MCEq and a universal treatment for systematic errors

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Diffuse workshop on Global Fit @ Earthquake Research Institute, University of Tokyo

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Origin of the series of models, methods and tools

Hans
Dembinski

Anatoli
Fedynitch

Ralph
Engel

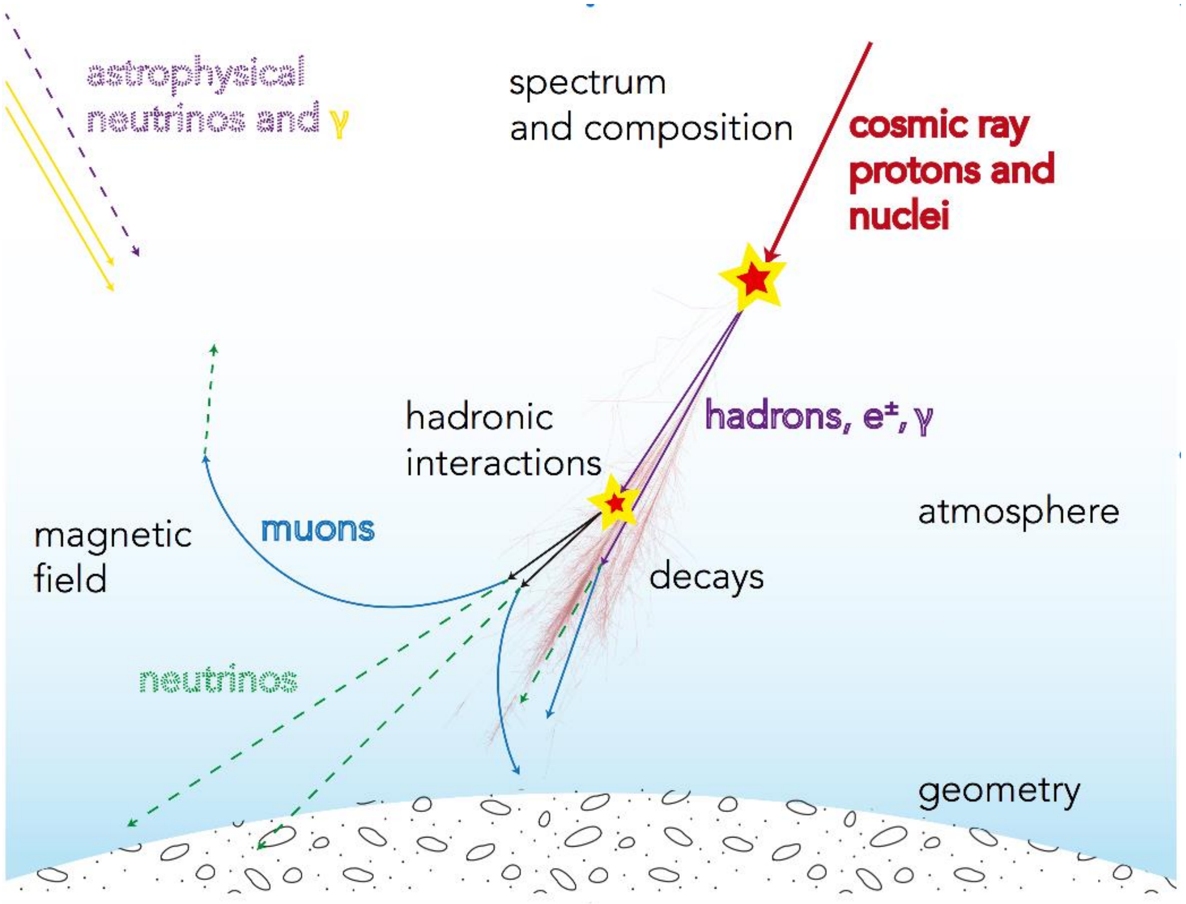
Thomas K.
Gaisser

Felix
Riehn

Todor
Stanev

Atmospheric neutrinos

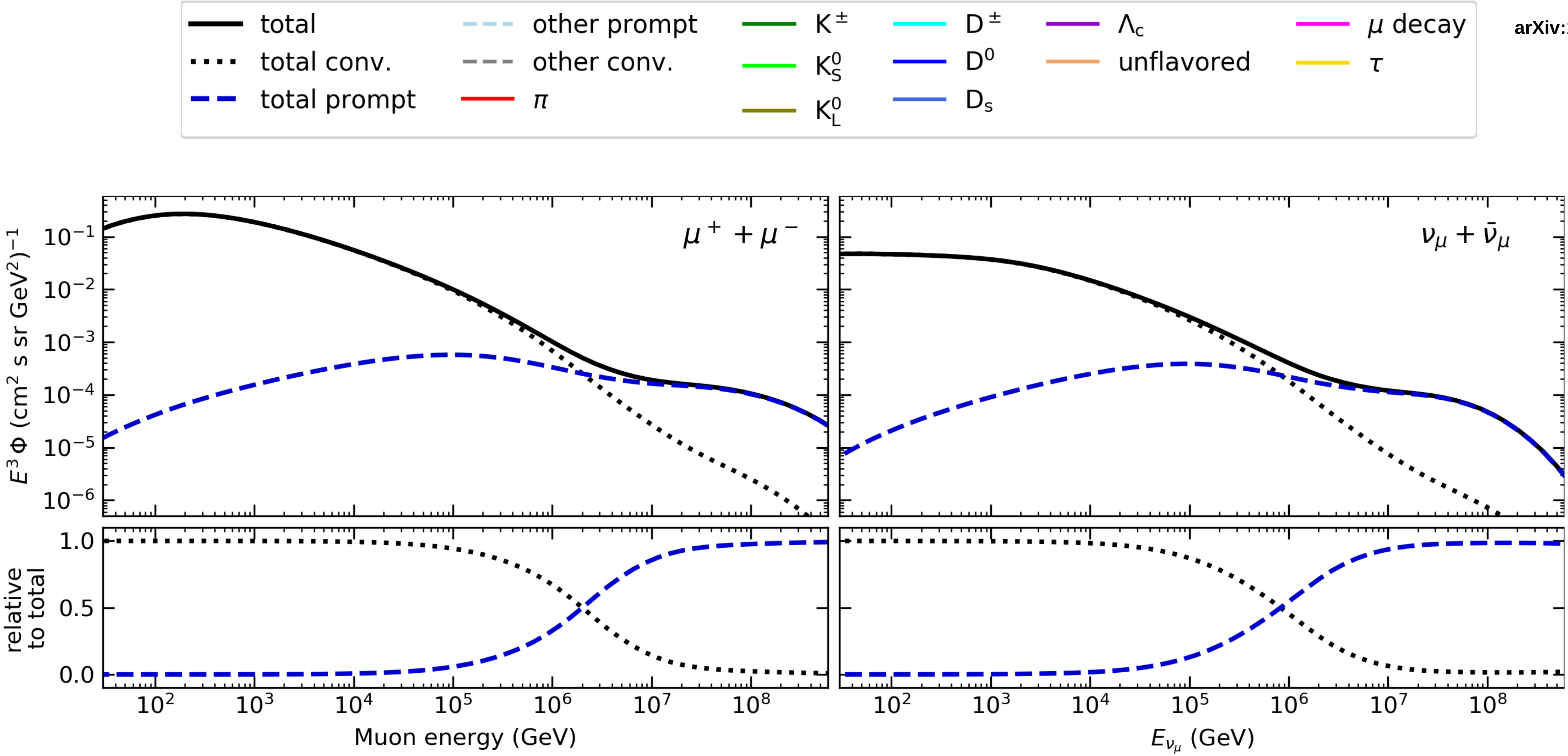
Ingredients for high-precision atmospheric neutrino flux calculation



- For <u>high precision</u> calculations all phenomena need accurate modeling
- Uncertain “ingredients”:
  - Cosmic ray spectrum and composition
  - Hadronic interactions
  - Atmosphere (dynamic, depends on use case)
  - (Rare) decays
  - Geometry, magnetic fields, solar modulation
- No clear prescription how to handle uncertainties.
- Energy range MeV – EeV!

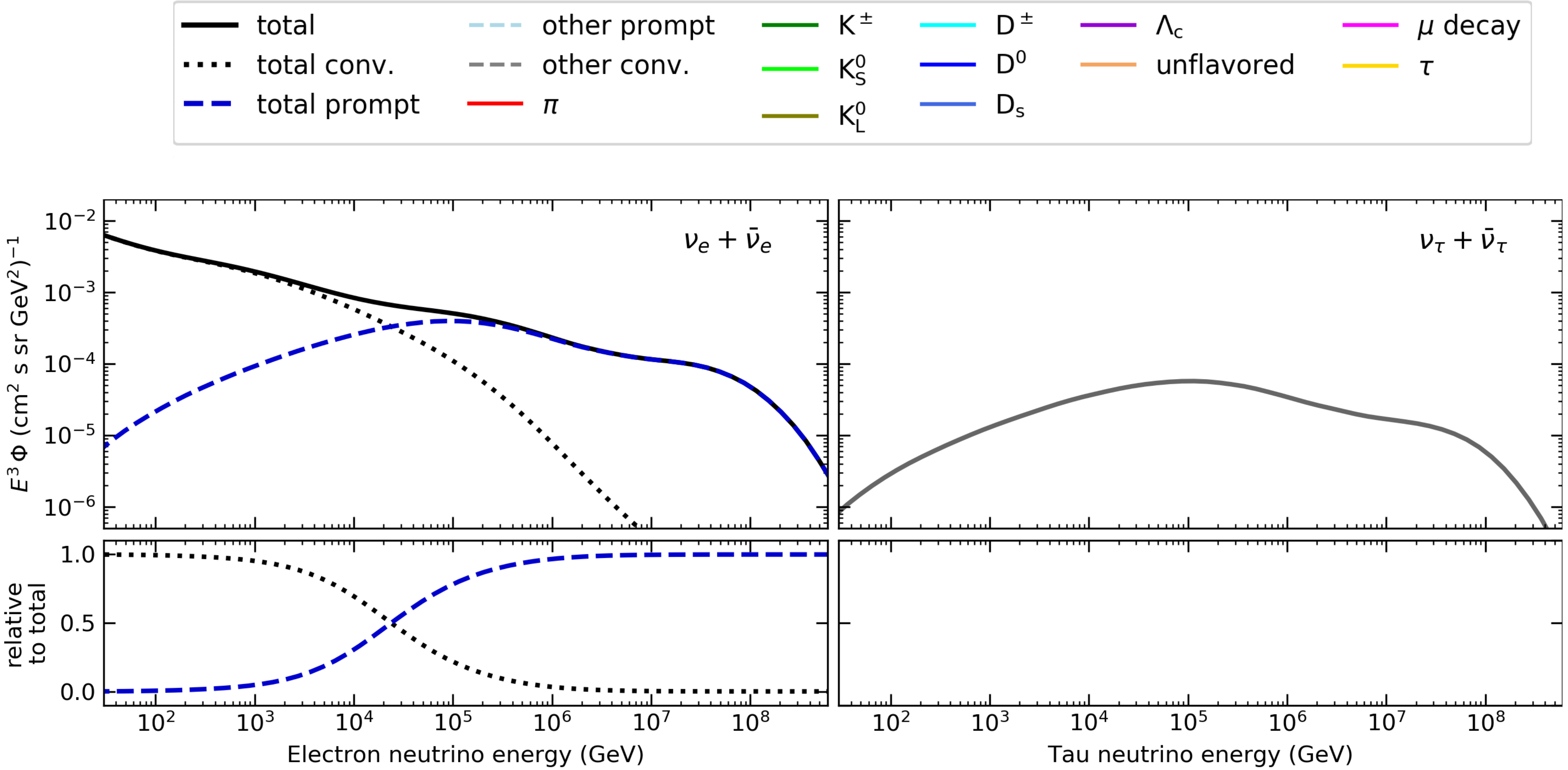
Hadrons contributing to muonic leptons

arXiv:1806.04140

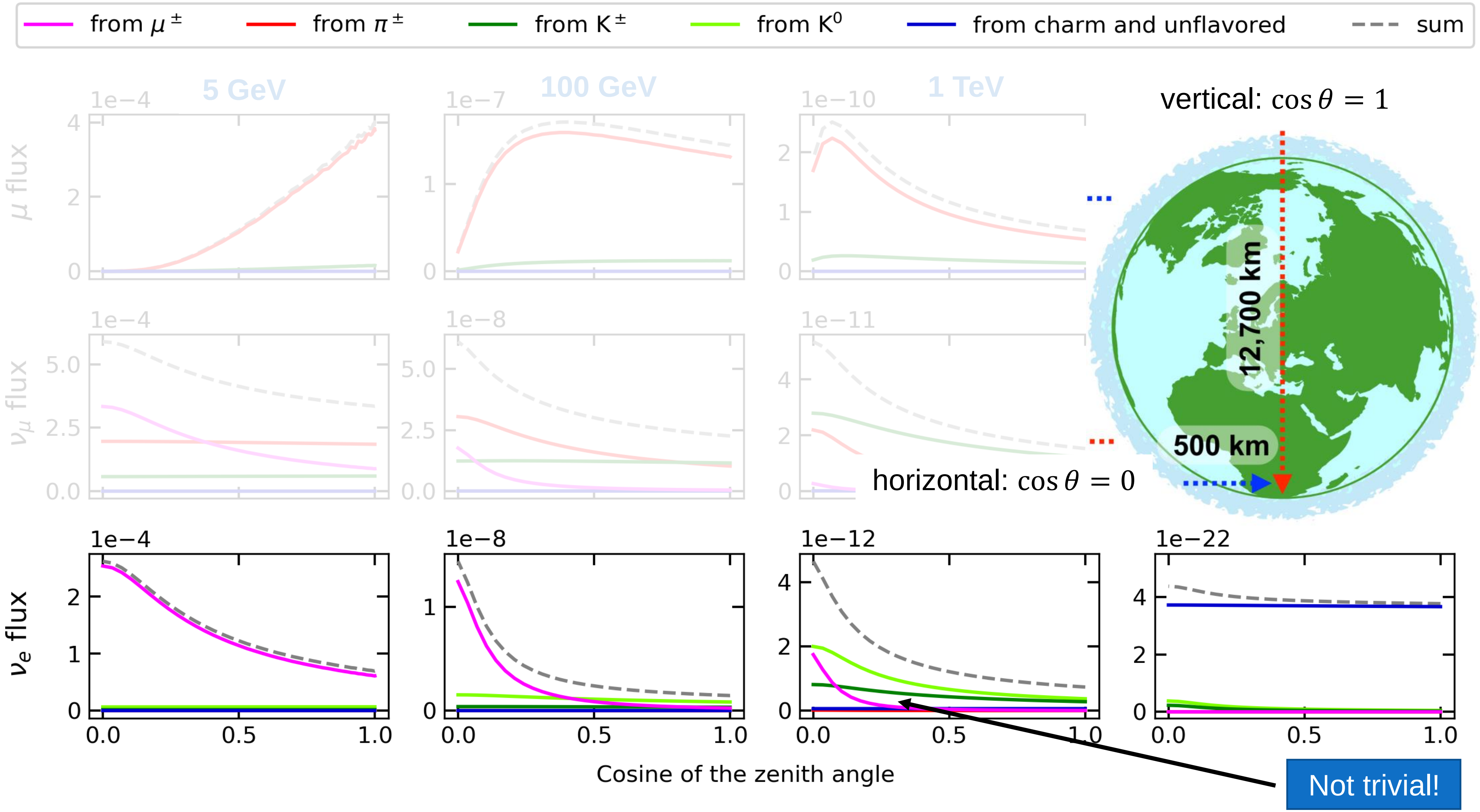


Hadrons contributing to electron and tau neutrinos

arXiv:1806.04140



Different hadronic components shape the zenith distribution



AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, 1806.04140 (2018)

Transport equations (hadronic cascade equations)

