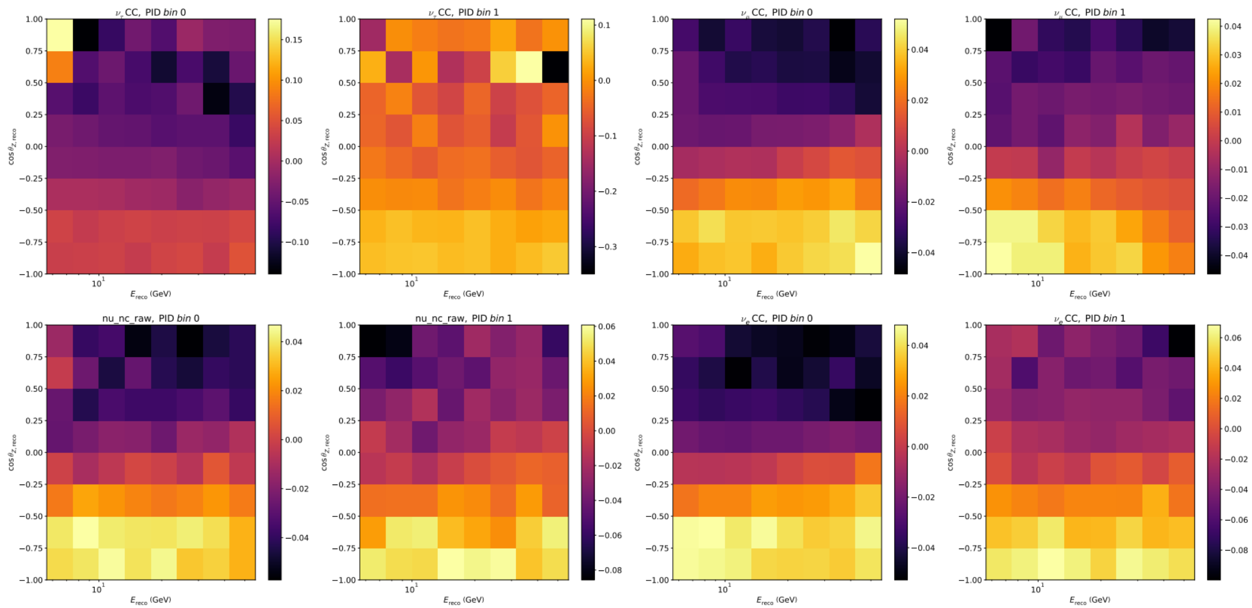






Slopes

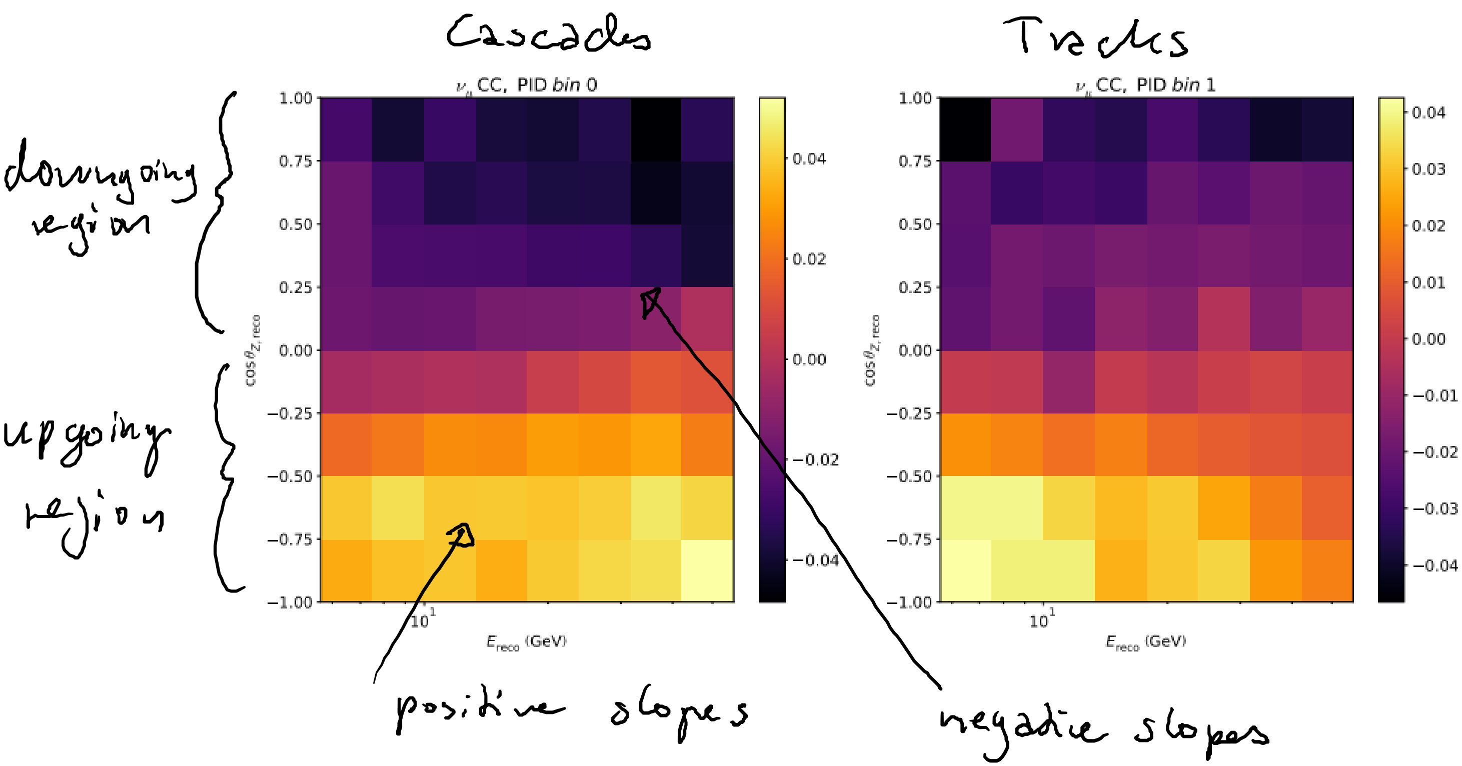
- Cannot plot the 4d hyperplanes, but the fit results
  - Here: fole_ice fwd slopes vs. (E,CZ) maps