System of coupled non-linear PDE for each particle species *h* :

$$\frac{\mathrm{d}\Phi_h(E,X)}{\mathrm{d}X} = -\frac{\Phi_h(E,X)}{\lambda_{\mathrm{int},h}(E)} - \frac{\Phi_h(E,X)}{\lambda_{\mathrm{dec},h}(E,X)} - \frac{\partial}{\partial E}(\mu(E)\Phi_h(E,X)) + \sum_k \int_E^\infty \mathrm{d}E_k \frac{\mathrm{d}N_{k(E_k)\to h(E)}}{\mathrm{d}E}\frac{\Phi_k(E_k,X)}{\lambda_{\mathrm{int},k}(E_k)} + \sum_k \int_E^\infty \mathrm{d}E_k \frac{\mathrm{d}N^{\mathrm{dec}}_{k(E_k)\to h(E)}}{\mathrm{d}E}\frac{\Phi_k(E_k,X)}{\lambda_{\mathrm{dec},k}(E_k,X}$$

cosmic ray physics

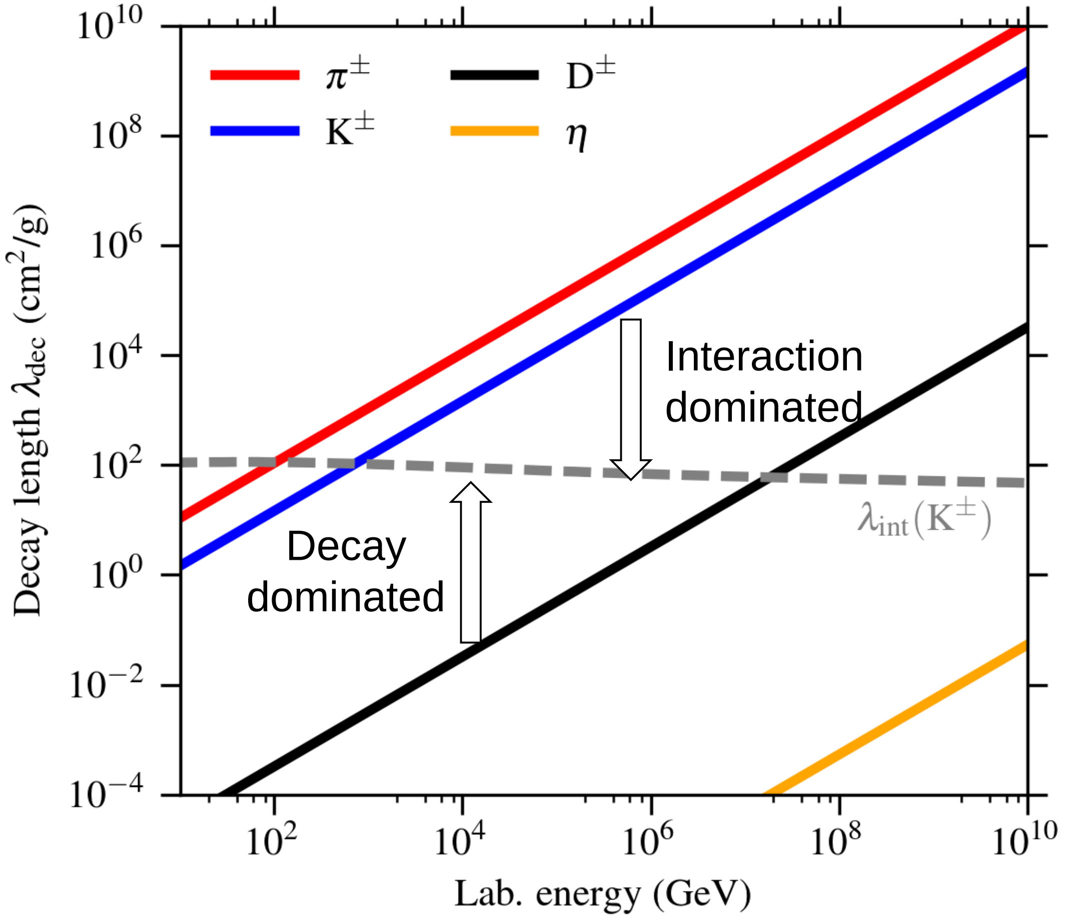
Interactions with air

Decays

atmospheric physics

Continuous losses

particle physics



$$X(h_0) = \int_0^{h_0} \mathrm{d}\ell\ \rho_{\mathrm{air}}(\ell)$$

MCEq: Matrix Cascade Equations

A. Fedynitch, R. Engel, T. K. Gaisser, F. Riehn and S. Todor
PoS ICRC 2015, 1129 (2015), EPJ Web Conf. 99, 08001 (2015)
and EPJ Web Conf. 116, 11010 (2016)

$$\frac{\mathrm{d}\Phi_h(E,X)}{\mathrm{d}X} = -\frac{\Phi_h(E,X)}{\lambda_{\mathrm{int},h}(E)} - \frac{\Phi_h(E,X)}{\lambda_{\mathrm{dec},h}(E,X)} - \frac{\partial}{\partial E}(\mu(E)\Phi_h(E,X)) + \sum_\ell \int_E^\infty \mathrm{d}E_\ell \, \frac{\mathrm{d}N_{\ell(E_\ell)\to h(E)}}{\mathrm{d}E} \frac{\Phi_\ell(E_\ell,X)}{\lambda_{\mathrm{int},l}(E_\ell)} + \sum_\ell \int_E^\infty \mathrm{d}E_\ell \, \frac{\mathrm{d}N^{\mathrm{dec}}_{\ell(E_\ell)\to h(E)}}{\mathrm{d}E} \frac{\Phi_\ell(E_\ell,X)}{\lambda_{\mathrm{dec},l}(E_\ell,X)}$$

$$\frac{\mathrm{d}\Phi^h_{E_i}}{\mathrm{d}X} = -\frac{\Phi^h_{E_i}}{\lambda^h_{\mathrm{int},E_i}} - \frac{\Phi^h_{E_i}}{\lambda^h_{\mathrm{dec},E_i}(X)} - \vec{\nabla}_i(\mu^h_{E_i}\Phi^h_{E_i}) + \sum_{E_k \geq E_i}^{E_N} \sum_\ell \frac{c_{\ell(E_k)\to h(E_i)}}{\lambda^\ell_{\mathrm{int},E_k}} \Phi^\ell_{E_k} + \sum_{E_k \geq E_i}^{E_N} \sum_\ell \frac{d_{\ell(E_k)\to h(E_i)}}{\lambda^\ell_{\mathrm{dec},E_k}(X)} \Phi^\ell_{E_k}$$

State (or flux) vector

$$\vec{\Phi} = \left(\vec{\Phi}^{\mathrm{p}} \quad \vec{\Phi}^{\mathrm{n}} \quad \vec{\Phi}^{\pi^+} \quad \ldots \quad \vec{\Phi}^{\bar{\nu}_\mu} \quad \ldots\right)^T$$

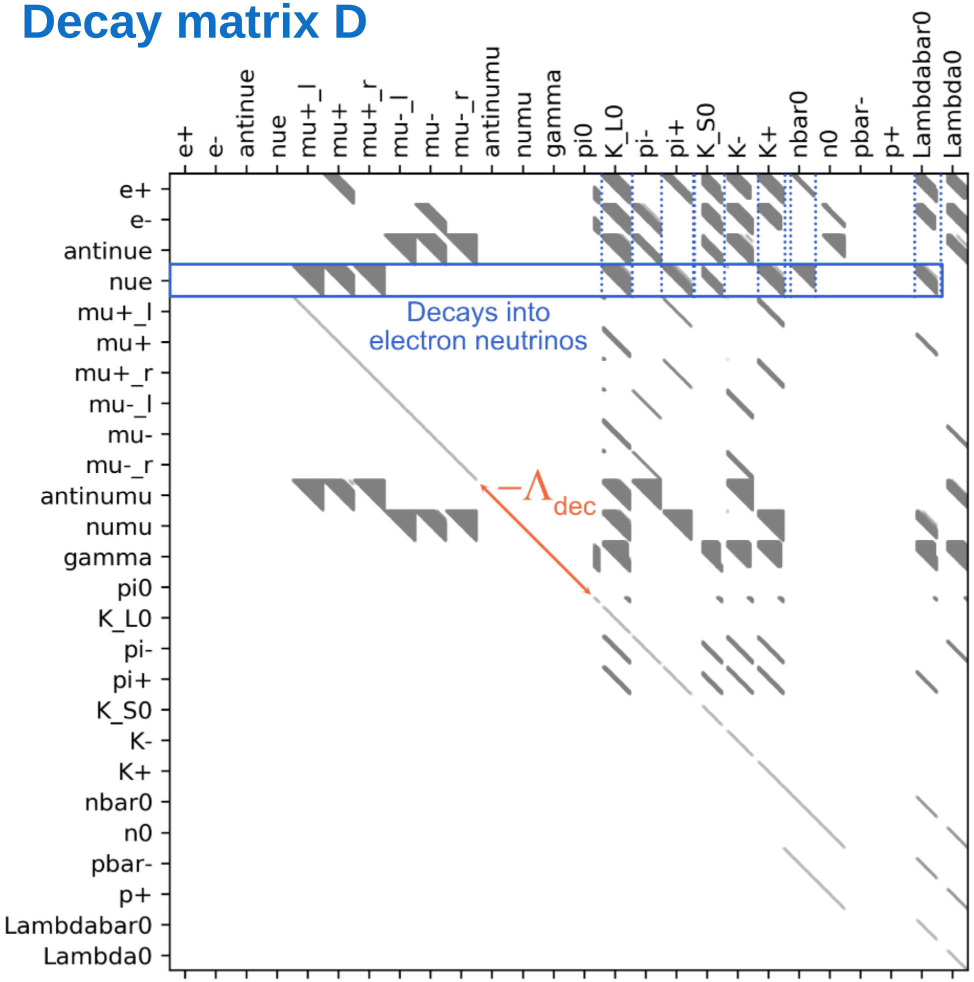
$$\vec{\Phi}^{\mathrm{p}} = \left(\Phi^{\mathrm{p}}_{E_0} \quad \Phi^{\mathrm{p}}_{E_1} \quad \cdots \quad \Phi^{\mathrm{p}}_{E_N}\right)^T$$

"Matrix form"

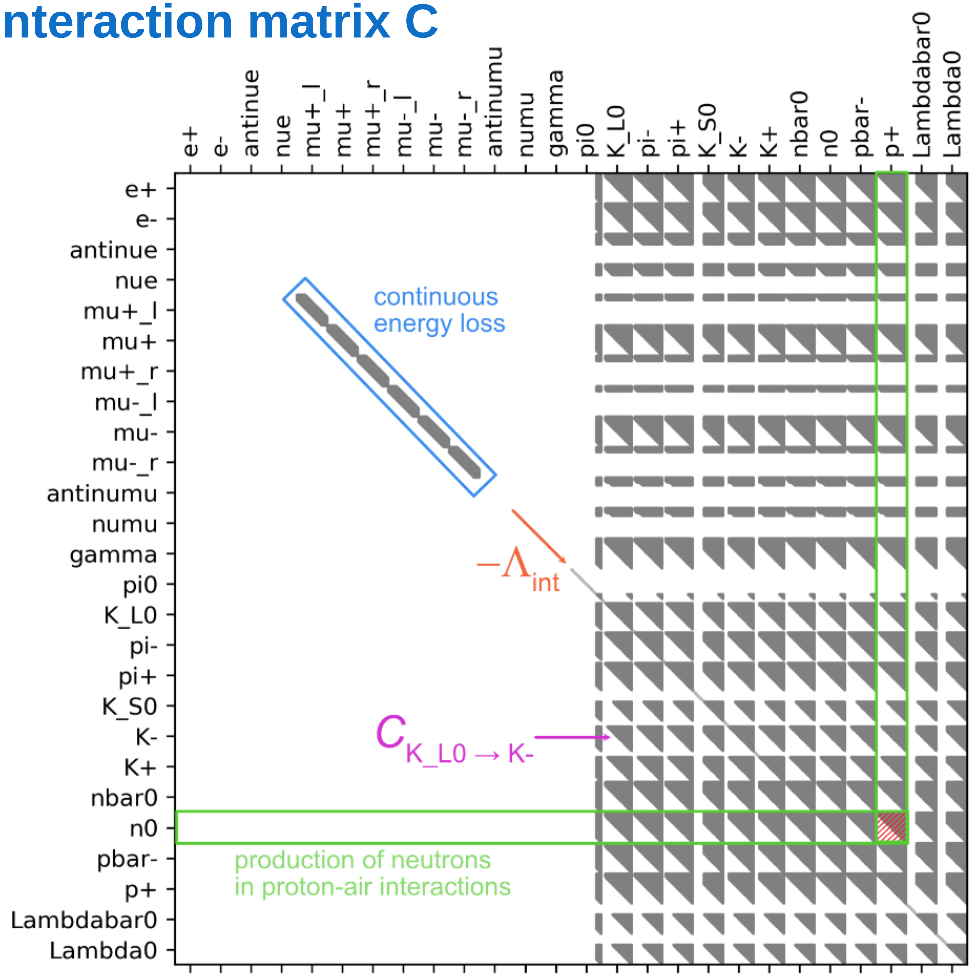
$$\frac{\mathrm{d}}{\mathrm{d}X}\vec{\Phi} = -\vec{\nabla}_E(\mathrm{diag}(\vec{\mu})\vec{\Phi}) + (-\mathbf{1} + \mathbf{C})\mathbf{\Lambda}_{\mathrm{int}}\vec{\Phi} + \frac{1}{\rho(X)}(-\mathbf{1} + \mathbf{D})\mathbf{\Lambda}_{\mathrm{dec}}\vec{\Phi}$$

Sparse matrix structure

Decay matrix D

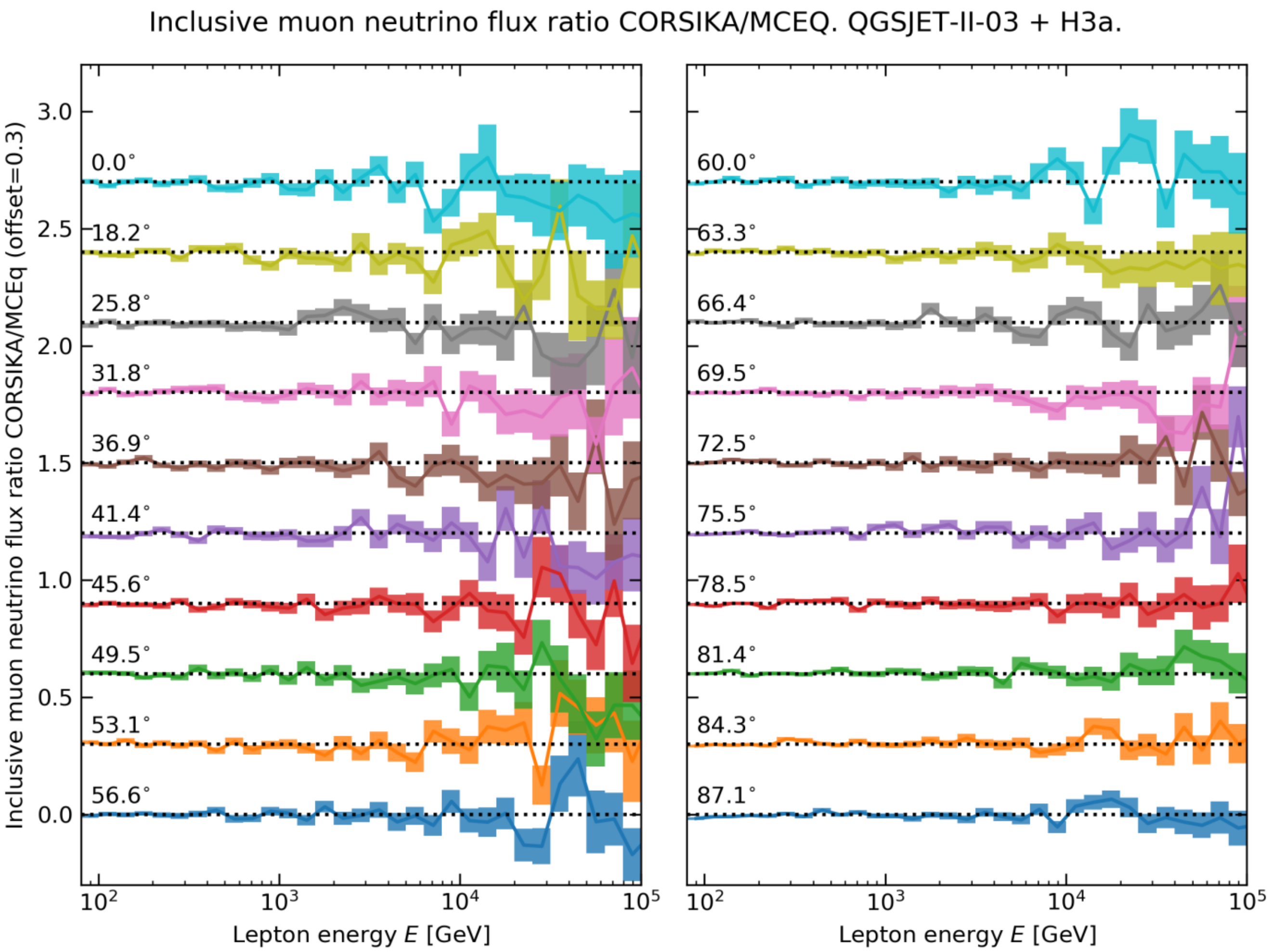


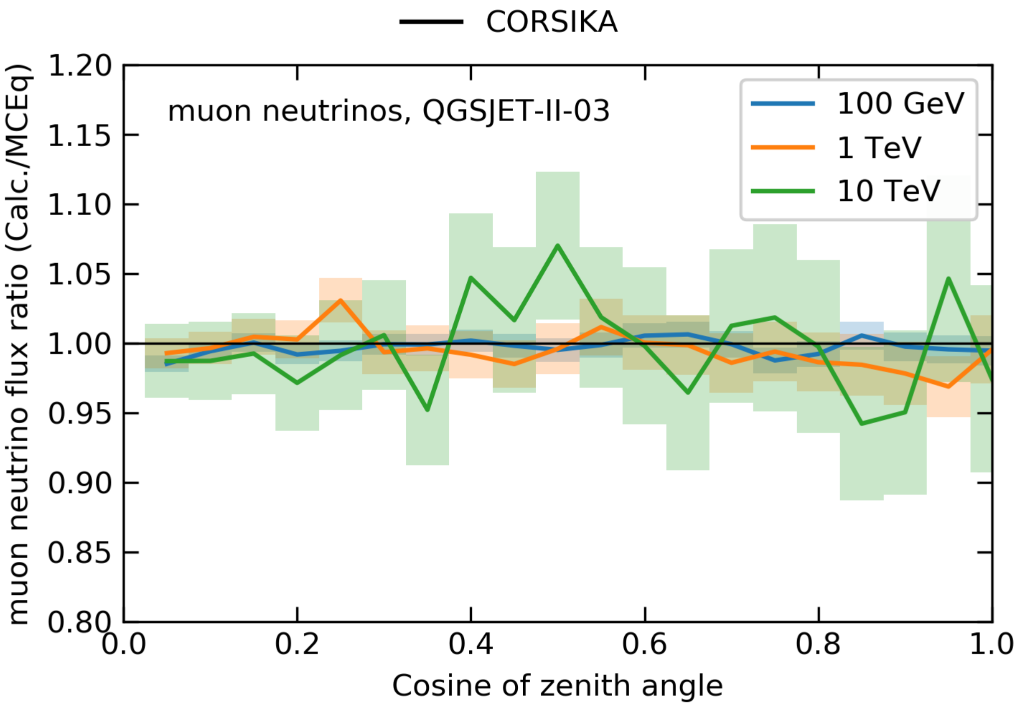
Interaction matrix C



matrices are sparse → high performance

MCEq vs (thinned) CORSIKA calculation in 1D



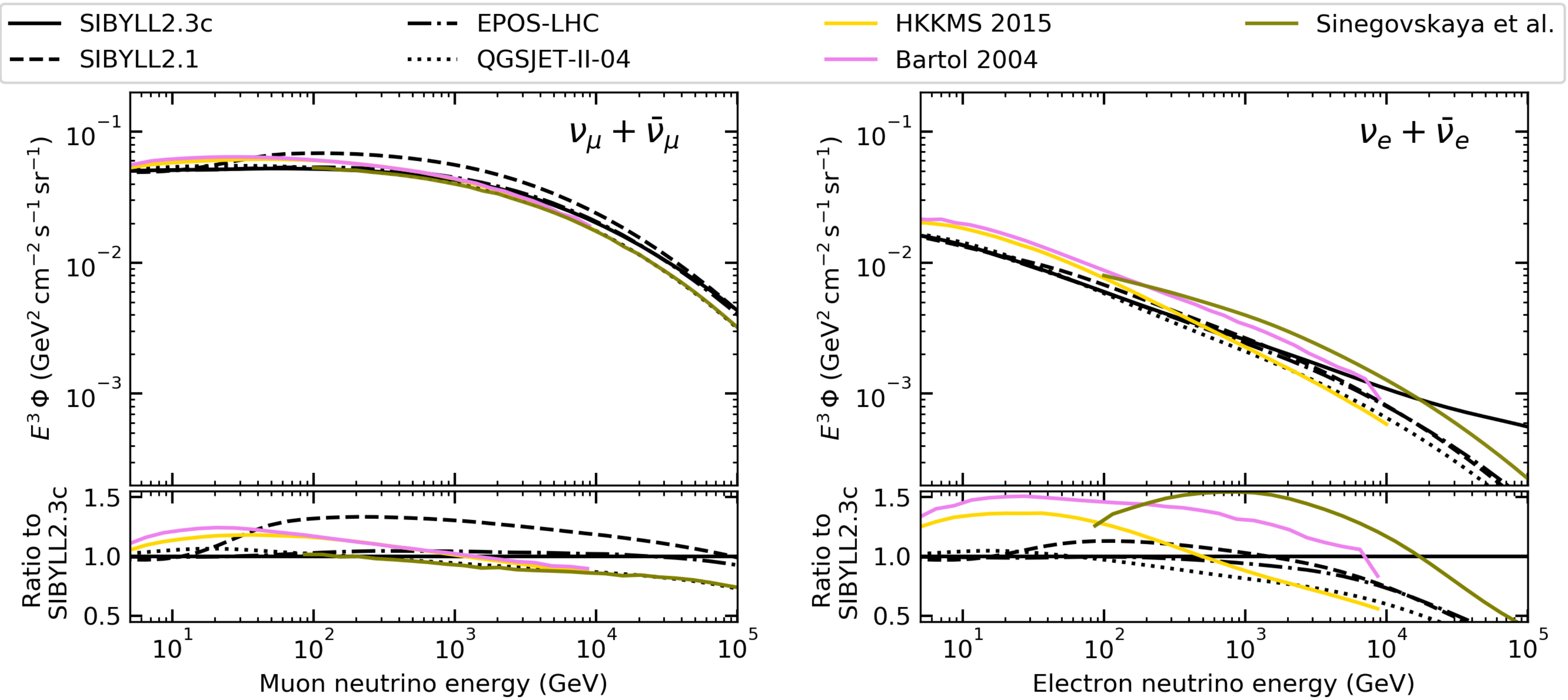


> BSD licensed @ https://github.com/afedynitch/MCEq

CORSIKA: A. Fedynitch, J. Becker Tjus and P. Desiati, PRD 2012
MCEq: Code paper, AF, R. Engel, in prep. for submission

MCEq vs. traditional calculations

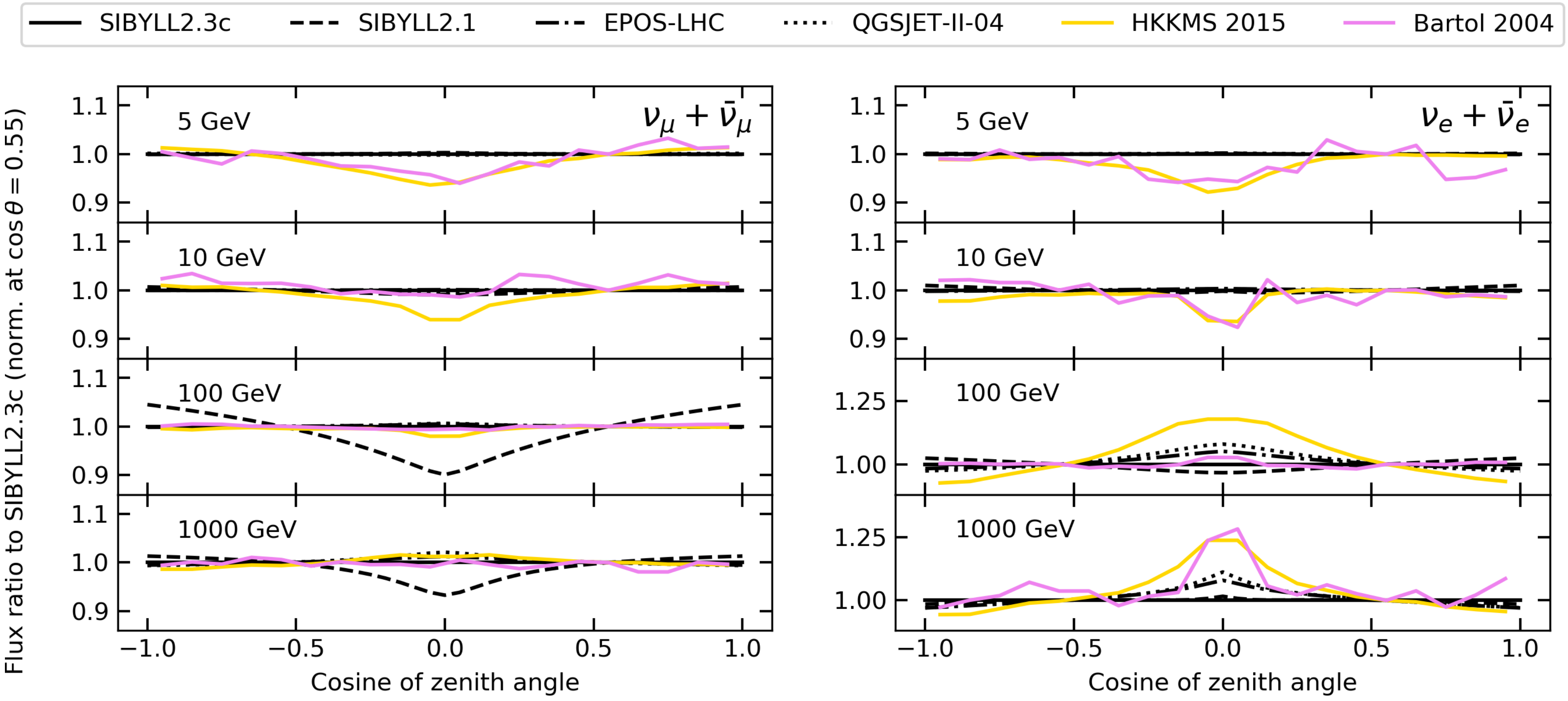
HKKMS: M. Honda et al., PRD 92 (2015)
Bartol: G. Barr et al., PRD 70 (2004)
Sinegovskaya et al. PRD 91 (2015)
MCEq: AF, R. Engel in prep.



- Old 2002 (GH) primary model for HKKMS and Bartol, H3a for the rest
- Data can not discriminate between calculations
- Shown are zenith and azimuth averages

AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, 1806.04140 (2018)

Hadronic model dependence of zenith distributions



- Good agreement above tens of GeV for muon neutrinos
- Some tension between calculations at the horizon in electron neutrinos
- Affected by K/Pi, K^{+-}/K^0_L ratios

AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, 1806.04140 (2018)

Hadronic uncertainties: re-spin of Barr et al. approach

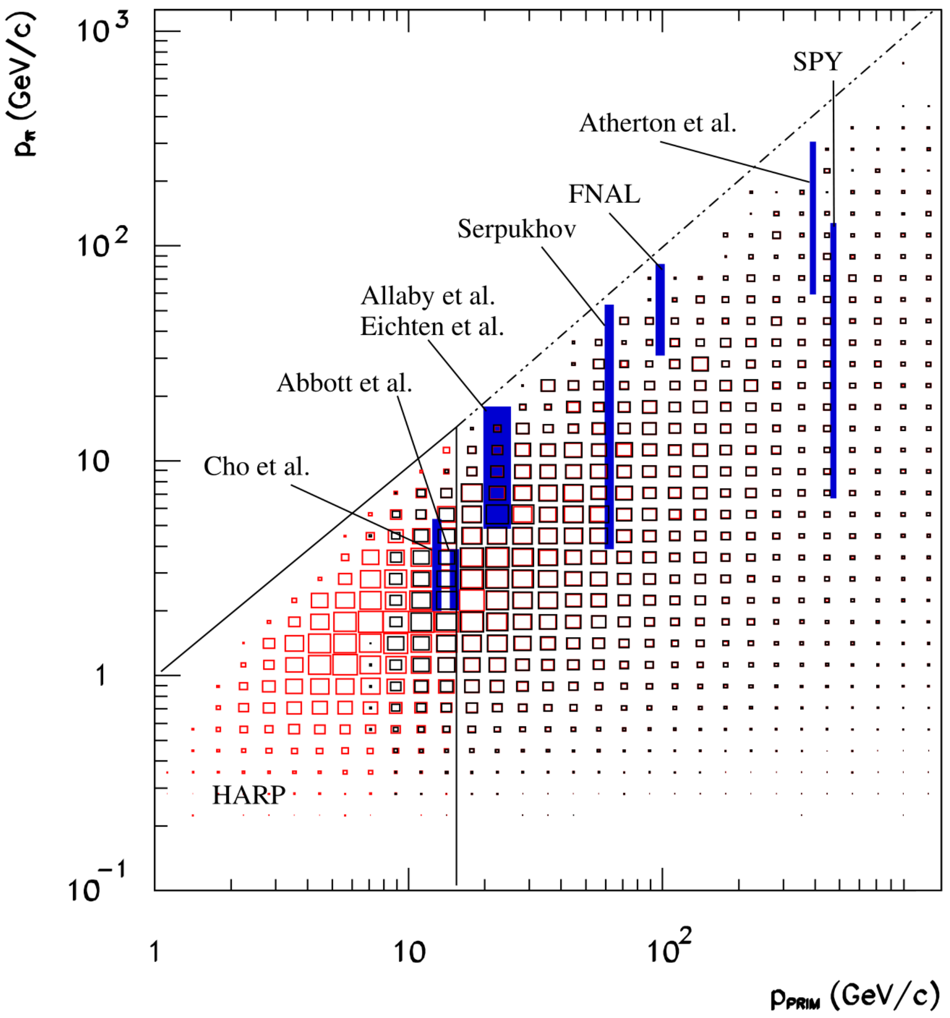
- “*Uncertainties in atmospheric neutrino fluxes*”, G. D. Barr, S. Robbins, T. K. Gaisser, and T. Stanev, Phys. Rev. D 74, 094009 (2006) (extensive discussion also in Sanuki et al. PRD 75 (2007)

- Cut phase-space in regions/slices in E_{lab} and x_{lab} and **assign** uncertainty to each slice (uncorrelated)

- Uncertainty assigned by hand and not derived from data. Assignment based on availability of data, not how well the model [TARGET2.1] describes it

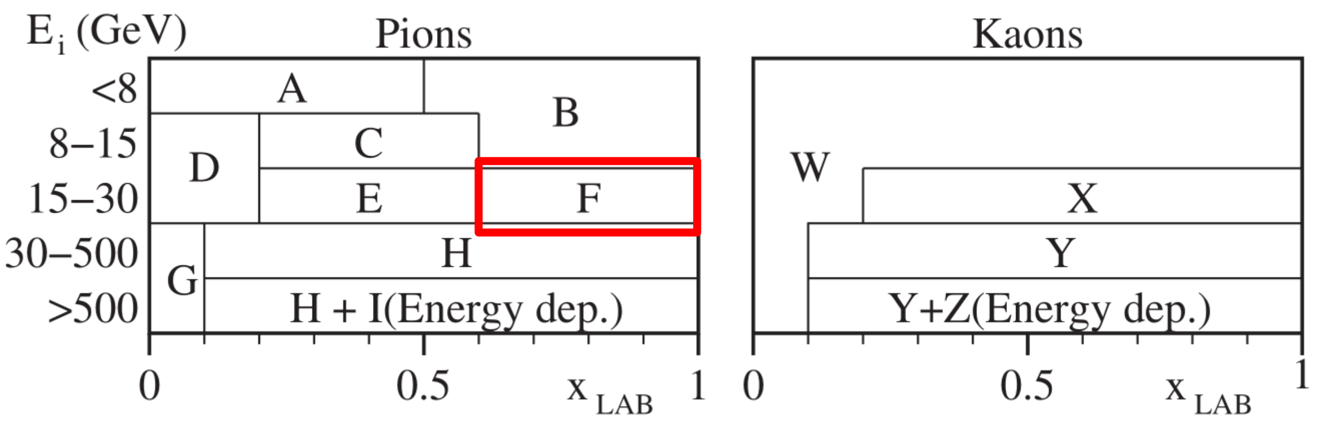
- Many “free” parameters with unclear correlations

PHYSICAL REVIEW D **74,** 094009 (2006)

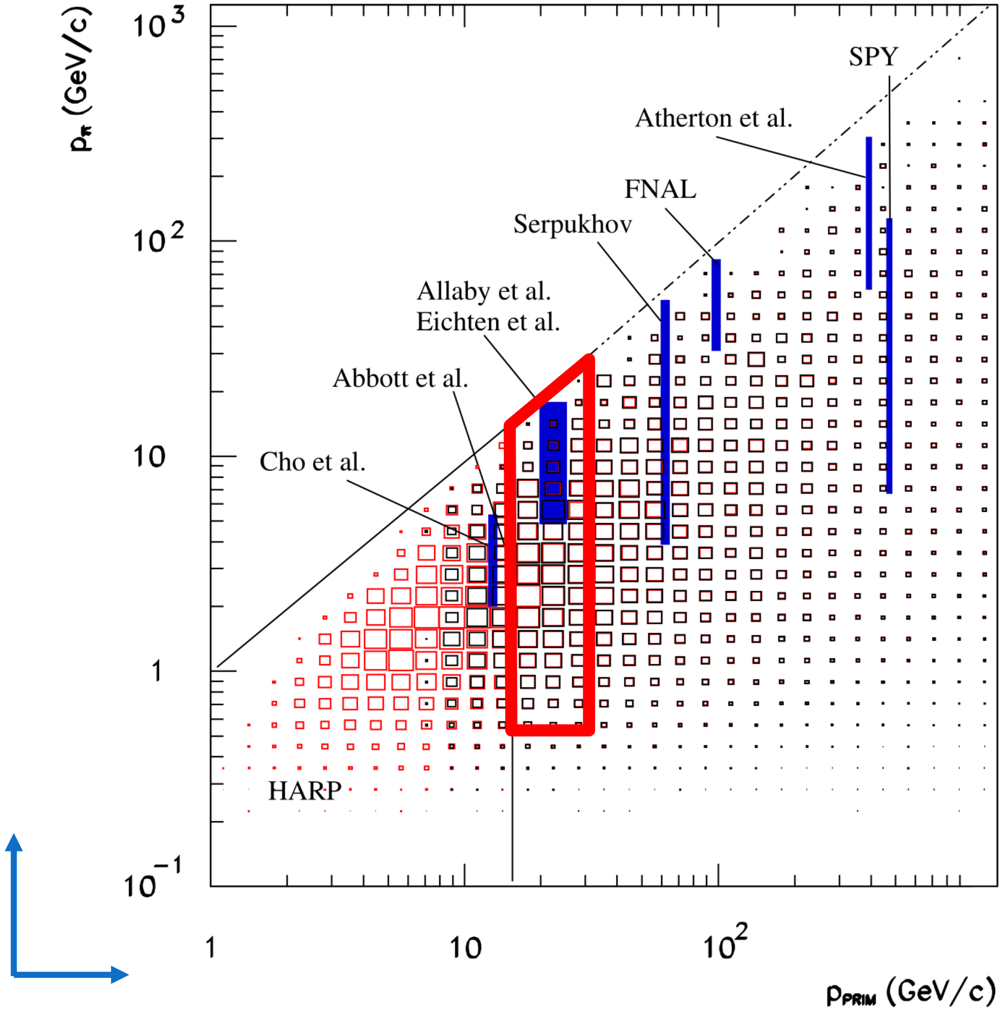


Phase space regions

“Barr regions”