https://wiki.icecube.wisc.edu/index.php/Nutau_Appearance_Analysis_with_PISA

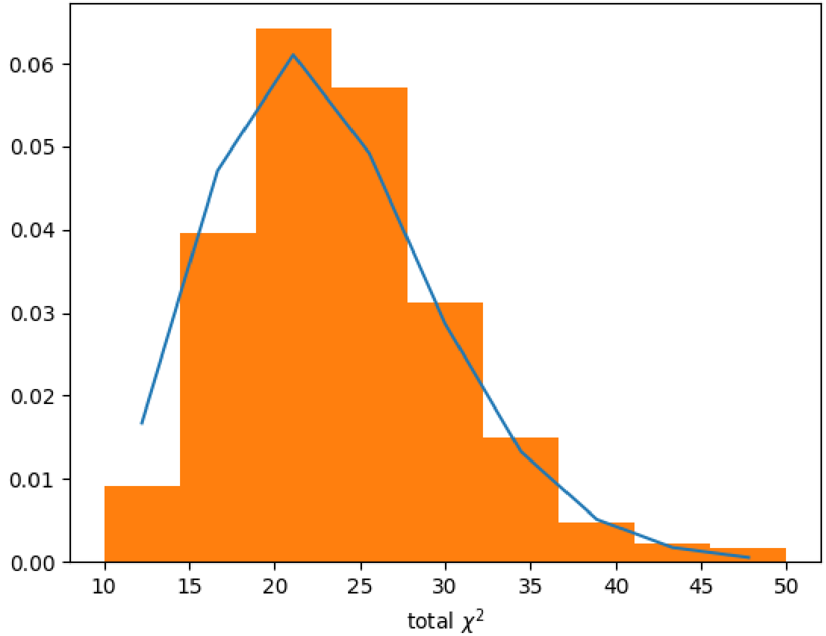
A closer look

- Example: numu CC events



Goodness of fit

- In our case, the simple linear function gives good results (28 MC points – 5 function parameters)



- blue line: expected chi2 dist
- orange hist: actual chi2 from DRAGON hyperplane fits

Smoothing

- Some efforts were made to have smoother fit results (there is currently some statistical noise that can be seen in the maps before)
- Option 1: smooth event distribution prior to fitting the hyperplanes (e.g KDEs)
  - Andrii is working on that
- Option 2: smooth the resulting functions
  - This smoothing makes the assumption that the discrete systematics don’t create unsmooth shapes….which not everybody in the LowEn WG agrees with
  - Maybe experts could comment on that if these properties are expected to rapidly change vs. energy etc?

Summary

- Parameterization allows to use a bunch of different simulations to be treated as one or more continuous nuisance parameters in the analysis
- If we have more than one parameter, hyperplanes are the way to go
- Currently used in the DRAGON nutau analysis and others started using it, e.g. 6y LEESARD osc. analysis