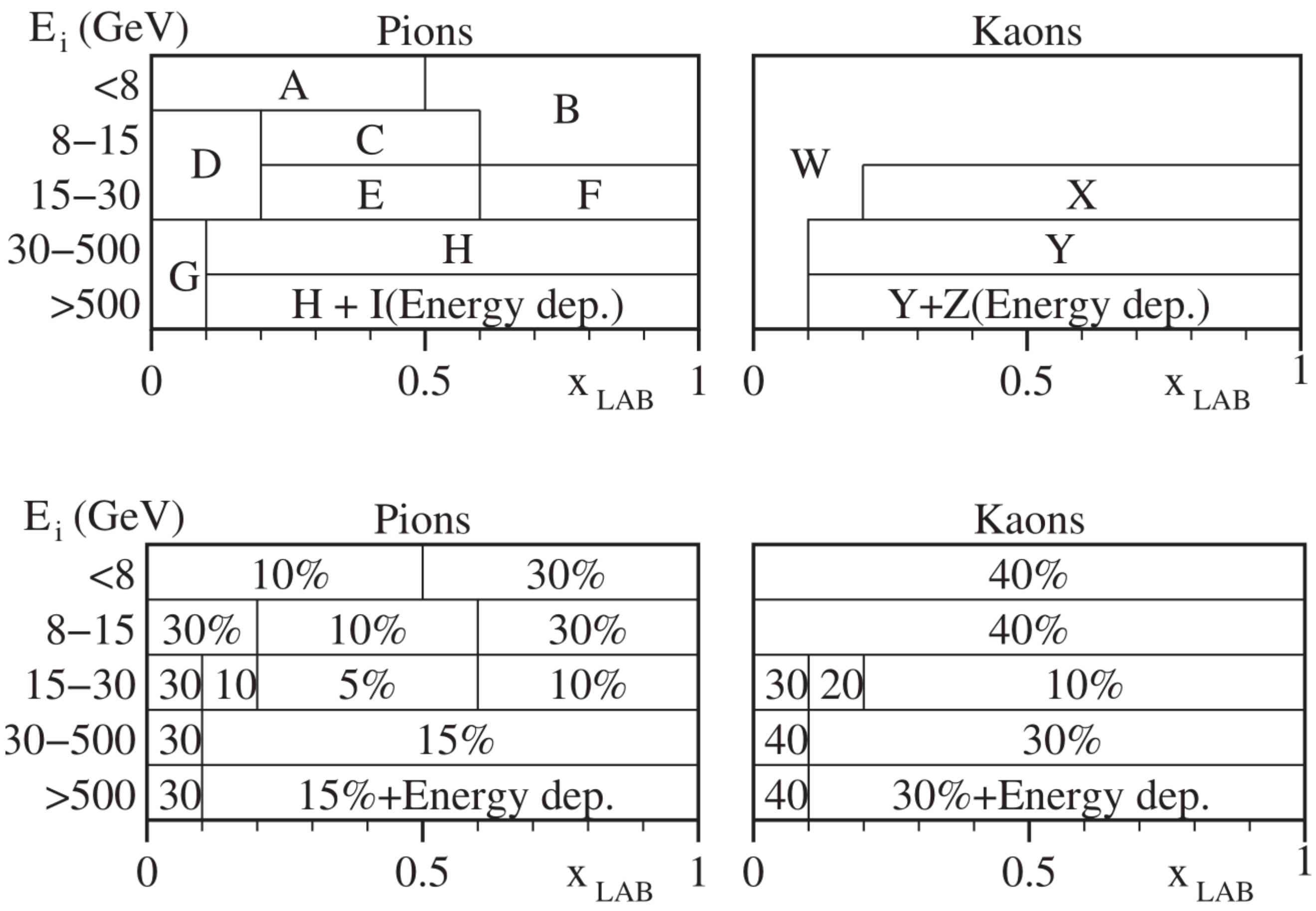
PHYSICAL REVIEW D **74,** 094009 (2006)



Same axes as in the MCEq matrices

MCEq-based implementation

"Barr regions"



- Compute partial derivatives wrt. phase-space regions (Taylor expansion), i.e. $\frac{\partial \Phi_\nu}{\partial W}$
- No correlations between phase-space regions (as in Barr et al.) or add. correlations

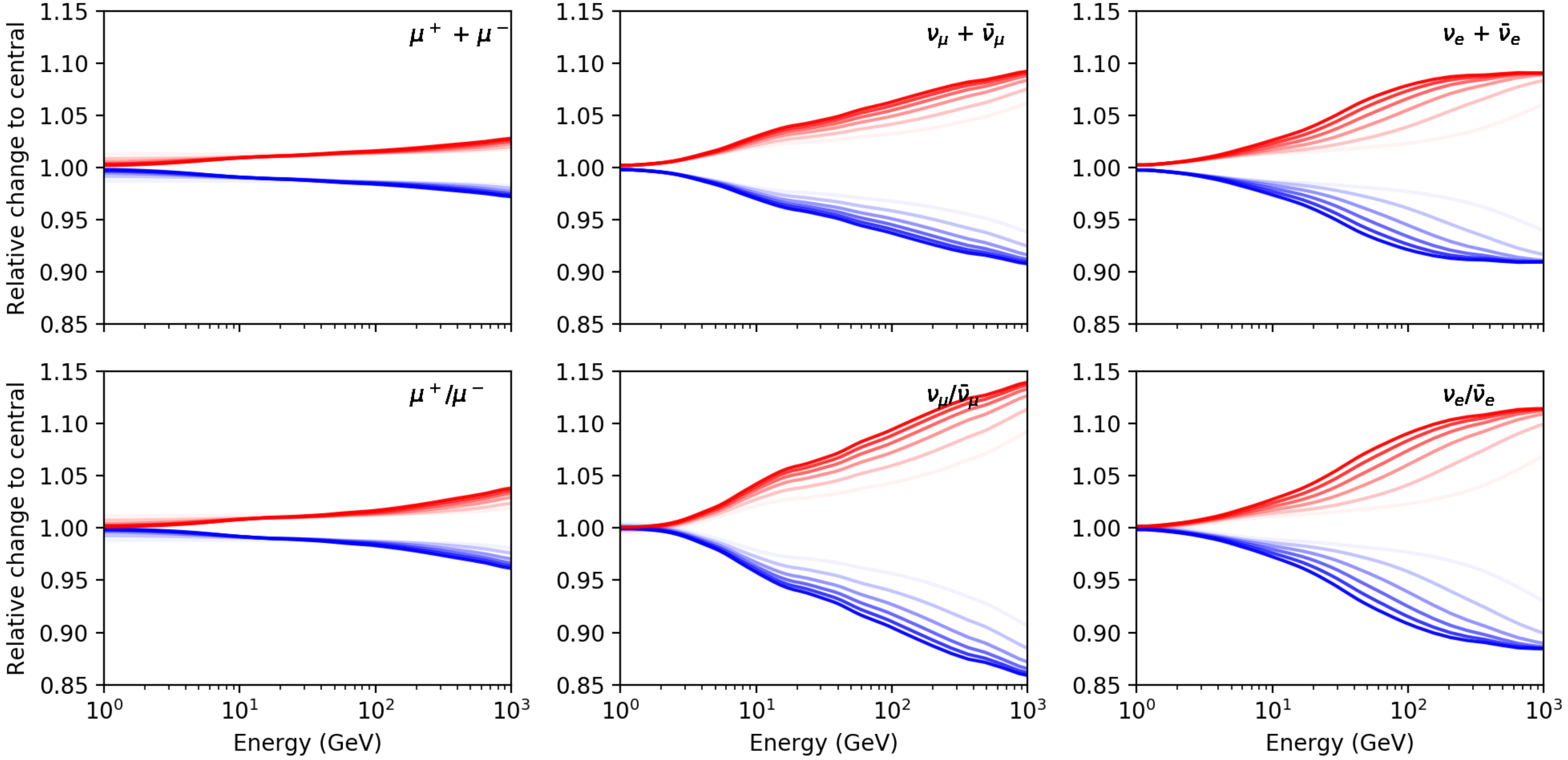
Elements of Jacobian (numerical)

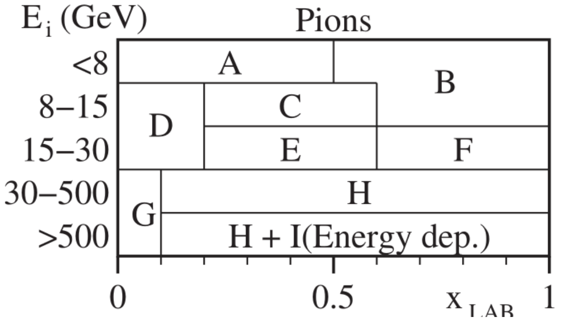
$$J_{E_i j} = \frac{\partial \Phi_\nu(E_i)}{\partial p} = \frac{\Phi_\nu(\delta p_j +) - \Phi_\nu(\delta p_j -)}{2\delta p_j}$$

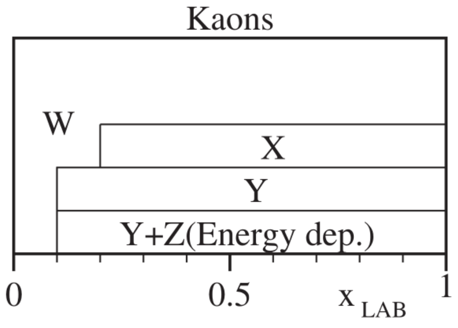
Error propagation

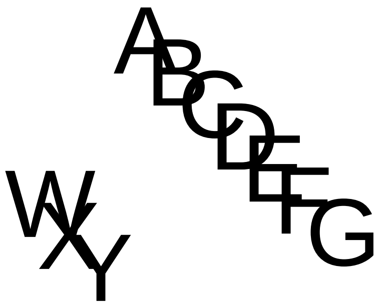
$$\mathrm{cov}[\Phi_\nu(E_i), \Phi_\nu(E_j)] = \sum_{mn} J_{E_i m} J_{E_j n} \mathrm{cov}[p_m, p_l]$$

… impact on flux

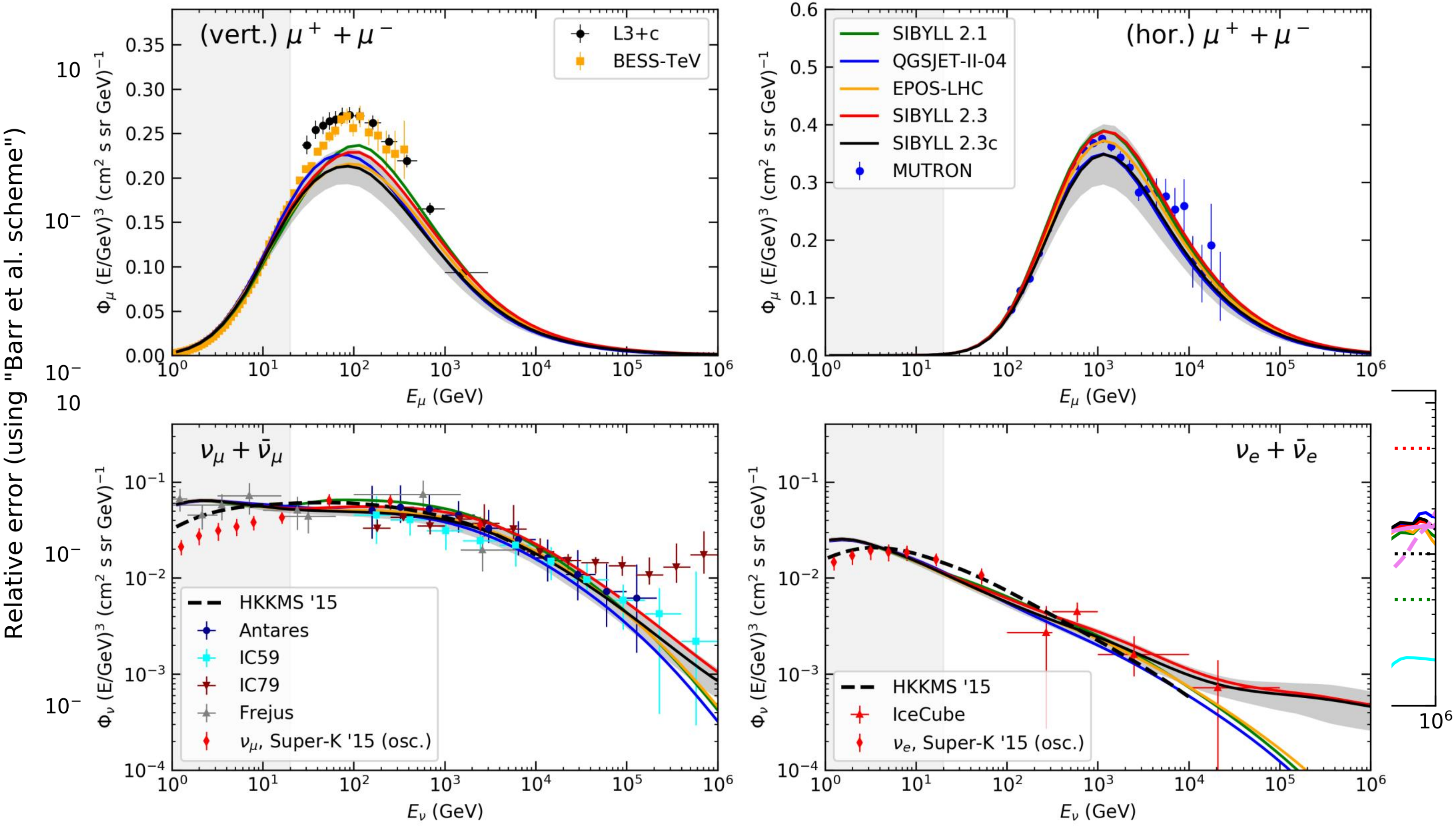




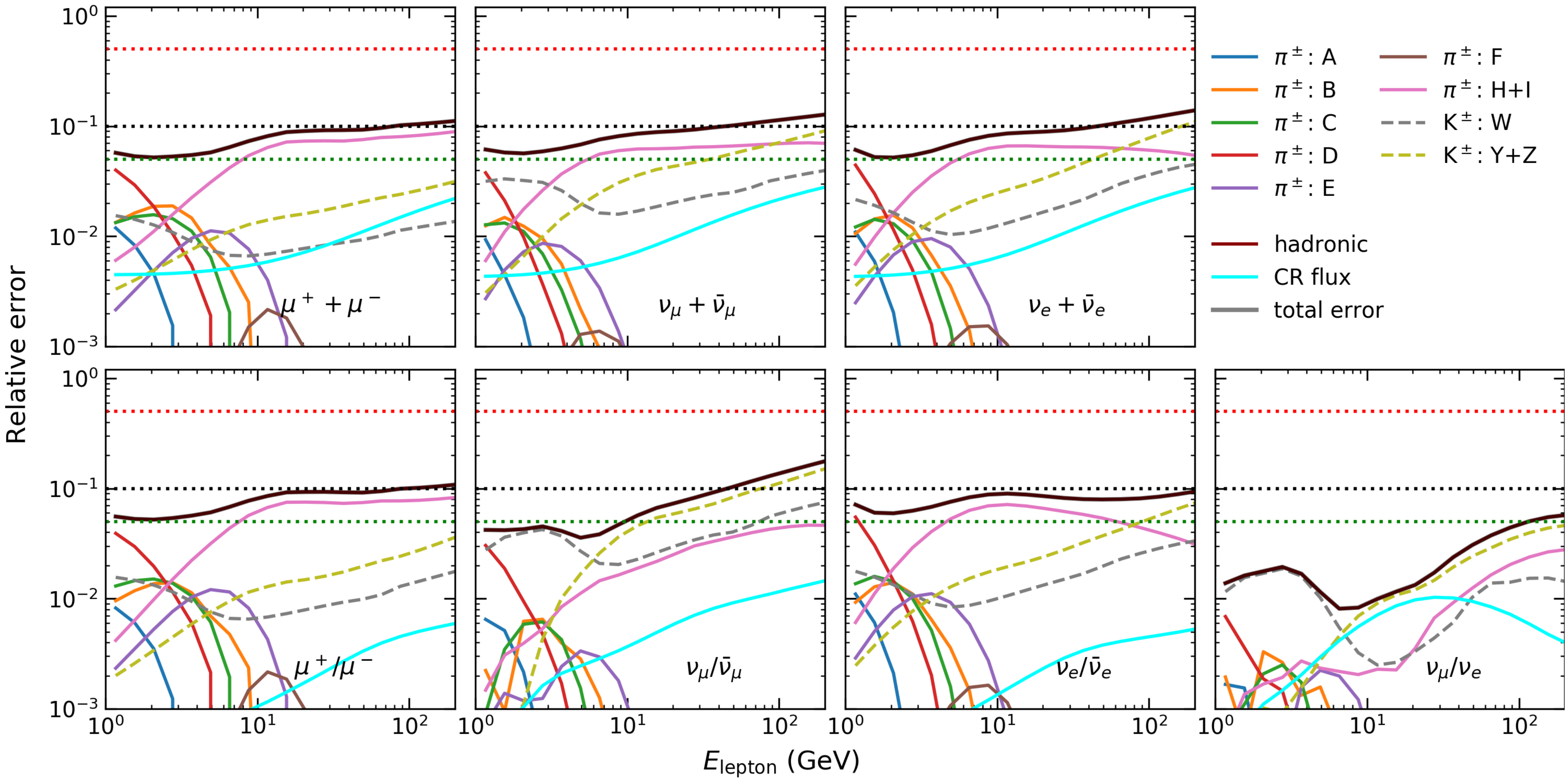




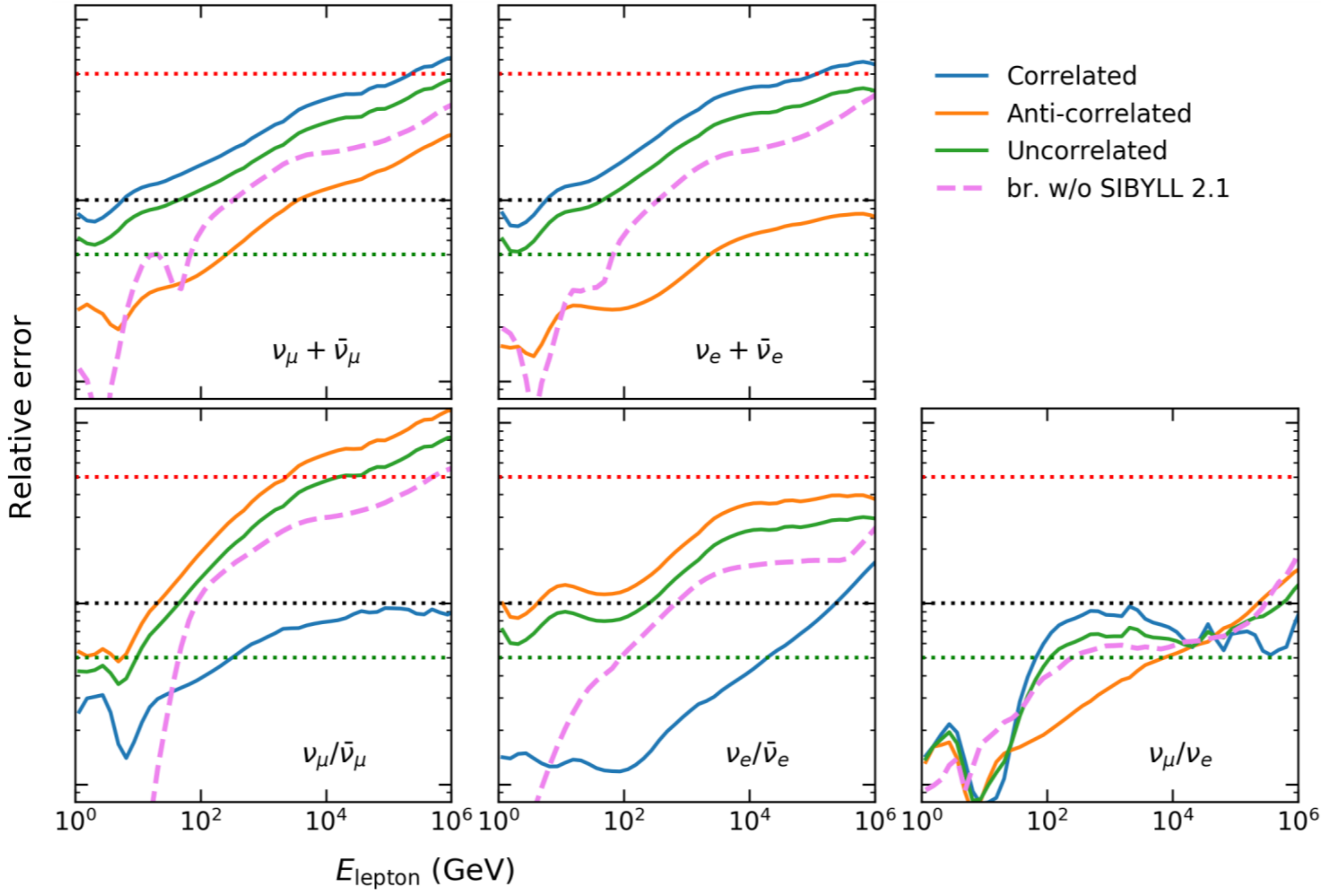
Computation of error bands through error propagation



Contribution of individual "Barr groups"



Correlations between phase-space patches unclear



Examples	For one "Barr" - parameters
symmetric	$p\pi^+\uparrow$ $n\pi^+\uparrow$ $p\pi^-\uparrow$ $n\pi^-\uparrow$
asymmetric	$p\pi^+\uparrow$ $n\pi^+\uparrow$ $p\pi^-\downarrow$ $n\pi^-\downarrow$
uncorrelated	$p\pi^+\uparrow$ $n\pi^+0$ $p\pi^-0$ $n\pi^-0$

- The production of charged secondaries is physically not independent
- It is very difficult to extract this information from hadronic interaction models directly

Calibration of ν uncertainties with “global fit” to μ data

Cosmic ray flux: Global Spline Fit [7] + Atmospheric conditions + Hadronic interactions: SYBILL 2.3c = Flux of muons and neutrinos

Experiment	Energy (GeV)	Measurements	Reported unit	Location	Altitude	Zenith range
AMS-02	0.1-2500	Flux & charge ratio	rigidity	28.57°N , 80.65° W	5 m (sea level)	
BESS-TeV	0.6-400	Flux	momentum	36.2°N, 140.1°W	30 m	0-25.8°
CMS	5-1000	Charge ratio	momentum	46.31°N, 6.071°E	420 m	$p\cos\theta_z$
L3+C	20-3000	Flux & charge ratio	momentum	46.25°N, 6.02°E	450 m	0-58°
MINOS	1000-7000	Charge ratio	total energy	47.82°N, 92.24°W	5 m (sea level)	unfolded
OPERA	891-7079	Charge ratio	total energy	42.42°N, 13.51°E	5 m (sea level)	$E\cos\theta^*$

(GeV)

DEIS

Juan Pablo Yanez and AF, Neutrino 2018 + ICRC 2019

How we did it

- New version the cascade code MCEq with improved accuracy at low E
- Cut secondary particle phase-space according to parameters B_i from Barr et al.
- Generate database of fluxes $\Phi(E_\mu)$ and Jacobians

$$\frac{\partial \Phi(E_\mu)}{\partial \mathscr{B}_i} = \frac{\Phi(E_\mu, \mathscr{B}_i = 1+\delta) - \Phi(E_\mu, \mathscr{B}_i = 1-\delta)}{2\delta}$$

- Fluxes with modifications to B_i can be quickly evaluated in the fit:

$$\Phi(E_\mu, \mathscr{B}_a, \mathscr{B}_b, \dots) = \Phi(E_\mu) + \sum_i \mathscr{B}_i \frac{\partial \Phi(E_\mu)}{\partial \mathscr{B}_i}$$

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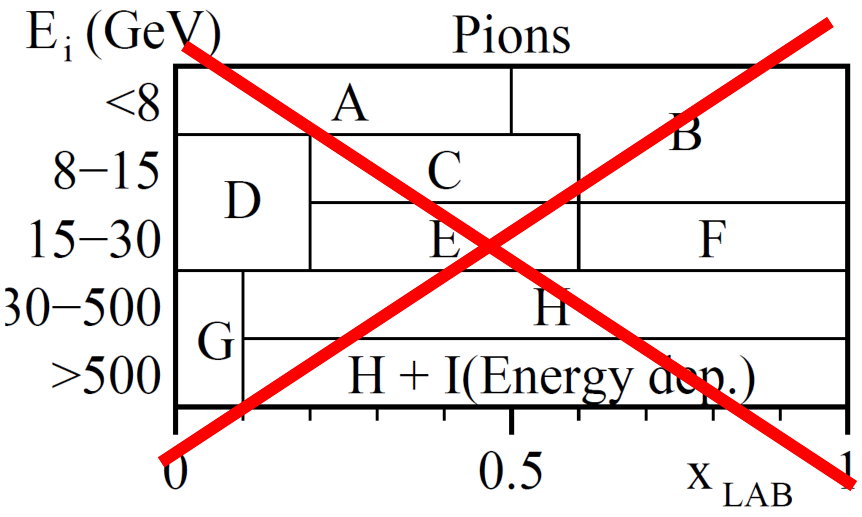
What we found

- Original attempt was to use the parameterization of Barr et al.
  1. Found data to be insensitive
  2. Too many correlations
  3. Impossible to constrain



- Simplified to four parameters
  - Yields of each meson species
  - Global, energy-independent scales
  - Enough to describe data

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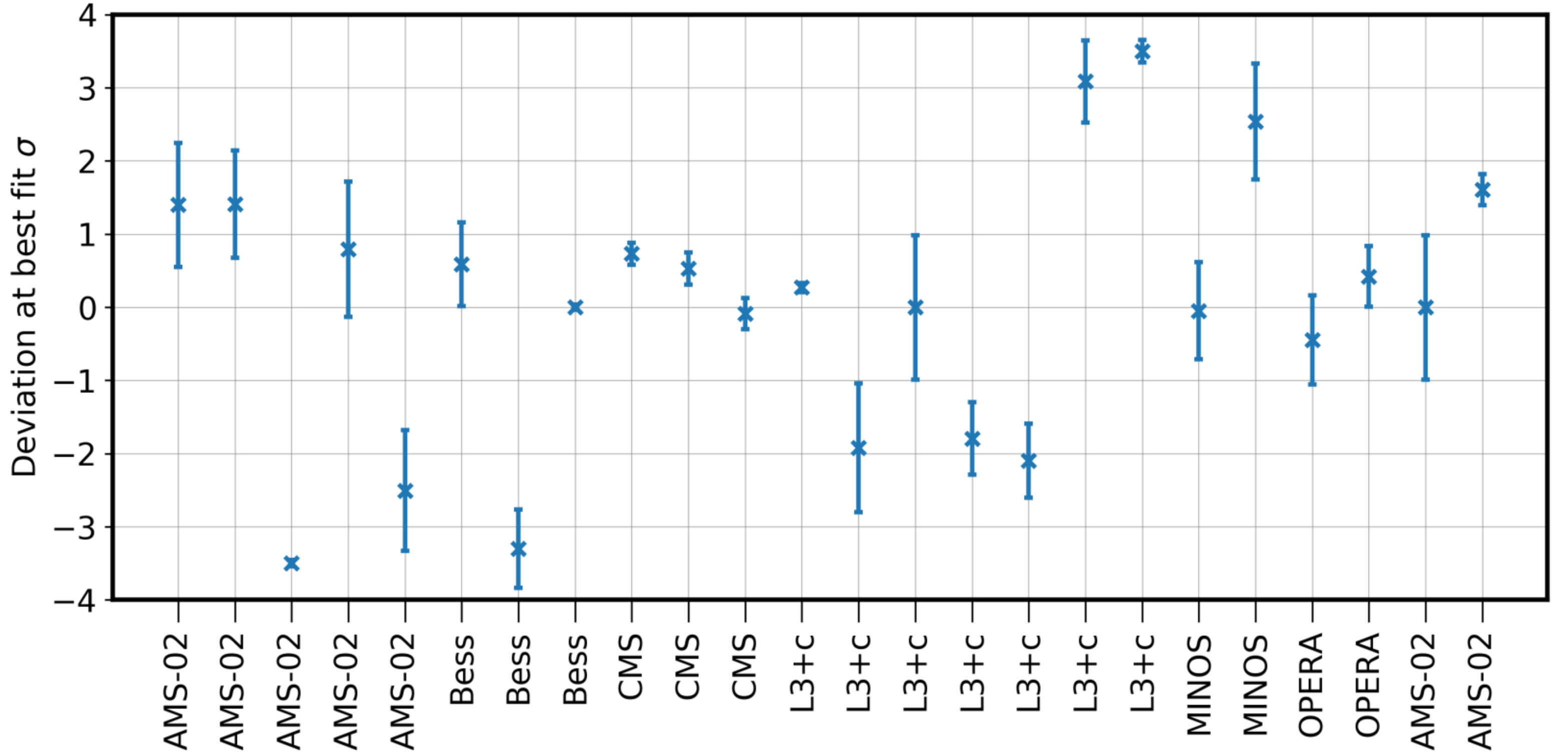
Some experiments are hard to fit

- Some experiments are hard to fit regardless of modifications
- Possible systematic effects not reported
- Additional modifications will be included in next iterations of the study

Deviations of experimental parameters at best fit point



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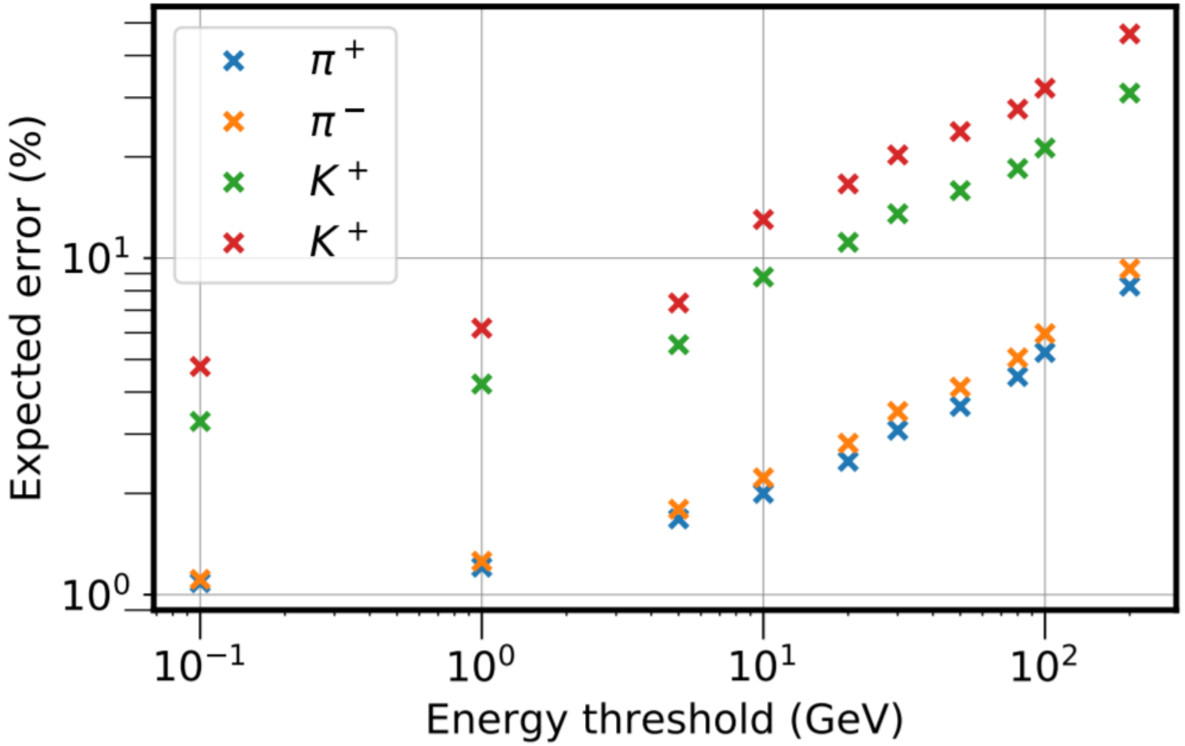
Impact of energy threshold for the fit

- High energy data less sensitive
- This is because the features in the muon spectrum are smooth
- and fit variables become strongly correlated
- More angles are needed
- We're investigating horizontal and high-altitude balloon data



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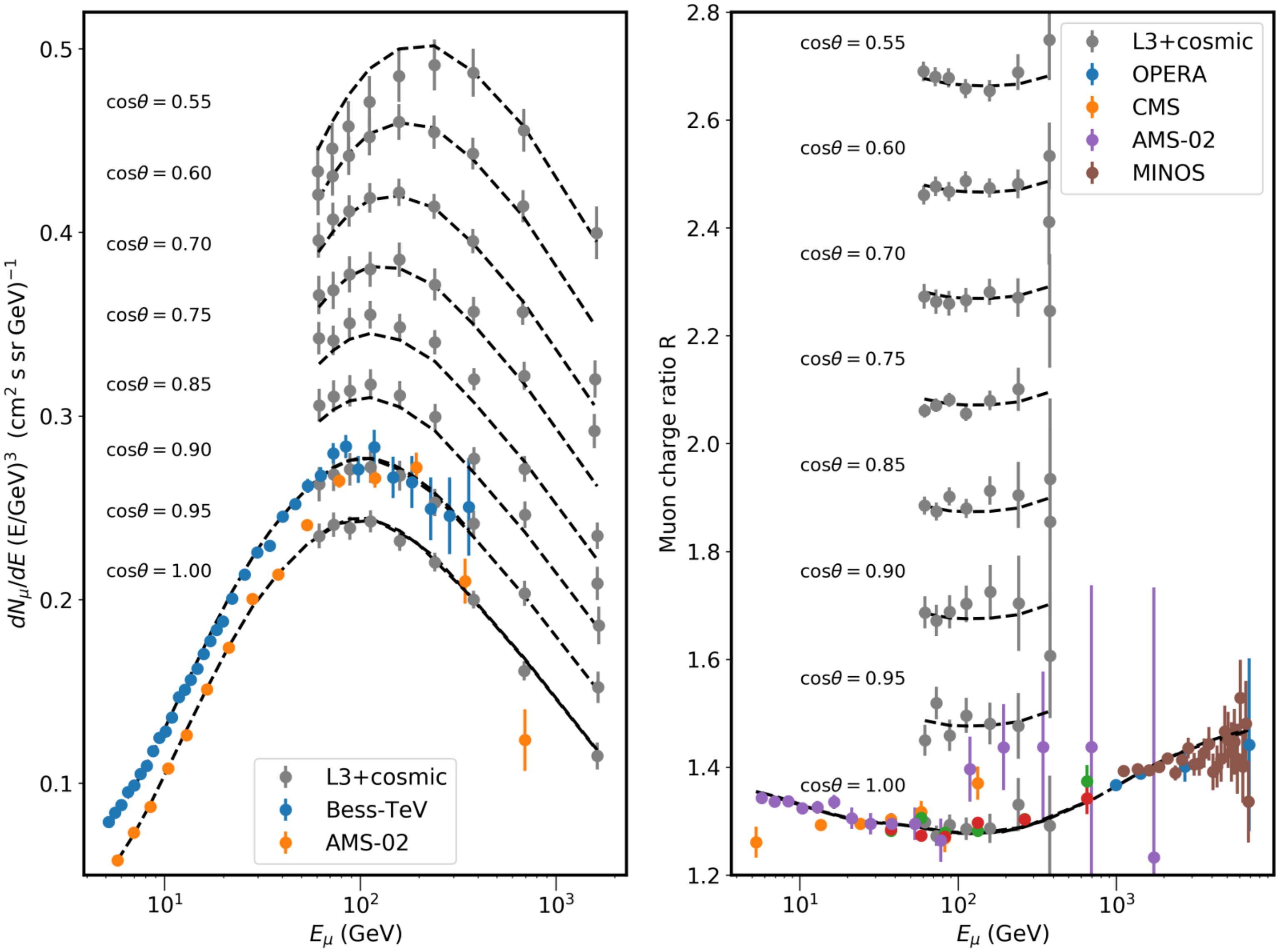
Fit results

- Some experiments are hard to fit regardless of modifications
- Possible systematic effects not reported
- L3+C – previously “the reference dataset”– is not as good as we thought
- We will include more data and CR flux uncertainties in the next iteration and report later this year



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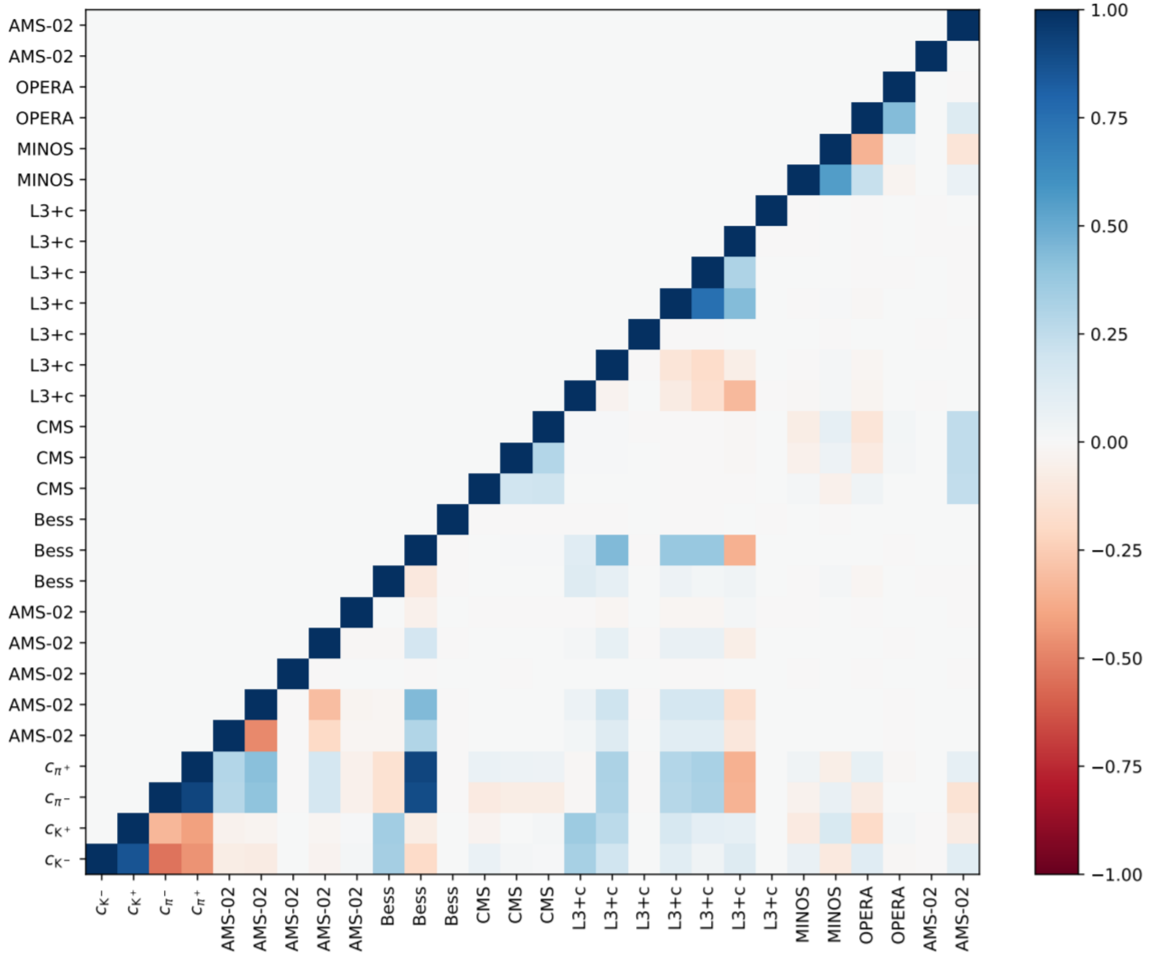
Fit parameters and correlations

- With sufficiently low threshold (5 GeV) the correlations are reduced
- Errors between a few to ten %
- Neutrino flux errors in the range covered by fit comparable to kaon errors

Parameter	Best fit	Error
c_{π^-}	+0.141	±0.017
c_{π^+}	+0.116	±0.016
c_{K^-}	+0.402	±0.073
c_{K^+}	+0.583	±0.055



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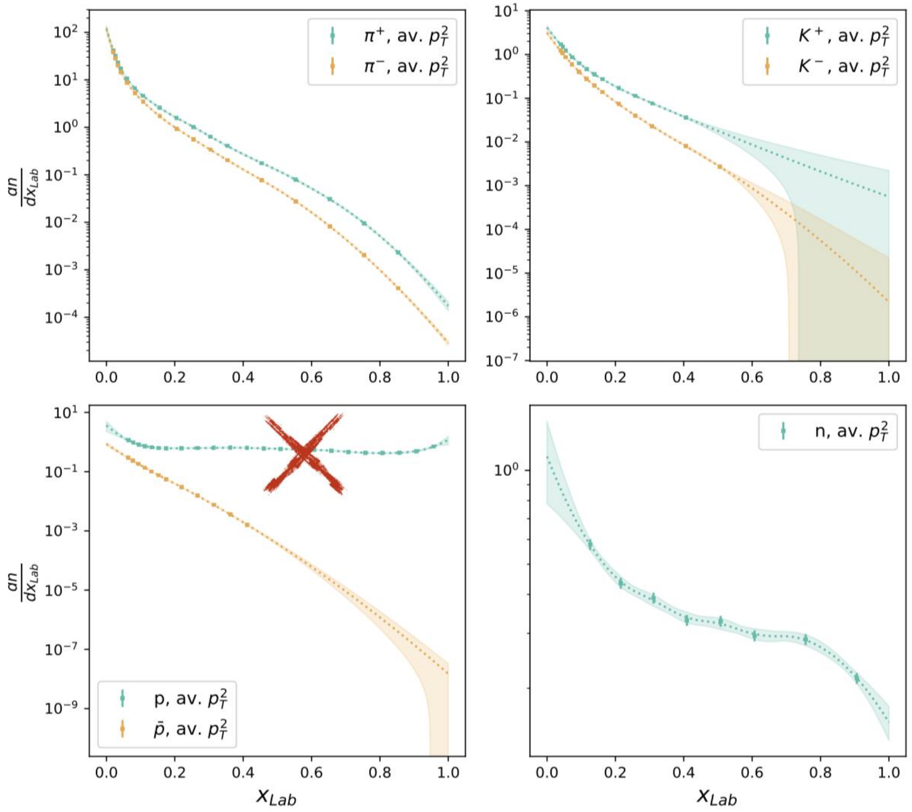
Alternative under investigation: data-driven inclusive interaction model

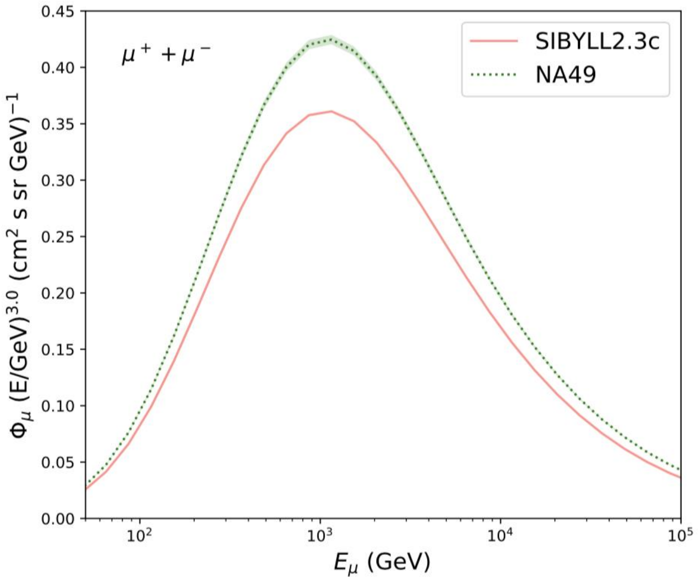
"The SHIn-project"

NA49 pp data (158GeV)



p+p → part+X

Experiment	Interaction	E_p [GeV]	yields
NA49	pp	158	$\pi^\pm, K^\pm, \overset{(-)}{p}, n$
NA49	pC	158	$\pi^\pm, \overset{(-)}{p}, n$
NA61/SHINE	pC	31	$\pi^\pm, K^\pm, K^0_S, \Lambda$
NA61/SHINE	pp	20, 31, 40, 80, 158	$\pi^\pm, K^\pm, \overset{(-)}{p}$
NA61/SHINE	$\pi^- C$	158, 350	ρ^0, ω, K^{*0}
NA61/SHINE (upcoming)	$\pi^- C$	158, 350	$\pi^\pm, K^\pm, \overset{(-)}{p}$

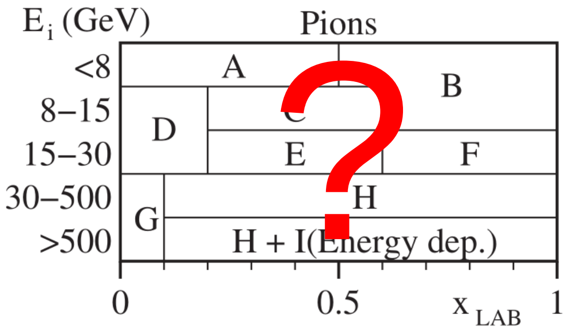


Matthias Huber

Conclusions and future path

- Current atmospheric neutrino detectors cover **9 orders of magnitude** in energy (MeV-PeV) → **challenge** for modeling!
- **High-precision** (and high-performance) calculations available through MCEq that well match full Monte Carlo
- **Unsolved problems remain**, in particular hadronic interactions, but data-driven techniques can improve the precision as in the HKKM calculations or our muon fit. However, the **parameterization** has to be **revised**
- Work is progressing on building a purely accelerator data driven model. Delays because NA61 presents data in different, **incompatible formats** and **communication is not working**.
- The tools allow to **handle flux systematics** in data analysis, replacing effective parameters with more physical (but not perparameters

Atmospheric flux		
ν flux template	discrete (7)	
ν / $\bar{\nu}$ ratio	continuous	0.025
π / K ratio	continuous	0.1
Normalization	continuous	none[1]
Cosmic ray spectral index	continuous	0.05
Atmospheric temperature	continuous	model tuned

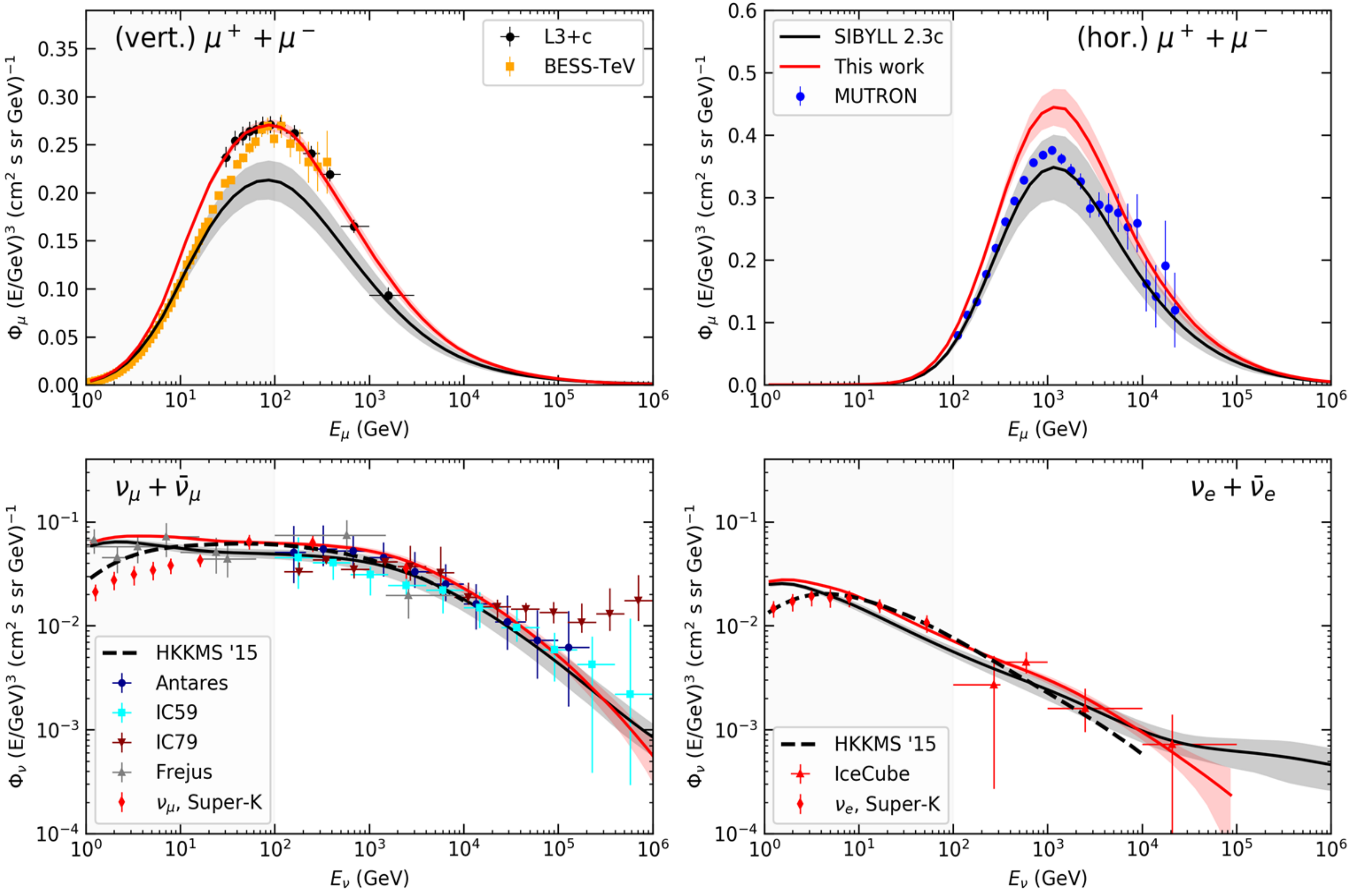


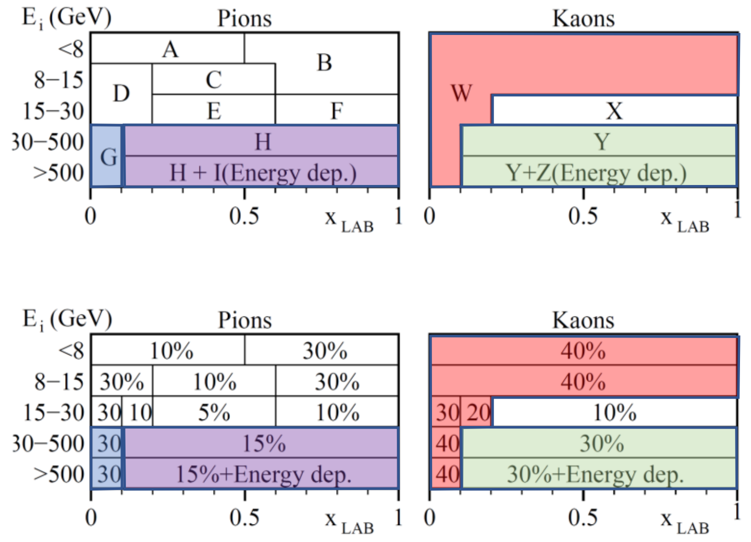
?!

Likely something new needed


Enjoy your stay in Tokyo
and happy discoveries!
近 海 郵 船

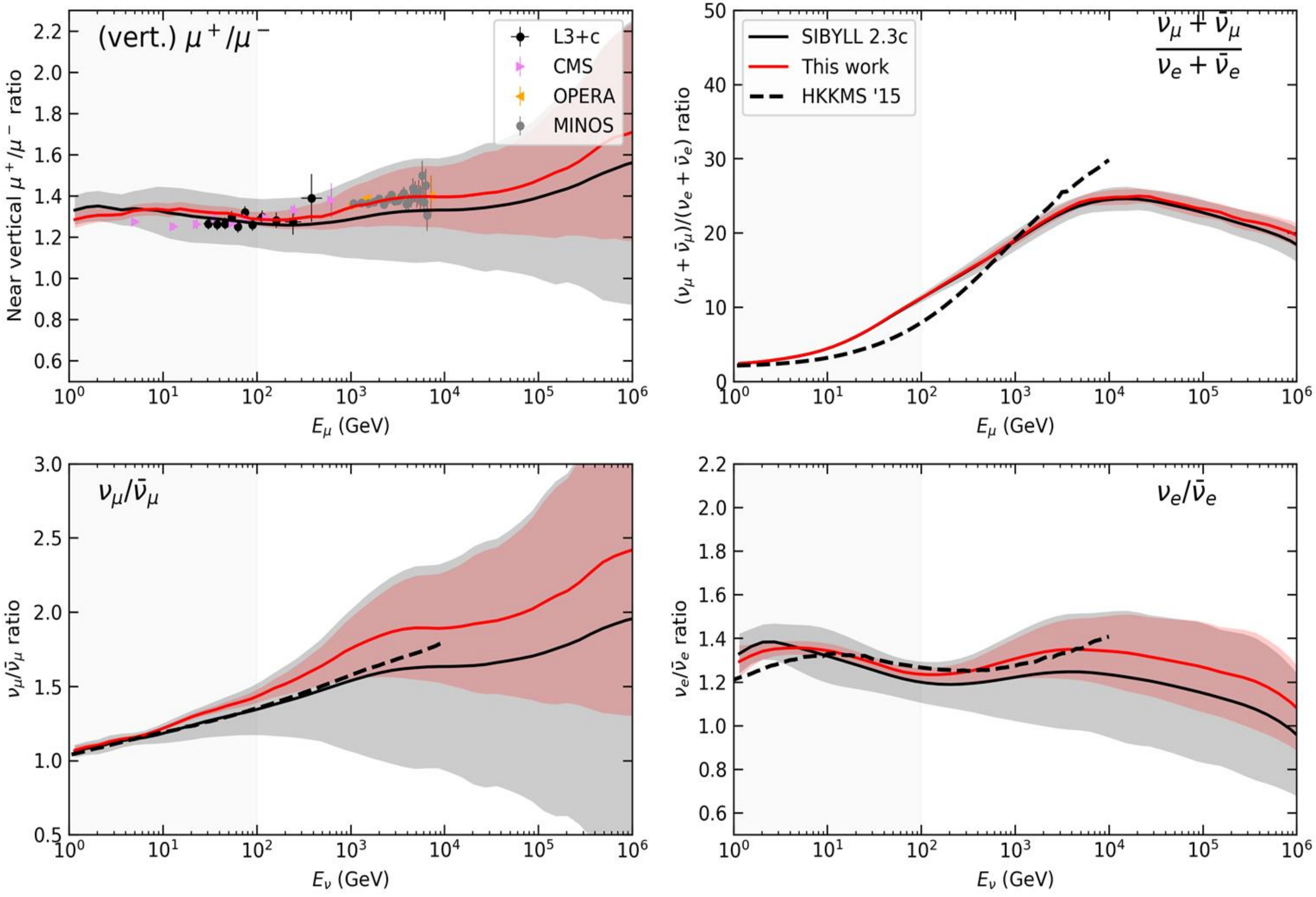
Results of the fit on fluxes

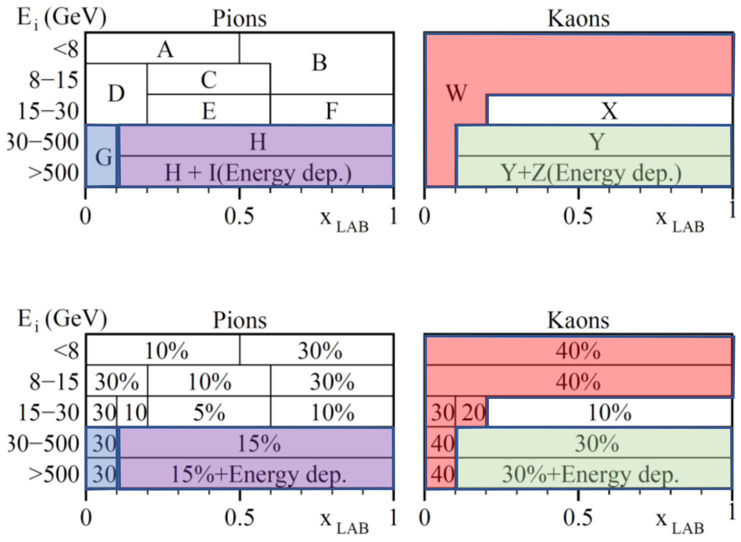




Name	value, error
π⁺: G	0.13±0.10
π⁺: H	0.30±0.03
K⁺: W	0.14±0.08
K⁺: Y	0.47±0.07
π⁻: G	0.44±0.08
π⁻: H	0.16±0.04
K⁻: W	0.20±0.10
K⁻: Y	0.11±0.07

Results of the fit

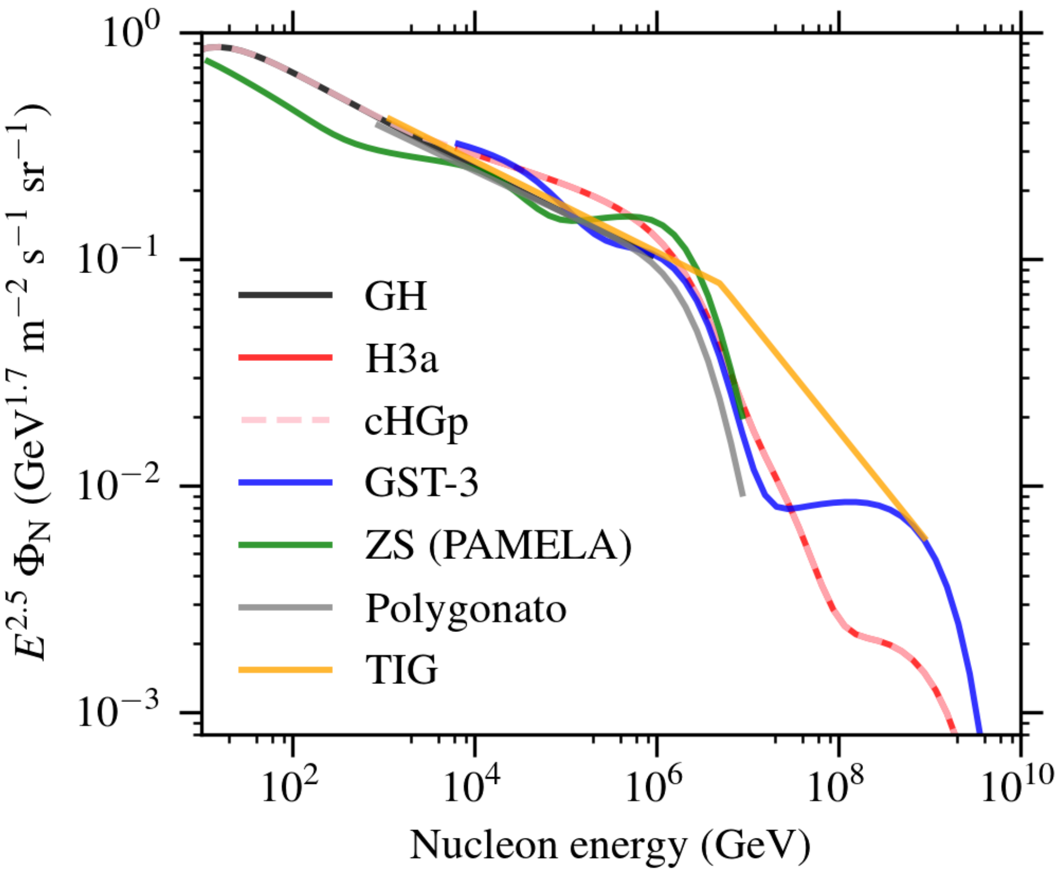




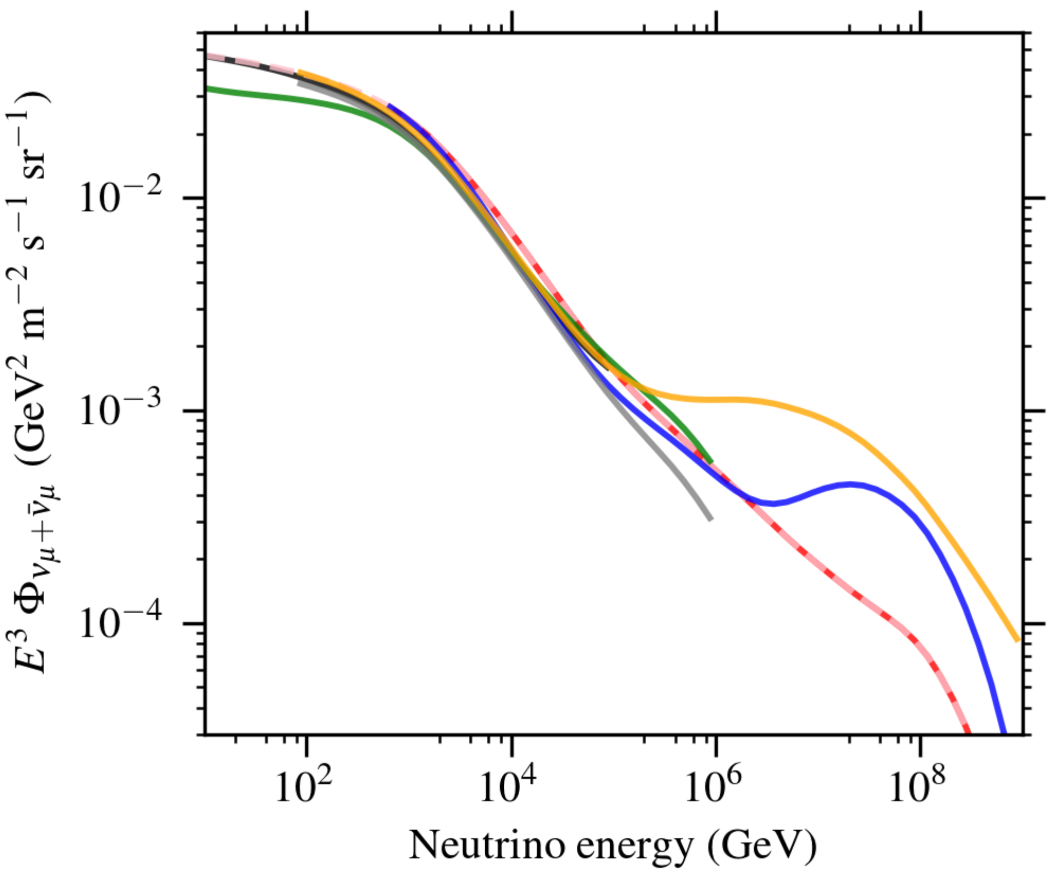
Name	value, error
π^+: G	0.13±0.10
π^+: H	0.30±0.03
K^+: W	0.14±0.08
K^+: Y	0.47±0.07
π^-: G	0.44±0.08
π^-: H	0.16±0.04
K^-: W	0.20±0.10
K^-: Y	0.11±0.07

Cosmic ray flux uncertainties – ‘bracketing’ overestimates

Cosmic ray nucleon flux



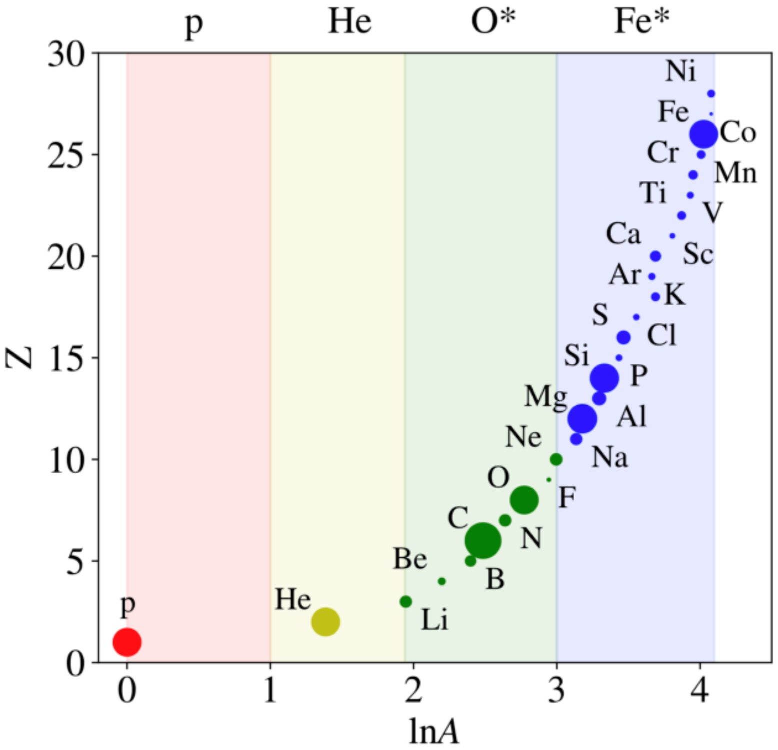
Atmospheric neutrino flux

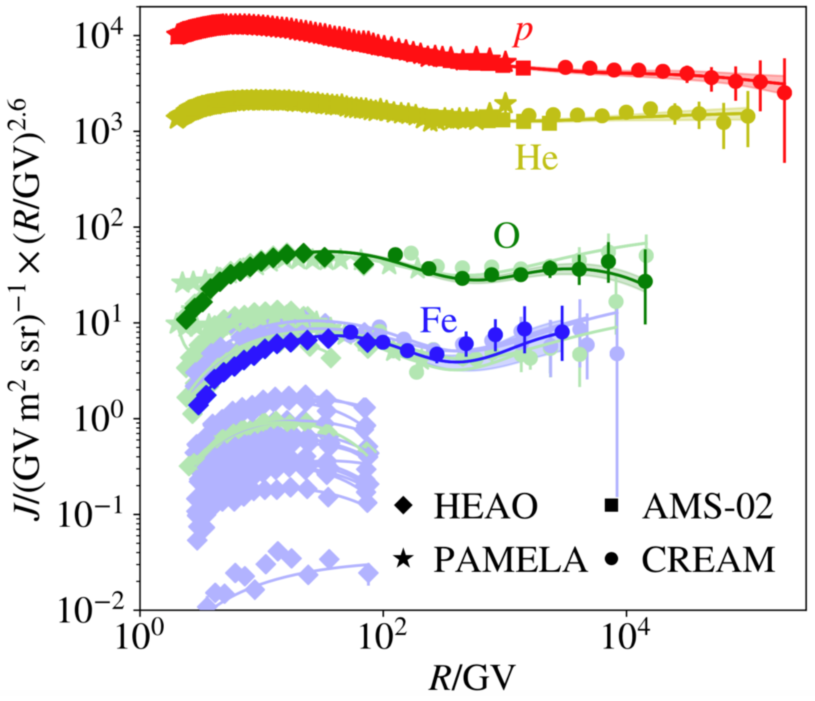


Global Spline Fit – fit to direct & indirect observations

H. Dembinski, AF, T. Gaisser
PoS(ICRC2017)533

- Fit **four** independent mass groups, which cover equal ranges in lnA: proton (p), helium (He), oxygen group (O*), and iron group (Fe*)
- Assumption: this holds **at all energies**
- One leading element *L* per group described by smooth spline curve
- Other elements *j* in a group kept in constant ratio: $J_j(R)/ J_L(R) = const.$



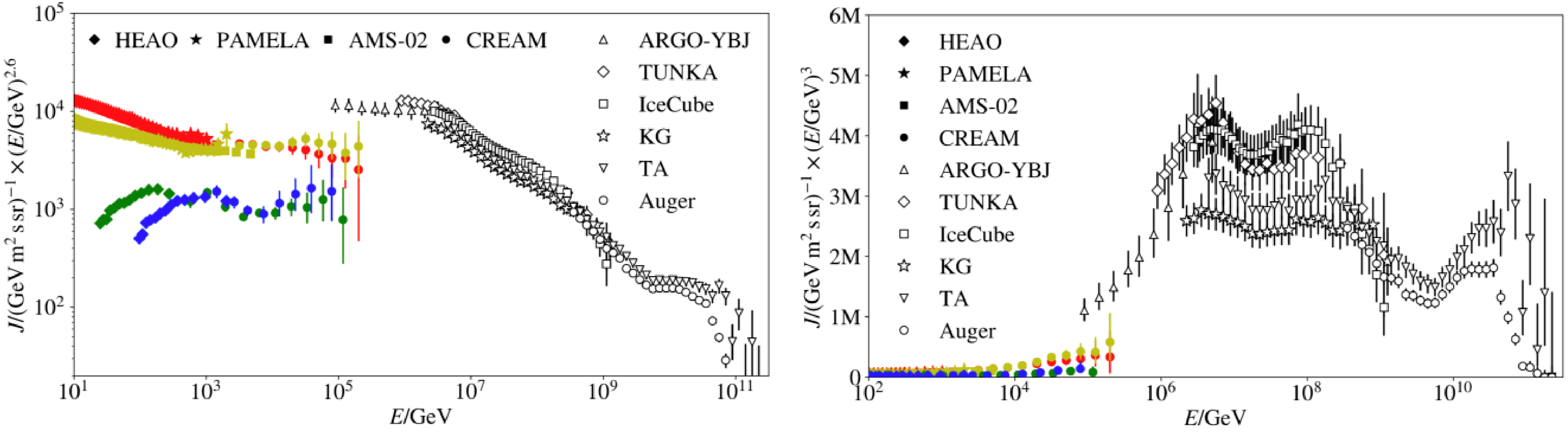


Mass sensitivity of air-shower experiments is ~ lnA

Handling energy-scale uncertainty

H. Dembinski, AF, T. Gaisser
PoS(ICRC2017)533

Original data



- The determination of **energy scale in air-shower experiments is uncertain**
- This is caused by inconsistencies of **hadronic interaction models**
- Fit adjusts energy scales **within systematic uncertainties** of the experiment

$$\tilde{J}(\tilde{E}) = J(E)\frac{\mathrm{d}E}{\mathrm{d}\tilde{E}} = J\left(\frac{\tilde{E}}{1+z_E}\right)\frac{1}{1+z_E}$$

Flux distortion caused by energy-scale offset z_E

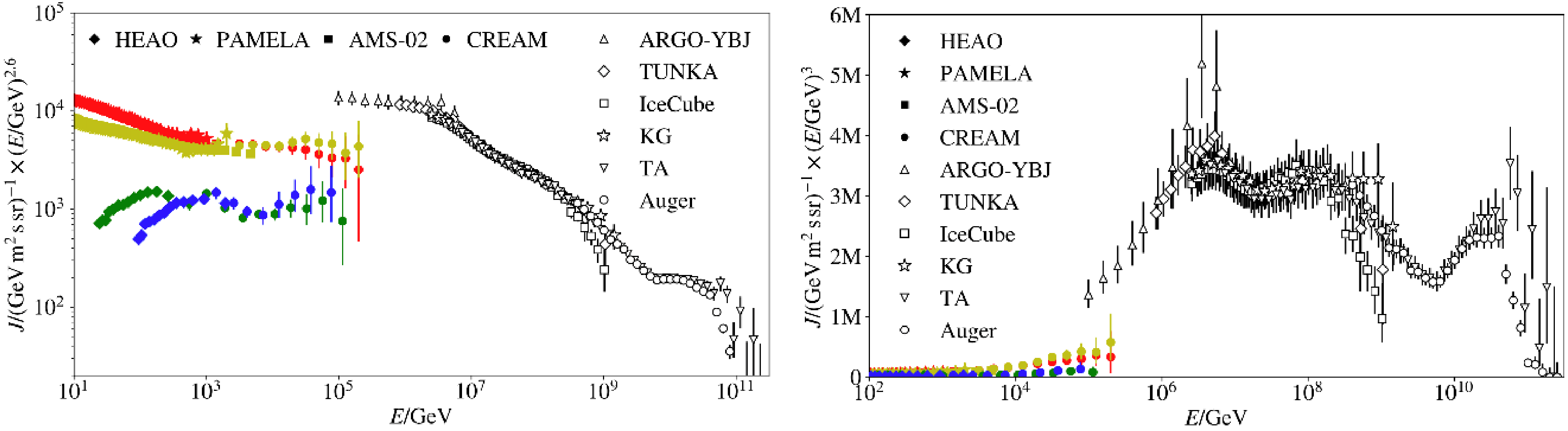
$$S = \sum_i z_i^2 + \sum_j \left(\frac{z_{Ej}}{(\sigma[E]/E)_j}\right)^2$$

Flux residuals   Energy-scale offset residuals

Handling energy-scale uncertainty

H. Dembinski, AF, T. Gaisser
PoS(ICRC2017)533

Adjusted data



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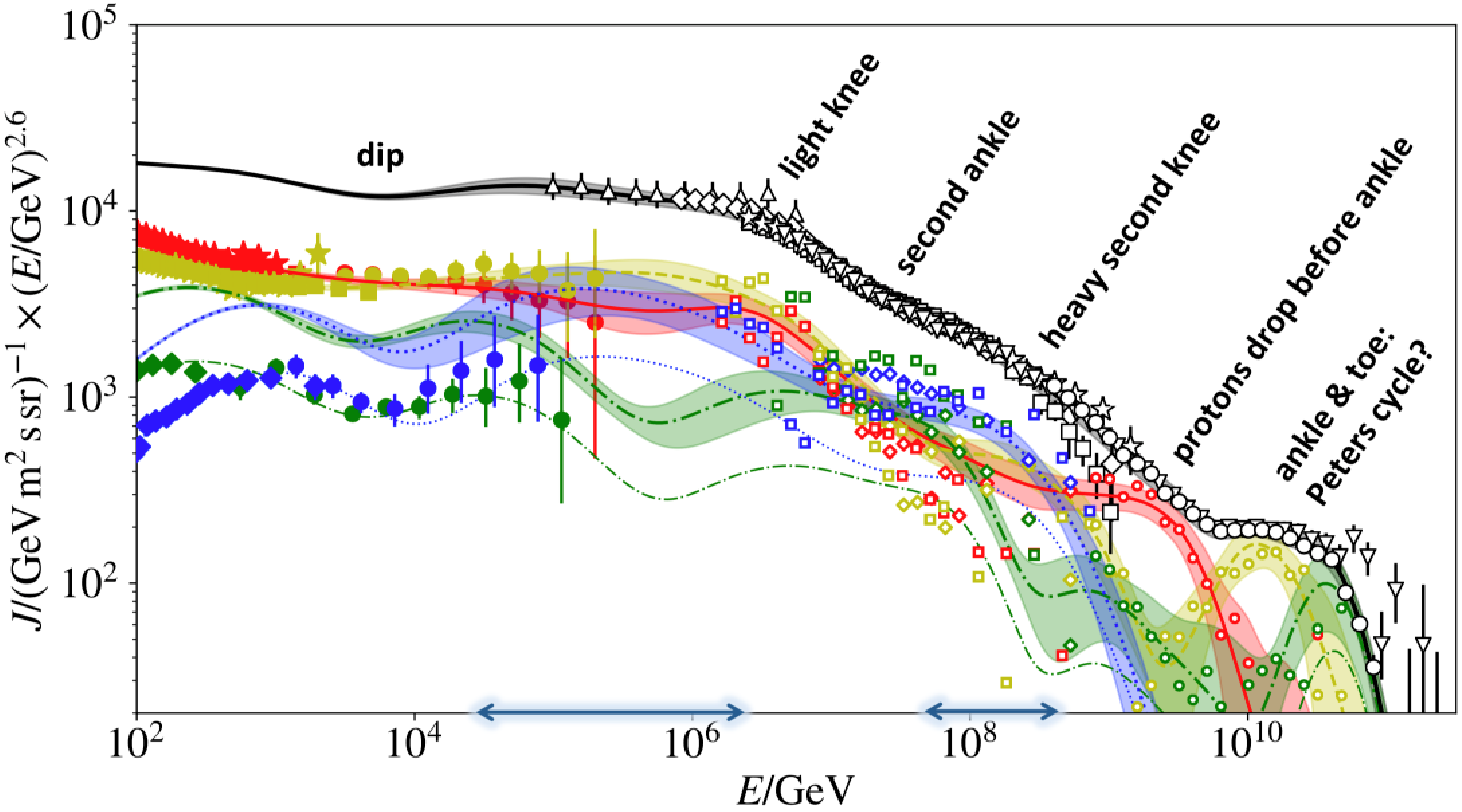
$$S = \sum_i z_i^2 + \sum_j \left(\frac{z_{Ej}}{(\sigma[E]/E)_j}\right)^2$$

Flux residuals Energy-scale offset residuals

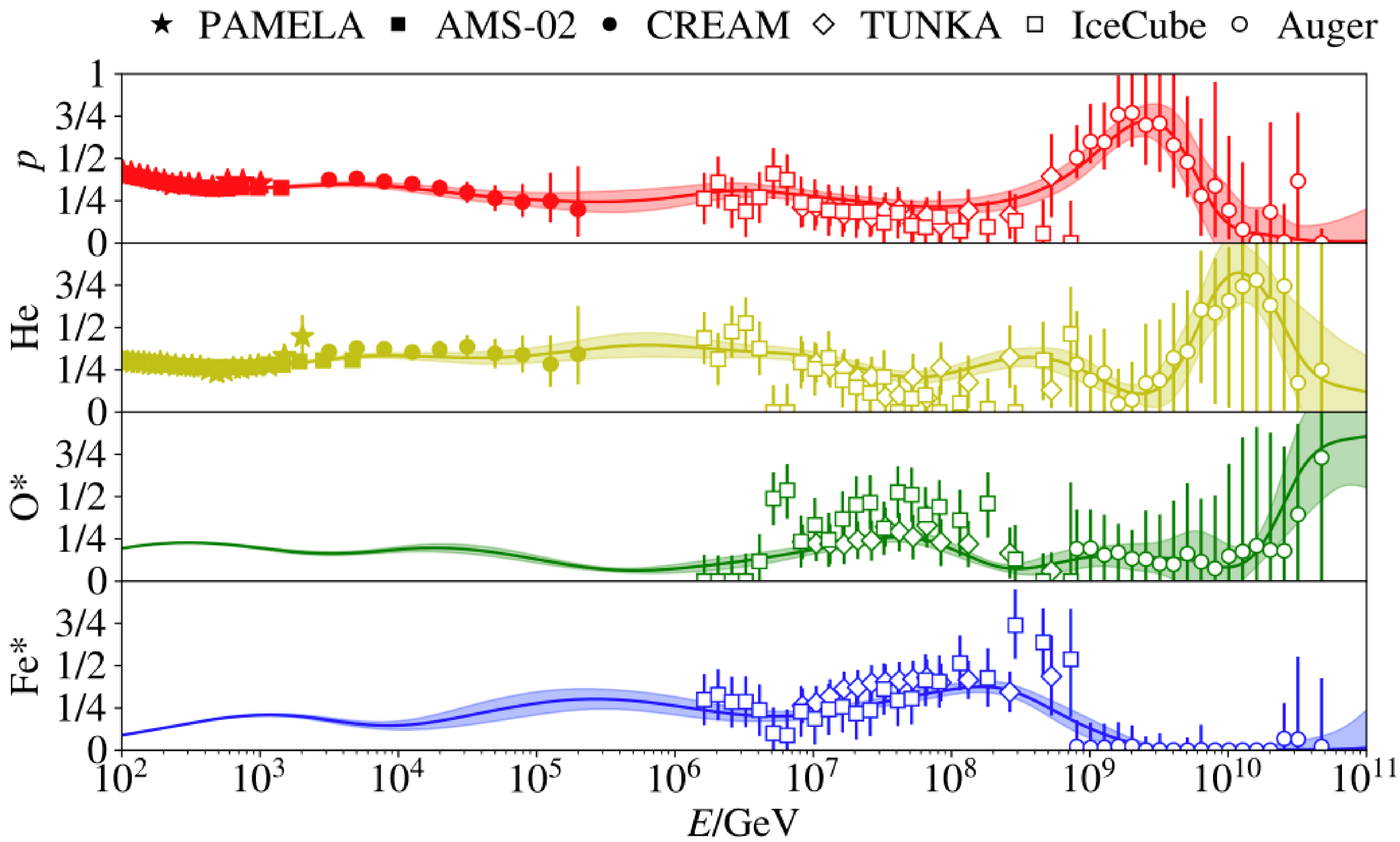
The Global Spline Fit



More composition data needed

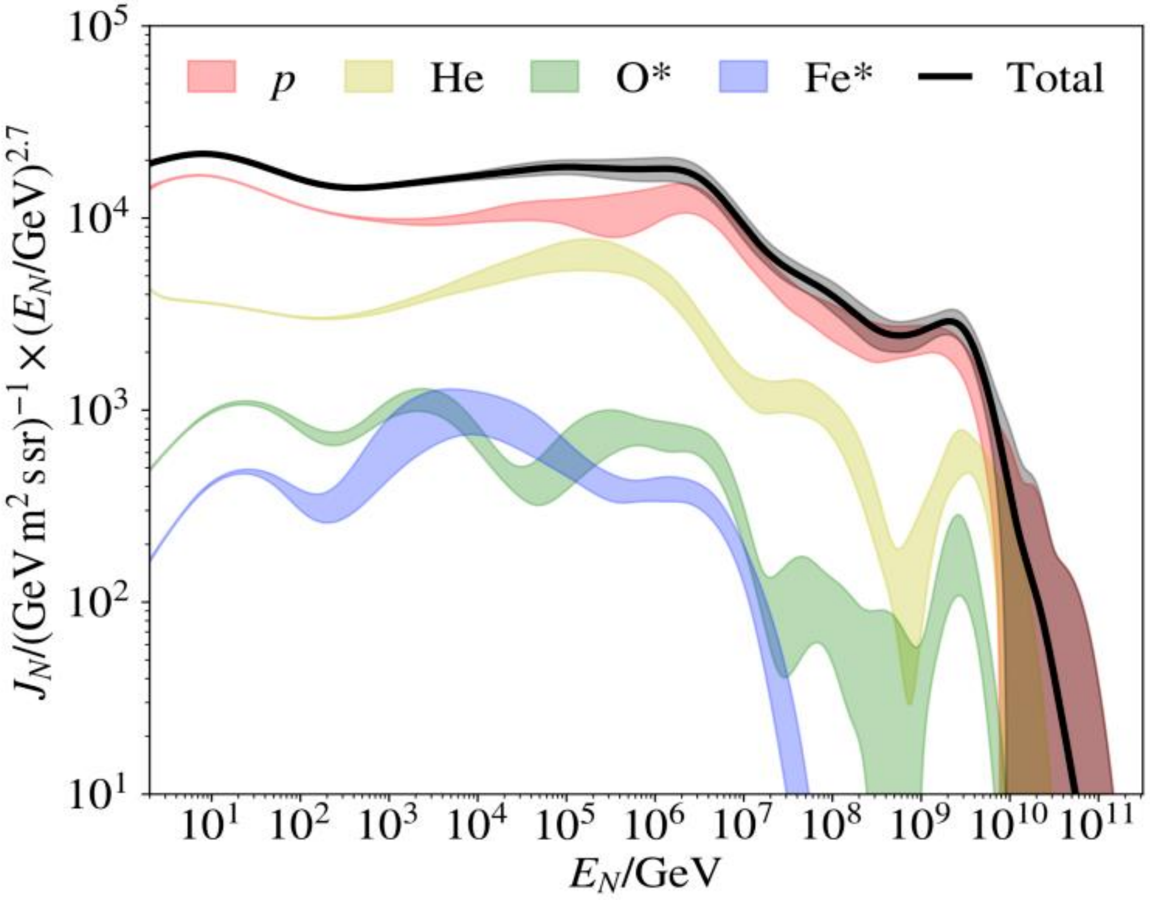
Fitted composition data

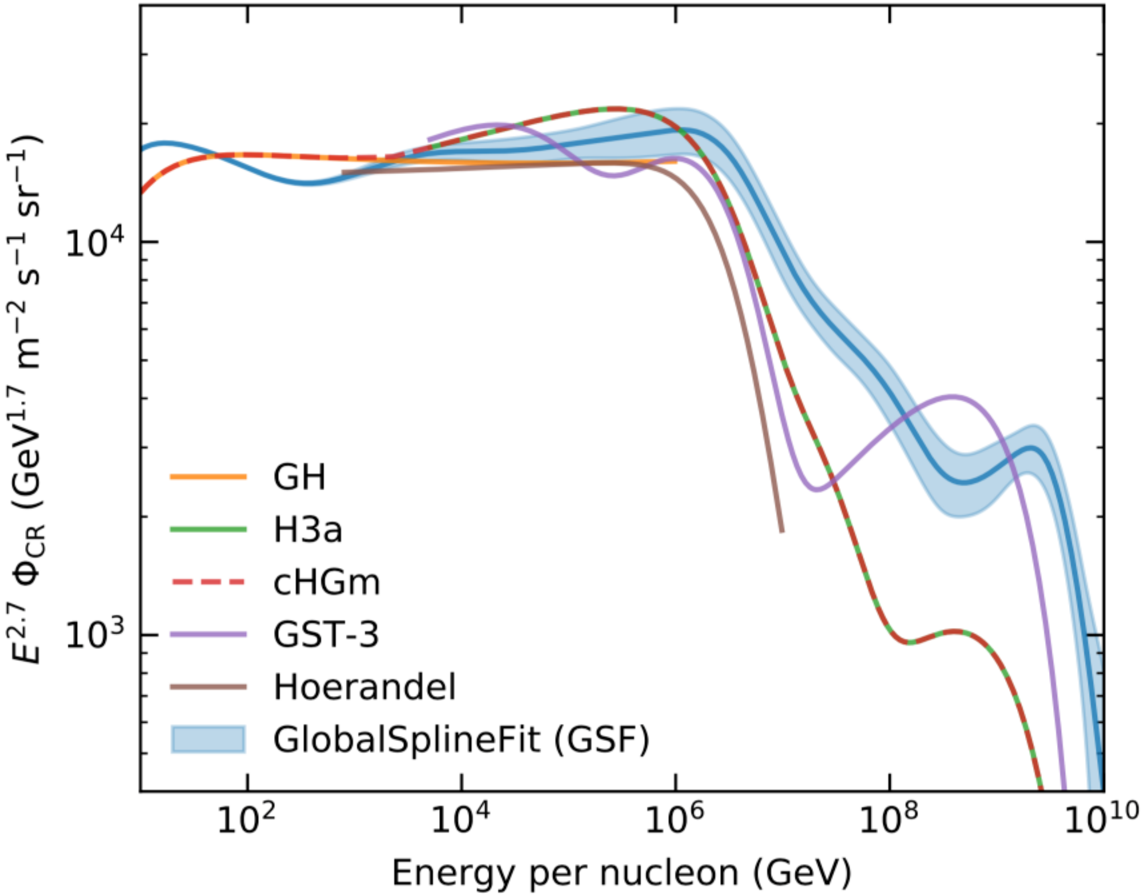
4-mass group experiments



Derived result: nucleon flux

AF et al, PoS(ICRC2017)1019





Dominated by proton flux. Details of sub-leading elements not important.

Harder spectrum at the knee due to lighter composition as assumed by 3-population models